***Additive-Contingent Nonlinearity*, Asymptotic Behaviors And Quantum-Causality In A Group Of Covariant Systems.**

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Abstract.
Some properties of the equations $x^2+y^2+z^2+v^2= rXYZ$, $x^2+y^2+z^2= rXYZ$, $x^2+y^2+z^2+v^2+u^2=rXYZ$, $X^2+Y^2+Z^2+V^2= rXYZ$, $X^2+Y^2+Z^2 = rXYZ$, $X^2+Y^2+Z^2+V^2 +U^2 = rXYZ$, $X^i+Y^i+Z^i+V^i= rXYZ$, $x^3+y^3+z^3=rXYZ$, $x^3+y^3+z^3+x^6+y^6+z^6=rXYZ$, $x^6+y^6+z^6=rXYZ$, $[(x^{12}+y^{12}+z^{12})-(x^6+y^6+z^6)]=rXYZ$, and $x^i+y^i+z^i=rXYZ$, (*i* is a positive integer), where $x\,|\,X$ (ie. X is a multiple of x), $y\,|\,Y$, and $z\,|\,Z$ are real numbers. This article also summarizes the relationships to *Homotopy Theory*, PDEs, Mathematical Cryptography and Analysis. The proofs are within the context of Sub-Rings. The additional common factor is that each of the variables x,y,z, v and dXYZ are multiples of (n-f), where *n* and *f* are real numbers. The solutions derived herein can be extended to other problems wherein (n-f) can take the form of polynomials/functions such as (6^d-3), $(14-5^c)$, (a^i-b^{2i}), etc.. Some of the results are applicable where all variables are Integers.

Keywords: *Nonlinearity*; Prime Numbers; Sub-Rings And Ring Theory; Mathematical Cryptography; *Beal Conjecture*; Dynamical Systems; Group Theory; *Homotopy Theory*; Partial Differential Equations; Multicollinearity; Ill-posed Problems.

1. Introduction.

The Markoff equation M_a: $X^2+Y^2+Z^2 =aXYZ$ is not new in the literature - during 1779, Euler studied the equation $X^2+Y^2+Z^2$, and derived a solution that was somewhat different from Markoff's solution. This article analyzes the properties of the equations $x^2+y^2+z^2+v^2= rXYZ$, $x^2+y^2+z^2= rXYZ$, $x^2+y^2+z^2+v^2+u^2=rXYZ$, $X^2+Y^2+Z^2+V^2= rXYZ$, $X^2+Y^2+Z^2 = rXYZ$, $X^2+Y^2+Z^2+V^2 +U^2 = rXYZ$, $X^i+Y^i+Z^i+V^i= rXYZ$, $x^3+y^3+z^3=rXYZ$, $x^3+y^3+z^3+x^6+y^6+z^6=rXYZ$, $x^6+y^6+z^6=rXYZ$, $[(x^{12}+y^{12}+z^{12})-(x^6+y^6+z^6)]=rXYZ$, and $x^i+y^i+z^i=rXYZ$, (*i* is a positive integer), where $x\,|\,X$ (ie. X is a multiple of x), $y\,|\,Y$, and $z\,|\,Z$ are real numbers. This group of equations have not been studied in detail in the literature. The second novelty in this study is that the scope of the solutions is real numbers and not only positive integers, and each of the equations is an ill-posed problem because their behavior can change drastically over any range of real numbers. The third novelty in this study is that taken together the equations $x^2+y^2+z^2+v^2= rXYZ$, $x^2+y^2+z^2= rXYZ$, $x^2+y^2+z^2+v^2+u^2=rXYZ$, $X^2+Y^2+Z^2+V^2= rXYZ$, $X^2+Y^2+Z^2 = rXYZ$, $X^2+Y^2+Z^2+V^2 +U^2 = rXYZ$, $X^i+Y^i+Z^i+V^i= rXYZ$, $x^3+y^3+z^3=rXYZ$, $x^3+y^3+z^3+x^6+y^6+z^6=rXYZ$, $x^6+y^6+z^6=rXYZ$, $[(x^{12}+y^{12}+z^{12})-(x^6+y^6+z^6)]=rXYZ$, and $x^i+y^i+z^i=rXYZ$, (*i* is a positive integer), exhibit or can exhibit:

i) *Super-Additive Horizontal Nonlinearity* and *Homomorphisms* – wherein as more variables are added to the left side of each equation, the greater the absolute amount of, and probability of Nonlinearity.

ii) *Contingent Vertical Nonlinearity* and *Homomorphisms* – wherein for each equation, the greater the absolute magnitudes of the independent variables (on the left side of each equation), the greater the Nonlinearity of the equation. Absolute Magnitude refers to magnitude of a variable without regard to its sign.

2. Existing Literature.

Pienaar (2017) discussed Causality in the Quantum World. Allen, Barrett, et. al. (2017). proposed a quantum causal model based on a generalization of Reichenbach's common cause principle.

Abram, Lapointe & Reutenauer (2020), Jiang, Gao & Cao (2020) and Togbe, Kafle & Srinivasan (2020) analyzed the *Markoff Equation* $X^2+Y^2+Z^2= aXYZ$ (which perhaps is the most popular equation that is structurally similar to the equations studied in this article; but the properties and methods introduced herein are new). MacHale (1991) studied the equation $X^3+Y^3+Z^3 = 3XYZ$.

Fang (2011) analyzed the equation f(p+q+r)=f(p)+f(q)+f(r). Lindqvist (2018) studied generalized Fermat Equations (sums of three powers). Andreescu (2002) analyzed the equation $(x+y+z)^2=xyz$. Ward (1948) analyzed sums of three fourth powers.

Hayakawa & Takeuchi (1987) and Vijayalakshmi & Karpagam (2019) analyzed and developed solutions for singularities using Algebraic methods. Brudno (1970) studied the solutions of the diophantine equation $A^6B^6+C^6D^6+E^6+F^6$. Guy (2004) and Browning (2003) reviewed and analyzed respectively, solutions for equations of the type $X^4+Y^4+Z^4+U^4=V^4$. Gar-el & Vaserstein (2002) studied the Diophantine Equation $a^3+b^3+c^3+d^3= 0$. Bremner (1981) discussed solutions for the equations of equal sums of fifth powers.

Resta & Meyrignac (2003) studied the smallest solutions to the Diophantine equation $x^6+y^6 = a^6+b^6+c^6+d^6+e^6$. Gerbicz, Meyrignac & Beckert (August, 2011) analyzed solutions of the Diophantine equation $(a^6+b^6)=(c^6+d^6+e^6+f^6+g^6)$ for a, b, c, d, e, f, g <250,000 (used a distributed Boinc project; and also listed primitive solutions up to 250,000 and the discoverer's name, sorted in lexicographical order).

Guy (2004) noted the following:

i) Norrie (1911) discovered the equation: $30^4+120^4+272^4+315^4=353^4$.

ii) Lander & Parkin (____) discovered the equation: $27^5+84^5+110^5+133^5=144^5$.

The equation $x^3+y^3+z^3=k$ in positive/negative integers has remained a mathematical puzzle for decades. For the same equation $x^3+y^3+z^3=k$, Huisman (2016) stated that as of 2016, solutions were known for all but thirteen values of k <1000 (the thirteen values were: 33,42,114,165,390,579,627,633,732,795,906,921,975). See Booker (2019) and note that during 2019, using computer simulations, Prof. Andrew Booker (Reader of Pure Mathematics from the Bristol University's School of Mathematics), found the solution[1] for the equation $x^3+y^3+z^3=33$; which is: $(8,866,128,975,287,528)^3 + (–8,778,405,442,862,239)^3 + (–2,736,111,468,807,040)^3$. Furthermore, in 2019 and using computer simulations[2], Prof. Booker and a research team also found the solution for the Diophantine Equation $x^3+y^3+z^3=42$, which is: x=-80538738812075974, y=80435758145817515, and z=12602123297335631.

Huisman (2016), Miller & Woollett (1955), Mordell (1953), Montgomery (1985), Koyama Tsuruoka & Sekigawa (1997), Bremner (1995), Cassels (1985), Elsenhans & Jahnel (2009), Beck, Pine, et. al. (2007), Lehmer (1956), Scarowsky & Boyarsky (1984), and Heath-Brown, Lioen & te Riele (1993) analyzed equations of the type $x^3+y^3+z^3=k$ in real numbers.

MacHale (1991) studied the equation $X^3+Y^3+Z^3=3XYZ$. Miyake (2009) analyzed Hesse's elliptic curves of the type: $U^3+V^3+W^3=3\mu UVW$. Dofs (1995) and Halbeisen & Hungerbuhler (2019) analyzed equations of the type: $x^3+y^3+z^3=nxyz$. Mordell (1955) developed solutions of $ax^3+ay^3+bz^3=bc^3$.

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[1] *See*: University of Bristol (April 2, 2019). *Bristol Mathematician Cracks Diophantine Puzzle*. https://phys.org/news/2019-04-bristol-mathematician-diophantine-puzzle.html. This same article in phys.org reported that as of 2019, the solutions of $x^3+y^3+z^3=k$ in the interval where 0<k<100, had been found except for the number k=42.

[2] *See*: Miller, S. (2019). *Sutherland Helps Solve Decades-Old Sum-Of-Three-Cubes Puzzle*. Available at: https://science.mit.edu/sutherland-helps-solve.../....

See: Phys.org. (2019). *Sum Of Three Cubes For 42 Finally Solved—Using Real Life Planetary Computer*. Available at: <https://phys.org/.../2019-09-sum-cubes-solvedusing-real...> .

Gundersen (1998) analyzed the equation $f^6+g^6+h^6\equiv 1$. Brudno (1976) studied equations of the type $X^6+Y^6+Z^6=k$. Elkies (1988) and Yuan (1996) analyzed equations of the type $A^4+B^4+C^4=D^4$.

On Diophantine Equations in *Analysis*, see Zaidenberg (1988), Tohge (2011), Zadeh (2019), Bitim & Keskin (2013) and Gundersen (1998). On solutions to Diophantine Equations in Computer Mathematics, Mathematical Physics and Mathematical Chemistry, see: Ren & Yang (2012), Bremner (1986), Papp & Vizvari (2006), Ibarra & Dang (2006), and Rahmawati, Sugandha, et. al. (2019).

The *Beal Conjecture* states that if *a*, *b*, *c*, *x*, *y*, and *z* are positive integers where $a^x+b^y=c^z$, and $x, y, z > 2$, then *a*, *b* and *c* have a common prime factor. The methods introduced in this article may help resolve the *Beal Conjecture* and similar problems.

3. Relationships To Mathematical Cryptography, Analysis, Group Theory And Prime Numbers.

On Homomorphisms, see: Wang & Chin (2012). Chu (2008) and Lu & Wu (2016) studied dynamical systems pertaining to Diophantine equations (and equations such as $x^2+y^2+z^2+v^2=rXYZ$, and $x^2+y^2+z^2+v^2+u^2=rXYZ$, and the *Markoff Equation* $X^2+Y^2+Z^2 = rXYZ$, $x^3+y^3+z^3=rXYZ$, and $x^6+y^6+z^6=rXYZ$, and $x^i+y^i+z^i=rXYZ$ and the *Markoff Equation* $X^2+Y^2+Z^2= aXYZ$ can approximate Dynamical Systems).

Luca, Moree & Weger (2011) discussed *Group Theory* as it relates to Diophantine Equations. Elia (2005), Jones, Sato, et. al. (1976) and Matijasevič (1981) noted that primes can also be represented as Diophantine equations or as polynomials (ie. each of the equations $x^2+y^2+z^2+v^2=rXYZ$, and $x^2+y^2+z^2+v^2+u^2=rXYZ$, and the *Markoff Equation* $X^2+Y^2+Z^2 = rXYZ$, $x^3+y^3+z^3=rXYZ$, and $x^6+y^6+z^6=rXYZ$, and $x^i+y^i+z^i=rXYZ$ and the *Markoff Equation* $X^2+Y^2+Z^2=aXYZ$ can represent a prime).

On uses of Diophantine Equations in Cryptography, see: Ding, Kudo, et. al. (2018), Okumura (2015), and Ogura (2012) (ie. the equations $x^2+y^2+z^2+v^2=rXYZ$, and $x^2+y^2+z^2+v^2+u^2=rXYZ$, and the *Markoff Equation* $X^2+Y^2+Z^2 = rXYZ$, $x^3+y^3+z^3=rXYZ$, and $x^6+y^6+z^6=rXYZ$, and $x^i+y^i+z^i=rXYZ$ can be used in cryptoanalysis, and in the creation of public-keys).

On solutions to Diophantine Equations in *Analysis*, see Zaidenberg (1988), Tohge (2011), and Zadeh (2019).

On solutions to Diophantine Equations in Mathematical Physics, Mathematical Chemistry and Computer Science, see: Ren & Yang (2012), Bremner (1986), Papp & Vizvari (2006), Ibarra & Dang (2006), Abram, Lapointe & Reutenauer (2020) and Rahmawati, Sugandha, et. al. (2019).

