Energy flow and photons from primary coil to secondary coil of transformer

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***Abstract*—The author proposed the mutual energy theorem in 1987 and the mutual energy theory since 2017. The theory of mutual energy has an axiom more than Maxwell's equations, that is, the law of conservation of energy. The conservation of energy law is not Poynting's theorem, but an extension of the mutual energy theorem. Because of this new axiom, the new axiom conflicts with Poynting's theorem. In fact, it is also a conflict with Maxwell's electromagnetic theory. The author believes that the law of conservation of energy should be adhered to, so Poynting's theorem or Maxwell's classical electromagnetic theory should be appropriately modified. These works have been completed by the author in other papers. Recently, when studying the magnetic quasi-static electromagnetic field, the author found that the electromagnetic energy conservation law proposed by the author can be strictly proved in the magnetic quasi-static electromagnetic field environment. This is a great inspiration to the author. Based on the theory of magnetic quasi-static electromagnetic field, the author strictly proves the law of conservation of energy (mutual energy theorem) and the mutual energy flow theorem proposed by the author, and applies this theory to calculate the energy flow from the primary coil to the secondary coil of transformer. In order to simplify the problem, the author only calculates the mutual energy flow between two parallel current elements. The calculation results show that the mutual energy flow is generated on the primary coil and annihilated on the secondary coil. The properties of mutual energy flow are completely consistent with those of photons. It is also close to the photon model in Cramer's transactional interpretation of quantum mechanics. The author believes that the transmitting antenna is consistent with the primary coil of the transformer, and the receiving antenna is consistent with the secondary coil of the transformer. The author believes that for the theoretical generalization from magnetic quasi-static electromagnetic field to radiated electromagnetic field, the law of conservation of energy, the theorem of mutual energy flow and the time average value of self energy flow should be kept at zero. In this way, a new electromagnetic field theory which is different from Maxwell's electromagnetic theory is obtained. Maxwell's electromagnetic field theory adds displacement current, but it makes formulas such as the energy conservation law problematic in the case of radiation field. Maxwell's electromagnetic field theory and mutual energy theory provide the same electric field, but the far-field part of the magnetic field (i.e. radiated electromagnetic field) is different. In addition, the electromagnetic mutual energy theory proposed by the author believes that the charge of the receiver produces an advanced wave. Advanced wave violates today's causality, but it is an objective physical existence.**

***Index Terms*—Maxwell equation; Reciprocity theorem; conservation of energy; Poynting theorem; Energy flow; transformer; Primary coil; Secondary coil; Transmitting antenna; Receiving antenna; retarded wave; retarded potential; advanced wave; Advanced potential; Absorber; Radiator; Photons; Quantum; Electromagnetic wave; Electromagnetic field;**

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I. Introduction

A. Mutual energy theory of electromagnetic field

In 1987, the author proposed the mutual energy theorem [10], [21], [20], and in 2017, he proposed the mutual energy flow theorem and the self energy principle [11], [12], [13], [14]. The self energy principle shows that there is a time reversal wave, which can offset the self energy flow. Because the author found that the mutual energy flow has actually transferred all the energy. If the self energy flow also transfers energy, the energy transferred by the self energy flow must collapse according to the wave, but this will produce two different photons, the photons of mutual energy flow and the photons of self energy flow collapse. We did not observe two different photons. Therefore, the author assumes that the time reversal of the electromagnetic wave generated by the transmitting antenna method returns. Or the wave collapses in the opposite direction. In this way, the author proposes a method to solve the problem of wave particle duality.

However, time reversal wave must be introduced. We did not observe the time reversal wave in the experiment. Recently, the author found that time reversal wave is unnecessary to introduce. If the phase difference between the magnetic field and the electric field is 90 degrees, this electromagnetic wave constitutes the so-called reactive power wave. The Poynting vector of reactive power wave takes a pure virtual value, so the average value of Poynting energy flow in a time period is zero. In this way, the self energy flow will not transfer energy. Energy is transferred by mutual energy flow. This is in line with the viewpoint of mutual energy theory put forward by the author. However, Maxwell's radiated electric field theory does not support the electromagnetic wave of reactive power. According to Maxwell's electromagnetic theory, the electromagnetic field radiated by the antenna is active power. The electric field and magnetic field of the radiated electromagnetic field are in phase.

The author found that under the condition of magnetic quasi-static electromagnetic field, the electric field and magnetic field have a phase difference of 90 degrees. For example, the phase difference between the surface induced electric field and magnetic field of a wire of AC current is 90 degrees. The author found that under the condition of magnetic quasi-static electromagnetic field, we can completely prove the law of conservation of energy. The law of conservation of energy here is not Poynting's theorem. It is the generalization of the mutual energy theorem proposed by the author. But when it comes to Maxwell's radiated electromagnetic field, due to the appearance of displacement current, this proof cannot be completed. This paper believes that this is because there is a problem in the definition of magnetic field in Maxwell's theory, so this paper defines a new magnetic field. According to the new magnetic field, the electric field and the magnetic field maintain a phase of 90 degrees, so the self energy flow is still zero even for the radiated electromagnetic field. Even under the condition of radiated electromagnetic field, energy is completely transferred by mutual energy flow.

In this paper, the law of conservation of electromagnetic field energy is derived under the condition of magnetic quasi-static electromagnetic field. For the radiated electromagnetic field, the author found that Maxwell extended the theory by introducing the displacement current method. Lorenz uses the method of retarded potential. Kirchhoff is using the current continuity equation. The author believes that even for the radiated electromagnetic field, the law of conservation of energy still holds, so as to obtain the retarded wave radiated by the transmitting antenna and the advanced wave radiated by the receiving antenna. In this way, the author's theory covers the retarded wave and the advanced wave. This proves that the advanced wave is an objective physical existence.

B. Advanced wave

Prior to the author, Wheeler and Feynman proposed the absorber theory in 1945 [1], [2]. The absorber theory is based on the theory of long-distance interaction [16], [18], [6]. Stephenson also proposed his own leading wave theory [17]. Cramer has established the transactional interpretation of quantum mechanics based on the absorber theory, and these interpretations also support the advanced wave [3], [4]. However, the advanced wave above is a qualitative theory. The advanced wave proposed by the author is a quantitative theory, which is proved by the law of conservation of energy in electromagnetic field.