4. Relationships To *Homotopy Theory*

As noted herein and above, a common factor in the proofs introduced for the equations studied herein (where $x \mid X$ (ie. X is a multiple of x), $y \mid Y$, and $z \mid Z$ are real numbers) is that each of the variables x, y, z and dXYZ are multiples of (n-f), all of which are real numbers. Thus, the solutions derived herein can be extended to other problems wherein (n-f) can be a function or polynomial such as (15^c-3), $(5^d-\sqrt{f})$, (a^s-b^s), etc.; or where (x^i, [y^i+z^i], [z^i+x^i], etc.) are individual systems. More importantly, where (n-f) is a function, then in the solutions introduced herein, (n-f) can merge/map into, and create a *Homotopy* with each of the equations $x^2+y^2+z^2+v^2=rXYZ$, $x^i+y^i+z^i +v^i= rXYZ$ (where i is an integer), $x^2+y^2+z^2= rXYZ$, $x^2+y^2+z^2+v^2+u^2=rXYZ$, $x^3+y^3+z^3=rXYZ$, and $x^6+y^6+z^6=rXYZ$, and $x^i+y^i+z^i=rXYZ$ and the *Markoff Equation* $X^2+Y^2+Z^2=aXYZ$

5. Partial Differential Equations (PDEs), Invalidity of The "*Variance-Inflation-Factor*", And (n-f) As A Measure Of Multicollinearity.

Nwogugu (2012: 324-330) and Nwogugu (2017: 280-284) explained why the core *differentiation* formulas are wrong (and thus many PDE solutions are or maybe wrong). In the realm of PDEs, (n-f) and the "multipliers" of x, y, z and v (a,b,c,j,k) can be used to find the sensitivity of each side (LHS and RHS) of the each of the equations $x^2+y^2+z^2+v^2=rXYZ$, and $x^2+y^2+z^2+v^2+u^2=rXYZ$, and the *Markoff Equation* $X^2+Y^2+Z^2 = rXYZ$, $x^3+y^3+z^3=rXYZ$, and $x^6+y^6+z^6=rXYZ$, and $x^i+y^i+z^i=rXYZ$ and the *Markoff Equation* $X^2+Y^2+Z^2=aXYZ$ to changes in any of the variables x, y, z, X, Y and Z.

With regards to (n-f) and in the realm of PDEs:

i) The common relationships are as follows:

$x = (n-f)a$; $X=lx$; and $X =(n-f)a*l$;

$y = (n-f)b$; $Y=oy$; and $Y=(n-f)b*o$;

$z = (n\text{-}f)c$; $Z=qz$; and $Z = (n\text{-}f)c*q$;

and the same for u, v, U and V.

ii) Use of (n-f) and the "*multipliers*" of x, y and z (l; o; q) converts each of the equations $x^2+y^2+z^2+v^2=rXYZ$, and $x^2+y^2+z^2+v^2+u^2=rXYZ$, and the *Markoff Equation* $X^2+Y^2+Z^2 = rXYZ$, $x^3+y^3+z^3=rXYZ$, and $x^6+y^6+z^6=rXYZ$, and $x^i+y^i+z^i=rXYZ$ and the *Markoff Equation* $X^2+Y^2+Z^2=aXYZ$ into a PDE. For example, $\boldsymbol{x^2+y^2+z^2=dXYZ}$ becomes $\mathbf{(X*\partial X/\partial x)^2+(Y*[\partial Y/\partial y])^2+(Z*[\partial Z/\partial z])^2=d*[x*[\partial X/\partial x)]*YZ}$ which is a PDE because $\partial X/\partial x = l$, and so on for the other LHS variables.

iii) (n-f) and the "*multipliers*" of x, y and z (l, o and q respectively) can be used to find the sensitivity of each side (LHS and RHS) of the equations $x^2+y^2+z^2+v^2=rXYZ$, and $x^2+y^2+z^2+v^2+u^2=rXYZ$, and the *Markoff Equation* $X^2+Y^2+Z^2 = rXYZ$, $x^3+y^3+z^3=rXYZ$, and $x^6+y^6+z^6=rXYZ$, and $x^i+y^i+z^i=rXYZ$ and the *Markoff Equation* $X^2+Y^2+Z^2=aXYZ$ to changes in any of the variables x, y, z, v, u, X, Y, Z, V and U. Following the above example, $\boldsymbol{x^2+y^2+z^2=rXYZ}$ becomes $\boldsymbol{(Xl)^2+(Yo)^2+(Zq)^2=rXYZ}$, which is a PDE because $\partial X/\partial x=l$, in which case $\partial(XYZr)/\partial x = l(a_2)$ where a_2 is a real number and is a function of (n-f).

The Nwogugu (2013) proof of the invalidity of Variance/Semi-variance and Correlation also invalidates "*Variance Inflation Factor*" (VIF) – that is, for VIF to be valid, the conditions in the Nwogugu (2013) proofs must simultaneously exist, which is impossible. VIF is the main generally-accepted measure of multicollinearity; and thus, most of the regression-based empirical research done during the last fifty years is unreliable, and that may also account for the ongoing *Replicability/Reproducibility Crisis* in academic research.

Given the foregoing, (n-f) quantifies and can serve as an indicator of multicollinearity in the following way. For the time series (of an equation such as $x^2+y^2+z^2+v^2=rXYZ$), (n-f) is calculated for each time-unit. If (n-f) is relatively "stable" over time (doesn't exceed stated upper and lower bounds), then "adjusted-average" (n-f) over the time-series can be a reliable indicator of the actual magnitude of multicollinearity.

6. The Special Case Of x=y=z: Some Simulated Solutions Of The Equations $x^3+y^3+z^3=rXYZ$, and $x^6+y^6+z^6=rXYZ$, In Integers.

For the special case where x=y=z, there appears to be infinitely many solutions for both $x^3+y^3+z^3=rXYZ$, and $x^6+y^6+z^6=rXYZ$, and Tables 1 & 2 below illustrates some of the solutions.

Theorem-A: For the equations $x^6+y^6+z^6$=rXYZ and $x^3+y^3+z^3$=rXYZ, where x│X, y│Y and z│Z are positive integers and x=y=z; each of the equations $x^6+y^6+z^6$=rXYZ and $x^3+y^3+z^3$=rXYZ has potentially and infinitely many solutions in positive integers; and the conditions (x+y+z) ≤r and (xyz)≤r exist in most of the solution-sets.

Proof: If x=y=z, $xa_1=X$, and $yb_1=Y$, and $zc_1=Z$, then: the equation $x^6+y^6+z^6=rXYZ$ is equivalent to: $3x^6=rx^3a_1b_1c_1$, and $3x^3=ra_1b_1c_1$, and $3x^3/[a_1b_1c_1]=d$. However, Table-2 in this article shows that the equation $x^6+y^6+z^6=rXYZ$ conforms to $3x^3=r$, only where x=y=z=X=Y=Z (in which case $a_1,b_1,c_1=1$). Also, the equation $x^6+y^6+z^6=rXYZ$ conforms to $3x^3/[a_1b_1c_1]=r$, only where x=y=z, $xa_1=X$, and $yb_1=Y$, and $zc_1=Z$; all of which is evidence that $x^6+y^6+z^6=rXYZ$ has potentially and infinitely many solutions for (x,y,z) in positive integers; and (x+y+z) ≤r and (xyz) ≤r in most of the solution-sets.

If x=y=z, $xa_1=X$, and $yb_1=Y$, and $zc_1=Z$, then: the equation $x^3+y^3+z^3=rXYZ$ is equivalent to: $3x^3=rx^3a_1b_1c_1$, and $3=ra_1b_1c_1$, and $3/[a_1b_1c_1]=d$. However, Table-1 in this article shows that the equation $x^3+y^3+z^3=rXYZ$ conforms to $3x^3=r$, only where x=y=z=X=Y=Z (in which case $a_1,b_1,c_1=1$). Also, the equation $x^3+y^3+z^3=rXYZ$ conforms to $3/[a_1b_1c_1]=r$, only where x=y=z, $xa_1=X$, and $yb_1=Y$, and $zc_1=Z$. All that is evidence that $x^3+y^3+z^3=rXYZ$ has potentially and infinitely many solutions for (x,y,z) in positive integers; and (x+y+z) ≤r and (xyz) ≤r exist in most of the solution-sets. ■

Table-1: Simulated Solutions Of $x^3+y^3+z^3$=rXYZ In Integers.

x	y	z	d
1	1	1	3
2	2	2	3
3	3	3	3
4	4	4	3
5	5	5	3
6	6	6	3
7	7	7	3
8	8	8	3
9	9	9	3
10	10	10	3
11	11	11	3
12	12	12	3
13	13	13	3
14	14	14	3
15	15	15	3
16	16	16	3
17	17	17	3
18	18	18	3
19	19	19	3
20	20	20	3
21	21	21	3
22	22	22	3
23	23	23	3
24	24	24	3
250	250	250	3
1,250.000	1,250.000	1,250.000	3
6,250.000	6,250.000	6,250.000	3
31,250.000	31,250.000	31,250.000	3
156,250.000	156,250.000	156,250.000	3
781,250.000	781,250.000	781,250.000	3
3,906,250.000	3,906,250.000	3,906,250.000	3
19,531,250.000	19,531,250.000	19,531,250.000	3
97,656,250.000	97,656,250.000	97,656,250.000	3
488,281,250.000	488,281,250.000	488,281,250.000	3
2,441,406,250.000	2,441,406,250.000	2,441,406,250.000	3
12,207,031,250.000	12,207,031,250.000	12,207,031,250.000	3
61,035,156,250.000	61,035,156,250.000	61,035,156,250.000	3
3.05176E+11	3.05176E+11	3.05176E+11	3
1.52588E+12	1.52588E+12	1.52588E+12	3
7.62939E+12	7.62939E+12	7.62939E+12	3

3.8147E+13	3.8147E+13	3.8147E+13	3
1.90735E+14	1.90735E+14	1.90735E+14	3
9.53674E+14	9.53674E+14	9.53674E+14	3
4.76837E+15	4.76837E+15	4.76837E+15	3
2.38419E+16	2.38419E+16	2.38419E+16	3
1.19209E+17	1.19209E+17	1.19209E+17	3
5.96046E+17	5.96046E+17	5.96046E+17	3
2.98023E+18	2.98023E+18	2.98023E+18	3
1.49012E+19	1.49012E+19	1.49012E+19	3
7.45058E+19	7.45058E+19	7.45058E+19	3
3.72529E+20	3.72529E+20	3.72529E+20	3
1.86265E+21	1.86265E+21	1.86265E+21	3
9.31323E+21	9.31323E+21	9.31323E+21	3
4.65661E+22	4.65661E+22	4.65661E+22	3
2.32831E+23	2.32831E+23	2.32831E+23	3
1.16415E+24	1.16415E+24	1.16415E+24	3
5.82077E+24	5.82077E+24	5.82077E+24	3
2.91038E+25	2.91038E+25	2.91038E+25	3
1.45519E+26	1.45519E+26	1.45519E+26	3
7.27596E+26	7.27596E+26	7.27596E+26	3
3.63798E+27	3.63798E+27	3.63798E+27	3
1.81899E+28	1.81899E+28	1.81899E+28	3
9.09495E+28	9.09495E+28	9.09495E+28	3
4.54747E+29	4.54747E+29	4.54747E+29	3
2.27374E+30	2.27374E+30	2.27374E+30	3
1.13687E+31	1.13687E+31	1.13687E+31	3
5.68434E+31	5.68434E+31	5.68434E+31	3
2.84217E+32	2.84217E+32	2.84217E+32	3
1.42109E+33	1.42109E+33	1.42109E+33	3
7.10543E+33	7.10543E+33	7.10543E+33	3
3.55271E+34	3.55271E+34	3.55271E+34	3
1.77636E+35	1.77636E+35	1.77636E+35	3
8.88178E+35	8.88178E+35	8.88178E+35	3
4.44089E+36	4.44089E+36	4.44089E+36	3
2.22045E+37	2.22045E+37	2.22045E+37	3
1.11022E+38	1.11022E+38	1.11022E+38	3
5.55112E+38	5.55112E+38	5.55112E+38	3
2.77556E+39	2.77556E+39	2.77556E+39	3
1.38778E+40	1.38778E+40	1.38778E+40	3
6.93889E+40	6.93889E+40	6.93889E+40	3
3.46945E+41	3.46945E+41	3.46945E+41	3
1.73472E+42	1.73472E+42	1.73472E+42	3
8.67362E+42	8.67362E+42	8.67362E+42	3

4.33681E+43	4.33681E+43	4.33681E+43	3
2.1684E+44	2.1684E+44	2.1684E+44	3
1.0842E+45	1.0842E+45	1.0842E+45	3
5.42101E+45	5.42101E+45	5.42101E+45	3
2.71051E+46	2.71051E+46	2.71051E+46	3
1.35525E+47	1.35525E+47	1.35525E+47	3
6.77626E+47	6.77626E+47	6.77626E+47	3
3.38813E+48	3.38813E+48	3.38813E+48	3
1.69407E+49	1.69407E+49	1.69407E+49	3
8.47033E+49	8.47033E+49	8.47033E+49	3
4.23516E+50	4.23516E+50	4.23516E+50	3
2.11758E+51	2.11758E+51	2.11758E+51	3
1.05879E+52	1.05879E+52	1.05879E+52	3
5.29396E+52	5.29396E+52	5.29396E+52	3
2.64698E+53	2.64698E+53	2.64698E+53	3
1.32349E+54	1.32349E+54	1.32349E+54	3
6.61744E+54	6.61744E+54	6.61744E+54	3
3.30872E+55	3.30872E+55	3.30872E+55	3
1.65436E+56	1.65436E+56	1.65436E+56	3
8.27181E+56	8.27181E+56	8.27181E+56	3
4.1359E+57	4.1359E+57	4.1359E+57	3
2.06795E+58	2.06795E+58	2.06795E+58	3
1.03398E+59	1.03398E+59	1.03398E+59	3
5.16988E+59	5.16988E+59	5.16988E+59	3
2.58494E+60	2.58494E+60	2.58494E+60	3
1.29247E+61	1.29247E+61	1.29247E+61	3
6.46235E+61	6.46235E+61	6.46235E+61	3
3.23117E+62	3.23117E+62	3.23117E+62	3
1.61559E+63	1.61559E+63	1.61559E+63	3
8.07794E+63	8.07794E+63	8.07794E+63	3
4.03897E+64	4.03897E+64	4.03897E+64	3
2.01948E+65	2.01948E+65	2.01948E+65	3
1.00974E+66	1.00974E+66	1.00974E+66	3
5.04871E+66	5.04871E+66	5.04871E+66	3
2.52435E+67	2.52435E+67	2.52435E+67	3
1.26218E+68	1.26218E+68	1.26218E+68	3
6.31089E+68	6.31089E+68	6.31089E+68	3
3.15544E+69	3.15544E+69	3.15544E+69	3
1.57772E+70	1.57772E+70	1.57772E+70	3
7.88861E+70	7.88861E+70	7.88861E+70	3
3.9443E+71	3.9443E+71	3.9443E+71	3
1.97215E+72	1.97215E+72	1.97215E+72	3
9.86076E+72	9.86076E+72	9.86076E+72	3