C. Reciprocity theorem

In addition, before the author proposed the mutual energy theorem, the theorems related to the mutual energy theorem were the reciprocity theorem. Here are Welch's time-domain reciprocity theorem [19] (1960), Rumsey's new reciprocity theorem [15] (1963), and after the author, De Hoop's cross-corelation reciprocity theorem [5](at the end of 1987). And the lost second Lorentz reciprocity theorem [9] These theorems are in the same form as the mutual energy theorem or differ by a Fourier transform. But we call it reciprocity theorem. It is generally believed that the reciprocity theorem is a mathematical formula, similar to Green's function. There are two quantities in Green's function and reciprocity theorem, one is real and the other is virtual. The energy theorem is different. For an energy theorem, both quantities in the formula must exist objectively in physics. The reciprocity theorem is not necessarily the energy theorem, but the energy theorem can be the reciprocity theorem. The law of conservation of energy is stronger than the theorem of energy. Before these reciprocity laws, there is Lorentz reciprocity theorem [8] (1900). The mutual energy theorem can be obtained from the Lorentz reciprocity theorem through conjugate transformation [7].

II. BACKGROUND

This paper studies the energy flow between the primary coil and the secondary coil of a transformer. Poynting vector is generally considered to represent energy flow in electromagnetic field theory. The author finds that this view is incomplete. Corresponding to the superposition electromagnetic field, Poynting vector can be divided into two parts: (1) self energy flow, (2) mutual energy flow. In fact, it is more accurate to say that what really transmits energy is a part of Poynting vector, that is, mutual energy flow. The author found that self energy flow corresponds to self induction and does not transmit energy. Mutual energy flow and mutual inductance transfer energy correspondingly. See figure 1.

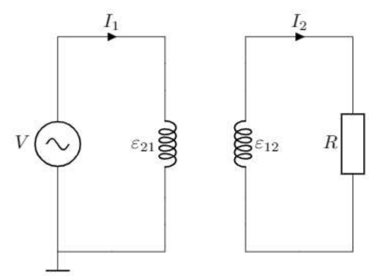

Fig. 1. The transformer has primary coil and secondary coil. The primary coil is connected with AC power supply and the secondary coil is connected with load resistance.

This section calculates the energy flow from the primary coil to the secondary coil of the transformer. The following is the magnetic quasi-static electromagnetic field equation,

$$\nabla \times \boldsymbol{E}_1 = -\frac{\partial}{\partial t}\boldsymbol{B}_1, \qquad \nabla \times \boldsymbol{H}_1 = \boldsymbol{J}_1 \tag{1}$$

$$\nabla \times \boldsymbol{E}_2 = -\frac{\partial}{\partial t}\boldsymbol{B}_2, \qquad \nabla \times \boldsymbol{H}_2 = \boldsymbol{J}_2 \tag{2}$$

The subscript (1) represents the primary coil and the subscript (2) represents the secondary coil. In the case of magnetic quasi-static electromagnetic field, the displacement current term $\frac{\partial}{\partial t}\boldsymbol{D}$ does not appear in the above equation. Magnetic quasi-static electromagnetic field is the electromagnetic field theory before Maxwell. The correctness of this part of electromagnetic field theory is based on Ampere's law, Faraday's law, Weber, Newman, Kirchhoff and others on the basis of experiment and theory. Maxwell's equation is obtained by considering the retarded effect of waves on this basis. Maxwell's electromagnetic theory is equivalent to Lorenz (not Lorentz) retarded potential theory. Maxwell's theory, adding displacement current $\frac{\partial}{\partial t}\boldsymbol{D}$ in ampere circuital law can be regarded as a retarded potential theory.

$$\boldsymbol{A} = \frac{\mu_0}{4\pi}\iiint_V \frac{\boldsymbol{J}}{r}dV \rightarrow \frac{\mu_0}{4\pi}\iiint_V \frac{\boldsymbol{J}(t-r/c)}{r}dV \tag{3}$$

$$\phi=\frac{1}{4\pi\epsilon_0}\iiint_V\frac{\rho}{r}dV\rightarrow\frac{1}{4\pi\epsilon_0}\iiint_V\frac{\rho(t-r/c)}{r}dV \quad (4)$$

$\rightarrow$ means promotion, $c=1/\sqrt{\epsilon_0\mu_0}$ is the speed of light. The electric field and magnetic field are obtained from the following formula,

$$\boldsymbol{E}=-\frac{\partial}{\partial t}\boldsymbol{A}-\nabla\phi \quad (5)$$

$$\boldsymbol{B}=\nabla\times\boldsymbol{A} \quad (6)$$

The correctness of the theory after the extension of the lag potential is no longer tested by experiments. Because for the radiated electromagnetic field, there is no effective method to detect the electric field and magnetic field respectively. What can be detected is actually the induced electromotive force on the receiving antenna,

$$\mathcal{E}=\int_C\boldsymbol{E}\cdot d\boldsymbol{l} \quad (7)$$

Or received power, etc

$$P=\mathcal{E}I \quad (8)$$

Therefore, the accuracy of Maxwell's theory with displacement current is actually lower than that of magnetic quasi-static electromagnetic field equation without displacement current. In this way, the Maxwell's path of magnetic quasi-static electromagnetic field should be regarded as an accurate electromagnetic field equation.

Generally, the electromagnetic theory believes that Maxwell's equation is an accurate electromagnetic field equation, which corresponds to the magnetic quasi-static electric field and magnetic field. Because the displacement current is ignored, it is regarded as an approximate equation. The author disagrees with this view. The author believes that the magnetic quasi-static electromagnetic field equation has been verified by experiments. It is more credible than the Maxwell's equation after considering the displacement current. Especially when the condition of the magnetic quasi-static equation is satisfied, that is, the size of the transformer is smaller than the wavelength. For example, the wavelength of 100 Hz alternating current is 3000 axiom,

$$\lambda=\frac{3\cdot 10^8 M/second}{100Hz}=3\cdot 10^6M=3000kM \quad (9)$$

The scale of our transformer is less than 1 meter. Therefore,

$$length\ of\ transformer\ll\lambda \quad (10)$$

In this case, the magnetic quasi-static electromagnetic field equation is very accurate for the transformer.