4.93038E+73	4.93038E+73	4.93038E+73	3
2.46519E+74	2.46519E+74	2.46519E+74	3
1.2326E+75	1.2326E+75	1.2326E+75	3
6.16298E+75	6.16298E+75	6.16298E+75	3
3.08149E+76	3.08149E+76	3.08149E+76	3
1.54074E+77	1.54074E+77	1.54074E+77	3
7.70372E+77	7.70372E+77	7.70372E+77	3
3.85186E+78	3.85186E+78	3.85186E+78	3
1.92593E+79	1.92593E+79	1.92593E+79	3
9.62965E+79	9.62965E+79	9.62965E+79	3
4.81482E+80	4.81482E+80	4.81482E+80	3
2.40741E+81	2.40741E+81	2.40741E+81	3
1.20371E+82	1.20371E+82	1.20371E+82	3
6.01853E+82	6.01853E+82	6.01853E+82	3
3.00927E+83	3.00927E+83	3.00927E+83	3
1.50463E+84	1.50463E+84	1.50463E+84	3
7.52316E+84	7.52316E+84	7.52316E+84	3
3.76158E+85	3.76158E+85	3.76158E+85	3
1.88079E+86	1.88079E+86	1.88079E+86	3
9.40395E+86	9.40395E+86	9.40395E+86	3
4.70198E+87	4.70198E+87	4.70198E+87	3
2.35099E+88	2.35099E+88	2.35099E+88	3
1.17549E+89	1.17549E+89	1.17549E+89	3
5.87747E+89	5.87747E+89	5.87747E+89	3
2.93874E+90	2.93874E+90	2.93874E+90	3
1.46937E+91	1.46937E+91	1.46937E+91	3
7.34684E+91	7.34684E+91	7.34684E+91	3
3.67342E+92	3.67342E+92	3.67342E+92	3
1.83671E+93	1.83671E+93	1.83671E+93	3
9.18355E+93	9.18355E+93	9.18355E+93	3
4.59177E+94	4.59177E+94	4.59177E+94	3
2.29589E+95	2.29589E+95	2.29589E+95	3
1.14794E+96	1.14794E+96	1.14794E+96	3
5.73972E+96	5.73972E+96	5.73972E+96	3
2.86986E+97	2.86986E+97	2.86986E+97	3
1.43493E+98	1.43493E+98	1.43493E+98	3
7.17465E+98	7.17465E+98	7.17465E+98	3
3.5873E+99	3.5873E+99	3.5873E+99	3
1.7937E+100	1.7937E+100	1.7937E+100	3
8.9683E+100	8.9683E+100	8.9683E+100	3
4.4842E+101	4.4842E+101	4.4842E+101	3
2.2421E+102	2.2421E+102	2.2421E+102	3

Table-2: Simulated Solutions Of $x^6+y^6+z^6$=rXYZ, In Integers.

x	y	z	d
1	1	1	3
2	2	2	24
3	3	3	81
4	4	4	192
5	5	5	375
6	6	6	648
7	7	7	1,029.000
8	8	8	1,536.000
9	9	9	2,187.000
10	10	10	3,000.000
11	11	11	3,993.000
12	12	12	5,184.000
13	13	13	6,591.000
14	14	14	8,232.000
15	15	15	10,125.000
16	16	16	12,288.000
17	17	17	14,739.000
18	18	18	17,496.000
19	19	19	20,577.000
20	20	20	24,000.000
21	21	21	27,783.000
22	22	22	31,944.000
23	23	23	36,501.000
24.000	24.000	24.000	41,472.000
250.000	250.000	250.000	46,875,000.000
1,250.000	1,250.000	1,250.000	5,859,375,000.000
6,250.000	6,250.000	6,250.000	7.32422E+11
31,250.000	31,250.000	31,250.000	9.15527E+13
156,250.000	156,250.000	156,250.000	1.14441E+16
781,250.000	781,250.000	781,250.000	1.43051E+18
3,906,250.000	3,906,250.000	3,906,250.000	1.78814E+20
19,531,250.000	19,531,250.000	19,531,250.000	2.23517E+22
97,656,250.000	97,656,250.000	97,656,250.000	2.79397E+24
488,281,250.000	488,281,250.000	488,281,250.000	3.49246E+26
2,441,406,250.000	2,441,406,250.000	2,441,406,250.000	4.36557E+28
12,207,031,250.000	12,207,031,250.000	12,207,031,250.000	5.45697E+30
61,035,156,250.000	61,035,156,250.000	61,035,156,250.000	6.82121E+32
3.05176E+11	3.05176E+11	3.05176E+11	8.52651E+34
1.52588E+12	1.52588E+12	1.52588E+12	1.06581E+37

7.62939E+12	7.62939E+12	7.62939E+12	1.33227E+39
3.8147E+13	3.8147E+13	3.8147E+13	1.66533E+41
1.90735E+14	1.90735E+14	1.90735E+14	2.08167E+43
9.53674E+14	9.53674E+14	9.53674E+14	2.60209E+45
4.76837E+15	4.76837E+15	4.76837E+15	3.25261E+47
2.38419E+16	2.38419E+16	2.38419E+16	4.06576E+49
1.19209E+17	1.19209E+17	1.19209E+17	5.0822E+51
5.96046E+17	5.96046E+17	5.96046E+17	6.35275E+53
2.98023E+18	2.98023E+18	2.98023E+18	7.94093E+55
1.49012E+19	1.49012E+19	1.49012E+19	9.92617E+57
7.45058E+19	7.45058E+19	7.45058E+19	1.24077E+60
3.72529E+20	3.72529E+20	3.72529E+20	1.55096E+62
1.86265E+21	1.86265E+21	1.86265E+21	1.9387E+64
9.31323E+21	9.31323E+21	9.31323E+21	2.42338E+66
4.65661E+22	4.65661E+22	4.65661E+22	3.02923E+68
2.32831E+23	2.32831E+23	2.32831E+23	3.78653E+70
1.16415E+24	1.16415E+24	1.16415E+24	4.73317E+72
5.82077E+24	5.82077E+24	5.82077E+24	5.91646E+74
2.91038E+25	2.91038E+25	2.91038E+25	7.39557E+76
1.45519E+26	1.45519E+26	1.45519E+26	9.24446E+78
7.27596E+26	7.27596E+26	7.27596E+26	1.15556E+81
3.63798E+27	3.63798E+27	3.63798E+27	1.44445E+83
1.81899E+28	1.81899E+28	1.81899E+28	1.80556E+85
9.09495E+28	9.09495E+28	9.09495E+28	2.25695E+87
4.54747E+29	4.54747E+29	4.54747E+29	2.82119E+89
2.27374E+30	2.27374E+30	2.27374E+30	3.52648E+91
1.13687E+31	1.13687E+31	1.13687E+31	4.4081E+93
5.68434E+31	5.68434E+31	5.68434E+31	5.51013E+95
2.84217E+32	2.84217E+32	2.84217E+32	6.88766E+97
1.42109E+33	1.42109E+33	1.42109E+33	8.6096E+99
7.10543E+33	7.10543E+33	7.10543E+33	1.0762E+102
3.55271E+34	3.55271E+34	3.55271E+34	1.3452E+104
1.77636E+35	1.77636E+35	1.77636E+35	1.6816E+106
8.88178E+35	8.88178E+35	8.88178E+35	2.1019E+108
4.44089E+36	4.44089E+36	4.44089E+36	2.6274E+110
2.22045E+37	2.22045E+37	2.22045E+37	3.2843E+112
1.11022E+38	1.11022E+38	1.11022E+38	4.1054E+114
5.55112E+38	5.55112E+38	5.55112E+38	5.1317E+116
2.77556E+39	2.77556E+39	2.77556E+39	6.4146E+118
1.38778E+40	1.38778E+40	1.38778E+40	8.0183E+120
6.93889E+40	6.93889E+40	6.93889E+40	1.0023E+123
3.46945E+41	3.46945E+41	3.46945E+41	1.2529E+125
1.73472E+42	1.73472E+42	1.73472E+42	1.5661E+127

8.67362E+42	8.67362E+42	8.67362E+42	1.9576E+129
4.33681E+43	4.33681E+43	4.33681E+43	2.447E+131
2.1684E+44	2.1684E+44	2.1684E+44	3.0587E+133
1.0842E+45	1.0842E+45	1.0842E+45	3.8234E+135
5.42101E+45	5.42101E+45	5.42101E+45	4.7793E+137
2.71051E+46	2.71051E+46	2.71051E+46	5.9741E+139
1.35525E+47	1.35525E+47	1.35525E+47	7.4676E+141
6.77626E+47	6.77626E+47	6.77626E+47	9.3345E+143
3.38813E+48	3.38813E+48	3.38813E+48	1.1668E+146
1.69407E+49	1.69407E+49	1.69407E+49	1.4585E+148
8.47033E+49	8.47033E+49	8.47033E+49	1.8231E+150
4.23516E+50	4.23516E+50	4.23516E+50	2.2789E+152

7. The Theorems.

Theorem-1: **For the equation $x^2+y^2+z^2+v^2= rXYZ$, in real numbers where x│X (ie. X is a multiple of x), y│Y, z│Z and v│V exist; and *a, b, c* and *j* are multiplicative components of X, Y, Z and V respectively (each of X, Y, Z and V are derived by multiplying *a, b, c* and *j* respectively by (*n-f*), another real number):**

i) If XYZg = (*n-f*), then XYZg = (ea)*(pb)*(hc)*(kj), for some real numbers g, e, p, h and k.

ii) If XYZr = (*n-f*), then XYZr = (ea)*(pb)*(hc) *(kj), for some real numbers e, p, h and k.

Proof:

This first section proves that XYZg =(ea)*(pb)*(hc)*(kj), for some real numbers g, e, p, h and k.

Let:

X= xl; Y=yo; Z=zq; V=vs; where l, o, q and s are real numbers.

X=(n-f)a; Y=(n-f)b; Z=(n-f)c; V=(n-f)j

XYZg = (n-f)

$XYZg = (n-f)^3(abc)g$

$XYZ = (n-f)^3(abc)$

$(n-f) = (n-f)^3(abc)g$

Thus $1 = (n-f)^2(abc)g$

If XYZg = (ea)*(pb)*(hc)*(kj),

Then: (XYZg)/(abcj) = ephk

but: $(ephk)(abcj) = (n-f)^3(abcj)g$

Thus, $(ephk) =(n-f)^3g = [(XYZ)/(abcj)]g$

And: $=(n-f)^3 = [(XYZ)/(abcj)]$

From above, if $XYZg = (n-f)^3(abcj)g$; then $XYZ = (n-f)^3(abcj)$

This second section proves that XYZr =(ea)*(pb)*(hc)*(kj), for some real numbers e, p, h and k.

Let:

X= xl; Y=yo; Z=zi; V=vs; where l, o, i and s are real numbers.

X=(n-f)a; Y=(n-f)b; Z=(n-f)c; V=(n-f)j

XYZr = (n-f)

The following are “Sub-Theorems” each of which completely proves this theorem and can stand-alone as an independent theorem.

Sub-Theorem-1A:
x= (n-f)a/l; y=(n-f)b/o; z=(n-f)c; v=(n-f)j/s
XYZr = $\mathbf{x^2+y^2+z^2+v^2=}$ $(n-f)^3$(abc)r
XYZ = $(n-f)^3$(abc)
(n-f) = $(n-f)^3$(abc)r
Thus: 1 = $(n-f)^2$(abc)r
If XYZr = (n-f) = (ea)*(pb)*(hc)*(kj) = (ephk)(abcj)/1:
(n-f) = (ephk)(abcj)/[$(n-f)^2$(abc)r]
$(n-f)^3$(abc)r = (ephk)(abcj)
XYZr = (ephk)(abcj) ■

Sub-Theorem-1B:
XYZr = $\mathbf{x^2+y^2+z^2+v^2=}$ $(n-f)^3$(abc)r
Therefore: XYZ = $(n-f)^3$(abc)
If XYZr = (ea)*(pb)*(hc)*(kj) = (ephk)(abcj):
Then: (XYZr)/(abcj) = ephk
but: (ephk)(abcj) = XYZr = $(n-f)^3$(abc)r
Thus, (ephk)j =$(n-f)^3$r = [(XYZrj)/(abcj)]
And: $(n-f)^3$ = [(XYZ)/(abc)]
And: XYZ = $(n-f)^3$(abc) ■

Sub-Theorem-1C:
X= xl; Y=yo; Z=zi; V=vs; where *l*, *o*, *i* and *t* are real numbers.
X=(n-f)a; Y=(n-f)b; Z=(n-f)c; V=(n-f)j;
XYZr = (n-f)
(n-f) = X/a=Y/b = Z/c = V/j = (ephk)(abcj)
X= (ephk)(abcj)a
Y= (ephk)(abcj)b
Z= (ephk)(abcj)c
XYZr = $(ephk)^3(abcj)^3$(abc)r = (ephk)(abcj)
abc = [(ephk)(abcj)]/[$(ephk)^3(abcj)^3$r] = [1/[$(ephk)^2(abcj)^2$r]]
Therefore, XYZr = [$(ephk)^3(abcj)^3$r]*[1/[$(ephk)^2(abcj)^2$r]]
XYZr = [(ephk)(abcj)] ■

Theorem-1: **For the equation $x^2+y^2+z^2+v^2$= rXYZ, in real numbers where x│X (ie. X is a multiple of x), y│Y, z│Z and v│V exist; and *a, b, c* and *j* are multiplicative components of X, Y, Z and V respectively (each of x, y, z and v are derived by multiplying *a, b, c* and *j* respectively by (*n-f*), another real number):**
i) If XYZg = (*n-f*), then XYZg = (ea)*(pb)*(hc)*(kj), for some real numbers g, e, p, h and k.
ii) If XYZr = (*n-f*), then XYZr = (ea)*(pb)*(hc) *(kj), for some real numbers e, p, h and k.
Proof:
This first section proves that XYZg =(ea)*(pb)*(hc)*(kj), for some real numbers g, e, p, h and k.

Let:
X= xl; Y=yo; Z=zq; V=vs; where l, o, q and s are real numbers.
x=(n-f)a; y=(n-f)b; z=(n-f)c; v(n-f)j; and thus:
X= la(n-f); Y=bo(n-f); z=qc(n-f); v=js(n-f);
XYZg = (n-f)

XYZg = $(n-f)^3$(abc)(loq)g
XYZ = $(n-f)^3$(abc) (loq)

$(n-f) = (n-f)^3(abc)(loq)g$
Thus $1 = (n-f)^2(abc)(loq)g$

If XYZg = (ea)*(pb)*(hc)*(kj),
Then: (XYZg)/(abcj) = ephk
but: $(ephk)(abcj) = (n-f)^3(abcj)(loq)g$
Thus, $(ephk) =(n-f)^3g = [(XYZ)/(abcj)(loq)]g$
And: $=(n-f)^3 = [(XYZ)/(abcj)(loq)]$
From above, if $XYZg = (n-f)^3(abcj)(loq)g$; then $XYZ = (n-f)^3(abcj)(loq)$

This second section proves that XYZr =(ea)*(pb)*(hc)*(kj), for some real numbers e, p, h and k.
Let:

X= xl; Y=yo; Z=zq; V=vs; where l, o, q and s are real numbers.
x=(n-f)a; y=(n-f)b; z=(n-f)c; v(n-f)j; and thus:
X= la(n-f); Y=bo(n-f); z=qc(n-f); v=js(n-f);
XYZr = (n-f)

The following are "Sub-Theorems" each of which completely proves this theorem and can stand-alone as an independent theorem.

Sub-Theorem-1A:
x= (n-f)a; y=(n-f)b; z=(n-f)c; v=(n-f)j
XYZr = $\mathbf{x^2+y^2+z^2+v^2=}$ $(n-f)^3(abc)(loq)r$
$XYZ = (n-f)^3(abc)(loq)$
$(n-f) = (n-f)^3(abc)(loq)r$
Thus: $1 = (n-f)^2(abc)(loq)r$
If XYZr = (n-f) = (ea)*(pb)*(hc)*(kj) = (ephk)(abcj)/1, then:

$(n-f) = (ephk)(abcj)/[(n-f)^2(abc)(loq)r]$
$(n-f)^3(abc)(loq)r = (ephk)(abcj)$
XYZr = (ephk)(abcj) ■

Sub-Theorem-1B:
XYZr = $\mathbf{x^2+y^2+z^2+v^2=}$ $(n-f)^3(abc)r$
Therefore: $XYZ = (n-f)^3(abc)(loq)$
If XYZr = (ea)*(pb)*(hc)*(kj) = (ephk)(abcj), then:

Then: (XYZr)/(abcj) = ephk
but: $(ephk)(abcj) = XYZr = (n-f)^3(abc)(loq)r$
Thus, $(ephk)j =(n-f)^3r(loq) = [(XYZrj)/(abcj)]$
And: $(n-f)^3 = [(XYZ)/(abc)(loq)]$
And: $XYZ = (n-f)^3(abc)(loq)$ ■

Sub-Theorem-1C:
X= xl; Y=yo; Z=zi; V=vs; where *l*, *o*, *i* and *t* are real numbers.

x=(n-f)a; y=(n-f)b; z=(n-f)c; v(n-f)j; and thus:
X= la(n-f); Y=bo(n-f); z=qc(n-f); v=js(n-f);

XYZr = (n-f)
(n-f) = X/la=Y/ob = Z/qc = V/js = (ephk)(abcj)
X= (ephk)(abcj)a
Y= (ephk)(abcj)b
Z= (ephk)(abcj)c
$XYZr = (ephk)^3(abcj)^3(abc)r = (ephk)(abcj)$
$abc = [(ephk)(abcj)]/[(ephk)^3(abcj)^3r] = [1/[(ephk)^2(abcj)^2r]]$

Therefore, $XYZr = [(ephk)^3(abcj)^3r]*[1/[(ephk)^2(abcj)^2r]]$

XYZr = [(ephk)(abcj)] ■

Theorem-2: **For the equation $x^2+y^2+z^2= rXYZ$ in real numbers where x│X (ie. X is a multiple of x), y│Y and z│Z exist, and *a, b and c* in real numbers are multiplicative components of X, Y and Z respectively (each of X, Y and Z are derived by multiplying *a*, *b* and *c* respectively by (*n-f*), another real number):**

i) If XYZg = (*n-f*), then XYZg = (ea)*(pb)*(hc), for some real numbers g, e, p and h.

ii) If XYZr = (*n-f*), then XYZr = (ea)*(pb)*(hc), for some real numbers e, p and h.

Proof:
This first section proves that XYZg = (ea)*(pb)*(hc), for some real numbers g, e, p and h.

Let:
X= xl; Y=yo; Z=zi; where l,o and i are real numbers.
X=(n-f)a; Y=(n-f)b; Z=(n-f)c
XYZg = (n-f)
Then: XYZg = $(n\text{-}f)^3$(abc)g; and XYZ = $(n\text{-}f)^3$(abc)
(n-f) = $(n\text{-}f)^3$(abc)g
Thus: 1 = $(n\text{-}f)^2$(abc)g

(ea)*(pb)*(hc) = (eph)(abc)
If XYZg = (ea)*(pb)*(hc); then:
(XYZg)/(abc) = eph
but: (eph)(abc) = $(n\text{-}f)^3$(abc)g
Thus, (eph) =$(n\text{-}f)^3$g = [(XYZ)/(abc)]g
And: $(n\text{-}f)^3$ = [(XYZ)/(abc)]
From above, if XYZg = $(n\text{-}f)^3$(abc)g; then XYZ = $(n\text{-}f)^3$(abc).

This second section proves that XYZr = (ea)*(pb)*(hc), for some real numbers e, p and h.
Let:
XYZr = (n-f)

The following are "Sub-Theorems" each of which completely proves this theorem and can stand-alone as an independent theorem.

Sub-Theorem-2A:
x= (n-f)a/l; y=(n-f)b/o; z=(n-f)c;
XYZr = $\mathbf{x^2+y^2+z^2=}$ $(n\text{-}f)^3$(abc)r
XYZ = $(n\text{-}f)^3$(abc)
(n-f) = $(n\text{-}f)^3$(abc)r
Thus: 1 = $(n\text{-}f)^2$(abc)r
If XYZr = (n-f) = (ea)*(pb)*(hc) = (eph)(abc)/1; then:
(n-f) = (eph)(abc)/[$(n\text{-}f)^2$(abc)r]
$(n\text{-}f)^3$(abc)r = (eph)(abc)
XYZr = (eph)(abc) ■

Sub-Theorem-2B:
XYZr = $\mathbf{x^2+y^2+z^2=}$ $(n\text{-}f)^3$(abc)r
Therefore: XYZ = $(n\text{-}f)^3$(abc)
If XYZr = (ea)*(pb)*(hc) = (eph)(abc):
Then: (XYZr)/(abc) = eph
but: (eph)(abc) = XYZr = $(n\text{-}f)^3$(abc)r
Thus, (eph) =$(n\text{-}f)^3$r = [(XYZr)/(abc)]
And: $(n\text{-}f)^3$ = [(XYZ)/(abc)]

And: $XYZ = (n-f)^3(abc)$
And: $XYZr = (n-f)^3(abc)r$ ■

Sub-Theorem-2C:
X= xl; Y=yo; Z=zi; where l, o and i are real numbers.
X=(n-f)a; Y=(n-f)b; Z=(n-f)c;
XYZr = (n-f)
(n-f) = X/a=Y/b = Z/c = (eph)(abc)
X= (eph)(abc)a
Y= (eph)(abc)b
Z= (eph)(abc)c
$XYZr = (eph)^3(abc)^3(abc)r = (eph)(abc)$
$abc= [(eph)(abc)]/[(eph)^3(abc)^3r] = [1/[(eph)^2(abc)^2r]]$
Therefore, $XYZr = [(eph)^3(abc)^3r]* [1/[(eph)^2(abc)^2r]]$
XYZr = [(eph)(abc)] ■

Theorem-3: For the equations $x^2+y^2+z^2+v^2+u^2=rXYZ$ and XYZ*g* = (*n-f*) in real numbers where x│X (ie. X is a multiple of x), y│Y, z│Z, v│V and u│U exist; and *a, b, c, j and m* are multiplicative components of X, Y, Z, V and U respectively (each of X, Y, Z, V and U are derived by multiplying each of *a*, *b*, *c*, *j* and *m* respectively by (*n-f*)), and:

i) If XYZg = (*n-f*), then XYZg = (ea)*(pb)*(hc)*(kj)*(qm), for some real numbers g, e, p, h, k and q.

ii) If XYZr = (*n-f*), then XYZr = (ea)*(pb)*(hc) *(kj)*(qm), for some real numbers e, p, h, k and q.

Proof:
This first section proves that XYZg = **(ea)*(pb)*(hc)*(kj)*(qm),** for some real numbers g, e, p, h, k and q.

Let:
X= xl; Y=yo; Z=zi; V=vs; U=ut; where l, o, i, t and s are real numbers.
X=(n-f)a; Y=(n-f)b; Z=(n-f)c; V=(n-f)j; U=(n-f)m
XYZg = (n-f)
Then: $XYZg = (n-f)^3(abc)g$; and $XYZ = (n-f)^3(abc)$
$(n-f) = (n-f)^3(abc)g$, or $1 = (n-f)^2(abc)g$; which implies that: $(abcg) \leq 1$
Thus: $1 = (n-f)^2(abc)g$

If XYZg = (ea)*(pb)*(hc) ***(kj)*(qm),** then:
(XYZg)/(abcjm) = ephkq
but: $(ephkq)(abcjm) = (n-f)^3(abc)g$
Thus, $(ephkq)jm = (n-f)^3g = [(XYZ)/(abc)]g$
And: $(n-f)^3 = [(XYZ)/(abc)]$
From above, if $XYZg = (n-f)^3(abc)g$; then $XYZ = (n-f)^3(abc)$.

This second section proves that XYZr =(ea)*(pb)*(hc)*(kj)*(qm), for some real numbers e, p, h, k and q.
Let:
X= xl; Y=yo; Z=zi; V=vs; U=ut; where l, o, s, t and s are real numbers.
X=(n-f)a; Y=(n-f)b; Z=(n-f)c; V=(n-f)j; U=(n-f)m
XYZr = (n-f)

The following are "Sub-Theorems" each of which completely and separately proves this theorem (and can stand-alone as an independent theorem).