III. Energy flow corresponding to Poynting's theorem in the case of magnetic quasi-static electromagnetic field

The mathematical formula of vector about curl is,

$$\nabla\cdot(\boldsymbol{E}_1\times\boldsymbol{H}_1)=\nabla\times\boldsymbol{E}_1\cdot\boldsymbol{H}_1-\boldsymbol{E}_1\cdot\nabla\times\boldsymbol{H}_1 \quad (11)$$

Substituting into the magnetic quasi-static Maxwell's equation (1,2), we get,

$$\nabla\cdot(\boldsymbol{E}_1\times\boldsymbol{H}_1)=-\frac{\partial}{\partial t}\boldsymbol{B}_1\cdot\boldsymbol{H}_1-\boldsymbol{E}_1\cdot\boldsymbol{J}_1 \quad (12)$$

Do volume integral, we get

$$\iint_{\Gamma_1}(\boldsymbol{E}_1\times\boldsymbol{H}_1)\cdot\hat{n}d\Gamma$$
$$=-\iiint_{V_1}\frac{\partial}{\partial t}\boldsymbol{B}_1\cdot\boldsymbol{H}_1dV-\iiint_{V_1}\boldsymbol{E}_1\cdot\boldsymbol{J}_1dV \quad (13)$$

Considering,

$$\int_{t=-\infty}^{\infty}dt\iiint_{V_1}\frac{\partial}{\partial t}\boldsymbol{B}_1\cdot\boldsymbol{H}_1dV$$
$$=\int_{t=-\infty}^{\infty}\frac{\partial}{\partial t}Udt=U(\infty)-U(-\infty)=0 \quad (14)$$

Where

$$U=\frac{1}{2}\iiint_{V_1}\boldsymbol{B}_1\cdot\boldsymbol{H}_1dV \quad (15)$$

$U(t=-\infty)$ is the energy at the negative infinity of time. At this time, the process of our research has not yet begun. So the energy is zero.

$U(t=\infty)$ is the energy when time is infinite. At that time, the process of our research has ended, so the energy is zero. have

$$\int_{t=-\infty}^{\infty}dt\iint_{\Gamma_1}(\boldsymbol{E}_1\times\boldsymbol{H}_1)\cdot\hat{n}d\Gamma=-\int_{t=-\infty}^{\infty}dt\iiint_{V_1}\boldsymbol{E}_1\cdot\boldsymbol{J}_1dV \quad (16)$$

When converting to frequency Fourier frequency domain, there is a similar method to the above ,

$$\iint_{\Gamma_1}(\boldsymbol{E}_1\times\boldsymbol{H}_1^*)\cdot\hat{n}d\Gamma=-\iiint_{V_1}\boldsymbol{E}_1\cdot\boldsymbol{J}_1^*dV \quad (17)$$

Similarly,

$$\iint_{\Gamma_2}(\boldsymbol{E}_2\times\boldsymbol{H}_2^*)\cdot\hat{n}d\Gamma=-\iiint_{V_2}\boldsymbol{E}_2\cdot\boldsymbol{J}_2^*dV \quad (18)$$

The above two formulas are Poynting's theorem of primary coil and secondary coil in frequency domain. Among them,

$$\iiint_{V_1}\boldsymbol{E}_1\cdot\boldsymbol{J}_1^*dV\rightarrow\int_{C_1}\boldsymbol{E}_1\cdot d\boldsymbol{l}I_1^*=\mathcal{E}_1I_1^*=-L_1j\omega I_1I_1^* \quad (19)$$

Among them,

$$\mathcal{E}_1=-L_1\frac{d}{dt}I_1=-L_1j\omega I_1 \quad (20)$$

Is the induced electromotive force of the primary coil. And change the volume current into line current,

$$\boldsymbol{J}_1dV\rightarrow d\boldsymbol{l}I_1 \quad (21)$$

Here L_1 Is the self inductance of the primary coil, and the self inductance is a real value,

$$\Re \iint_{\Gamma_1} (\boldsymbol{E}_1 \times \boldsymbol{H}_1^*) \cdot \hat{n} d\Gamma = \Re(-L_1 \frac{d}{dt} I_1) = \Re(-L_1 j\omega I_1) = 0 \tag{22}$$

$\Re$ is the real part of the complex number. In the above formula, L_1 Is the self inductance of the primary coil, and the self inductance is a real number. $\boldsymbol{S}_{11} = \boldsymbol{E}_1 \times \boldsymbol{H}_1^*$ is the self energy flow density of the primary coil. Similarly

$$\Re \iint_{\Gamma_2} (\boldsymbol{E}_2 \times \boldsymbol{H}_2^*) \cdot \hat{n} d\Gamma = \Re(-L_2 \frac{d}{dt} I_2) = \Re(-L_2 j\omega I_2) = 0 \tag{23}$$

Is the self energy flow density of the secondary coil. Therefore, the energy flow corresponding to Poynting vector is reactive power. Does not transmit energy. That is to say, self energy flow does not transmit energy. We see self energy flow and self inductance L_1, L_2 Corresponding, L_1, L_2 are the inductance of primary coil and secondary coil. The above formula tells us that the self energy flow, or the self energy flow corresponding to Poynting's theorem, is reactive power, so it does not transmit energy.

IV. Energy conservation law of transformer

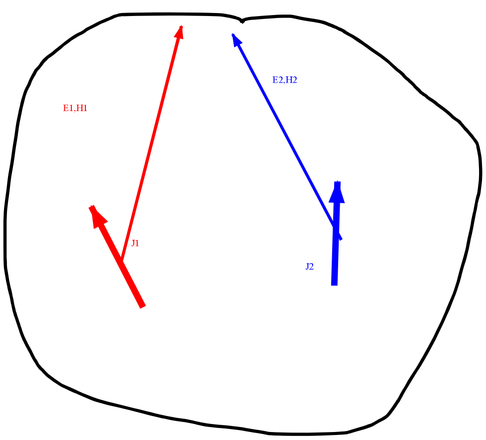

Fig. 2. Suppose there are two current elements, a primary coil equivalent to a transformer, releasing energy, and a secondary coil equivalent to a transformer, absorbing energy. Both current elements generate their own electromagnetic fields.