Sub-Theorem-3A:
x= (n-f)a/l; y=(n-f)b/o; z=(n-f)c; v=(n-f)j/s
$XYZr = x^2+y^2+z^2+v^2+u^2 = (n-f)^3(abc)r$
$XYZ = (n-f)^3(abc)$
$(n-f) = (n-f)^3(abc)r$
Thus: $1 = (n-f)^2(abc)r$
If XYZr = (n-f) = (ea)*(pb)*(hc)*(kj)*(qm) = (ephkq)(abcjm)/1; then:
$(n-f) = (ephkq)(abcjm)/[(n-f)^2(abc)r]$
$(n-f)^3(abc)r = (ephkq)(abcjm)$
XYZr = (ephkq)(abcjm) ■

Sub-Theorem-3B:
$XYZr = x^2+y^2+z^2+v^2+u^2 = (n-f)^3(abc)r$
Therefore: $XYZ = (n-f)^3(abc)$
If XYZr = (ea)*(pb)*(hc)*(kj)*(qm) = (ephkq)(abcjm):
Then: (XYZr)/(abcjm) = ephkq
but: $(ephkq)(abcjm) = XYZr = (n-f)^3(abc)r$
Thus, $(ephkq)jm =(n-f)^3r = [(XYZrjm)/(abcjm)]$
And: $(n-f)^3 = [(XYZ)/(abc)]$
And: $XYZ = (n-f)^3(abc)$ ■

*Sub-Theorem-*3C:
X= xl; Y=yo; Z=zi; V=vs; U=ut; where l, o, i, t and s are real numbers.
X=(n-f)a; Y=(n-f)b; Z=(n-f)c; V=(n-f)j; U=(n-f)m
XYZr = (n-f)
(n-f) = X/a=Y/b = Z/c = V/j = U/m = (ephkq)(abcjm)
X= (ephkq)(abcjm)a
Y= (ephkq)(abcjm)b
Z= (ephkq)(abcjm)c
$XYZr = (ephkq)^3(abcjm)^3(abc)r = (ephkq)(abcjm)$
$abc= [(ephkq)(abcjm)]/ [(ephkq)^3(abcjm)^3r] = [1/ [(ephkq)^2(abcjm)^2r]]$
Therefore, $XYZr = [(ephkq)^3(abcjm)^3r]* [1/ [(ephkq)^2(abcjm)^2r]]$
XYZr = [(ephkq)(abcjm)] ■

Theorem-4: **For the equations $x^2+y^2+z^2+v^2= rXYZ$ and XYZg = (n-f) in real numbers where x│X (ie. X is a multiple of x), y│Y, z│Z and v│V exist; if *a, b, c and j* are multiplicative components of X, Y, Z and V respectively (each of X, Y, Z and V are derived by multiplying each of *a, b, c and j* respectively by (*n-f*)), then the upper-bounds and lower-bounds of both *g* and *(n-f)* can be defined.**
Proof:
Let:
X= xl; Y=yo; Z=zq; V=vs; where l, o, q and s are real numbers.
X=(n-f)a; Y=(n-f)b; Z=(n-f)c; V=(n-f)j
XYZg = (n-f)
a = 1/YZg; and b =1/XZg; and c=1/XYg; and j= V/XYZg

$XYZg = (n-f)^3(abc)g$
$XYZ = (n-f)^3(abc)$
$(n-f) = (n-f)^3(abcg)$, and $1= (n-f)^2(abcg)$; and $g = 1/[(n-f)^2(abc)]$; which implies that:
1) $(abcg)\leq 1\leq [(n-f)^2, (n-f)]$ (hereafter, "$LB_{(n-f)}$" or the "Lower-Bound of [n-f]).
2) g<1.
3) (n-f) ≤ XYZr (hereafter, "$UB_{(n-f)}$" or the "Upper-Bound of [n-f]).
4) As (n-f) →+∞, (abc)g→-∞;

In $\mathbf{x^2+y^2+z^2+v^2= rXYZ}$, *r* varies vary primarily with the magnitudes (and to a lesser extent, the signs) of X,Y, V and Z. However, given X, Y, Z and V, then a,b, c and j can be determined by substituting a = 1/YZg, b = 1/XZg and c = 1/XYg, and j= V/XYZ, into X/a = Y/b = Z/c = V/j = (n-f) = XYZg

In $\mathbf{x^2+y^2+z^2+v^2= rXYZ}$, both *n* and *f* vary primarily with the magnitudes (and to a lesser extent, the signs) of X,Y and Z.

$XYZg = (n-f)^3(abc)g$
n = XYZg+f
$n = [XYZ/(abc)]^{1/3}+f$
Thus $XYZg = [XYZ/(abc)]^{1/3}$
$g = \{[XYZ/(abc)]^{1/3}\}/XYZ$ (referred to as "LB_g" or "Lower Bound of g")
but also $g = 1/[(n-f)^2(abc)]$ (referred to as "UB_g" or "Upper Bound of g")
As the denominator in UB_g tends to zero, *g* in UB_g can become greater than one and significant – that can occur if 0<a, or b or c<1, and or if 0<(n-f)<1.
As the denominator in UB_g tends to minus infinity from zero, *g* in UB_g becomes smaller – that can occur if (a, or b or c)<0.

On the contrary, as the denominator in LB_g tends to zero, *g* in LB_g can become much smaller (unless 0<abc<1) – that can occur if 0<X, or Y or Z<1, and or if 0<a,b,c, or if (XYZ)<(abc).
As the denominator in LB_g tends to minus infinity from zero, g in LB_g can become smaller or bigger depending on the magnitude of abc.
Thus, its more likely that LB_g defines the lower bound of *g*, while UB_g defines upper bound of *g*. ■

Theorem-5: **For the equations $x^2+y^2+z^2= rXYZ$ and gXYZ = (n-f) in real numbers where x│X (ie. X is a multiple of x), y│Y and z│Z exist; if *a, b and c* are multiplicative components of X, Y and Z respectively (each of X, Y and Z are derived by multiplying each of *a, b and c* respectively by (*n-f*)), and given Theorems herein, the upper-bounds and lower-bounds of both *g* and *(n-f)* can be defined.**
Proof:
Let:

> X= xl; Y=yo; Z=zq; where l, o and q are real numbers.
> X=(n-f)a; Y=(n-f)b; Z=(n-f)c;
> XYZg = (n-f)
> a = 1/YZg; and b =1/XZg; and c=1/XYg;

$XYZg = (n-f)^3(abc)g$
$XYZ = (n-f)^3(abc)$
$(n-f) = (n-f)^3(abcg)$, and $1= (n-f)^2(abcg)$; and $g = 1/[(n-f)^2(abc)]$; which implies that:

1) $(abcg)\leq 1\leq [(n-f)^2, (n-f)]$ (hereafter, "$LB_{(n-f)}$" or the "Lower-Bound of (n-f)).
2) g<1.
3) $(n-f) \leq XYZr$ (hereafter, "$UB_{(n-f)}$" or the "Upper-Bound of (n-f)).
4) As $(n-f) \rightarrow +\infty$, $(abc)r \rightarrow -\infty$;

In $x^2+y^2+z^2 = rXYZ$, *r* varies with the magnitudes (and not the signs) of X, Y and Z. However, X, Y and Z, a, b, and c can be determined by substituting a = 1/YZr, b = 1/XZr and c = 1/XYr, into X/a = Y/b = Z/c = (n-f) = XYZg

In $x^2+y^2+z^2 = rXYZ$, both *n* and *f* vary primarily with the magnitudes (and to a much lesser extent, the signs) of X, Y and Z.

$XYZg = (n-f)^3(abc)g$

(n-f) = XYZg
n = XYZg + f
n = $[XYZ/(abc)]^{1/3}$+f
Thus: XYZg = $[XYZ/(abc)]^{1/3}$
g = $\{[XYZ/(abc)]^{1/3}\}$/XYZ; (referred to as "LB_g" or "Lower Bound of *g*")
but also g = $1/[(n-f)^2(abc)]$ (referred to as "UB_g" or "Upper Bound of *g*")

As the denominator in UB_g tends to zero, *g* in UB_g can become greater than one and significant – that can occur if 0<a, or b or c<1, and or if 0<(n-f)<1.
As the denominator in UB_g tends to minus infinity from zero, *g* in UB_g becomes smaller – that can occur if (a, or b or c)<0.

On the contrary, as the denominator in LB_g tends to zero, *g* in LB_g can become much smaller (unless 0<abc<1) – that can occur if 0<X, or Y or Z<1, and or if 0<a,b,c, or if (XYZ)<(abc).
As the denominator in LB_g tends to minus infinity from zero, *g* in LB_g can become smaller or bigger depending on the magnitude of abc.
Thus, LB_g defines the lower bound of *g*, while UB_g defines upper bound of *g*. ■

Theorem-6: **For the equations $x^2+y^2+z^2+v^2+u^2$= rXYZ and XYZg = (n-f) in real numbers where x│X (ie. X is a multiple of x), y│Y, z│Z, v│V and u│U exist; if *a, b, c, j and m* in real numbers are multiplicative components of X, Y, Z, V and U respectively (each of X, Y, Z, V and U are derived by multiplying each of *a, b, c, j and m* respectively by (*n-f*)), the upper-bounds and lower-bounds of both *g* and *(n-f)* can be defined.**
Proof:
Let:

X= xl; Y=yo; Z=zq; V=vs; U=ut; where l, o, q, t and s are real numbers.
X=(n-f)a; Y=(n-f)b; Z=(n-f)c; V=(n-f)j; U=(n-f)m
XYZg = (n-f)
a = 1/YZr; and b = 1/XZr; and c = 1/XYr; and j= V/XYZr; and m= U/XYZr

XYZg = $(n-f)^3$(abc)g
XYZ = $(n-f)^3$(abc)
(n-f) = $(n-f)^3$(abcg), and 1= $(n-f)^2$(abcg); and g = $1/[(n-f)^2(abc)]$; all of which implies that:

1) (abcg)≤ 1≤ $[(n-f)^2$, (n-f)] (hereafter, "$LB_{(n-f)}$" or the "Lower-Bound of (n-f)).
2) g<1.
3) (n-f) ≤ rXYZ (hereafter, "$UB_{(n-f)}$" or the "Upper-Bound of (n-f)).
4) As (n-f) →+∞, (abc)g→-∞;

In $\mathbf{x^2+y^2+z^2+v^2+u^2}$= rXYZ, r varies primarily with the magnitudes (and to a lesser extent, the signs) of x, y,z, v and u.

Given X, Y, Z, V and U; then a,b, and c can be determined by substituting a = 1/YZr, b = 1/XZr and c = 1/XYr, into X/a = Y/b = Z/c = V/j = U/m = (n-f) = XYZr

In $\mathbf{x^2+y^2+z^2+v^2+u^2}$= rXYZ, both *n* and *f* vary primarily with the magnitudes (and to a lesser extent, the signs) of X, Y and Z.

XYZg = $(n-f)^3$(abc)g
n = XYZg+f
n = $[XYZ/(abc)]^{1/3}$+f
Thus XYZg = $[XYZ/(abc)]^{1/3}$
g = $\{[XYZ/(abc)]^{1/3}\}$/XYZ (referred to as "LB_g" or "Lower-Bound of *g*")

but also $g = 1/[(n-f)^2(abc)]$ (referred to as "UB_g" or "Upper-Bound of *g*")
As the denominator in UB_g tends to zero, *g* in UB_g can become greater than one and significant – that can occur if 0<a, or b or c<1, and or if 0<(n-f)<1.
As the denominator in UB_g tends to minus infinity from zero, *g* in UB_g becomes larger – that can occur if (a, or b or c)<0.

On the contrary, as the denominator in LB_g tends to zero, *g* in LB_g can become much smaller (unless 0<abc<1) – that can occur if 0<X, or Y or Z<1, and or if 0<a,b,c, or if (XYZ)<(abc).
As the denominator in LB_g tends to negative-infinity from zero, *g* in LB_g can become smaller or bigger depending on the magnitude of abc.
Thus, LB_g defines the lower bound of *g*, while UB_g defines upper bound of *g*. ■

Theorem-7: **For the equation $X^2+Y^2+Z^2+V^2= rXYZ$ in real numbers, and given Theorems herein and above, if (n-f) is a multiplicative component of each of X,Y and Z (each of X, Y and Z are derived by multiplying (*n-f*) by another real number), then:**

1) If (n-f)=gXYZ, then for all n, f and *g* that are real numbers, g ϵ r.

2) rXYZ = (*n-f*), for some real numbers *n and f*.

Proof:
Let:

X=(n-f)a; Y=(n-f)b; Z=(n-f)c; V=(n-f)j;
XYZg = (n-f)
a = 1/YZg; and b = 1/XZg; and c = 1/XYg; and j= V/XYZg;

Thus: X=(gXYZ)(a); and Y=(gXYZ)(b); and Z=(gXYZ)(c); and V = (gXYZ)(j)

By substitution: $[(g^2X^2Y^2Z^2)(a^2)]+[(g^2X^2Y^2Z^2)(b^2)]+[(g^2X^2Y^2Z^2)(c^2)] + [(g^2X^2Y^2Z^2)(j^2)]= rXYZ$

Then by dividing both sides of the equation by rXYZ and substituting a=(1/YZg), b=(1/XZg), c=(1/XYg), and j= V/XYZg, the result is:
$\{[(g^2X^2Y^2Z^2)(1/(Y^2Z^2g^2))]/rXYZ\}+\{[(g^2X^2Y^2Z^2)(1/(X^2Z^2g^2))]/rXYZ\}+ \{[(g^2X^2Y^2Z^2)(1/(X^2Y^2g^2))]/rXYZ\} + \{[(g^2X^2Y^2Z^2)(V^2/(X^2Y^2Z^2g^2))]/rXYZ\}= 1$;
and thus: $[(X/rYZ)+(Y/rXZ)+(Z/rXY)] +(V^2/rXYZ)]= 1$

By taking a common denominator rXYZ for the left-hand side of the equation, the result is:
$[(X^2+Y^2+V^2+Z^2)/rXYZ] = 1$;
and by multiplying both sides of the equation by rXYZ, the result is: $X^2+Y^2+Z^2+V^2= rXYZ$

r can also be expressed solely in terms of X, Y, Z and V as follows:
$\{[((X^2Y^2Z^2)(1/(Y^2Z^2g^2)))/XYZ]+[((g^2X^2Y^2Z^2)(1/(X^2Z^2g^2)))/XYZ]+[((g^2X^2Y^2Z^2)(1/(X^2Y^2g^2)))/XYZ] + [((g^2X^2Y^2Z^2) (V^2/(X^2Y^2Z^2g^2)))/XYZ]\}= r = [(X/YZ)+(Y/XZ)+(Z/XY)+ (V^2/(XYZ))]$

Given the foregoing and since X=(gXYZ)(a); and Y=(gXYZ)(b); and Z=(gXYZ)(c) and V=(gXYZ)(j); and $X^2+Y^2+Z^2+ V^2= rXYZ$, for all n, f and *g* that are real numbers, *g*<*r*; and *g* ϵ *r*.

This second section proves that XYZ*r* = (*n-f*), for some real numbers *n and f*.