Refer to the above figure 2 and consider the superposition principle,

$$\boldsymbol{E} = \sum_{i=1}^{2} \boldsymbol{E}_i, \quad \boldsymbol{H} = \sum_{i=1}^{2} \boldsymbol{H}_i, \quad \boldsymbol{J} = \sum_{i=1}^{2} \boldsymbol{J}_i \tag{24}$$

We have Poynting theorem, which is the law of conservation of energy of magnetic quasi-static field. The proof method is the same as the formula (17).

$$\iint_{\Gamma} (\boldsymbol{E} \times \boldsymbol{H}^*) \cdot \hat{n} d\Gamma = -\iiint_V \boldsymbol{E} \cdot \boldsymbol{J}^* dV \tag{25}$$

By substituting the superposition principle into the Poynting theorem above, we can get the Poynting theorem of superposition field in the case of magnetic quasi-static electromagnetic field,

$$\sum_{i=1}^{2} \sum_{j=1}^{2} \iint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_j^*) \cdot \hat{n} d\Gamma = -\sum_{i=1}^{2} \sum_{j=1}^{2} \iiint_V \boldsymbol{E}_i \cdot \boldsymbol{J}_j^* dV \tag{26}$$

The real part of the above formula is obtained,

$$\Re \sum_{i=1}^{2} \sum_{j=1}^{2} \iint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_j^*) \cdot \hat{n} d\Gamma = -\Re \sum_{i=1}^{2} \sum_{j=1}^{2} \iiint_V \boldsymbol{E}_i \cdot \boldsymbol{J}_j^* dV \tag{27}$$

The above formula is the law of conservation of energy. I have previously proved (22,23), that is, the formula,

$$\Re \iint_{\Gamma_1} (\boldsymbol{E}_1 \times \boldsymbol{H}_1^*) \cdot \hat{n} d\Gamma = -\Re \iiint_{V_1} \boldsymbol{E}_1 \cdot \boldsymbol{J}_1^* dV = 0 \tag{28}$$

$$\Re \iint_{\Gamma_2} (\boldsymbol{E}_2 \times \boldsymbol{H}_2^*) \cdot \hat{n} d\Gamma = -\Re \iiint_{V_2} \boldsymbol{E}_2 \cdot \boldsymbol{J}_2^* dV = 0 \tag{29}$$

Since each term of the above formula is zero, you can subtract (28,29) from Poynting's theorem (27) of superposition field, which does not affect that the following formula is still the same as Poynting's theorem (27), which is the law of conservation of energy. Thus,

$$\Re \sum_{i=1}^{2} \sum_{j=1, j\neq i}^{2} \iint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_j^*) \cdot \hat{n} d\Gamma = -\Re \sum_{i=1}^{2} \sum_{j=1, j\neq i}^{2} \iiint_V \boldsymbol{E}_i \cdot \boldsymbol{J}_j^* dV \tag{30}$$

The summation sign of formula (30) is different from that of formula (27). If Γ is taken as a sphere with infinite radius, considering that the electric field and magnetic field change with $\frac{1}{r^2}$, the left curved area of the above formula is divided into zero, that is

$$\Re \sum_{i=1}^{2} \sum_{j=1, j\neq i}^{2} \iint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_j^*) \cdot \hat{n} d\Gamma = 0 \tag{31}$$

Hence, there is,

$$\Re \sum_{i=1}^{2} \sum_{j=1, j\neq i}^{2} \iiint_V \boldsymbol{E}_i \cdot \boldsymbol{J}_j^* dV = 0 \tag{32}$$

Or

$$-\Re \iiint_{V_1} \boldsymbol{E}_2^* \cdot \boldsymbol{J}_1 dV = \Re \iiint_{V_2} \boldsymbol{E}_1 \cdot J_2^* dV \tag{33}$$

The above formula is the energy conservation law of transformer. In the above formula, the volume current is replaced by the line current, and the real part of $\Re$ is taken (the real part of the formula is the real physical formula), which can be written as,

$$-\iiint_{V_1} \boldsymbol{E}_2^* \cdot \boldsymbol{J}_1 dV = \iiint_{V_2} \boldsymbol{E}_1 \cdot J_2^* dV$$

Replace the volume current density with the line current density,

$$-\int_{C_1} \boldsymbol{E}_2^* \cdot d\boldsymbol{l} I_1 = \int_{C_2} \boldsymbol{E}_1 \cdot d\boldsymbol{l} I_2^* \tag{34}$$

Or

$$-\mathcal{E}_{1,2}^* I_1 = \mathcal{E}_{2,1} I_2^* \tag{35}$$

Where $\mathcal{E}_{1,2} = \int_{C_1} \boldsymbol{E}_2 \cdot d\boldsymbol{l}$ is the induced electromotive force generated by the secondary coil current on the primary coil. $\mathcal{E}_{2,1} = \int_{C_2} \boldsymbol{E}_1 \cdot d\boldsymbol{l}$ is the induced electromotive force generated by the primary coil current on the secondary coil. Equation (35) shows that the power made by the primary coil current to overcome the induced electromotive force of the secondary coil is equal to the power provided by the primary coil current to the secondary coil current. It is assumed to be an ideal transformer. Next, we verify that the above formula is correct.

$$\mathcal{E}_{1,2} = -M_{1,2} j\omega I_2 \tag{36}$$

$$\mathcal{E}_{2,1} = -M_{2,1} j\omega I_1 \tag{37}$$

$M_{1,2}$ is the mutual inductance between the secondary coil and the primary coil. $M_{2,1}$ is the mutual inductance between the primary coil and the secondary coil. Substitute (35) to get,

$$-(-M_{1,2} j\omega I_2)^* I_1 = (-M_{2,1} j\omega I_1) I_2^* \tag{38}$$

Thus,

$$M_{1,2}^* = M_{2,1} \tag{39}$$

As long as the above formula is satisfied, the law of conservation of energy (34) is satisfied. We know that according to Neumann's definition of mutual inductance,

$$M_{2,1} = \frac{\mu_0}{4\pi} \int_{C_2} \int_{C_1} \frac{d\boldsymbol{l}_1 \cdot d\boldsymbol{l}_2}{r} \tag{40}$$

$$M_{1,2} = \frac{\mu_0}{4\pi} \int_{C_1} \int_{C_2} \frac{d\boldsymbol{l}_2 \cdot d\boldsymbol{l}_1}{r} \tag{41}$$