Let:

X=(n-f)a; Y=(n-f)b; Z=(n-f)c; V=(n-f)j; U=(n-f)m
XYZr = (n-f)
a = 1/YZr; and b = 1/XZr; and c = 1/XYr; and j= V/XYZr; and m= U/XYZr
X/a = Y/b = Z/c =V/j= (n-f) = XYZVr
a=1/YZr; and b=1/XZr; and c=1/XYr; and j=V/XYZr

Where $-\infty<n,f,a,b,c,j< +\infty$; and n,f,a,b, j and c are real numbers.

X = (n-f)a; and X=xl; and x=(n-f)(a/l);
Y = (n-f)b; and Y=yo; and y=(n-f)(b/o);
Z = (n-f)c; and Z=zq; and z=(n-f)(c/q);
V = (n-f)j; and V=vs; and v=(n-f)(j/s)

Where $-\infty<n,f,a,b,c,l,o,q,s<+\infty$ are real numbers.

$x^2 = (n-f)(a/l)*(n-f)(a/l) = (n-f)(n-f)(a/l)^2 = (n^2-nf-nf+f^2)(a/l)^2 = n^2(a/l)-2nf(a/l)+f^2(a/l)$
$y^2 = (n-f)(b/o)*(n-f)(b/o) = (n-f)(n-f)(b/o)^2 = (n^2-nf-nf+f^2)(b/o)^2 = n^2(b/o)^2-2nf(b/o)^2+f^2(b/o)^2$
$z^2 = (n-f)(c/q)*(n-f)(c/q) = (n-f)(n-f)(c/q)^2 = (n^2-nf-nf+f^2)(c/q)^2 = n^2(c/q)^2-2nf(c/q)^2+f^2(c/q)^2$
$v^2 = (n-f)(j/s)*(n-f)(j/s) = (n-f)(n-f)(j/s)^2 = (n^2-nf-nf+f^2)(j/s)^2 = n^2(j/s)^2-2nf(j/s)^2+f^2(j/s)^2$

Thus:
$x^2+y^2+z^2+v^2 = n^2((a/l)^2+(b/o)^2+(c/q)^2+)j/s)^2)-2nf((a/l)^2+(b/o)^2+(c/q)^2+)j/s)^2)-f^2((a/l)^2+(b/o)^2+(c/q)^2+)j/s)^2) = (n^2-2nf+f^2)((a/l)^2+(b/o)^2+(c/q)^2+)j/s)^2)$

If **rXYZ=(*n-f*)**, then:
$x^2+y^2+z^2+v^2 = (n^2-2nf+f^2)((a/l)^2+(b/o)^2+(c/q)^2+(j/s)^2)$
$= (n-f)(n-f)((a/l)^2+(b/o)^2+(c/q)^2+)j/s)^2)$
$= (XYZr)^2((a/l)^2+(b/o)^2+(c/q)^2+)j/s)^2)$
$= [(XYZr)^2(x/(XYZr)^2)]+[(XYZr)^2(y/(XYZr^2))]+[(XYZr)^2(z/(XYZr)^2)]+[(XYZr)^2(v/XYZr)^2)]$
$= x^2+y^2+z^2+v^2$ ■

Theorem-8: **For the equation $X^2+Y^2+Z^2$ = rXYZ in real numbers, and given Theorems herein and above, if (n-f) is a multiplicative component of each of X,Y and Z (each of X, Y and Z are derived by multiplying (*n-f*) by another real number), then:**

1) If (n-f)=gXYZ, then for all n, f and *g* that are real numbers, g є r.
2) XYZ*r* = (*n-f*), for some real numbers *n and f*.

Proof:
Let:

X=(n-f)a; Y=(n-f)b; Z=(n-f)c;
XYZg = (n-f)
a = 1/YZg; and b = 1/XZg; and c = 1/XYg;

Thus: X=(gXYZ)(a); and Y=(gXYZ)(b); and Z=(gXYZ)(c)

If: $X^2+Y^2+Z^2$= rXYZ;
Then by substitution: $[(g^2X^2Y^2Z^2)(a^2)]+[(g^2X^2Y^2Z^2)(b^2)]+[(g^2X^2Y^2Z^2)(c^2)]$ = rXYZ

Then by dividing both sides of the equation by rXYZ and substituting a=(1/YZg), b=(1/XZg) and c=(1/XYg), the result is: $\{[(g^2X^2Y^2Z^2)(1/(Y^2Z^2g^2))]/rXYZ\}+\{[(g^2X^2Y^2Z^2)(1/(X^2Z^2g^2))]/rXYZ\}+\{[(g^2X^2Y^2Z^2)(1/(X^2Y^2g^2))]/rXYZ\} = 1$;
and thus: [(X/rYZ)+(Y/rXZ)+(Z/rXY)] = 1

By taking a common denominator rXYZ for the left-hand side of the equation, the result is: $[(X^2+Y^2+Z^2)/rXYZ] = 1$;
and by multiplying both sides of the equation by rXYZ, the result is: $X^2+Y^2+Z^2$ = rXYZ

r can be expressed solely in terms of X, Y and Z as follows:

$\{[((X^2Y^2Z^2)(1/(Y^2Z^2g^2)))/XYZ]+[((g^2X^2Y^2Z^2)(1/(X^2Z^2g^2)))/XYZ]+[((g^2X^2Y^2Z^2)(1/(X^2Y^2g^2)))/XYZ]\} = r =$
$[(X/YZ)+(Y/XZ)+(Z/XY)]$

Given the foregoing and since X=(gXYZ)(a); and Y=(gXYZ)(b); and Z=(gXYZ)(c); and $X^2+Y^2+Z^2$ = rXYZ, for all n, f and *g* that are real numbers, *g*<*r*; and *g* ϵ *r*.

This second section proves that XYZ*r* = (*n-f*), for some real numbers *n and f*.

X = (n-f)a; Y = (n-f)b; and Z = (n-f)c
Thus: X/a = Y/b = Z/c = (n-f) = XYZr
Then: a = 1/YZr; and b = 1/XZr; and c = 1/XYr;
Where -∞<n,f,a,b,c,< +∞, are real numbers.

$X^2 = (n\text{-}f)a*(n\text{-}f)a = (n\text{-}f)(n\text{-}f)a^2 = (n^2\text{-}nf\text{-}nf+f^2)a^2 = n^2a^2\text{-}2nf(a^2)+f^2a^2$
$Y^2 = (n\text{-}f)b*(n\text{-}f)b = (n\text{-}f)(n\text{-}f)b^2 = (n^2\text{-}nf\text{-}nf+f^2)b^2 = n^2b^2\text{-}2nf(b^2)+f^2b^2$
$Z^2 = (n\text{-}f)c*(n\text{-}f)c = (n\text{-}f)(n\text{-}f)c^2 = (n^2\text{-}nf\text{-}nf+f^2)c^2 = n^2c^2\text{-}2nf(c^2)+f^2c^2$

$X^2+Y^2+Z^2= n^2a^2\text{-}2nf(a^2)+f^2a^2 + n^2b^2\text{-}2nf(b^2)+f^2b^2 + n^2c^2\text{-}2nf(c^2)+f^2c^2$
$= n^2a^2 + n^2b^2 + n^2c^2\text{-}2nf(a^2) \text{-}2nf(b^2) \text{-}2nf(c^2)+f^2a^2 +f^2b^2 +f^2c^2$
$= n^2(a^2+b^2+c^2)\text{-}2nf(a^2+b^2+c^2)\text{-}f^2(a^2+b^2+c^2) = (n^2\text{-}2nf+f^2)(a^2+b^2+c^2)$

Thus if **XYZ*r* = (*n-f*),** then:
$X^2+Y^2+Z^2 = (n^2\text{-}2nf+f^2)(a^2+b^2+c^2)$
$= (n\text{-}f)(n\text{-}f)(a^2+b^2+c^2)$
$= (XYZr)^2(a^2+b^2+c^2)$
$= [(XYZr)^2(1/(YZr)^2)]+[(XYZr)^2(1/(XZr^2))]+[(XYZr)^2(1/(XYr)^2)]$
$= [(XYZr)^2/(YZr)^2]+[(XYZr)^2/XZr^2)]+[(XYZr)^2/(XYr)^2)]$
$= X^2+Y^2+Z^2$ ■

Theorem-9: **For the equation $X^2+Y^2+Z^2+V^2 +U^2$ = rXYZ in real numbers, and given Theorems herein and above, if (n-f) is a multiplicative component of each of X,Y, V, Z and U (each of X,Y, V, U and Z are derived by multiplying (*n-f*) by another real number), then:**

1) If (n-f)=gXYZ, for all n, f and *g* that are real numbers, g ϵ r.
2) XYZ*r* = (*n-f*), for some real numbers *n and f*.

Proof:
Let:
X=(n-f)a; Y=(n-f)b; Z=(n-f)c; V=(n-f)j; U=(n-f)m
XYZg = (n-f)
a = 1/YZg; and b = 1/XZg; and c = 1/XYg; and j= V/XYZg; and m= U/XYZg

Thus: X=(gXYZ)(a); and Y=(gXYZ)(b); and Z=(gXYZ)(c) and V = (gXYZ)(j) and U= (gXYZ)(m)

If: $\mathbf{X^2+Y^2+Z^2+V^2 +U^2}$ = rXYZ;
Then by substitution: $[(g^2X^2Y^2Z^2)(a^2)]+[(g^2X^2Y^2Z^2)(b^2)]+[(g^2X^2Y^2Z^2)(c^2)] + [(g^2X^2Y^2Z^2)(j^2) +$
$[(g^2X^2Y^2Z^2)(m^2)]= rXYZ$

Then by dividing both sides of the equation by rXYZ and substituting a=(1/YZg), b=(1/XZg), c=(1/XYg), and j= V/XYZg, and m= U/XYZg, the result is:
$\{[(g^2X^2Y^2Z^2)(1/(Y^2Z^2g^2))]/rXYZ\}+\{[(g^2X^2Y^2Z^2)(1/(X^2Z^2g^2))]/rXYZ\}+ \{[(g^2X^2Y^2Z^2)(1/(X^2Y^2g^2))]/rXYZ\} +$
$\{[(g^2X^2Y^2Z^2)(V^2/(X^2Y^2Z^2g^2))]/rXYZ\}+ \{[(g^2X^2Y^2Z^2)(U^2/(X^2Y^2Z^2g^2))]/rXYZ\}= 1;$
and thus: $[(X/rYZ)+(Y/rXZ)+(Z/rXY)] +(V^2/rXYZ) +(U^2/rXYZ)]= 1$

By taking a common denominator rXYZ for the left-hand side of the equation, the result is:
$[(\mathbf{X^2+Y^2+Z^2+V^2+U^2})/rXYZ] = 1$;
and by multiplying both sides of the equation by rXYZ, the result is: $\mathbf{X^2+Y^2+Z^2+V^2+U^2} = rXYZ$

r can be expressed solely in terms of X, Y, Z, V and U as follows:
$\{[((X^2Y^2Z^2)(1/(Y^2Z^2g^2)))/XYZ]+[((g^2X^2Y^2Z^2)(1/(X^2Z^2g^2)))/XYZ]+[((g^2X^2Y^2Z^2)(1/(X^2Y^2g^2)))/XYZ] + [((g^2X^2Y^2Z^2)(V^2/(X^2Y^2Z^2g^2)))/XYZ] + [((g^2X^2Y^2Z^2)(U^2/(X^2Y^2Z^2g^2)))/XYZ]\} = d = [(X/YZ)+(Y/XZ)+(Z/XY)+(V^2/(XYZ))+(U^2/(XYZ))]$

Given the foregoing and since X=(gXYZ)(a); and Y=(gXYZ)(b); and Z=(gXYZ)(c); and V=(gXYZ)(j); and U=(gXYZ)(m); and $\mathbf{X^2+Y^2+Z^2+V^2+U^2} = rXYZ$, for all n, f and *g* that are real numbers, *g*<*r*; and *g* ϵ *r*.

This second section proves that XYZ*r* = (*n-f*), for some real numbers *n and f*.

Let:
X = (n-f)a; Y = (n-f)b; Z = (n-f)c; V = (n-f)j; U = (n-f)m
Thus: X/a = Y/b = Z/c =V/j=U/m= (n-f) = XYZr
Then: a=1/YZr; and b=1/XZr; and c=1/XYr; and j=V/XYZr; and m=U/XYZr
Where -∞<n,f,a,b,c,j, m< +∞, are real numbers.

$X^2 = (n-f)a*(n-f)a = (n-f)(n-f)a^2 = (n^2-nf-nf+f^2)a^2 = n^2a^2-2nf(a^2)+f^2a^2$
$Y^2 = (n-f)b*(n-f)b = (n-f)(n-f)b^2 = (n^2-nf-nf+f^2)b^2 = n^2b^2-2nf(b^2)+f^2b^2$
$Z^2 = (n-f)c*(n-f)c = (n-f)(n-f)c^2 = (n^2-nf-nf+f^2)c^2 = n^2c^2-2nf(c^2)+f^2c^2$
$V^2 = (n-f)j*(n-f)j = (n-f)(n-f)j^2 = (n^2-nf-nf+f^2)j^2 = n^2j^2-2nf(j^2)+f^2j^2$
$U^2 = (n-f)m*(n-f)m = (n-f)(n-f)m^2 = (n^2-nf-nf+f^2)m^2 = n^2m^2-2nf(m^2)+f^2m^2$

Thus:
If XYZr = (n-f), then: $X^2+Y^2+Z^2+V^2+U^2= n^2(a^2+b^2+c^2+j^2+m^2)-2nf(a^2+b^2+c^2+j^2+m^2)+f^2(a^2+b^2+c^2+j^2+m^2)$
$= (n^2-2nf+f^2)(a^2+b^2+c^2+j^2+m^2)$
$= (n-f)(n-f)(a^2+b^2+c^2+j^2+m^2)$
$= (XYZr)^2(a^2+b^2+c^2+j^2+m^2)$
$= [(XYZr)^2(1/(YZr)^2)]+[(XYZr)^2(1/(XZr^2))]+[(XYZr)^2(1/(XYr)^2)]+[(XYZr)^2(V/XYZr)^2)]$
$+[(XYZr)^2(U/XYZr)^2)]$
$= [(XYZr)^2/(YZr)^2]+[(XYZr)^2/XZr)^2]+[(XYZr)^2/(XYr)^2)]+[V^2(XYZr)^2/(XYZr)^2)]+[U^2(XYZr)^2/(XYZr)^2)]$
$= X^2+Y^2+Z^2+V^2+U^2$ ■

Theorem-10: *For the equation $X^i+Y^i+Z^i+V^i= rXYZ$, and given Theorems above, and for all values of X, Y, V and Z that are real numbers, if (n-f)=gXYZV, and (n-f) is a multiplicative component of each of X,Y, V and Z, then there exists a real number r such that $X^i+Y^i+Z^i+V^i= rXYZ$; where for all g, X, Y, V and Z that are real numbers, g ϵ r; and r can be expressed as $r = [(X^{(i-1)}/YZ)+(Y^{(i-1)}/XZ)+(Z^{(i-1)}/XY)+(V^{(i-1)}/XYZ)]$.*
Proof: The proof is straightforward and follows from the prior proofs herein and above. ■

Theorem-11: For the equation $x^3+y^3+z^3$=rXYZ in real numbers, where x│X (ie. X is a multiple of x), y│Y, and z│Z exist; if (*n-f*) is a multiplicative component of each of X, Y & Z (each of X,Y and Z are derived by multiplying (*n-f*) by another real number), then for all *g*, n and f that are real numbers:
1) **XYZ*g* = (*n-f*), and**
2) ***g* ϵ r.**

Proof:
Let:
X = (n-f)a; and X=xa_1; and x=(n-f)(a/a_1);
Y = (n-f)b; and Y=yb_1; and y=(n-f)(b/b_1);

$Z = (n-f)c$; and $Z=zc_1$; and $z=(n-f)(c/c_1)$;

Where $-\infty<n,f,a,b,c,a_1,b_1,c_1<+\infty$ are real numbers.

$x^3 = (n-f)(a/a_1)*(n-f)(a/a_1)*(n-f)(a/a_1) = (n-f)(n-f)(n-f)(a/a_1)^3= (n^2-nf-nf+f^2)(n-f)(a/a_1)^3 = n^3(a/a_1)^3-3n^2f(a/a_1)^3+3nf^2(a/a_1)^3+f^3(a/a_1)^3$
$y^3 = (n-f)(b/b_1)*(n-f)(b/b_1)*(n-f)(b/b_1) = (n-f)(n-f)(n-f)(b/b_1)^3= (n^2-nf-nf+f^2)(n-f)(b/b_1)^3 = n^3(b/b_1)^3-3n^2f(b/b_1)^3+3nf^2(b/b_1)^3+f^3(b/b_1)^3$
$z^3 = (n-f)(c/c_1)*(n-f)(c/c_1)*(n-f)(c/c_1) = (n-f)(n-f)(n-f)(c/c_1)^3= (n^2-nf-nf+f^2)(n-f)(c/c_1)^3 = n^3(c/c_1)^3-3n^2f(c/c_1)^3+3nf^2(c/c_1)^3+f^3(c/c_1)^3$
Thus:
$x^3+y^3+z^3 = n^3((a/a_1)^3+(b/b_1)^3+(c/c_1)^3)-3n^2f((a/a_1)^3+(b/b_1)^3+(c/c_1)^3) + 3nf^2((a/a_1)^3+(b/b_1)^3+(c/c_1)^3) + f^3((a/a_1)^3+(b/b_1)^3+(c/c_1)^3)= (n^3-3n^2f-3nf^2+f^3)((a/a_1)^3+(b/b_1)^3+(c/c_1)^3= (n-f)^3((a/a_1)^3+(b/b_1)^3+(c/c_1)^3)$

From above: $X/a = Y/b = Z/c = (n-f) = XYZg$
$a = 1/YZg$; and $(a/a_1) = x/(n-f) = x/XYZg$
$b = 1/XZg$; and $(b/b_1) = y/(n-f) = y/XYZg$
$c = 1/XYg$; and $(c/c_1) = z/(n-f) = z/XYZg$

If **XYZ*g* = (*n-f*)**, then: $x^3+y^3+z^3 = (n-f)^3((a/a_1)^3+(b/b_1)^3+(c/c_1)^3)$
$= (XYZg)^3((a/a_1)^3+(b/b_1)^3+(c/c_1)^3)$
$= [(XYZg)^3(x/XYZg)^3]+[(XYZg)^3(y/XYZg)^3]+[(XYZg)^3(z/XYZg)^3]$
$= x^3+y^3+z^3$

This second section proves that ***g* ϵ r.**
As stated herein and above:
$X = (n-f)a$; $X=a_1x$; and $x = (n-f)(X/a_1)$;
$Y = (n-f)b$; $Y=b_1y$; and $y = (n-f)(Y/b_1)$;
$Z = (n-f)c$; $Z=c_1z$; and $z = (n-f)(Z/c_1)$;
$(n-f) = XYZg$

$a = x/XYZg$
$b = y/XYZg$
$c = z/XYZg$

Thus, gXYZ is a multiplicative component of each of x, y and z. That is:
$x=(gXYZ)(a)$; and $y=(gXYZ)(b)$; and $z=(gXYZ)(c)$

If: $x^3+y^3+z^3 = dXYZ$, then by dividing both sides of the equation by rXYZ and substituting $a=(x/XYZg)$, $b=(y/XYZg)$ and $c=(z/XYZg)$, the result is:

$[(g^3X^3Y^3Z^3)(a^3)]+[(g^3X^3Y^3Z^3)(b^3)]+[(g^3X^3Y^3Z^3)(c^3)] = rXYZ$;
And: $\{[(g^3X^3Y^3Z^3)(x^3/X^3Y^3Z^3g^3))]/rXYZ\}+\{[(g^3X^3Y^3Z^3)(y^3/X^3Y^3Z^3g^3))]/rXYZ\} + \{[(g^3X^3Y^3Z^3)(z^3/X^3Y^3Z^3g^3))]/rXYZ\} = 1$;
And: $(x^3/rXYZ)+(y^3/rXYZ)+(z^3/rXYZ) = 1$;
And: $rXYZ = x^3+y^3+z^3$

Given the foregoing and since $x=(gXYZ)(a)$; and $y=(gXYZ)(b)$; and $z=(gXYZ)(c)$; and $x^3+y^3+z^3 =rXYZ$, for all X, Y, Z and *g* that are real numbers, *g* ϵ *r*. ■

Theorem-12: For the equation $x^3+y^3+z^3+x^6+y^6+z^6=rXYZ$ in real numbers, where x│X (ie. X is a multiple of x), y│Y, and z│Z exist; if (*n-f*) is a multiplicative component of each of X, Y & Z (each of X,Y and Z are derived by multiplying (*n-f*) by another real number), then for all *g*, n and f that are real numbers:

1) **XYZ*g* = (*n*-*f*), and**
2) ***g* ϵ r.**

Proof:
Let:
$X = (n-f)a$; and $X=xa_1$; and $x=(n-f)(a/a_1)$;
$Y = (n-f)b$; and $Y=yb_1$; and $y=(n-f)(b/b_1)$;
$Z = (n-f)c$; and $Z=zc_1$; and $z=(n-f)(c/c_1)$;

Where $-\infty<n,f,a,b,c,a_1,b_1,c_1< +\infty$ are real numbers.

$x^3 = (n-f)(a/a_1)*(n-f)(a/a_1)*(n-f)(a/a_1) = (n-f)(n-f)(n-f)(a/a_1)^3= (n^2-nf-nf+f^2)(n-f)(a/a_1)^3 = n^3(a/a_1)^3-3n^2f(a/a_1)^3+3nf^2(a/a_1)^3+f^3(a/a_1)^3$
$y^3 = (n-f)(b/b_1)*(n-f)(b/b_1)*(n-f)(b/b_1) = (n-f)(n-f)(n-f)(b/b_1)^3= (n^2-nf-nf+f^2)(n-f)(b/b_1)^3 = n^3(b/b_1)^3-3n^2f(b/b_1)^3+3nf^2(b/b_1)^3+f^3(b/b_1)^3$
$z^3 = (n-f)(c/c_1)*(n-f)(c/c_1)*(n-f)(c/c_1) = (n-f)(n-f)(n-f)(c/c_1)^3= (n^2-nf-nf+f^2)(n-f)(c/c_1)^3 = n^3(c/c_1)^3-3n^2f(c/c_1)^3+3nf^2(c/c_1)^3+f^3(c/c_1)^3$
Thus:
$x^3+y^3+z^3 = n^3((a/a_1)^3+(b/b_1)^3+(c/c_1)^3)-3n^2f((a/a_1)^3+(b/b_1)^3+(c/c_1)^3) + 3nf^2((a/a_1)^3+(b/b_1)^3+(c/c_1)^3) + f^3((a/a_1)^3+(b/b_1)^3+(c/c_1)^3)= (n^3-3n^2f-3nf^2+f^3)((a/a_1)^3+(b/b_1)^3+(c/c_1)^3= (n-f)^3((a/a_1)^3+(b/b_1)^3+(c/c_1)^3)$

$x^6 = (n-f)^2(a/a_1)^2*(n-f)^2(a/a_1)^2*(n-f)^2(a/a_1)^2 = (n-f)^2(n-f)^2(n-f)^2(a/a_1)^6= (n^2-2nf+f^2)(n^2-2nf+f^2)(n^2-2nf+f^2)(a/a_1)^6$
$=(n^4-4n^3f+6n^2f^2-4nf^3+f^4)(n^2-2nf+f^2)(a/a_1)^6$
$= (n^6-4n^5f+6n^4f^2-4n^3f^3+n^2f^4-2n^5f+8n^4f^2-12n^3f^3+8n^2f^4+2nf^5+n^4f^2-4n^3f^3+6n^2f^4-4nf^5+f^6) (a/a_1)^6$
$= (n^6-8n^5f+15n^4f^2-20n^3f^3+15n^2f^4+f^6)(a/a_1)^6$.

$y^6 = (n^6-8n^5f+15n^4f^2-20n^3f^3+15n^2f^4+f^6)(b/b_1)^6$
$z^6 = (n^6-8n^5f+15n^4f^2-20n^3f^3+15n^2f^4+f^6)(c/c_1)^6$
Thus:
$x^6+y^6+z^6 = (n^6-8n^5f+15n^4f^2-20n^3f^3+15n2f^4+f^6)((a/a_1)^6+(b/b_1)^3+(c/c_1)^6) = (n-f)^6((a/a_1)^6+(b/b_1)^6+(c/c_1)^6)$

From above: $X/a = Y/b = Z/c = (n-f) = XYZg$
$a = 1/YZg$; and $(a/a_1) = x/(n-f) = x/XYZg$
$b = 1/XZg$; and $(b/b_1) = y/(n-f) = y/XYZg$
$c = 1/XYg$; and $(c/c_1) = z/(n-f) = z/XYZg$

If **XYZ*g* = (*n*-*f*)**, then: $x^3+y^3+z^3+ x^6+y^6+z^6 = [(n-f)^3((a/a_1)^3+(b/b_1)^3+(c/c_1)^3)]+ [(n-f)^6((a/a_1)^6+(b/b_1)^6+(c/c_1)^6)]$
$= [(XYZg)^3((a/a_1)^3+(b/b_1)^3+(c/c_1)^3)]+ [(XYZg)^6((a/a_1)^6+(b/b_1)^6+(c/c_1)^6)]$
$= [(XYZg)^3(x/XYZg)^3]+[(XYZg)^3(y/XYZg)^3]+[(XYZg)^3(z/XYZg)^3] + [(XYZg)^6(x/XYZg)^6] + [(XYZg)^6(y/XYZg)^6] + [(XYZg)^6(z/XYZg)^6]$
$= x^3+y^3+z^3+x^6+y^6+z^6$.