Therefore (39) is satisfied. The formula (39) seems redundant, but it is not. In the case of radiation field, when considering retardation, if the secondary coil of the transformer is moved beyond a certain distance from the primary coil, see figure 3. When the retarded wave has to be considered, $M_{2,1}$ needs to consider the retarded factor, so there are,

Fig. 3. When the secondary coil moves to a place far away from the primary coil, in fact, the primary coil becomes a transmitting antenna and the secondary coil becomes a receiving antenna.

$$M_{2,1} = \frac{\mu_0}{4\pi} \int_{C_2} \int_{C_1} \exp(-jkr) \frac{d\boldsymbol{l}_1 \cdot d\boldsymbol{l}_2}{r} \tag{42}$$

In this case, formula (39) leads to advanced mutual inductance,

$$M_{1,2} = \frac{\mu_0}{4\pi} \int_{C_1} \int_{C_2} \exp(+jkr) \frac{d\boldsymbol{l}_2 \cdot d\boldsymbol{l}_1}{r} \tag{43}$$

The above formula is the advanced potential, which indicates that the advanced wave exists. However, these are discussed in the last chapter of this article. In this chapter and next chapter, only the magnetic quasi-static field is considered.

V. Energy flow law of transformer

We have obtained the formula (30)

$$\sum_{i=1}^{2} \sum_{j=1, j\neq i}^{2} \iint_\Gamma (\boldsymbol{E}_i \times \boldsymbol{H}_j^*) \cdot \hat{n} d\Gamma = -\sum_{i=1}^{2} \sum_{j=1, j\neq i}^{2} \iiint_V \boldsymbol{E}_i \cdot \boldsymbol{J}_j^* dV \tag{44}$$

Note that we saved the real part $\Re$, but this step should be kept in mind. The above formula can be rewritten as,

$$-\iint_\Gamma (\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma = \iiint_V (\boldsymbol{E}_1 \cdot \boldsymbol{J}_2^* + \boldsymbol{E}_2^* \cdot \boldsymbol{J}_1) dV \tag{45}$$

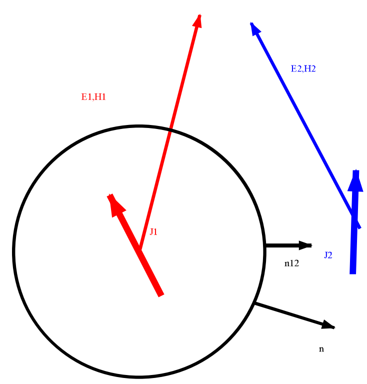

Fig. 4. Γ surface surrounds current element $\boldsymbol{J}_1$. Current element $\boldsymbol{J}_2$ is at the outside zone V_1. Normal vector $\hat{n}_{12}$ Consistent with the direction of normal vector $\hat{n}$.

Select V as V_1 Only current $\boldsymbol{J}_1$,Γ_1 is boundary of V_1.

$$-\iint_{\Gamma_1} (\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1) \cdot \hat{n}_{12} d\Gamma = \iiint_{V_1} (\boldsymbol{E}_2^* \cdot \boldsymbol{J}_1) dV \tag{46}$$

Select V as V_2. Inside V_2 there is only current $\boldsymbol{J}_2, \Gamma_2$ is the boundary of V_2. So there is,

$$\iint_{\Gamma_2} (\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1) \cdot \hat{n}_{12} d\Gamma = \iiint_{V_2} (\boldsymbol{E}_1 \cdot \boldsymbol{J}_2^*) dV \tag{47}$$

I have already proved the law of conservation of energy of transformer (33)

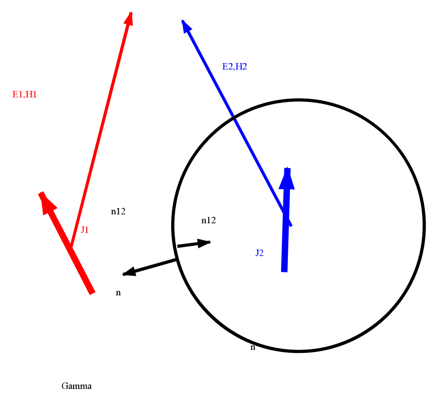

Fig. 5. Γ surface surrounds current element $\boldsymbol{J}_2$. Current element $\boldsymbol{J}_1$ Outside zone v. Normal vector $\hat{n}_{12}$ It is inconsistent with the direction of normal vector $\hat{n}$. So there is an extra minus sign.

$$-\iiint_{V_1} \boldsymbol{E}_2^*(\omega)\cdot\boldsymbol{J}_1(\omega)dV=\iiint_{V_2} \boldsymbol{E}_1(\omega)\cdot J_2^*(\omega)dV \quad (48)$$

It is obtained from the first three formulas (46,47,48),

$$-\iiint_{V_1} \boldsymbol{E}_2^*(\omega)\cdot\boldsymbol{J}_1(\omega)dV=(\xi_1,\xi_2)=\iiint_{V_2} \boldsymbol{E}_1(\omega)\cdot J_2^*(\omega)dV \quad (49)$$

Where

$$(\xi_1,\xi_2)=\iint_{\Gamma}(\boldsymbol{E}_1(\omega)\times\boldsymbol{H}_2^*(\omega)+\boldsymbol{E}_2^*(\omega)\times\boldsymbol{H}_1(\omega))\cdot\hat{n}_{12}d\Gamma \quad (50)$$

The above is mutual energy flow, and word "mutual" can be removed. Because self energy flow does not transmit energy. Where Γ can be Γ_1 or Γ_2. They enclose the current elements $\boldsymbol{J}_1$ or $\boldsymbol{J}_2$. The closed surface Γ can also be infinite plane between two current elements. In short, Γ constitutes the division of two current elements. If in the time domain,

$$\begin{aligned}-\int_{t=-\infty}^{\infty}dt\iiint_{V_1}\boldsymbol{E}_2(t)\cdot\boldsymbol{J}_1(t)dV\\=(\xi_1,\xi_2)\\=\int_{t=-\infty}^{\infty}dt\iiint_{V_2}\boldsymbol{E}_1(t)\cdot J_2(t)dV\end{aligned} \quad (51)$$

where,

$$(\xi_1,\xi_2)=\int_{t=-\infty}^{\infty}dt\iint_{\Gamma}(\boldsymbol{E}_1(t)\times\boldsymbol{H}_2(t)+\boldsymbol{E}_2(t)\times\boldsymbol{H}_1(t))\cdot\hat{n}_{12}d\Gamma \quad (52)$$

VI. The energy flow from the primary coil to the secondary coil of the transformer is mutual energy flow

Traditionally, the energy flow of transformer has been considered as Poynting vector. There are many literatures that attempt to use Poynting's theorem to interpret the energy flow from the primary coil to the secondary coil, but the author has not found any examples of real results. Although some people claim that he has explained the energy flow from the primary coil to the secondary coil of the transformer by using the Poynting theorem, most of them are very far fetched. In this chapter, we prove that the mutual energy flow transfers energy from the primary coil to the secondary coil strictly under the condition of magnetic quasi-static electromagnetic field. This energy flow is generated on the primary coil and annihilated on the secondary coil. Therefore, the energy flow from the primary coil to the secondary coil has the shape and properties of photons.