This following second section proves that ***g* ϵ r.**

As stated herein and above:
$X = (n-f)a$; $X=a_1x$; and $x = (n-f)(X/a_1)$;
$Y = (n-f)b$; $Y=b_1y$; and $y = (n-f)(Y/b_1)$;
$Z = (n-f)c$; $Z=c_1z$; and $z = (n-f)(Z/c_1)$;
$(n-f) = XYZg$

$a = x/XYZg$
$b = y/XYZg$
$c = z/XYZg$

Thus, gXYZ is a multiplicative component of each of x, y and z. That is:
x=(gXYZ)(a); and y=(gXYZ)(b); and z=(gXYZ)(c)

If: $x^3+y^3+z^3+x^6+y^6+z^6$= rXYZ, then by substituting a=(x/XYZg), b=(z/XYZg) and c=(z/XYZg), and by dividing both sides of the equation by rXYZ, the result is:

$[(g^3X^3Y^3Z^3)(a^3)]+[(g^3X^3Y^3Z^3)(b^3)]+[(g^3X^3Y^3Z^3)(c^3)] + [(g^6X^6Y^6Z^6)(a^6)] + [(g^6X^6Y^6Z^6)(b^6)] + [(g^6X^6Y^6Z^6)(c^6)] = rXYZ$;

and: $\{[(g^3X^3Y^3Z^3)(x^3/X^3Y^3Z^3g^3))]/rXYZ\}+\{[(g^3X^3Y^3Z^3)(y^3/X^3Y^3Z^3g^3))]/rXYZ\} + \{[(g^3X^3Y^3Z^3)(z^3/X^3Y^3Z^3g^3))]/rXYZ\}+\{[(g^6X^6Y^6Z^6)(x^6/X^6Y^6Z^6g^6))]/rXYZ\} + \{[(g^6X^6Y^6Z^6)(y^6/X^6Y^6Z^6g^6))]/rXYZ\}+\{[(g^6X^6Y^6Z^6)(z^6/X^6Y^6Z^6g^6))]/rXYZ\} = 1$;

and thus: $(x^3/rXYZ)+(y^3/rXYZ)+(z^3/rXYZ) +(x^6/rXYZ)+(y^6/rXYZ)+(z^6/rXYZ) = 1$;

and: $x^3+y^3+z^3+x^6+y^6+z^6 = rXYZ$

Given the foregoing and since X=(gXYZ)(a); and Y=(gXYZ)(b); and Z=(gXYZ)(c); and $x^3+y^3+z^3+x^6+y^6+z^6$ = rXYZ, for all X, Y, Z and *g* that are real numbers, $g<r$; and $g \in r$. ■

Theorem-13: **For the equation $x^6+y^6+z^6$=rXYZ in real numbers, where x│X (ie. X is a multiple of x), y│Y, and z│Z exist; if (*n-f*) is a multiplicative component of each of X, Y & Z (each of X,Y and Z are derived by multiplying (*n-f*) by another real number), then for all *g*, n and f that are real numbers:**

1) **XYZ*g* = (*n-f*), and**
2) ***g* ϵ r.**

Proof:
Let:
X = (n-f)a; and X=xa_1; and x=(n-f)(a/a_1);
Y = (n-f)b; and Y=yb_1; and y=(n-f)(b/b_1);
Z = (n-f)c; and Z=zc_1; and z=(n-f)(c/c_1);

Where $-\infty<n,f,a,b,c,a_1,b_1,c_1<+\infty$ are real numbers.

$x^6 = (n-f)^2(a/a_1)^2*(n-f)^2(a/a_1)^2*(n-f)^2(a/a_1)^2 = (n-f)^2(n-f)^2(n-f)^2(a/a_1)^6= (n^2-2nf+f^2)(n^2-2nf+f^2)(n^2-2nf+f^2)(a/a_1)^6$
$=(n^4-4n^3f+6n^2f^2-4nf^3+f^4)(n^2-2nf+f^2)(a/a_1)^6$
$= (n^6-4n^5f+6n^4f^2-4n^3f^3+n^2f^4-2n^5f+8n^4f^2-12n^3f^3+8n^2f^4+2nf^5+n^4f^2-4n^3f^3+6n^2f^4-4nf^5+f^6)(a/a_1)^6$
$= (n^6-8n^5f+15n^4f^2-20n^3f^3+15n^2f^4+f^6)(a/a_1)^6$

$y^6 = (n^6-8n^5f+15n^4f^2-20n^3f^3+15n^2f^4+f^6)(b/b_1)^6$
$z^6 = (n^6-8n^5f+15n^4f^2-20n^3f^3+15n^2f^4+f^6)(c/c_1)^6$
Thus:
$x^6+y^6+z^6 = (n^6-8n^5f+15n^4f^2-20n^3f^3+15n^2f^4+f^6)((a/a_1)^6+(b/b_1)^6+(c/c_1)^6) = (n-f)^6((a/a_1)^6+(b/b_1)^6+(c/c_1)^6)$

From above: X/a = Y/b = Z/c = (n-f) = XYZg
a = 1/YZg; and (a/a_1) = x/(n-f) = x/XYZg
b = 1/XZg; and (b/b_1) = y/(n-f) = y/XYZg
c = 1/XYg; and (c/c_1) = z/(n-f) = z/XYZg

If **XYZ*g* = (*n-f*)**, then: $x^6+y^6+z^6 = (n-f)^6((a/a_1)^6+(b/b_1)^6+(c/c_1)^6)$
$= (XYZg)^6((a/a_1)^6+(b/b_1)^6+(c/c_1)^6)$
$= [(XYZg)^6(x/XYZg)^6]+[(XYZg)^6(y/XYZg)^6]+[(XYZg)^6(z/XYZg)^6]$
$= x^6+y^6+z^6$

This following second section proves that ***g* ϵ r.**
X = (n-f)a; X=a_1x; and X = (n-f)a_1a;
Y = (n-f)b; Y=b_1y; and Y = (n-f)b_1b;

$Z = (n\text{-}f)c$; $Z=c_1z$; and $Z = (n\text{-}f)c_1c$;
$(n\text{-}f) = XYZg$

$a = x/XYZg$
$b = y/XYZg$
$c = z/XYZg$

Thus, gXYZ is a multiplicative component of each of x, y and z. That is:
x=(gXYZ)(a); and y=(gXYZ)(b); and z=(gXYZ)(c)

If: $x^6+y^6+z^6 = dXYZ$; then by dividing both sides of the equation by rXYZ and substituting a=(x/XYZg), b=(z/XYZg) and c=(z/XYZg), the result is:

$$[(g^6X^6Y^6Z^6)(a^6)]+[(g^6X^6Y^6Z^6)(b^6)]+[(g^6X^6Y^6Z^6)(c^6)] = rXYZ$$

And: $\{[(g^6X^6Y^6Z^6)(x^6/X^6Y^6Z^6g^6))]/rXYZ\}+\{[(g^6X^6Y^6Z^6)(y^6/X^6Y^6Z^6g^6))]/rXYZ\}+\{[(g^6X^6Y^6Z^6)(z^6/X^6Y^6Z^6g^6))]/rXYZ\} = 1$;

And: $(x^6/rXYZ)+(y^6/rXYZ)+(z^6/rXYZ) = 1$;

And: $x^6+y^6+z^6 = rXYZ$

Given the foregoing and since X=(gXYZ)(a); and Y=(gXYZ)(b); and Z=(gXYZ)(c); and $x^6+y^6+z^6 = rXYZ$, for all X, Y, Z and *g* that are real numbers, $g<r$; and $g \in r$. ■

Theorem-14: **For the equation $[(x^{12}+y^{12}+z^{12})-(x^6+y^6+z^6)]=rXYZ$ in real numbers, where x│X (ie. X is a multiple of x), y│Y, and z│Z exist; if (*n-f*) is a multiplicative component of each of X, Y & Z (each of X,Y and Z are derived by multiplying (*n-f*) by another real number), then for all *g*, n and f that are real numbers:**

1) **XYZ*g* = (*n-f*), and**
2) ***g* ϵ r.**

Proof:
Let:
$X = (n\text{-}f)a$; $X=a_1x$; and $X = (n\text{-}f)a_1a$;
$Y = (n\text{-}f)b$; $Y=b_1y$; and $Y = (n\text{-}f)b_1b$;
$Z = (n\text{-}f)c$; $Z=c_1z$; and $Z = (n\text{-}f)c_1c$;
$(n\text{-}f) = XYZg$

Where $-\infty<n,f,a,b,c,a_1,b_1,c_1<+\infty$ are real numbers.

$$x^6 = (n-f)^2(a/a_1)^2*(n-f)^2(a/a_1)^2*(n-f)^2(a/a_1)^2 = (n-f)^2(n-f)^2(n-f)^2(a/a_1)^6= (n^2-2nf+f^2)(n^2-2nf+f^2)(n^2-2nf+f^2)(a/a_1)^6$$
$$=(n^4-4n^3f+6n^2f^2-4nf^3+f^4)(n^2-2nf+f^2)(a/a_1)^6$$
$$= (n^6-4n^5f+6n^4f^2-4n^3f^3+n^2f^4-2n^5f+8n^4f^2-12n^3f^3+8n^2f^4+2nf^5+n^4f^2-4n^3f^3+6n^2f^4-4nf^5+f^6)(a/a_1)^6$$
$$= (n^6-8n^5f+15n^4f^2-20n^3f^3+15n^2f^4+f^6)(a/a_1)^6$$
$$y^6 = (n^6-8n^5f+15n^4f^2-20n^3f^3+15n^2f^4+f^6)(b/b_1)^6$$
$$z^6 = (n^6-8n^5f+15n^4f^2-20n^3f^3+15n^2f^4+f^6)(c/c_1)^6$$

Thus:
$$x^6+y^6+z^6 = (n^6-8n^5f+15n^4f^2-20n^3f^3+15n^2f^4+f^6)((a/a_1)^6+(b/b_1)^6+(c/c_1)^6) = (n-f)^6[(a/a_1)^6+(b/b_1)^6+(c/c_1)^6]$$
Similarly, $x^{12}+y^{12}+z^{12} = (n-f)^{12}[(a/a_1)^{12}+(b/b_1)^{12}+(c/c_1)^{12}]$

From above: $X/a = Y/b = Z/c = (n\text{-}f) = XYZg$
$a = 1/YZg$; and $(a/a_1) = x/(n\text{-}f) = x/XYZg$
$b = 1/XZg$; and $(b/b_1) = y/(n\text{-}f) = y/XYZg$
$c = 1/XYg$; and $(c/c_1) = z/(n\text{-}f) = z/XYZg$

If **XYZ*g* = (*n-f*)**, then: $(x^{12}+y^{12}+z^{12})-(x^6+y^6+z^6) = [(n-f)^{12}((a/a_1)^{12}+(b/b_1)^{12}+(c/c_1)^{12})]-[(n-f)^6((a/a_1)^6+(b/b_1)^6+(c/c_1)^6)]$

$= [(XYZg)^{12}((a/a_1)^{12}+(b/b_1)^{12}+(c/c_1)^{12})]-[(XYZg)^6((a/a_1)^6+(b/b_1)^6+(c/c_1)^6)]$

$= ([(XYZg)^{12}(x/XYZg)^{12}]+[(XYZg)^{12}(y/XYZg)^{12}] + [(XYZg)^{12}(z/XYZg)^{12}]) -([(XYZg)^6(x/XYZg)^6] + [(XYZg)^6(y/XYZg)^6] + [(XYZg)^6(z/XYZg)^6])$

$= (x^{12}+y^{12}+z^{12})-(x^6+y^6+z^6)$

This following second section proves that ***g* ϵ r.**

As stated herein and above:
$x = (n-f)a$; $X=a_1x$; and $x = (n-f)(X/a_1)$;
$y = (n-f)b$; $Y=b_1y$; and $y = (n-f)(Y/b_1)$;
$z = (n-f)c$; $Z=c_1z$; and $z = (n-f)(Z/c_1)$;
$(n-f) = XYZg$

$a = x/XYZg$
$b = y/XYZg$
$c = z/XYZg$

Thus, gXYZ is a multiplicative component of each of x, y and z. That is:
$x=(gXYZ)(a)$; and $y=(gXYZ)(b)$; and $z=(gXYZ)(c)$

If $(x^{12}+y^{12}+z^{12})-(x^6+y^6+z^6)= dXYZ$; then by substituting $a=(x/XYZg)$, $b=(z/XYZg)$ and $c=(z/XYZg)$, and dividing both sides of the equation by rXYZ and the result is:

$[(g^{12}X^{12}Y^{12}Z^{12}a^{12})+(g^{12}X^{12}Y^{12}Z^{12}b^{12})+(g^{12}X^{12}Y^{12}Z^{12}c^{12})] - [(g^6X^6Y^6Z^6a^6) + (g^6X^6Y^6Z^6b^6) + (g^6X^6Y^6Z^6c^6)] = rXYZ$;

And: $\{[(g^{12}X^{12}Y^{12}Z^{12})(x^{12}/g^{12}X^{12}Y^{12}Z^{12}))]/rXYZ\}+\{[(g^{12}X^{12}Y^{12}Z^{12}) (y^{12}/g^{12}X^{12}Y^{12}Z^{12}))]/rXYZ\} + \{[(g^{12}X^{12}Y^{12}Z^{12})(z^{12}/g^{12}X^{12}Y^{12}Z^{12}))]/rXYZ\} - \{[(g^6X^6Y^6Z^6)(x^6/X^6Y^6Z^6g^6))]/rXYZ\} - \{[(g^6X^6Y^6Z^6)(y^6/X^6Y^6Z^6g^6))]/rXYZ\} - \{[(g^6X^6Y^6Z^6)(z^6/X^6Y^6Z^6g^6))]/rXYZ\} = 1$;

and: $[(x^{12}/rXYZ)+(y^{12}/rXYZ)+(z^{12}/rXYZ)]-[(x^6/rXYZ)+(y^6/rXYZ)+(z^6/rXYZ)] = 1$;

And: $[(x^{12}+y^{12}+z^{12})-(x^6+y^6+z^6)]/rXYZ = 1$; and $[(x^{12}+y^{12}+z^{12})-(x^6+y^6+z^6)] = rXYZ$.

Given the foregoing and since $X=(gXYZ)(a)$; and $Y=(gXYZ)(b)$; and $Z=(gXYZ)(c)$; and $x^{12}+y^{12}+z^{12}-x^6-y^6-z^6= rXYZ$, for all n, f and *g* that are real numbers, $g<r$; and $g \in r$. ■

Theorem-15: **For the equation $x^i+y^i+z^i=rXYZ$, and where x=X, y=Y, z=Z, g, n and f are real numbers; and given Theorems herein and above, if (n-f)=gXYZ, and (n-f) is a multiplicative component of each of X,Y and Z, then g ϵ r; and r can be expressed as $r = [(X^{(i-1)}/YZ)+(Y^{(i-1)}/XZ)+(Z^{(i-1)}/XY)]$.**
Proof: The proof is straightforward and follows from prior proofs above. ■

Conclusion.
The equations studied herein exhibit patterns of Nonlinearity that have potential applications in Applied Math, Computer Science, Economics and Physics.

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