A. Energy flow of two coils

Figure 6 shows the primary coil and secondary coil of a transformer in the space. We don't consider the iron core, so the problem is relatively simple.

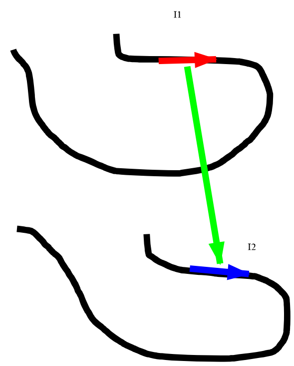

Fig. 6. Calculate the energy flow from the current element on the primary coil to the current element on the secondary coil.

There is a current element (red) on the primary coil. There is a current element on the secondary coil, blue. We are concerned with the energy flow from the primary coil current element to the secondary coil current source. This energy flow is represented by a green arrow, as shown in figure 6.

To calculate the energy flow from the primary coil to the secondary coil, you can first calculate the energy flow from a current element on the primary coil to a current element on the secondary coil. Then add up all the energy flows to get the total energy flow from the primary coil to the secondary coil. The next section calculates the energy flow between two current elements.

B. Energy flow between two current elements

In the figure 7 below, there are two current elements, one is the primary coil, the other is the secondary coil, and the primary coil is followed by a power supply, which is represented by a battery here, but it may also be an AC power supply. The secondary coil is followed by a large load resistance, so that the impedance of the secondary circuit can ignore the inductive reactance of the secondary coil inductance. The secondary impedance is

$$Z_2=j\omega L_2+R_2\simeq R_2\sim 1 \quad (53)$$

Next, calculate the Poynting vector and mutual energy flow density from the primary coil current element to the secondary coil current element, as shown in figure 7. To simplify the problem, we only calculate the amount along the x axis. In the following, we use the $\sim$ symbol to express the proportional. This symbol allows us to only consider the phase and the direction of the vector, and ignore the value. In this paper, the calculation of mutual energy flow is only concerned with its direction and phase. The size of the value is not interested.

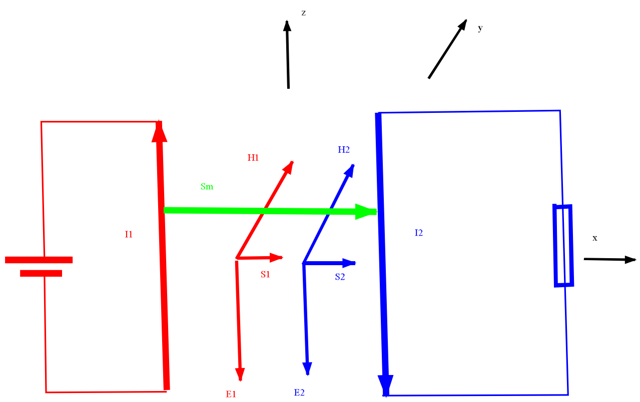

Fig. 7. For a transformer with two current elements, the two current elements are relatively close. Primary coil connects an AC power supply. Secondary coil connects a resistance.

The induced electromotive force of the primary coil current on the secondary coil is,

$$\mathcal{E}_{2,1} = -j\omega M_{2,1} I_1 \tag{54}$$

So the electric field of the primary coil is (here, it is assumed that the two current elements are very close and the gap is almost zero), so we can know the direction of the electric field,

$$\boldsymbol{E}_1 \sim jI_1(-\hat{z}) \tag{55}$$

In fact, the electric field can also be determined by,

$$\boldsymbol{E}_1 = -\frac{\partial}{\partial t}\boldsymbol{A}_1 = -j\omega\frac{\mu_0}{4\pi}I_1\int_C \frac{d\boldsymbol{l}}{r} \sim -jI_1\int_C \frac{dl}{r}\hat{z} \sim jI_1(-\hat{z}) \tag{56}$$

The magnetic field is obtained from the ampere circuital law,

$$\boldsymbol{H}_1 = \frac{I_1}{2}\hat{y} \sim I_1\hat{y} \tag{57}$$

The induced electromotive force of the secondary coil current on the primary coil is

$$\mathcal{E}_{1,2} = -j\omega M_{1,2} I_2 \tag{58}$$

So the electric field of the secondary coil is,

$$\boldsymbol{E}_2 \sim -jI_2(-\hat{z}) \tag{59}$$

The above formula is one more negative sign than (55), because the current of the secondary coil is head down (see figure 6). The magnetic field is obtained from the ampere circuital law,

$$\boldsymbol{H}_2 = \frac{I_2}{2}\hat{y} \sim I_2\hat{y} \tag{60}$$

Calculate the self energy flow Poynting vector of the primary coil,

$$\boldsymbol{S}_{11} = \boldsymbol{E}_1 \times \boldsymbol{H}_1^* \sim jI_1(-\hat{z}) \times (I_1\hat{y})^* = jI_1I_1^*\hat{x} \sim j \tag{61}$$

Calculate the self energy Poynting vector of the secondary coil,

$$\boldsymbol{S}_{22} = \boldsymbol{E}_2 \times \boldsymbol{H}_2^* \sim -jI_2(-\hat{z}) \times (I_2\hat{y})^* = -jI_2I_2^*\hat{x} \sim -j \tag{62}$$

The self energy flow represented by Poynting vector is pure imaginary number, so it is reactive power, so the average value of the transmitted energy flow is zero. Next, calculate the mutual energy flow,

$$\boldsymbol{S}_m = \boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1$$

$$= ((jI_1)(I_2)^* + (-jI_2)^*(I_1))\hat{x}$$

$$= jI_1I_2^*(1+1)\hat{x} \sim jI_1I_2^*\hat{x} \tag{63}$$

Considering,

$$I_2 = \frac{\mathcal{E}_{1,2}}{Z_2} \tag{64}$$

$\mathcal{E}_{1,2}$ is the electromotive force on the secondary coil.

$$\mathcal{E}_{1,2} \sim E_1 \sim jI_1 \tag{65}$$

Assuming that the load resistance of the secondary circuit is much larger than the reactance, there are,

$$Z_2 = j\omega L_2 + R_2 \sim R_2 \tag{66}$$

Hence,

$$I_2 = \frac{\mathcal{E}_{1,2}}{Z_2} \sim \frac{jI_1}{R_2} \sim jI_1 \tag{67}$$

Substitute to (63), we get,

$$\boldsymbol{S}_m \sim jI_1(jI_1)^*\hat{x} = I_1I_1^*\hat{x} \sim \hat{x} \tag{68}$$

It can be seen that the mutual energy flow is a real number, so it is active power. The primary coil of the transformer is at position $x = 0$, and the secondary coil of the transformer is at position $x = L$. we assume that L is a very small value. Therefore,

$$\boldsymbol{S}_m \sim \begin{cases} 0 & x < 0 \\ \hat{x} & 0 \leq x \leq L \\ 0 & x > L \end{cases} \tag{69}$$

In the above formula, it is considered that,

$$S_m = \boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1 = S_{12} + S_{21} \tag{70}$$

Magnetic field $\boldsymbol{H}_1$ change the symbol at $x = 0$. Magnetic field $\boldsymbol{H}_2$ change the symbol at $x = L$. So outside the area $0 \leq x \leq L, \boldsymbol{S}_{12}$ and $\boldsymbol{S}_{2,1}$ offset. The above formula (69) shows that the mutual energy current density is generated on the primary coil and annihilated on the secondary coil. Therefore, the mutual energy flow density $\boldsymbol{S}_m$ has the property of photon. Photons are generated at the source and annihilated at the sink. Therefore, we can interpret photons as mutual energy flow. In other words, photons are constantly generated on the primary coil and annihilated on the secondary coil.

With the energy flow between two current elements, the total energy flow on the two coils is a problem of summation or integration. In principle, there is no difficulty, so this paper can be ignored. In a word, it can be seen from this example that the energy flow of primary coil and secondary coil is mutual energy flow, not self energy flow. This conclusion seems very common, because the energy transfer of transformer is also completed by mutual inductance rather than self inductance. Our conclusion is completed under the condition of magnetic quasi-static electromagnetic field. However, the author mentioned earlier that the magnetic quasi-static electromagnetic field is a fairly accurate electromagnetic field theory, which is more credible than Maxwell's electromagnetic field theory containing displacement current.

It is worth mentioning that the formula (69) is very close to the photon model in Cramer quantum mechanics transaction interpretation [3], [4]. In Cramer's model, it is the superposition of retarded wave and advanced wave, and in the author's theory, it is superposition of the mixed Poynting vector $\boldsymbol{S}_{12} = \boldsymbol{E}_1 \times \boldsymbol{H}_2$ and $\boldsymbol{S}_{21} = \boldsymbol{E}_2 \times \boldsymbol{H}_1$. In the Cramer model, the retarded wave and the advanced wave are offset by a 180 degree phase, except between the light source and the light sink. But this 180 degree is very abrupt. In the author's theory, the magnetic field is caused by the reverse direction of the current element. So it's very natural.

VII. From magnetic quasi-static electromagnetic field to radiated electromagnetic field

Neumann gave the law of electromagnetic induction in 1845. When the position of two coils changes, their potential energy is,

$$W = -\int_{C_2} \frac{\mu_0}{4\pi} \int_{C_1} I_1 \frac{d\boldsymbol{l}_1}{r} \cdot d\boldsymbol{l}_2 \tag{71}$$

Or the induced electromotive force is,

$$\mathcal{E} = -\frac{\partial}{\partial t} \int_{C_2} \frac{\mu_0}{4\pi} \int_{C_1} I_1 \frac{d\boldsymbol{l}_1}{r} \cdot d\boldsymbol{l}_2 \tag{72}$$

Thus, the vector potential can be defined,

$$\boldsymbol{A} = \frac{\mu_0}{4\pi} \int_{C_1} I_1 \frac{d\boldsymbol{l}_1}{r} \tag{73}$$

Hence,

$$\mathcal{E} = -\frac{\partial}{\partial t} \int_{C_2} \boldsymbol{A} \cdot d\boldsymbol{l}_1 \tag{74}$$

or

$$\int_{C_2} \boldsymbol{E} \cdot d\boldsymbol{l}_1 = -\frac{\partial}{\partial t} \int_{C_2} \boldsymbol{A} \cdot d\boldsymbol{l}_1 \tag{75}$$

or

$$\int_{C_2} \boldsymbol{E} \cdot d\boldsymbol{l}_1 = -\int_{C_2} \frac{\partial}{\partial t} \boldsymbol{A} \cdot d\boldsymbol{l}_1 \tag{76}$$

or

$$\int_{C_2} (\boldsymbol{E} + \frac{\partial}{\partial t} \boldsymbol{A}) \cdot d\boldsymbol{l}_1 = 0 \tag{77}$$

or

$$\boldsymbol{E} + \frac{\partial}{\partial t} \boldsymbol{A} = -\nabla \phi \tag{78}$$

or

$$\boldsymbol{E} = -\frac{\partial}{\partial t} \boldsymbol{A} - \nabla \phi \tag{79}$$

Where

$$\boldsymbol{A} = \frac{\mu_0}{4\pi} \iiint_V \frac{\boldsymbol{J}}{r} dV \tag{80}$$

These can be regarded as the known magnetic quasi-static electromagnetic field theory. Now let's see how Kirchhoff, Maxwell and Lorenz generalized the theory of radiated electromagnetic field from the magnetic quasi-static electromagnetic field theory.

A. *Kirchhoff's method*

In 1857, Kirchhoff knew that the current continuity equation in the conductor was,

$$\boldsymbol{J} = -\sigma(\frac{\partial}{\partial t} \boldsymbol{A} + \nabla \phi) \tag{81}$$

σ is conductivity. Take the divergence of current,

$$\nabla \cdot \boldsymbol{J} \quad = \quad -\sigma(\frac{\partial}{\partial t} \nabla \cdot \boldsymbol{A} + \nabla \cdot \nabla \phi) \tag{82}$$

Considering the current continuity equation,

$$\nabla \cdot \boldsymbol{J} = -\frac{\partial}{\partial t} \rho \tag{83}$$

If Kirchhoff adopts the Lorenz gauge,

$$\nabla \cdot \boldsymbol{A} = -\mu_0 \epsilon_0 \frac{\partial}{\partial t} \phi \tag{84}$$

There is

$$-\frac{\partial}{\partial t} \rho = -\sigma((-\mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} \phi) + \nabla \cdot \nabla \phi) \tag{85}$$

or

$$\nabla^2 \phi - \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} \phi = \frac{1}{\sigma} \frac{\partial}{\partial t} \rho \tag{86}$$

In this way, Kirchhoff is the first to get the wave equation of scalar potential. It's in the conductor, of course. Unfortunately, Kirchhoff chose Weber vector potential,

$$\boldsymbol{A}_W = \frac{\mu_0}{4\pi} \iiint_V \frac{(\boldsymbol{r} \cdot \boldsymbol{J}) \cdot \boldsymbol{r}}{r} dV \tag{87}$$

And the so-called Kirchhoff specification,

$$\nabla \cdot \boldsymbol{A}_W = \mu_0 \epsilon_0 \frac{\partial}{\partial t} \phi \tag{88}$$

So he got,

$$\nabla\cdot\boldsymbol{J}=-\sigma(\mu_0\epsilon_0\frac{\partial^2}{\partial t^2}\phi+\nabla\cdot\nabla\phi) \tag{89}$$

$$-\frac{\partial}{\partial t}\rho=-\sigma(\mu_0\epsilon_0\frac{\partial^2}{\partial t^2}\phi+\nabla^2\phi) \tag{90}$$

or

$$\nabla^2\phi+\mu_0\epsilon_0\frac{\partial^2}{\partial t^2}\phi=\frac{1}{\sigma}\frac{\partial}{\partial t}\rho \tag{91}$$

Kirchhoff did not derive the correct equation of scalar potential, which is a pity. But his method is still correct, so the author assumes that Kirchhoff used Neumann vector potential and Lorenz gauge. In addition, like Maxwell, he also knows that the electromagnetic field in space is,

$$\boldsymbol{E}=-\frac{\partial}{\partial t}\boldsymbol{A}-\nabla\phi \tag{92}$$

Find the divergence of the above formula,

$$\nabla\cdot\boldsymbol{E}=-\frac{\partial}{\partial t}\nabla\cdot\boldsymbol{A}-\nabla\cdot\nabla\phi \tag{93}$$

Consider substituting Gauss law, Lorentz gauge,

$$\nabla\cdot\boldsymbol{E}=\rho/\epsilon_0 \tag{94}$$

$$\nabla\cdot\boldsymbol{A}=-\mu_0\epsilon_0\frac{\partial}{\partial t}\phi \tag{95}$$

get

$$\rho/\epsilon_0=-\frac{\partial}{\partial t}(-\mu_0\epsilon_0\frac{\partial}{\partial t}\phi)-\nabla\cdot\nabla\phi \tag{96}$$

or

$$\nabla^2\phi-\mu_0\epsilon_0\frac{\partial^2}{\partial t^2}\phi=-\rho/\epsilon_0 \tag{97}$$

The wave equation of scalar potential is obtained. From this, we can guess that there should be,

$$\nabla^2\boldsymbol{A}-\mu_0\epsilon_0\frac{\partial^2}{\partial t^2}\boldsymbol{A}=-\mu_0\boldsymbol{J} \tag{98}$$

This is because the divergence of the above formula is obtained,

$$\nabla^2\nabla\cdot\boldsymbol{A}-\mu_0\epsilon_0\frac{\partial^2}{\partial t^2}\nabla\cdot\boldsymbol{A}=-\mu_0\nabla\cdot\boldsymbol{J} \tag{99}$$

Considering the Lorenz gauge and current continuity equation,

$$\nabla^2(-\mu_0\epsilon_0\frac{\partial}{\partial t}\phi)-\mu_0\epsilon_0\frac{\partial^2}{\partial t^2}(-\mu_0\epsilon_0\frac{\partial}{\partial t}\phi)=-\mu_0(-\frac{\partial}{\partial t}\rho) \tag{100}$$

or

$$\frac{\partial}{\partial t}(\nabla^2\phi-\mu_0\epsilon_0\frac{\partial^2}{\partial t^2}\phi)=-\frac{\partial}{\partial t}\frac{1}{\epsilon_0}\rho \tag{101}$$

or

$$\nabla^2\phi-\mu_0\epsilon_0\frac{\partial^2}{\partial t^2}\phi=-\frac{1}{\epsilon_0}\rho+C \tag{102}$$

Constant C is not important. It can be ignored,

$$\nabla^2\phi-\mu_0\epsilon_0\frac{\partial^2}{\partial t^2}\phi=-\frac{1}{\epsilon_0}\rho \tag{103}$$

In this way, the wave equation of vector potential is obtained, and the retarded potential can be obtained from the wave equation,

$$\begin{cases}\boldsymbol{A}=\frac{\mu_0}{4\pi}\iiint_V\frac{\boldsymbol{J}(\boldsymbol{x}',t-r/c)}{r}dV\\ \boldsymbol{\phi}=\frac{1}{4\pi\epsilon_0}\iiint_V\frac{\boldsymbol{J}(\boldsymbol{x}',t-r/c)}{r}dV\end{cases} \tag{104}$$

B. Maxwell's method

Maxwell's theory of radiated electromagnetic field is to increase the displacement current $\frac{\partial}{\partial t}\boldsymbol{D}$ (in 1862),

$$\begin{cases}\nabla\times\boldsymbol{H}=\boldsymbol{J}\\ \nabla\times\boldsymbol{E}=-\frac{\partial}{\partial t}\boldsymbol{B}\end{cases}\Rightarrow\begin{cases}\nabla\times\boldsymbol{H}=\boldsymbol{J}+\frac{\partial}{\partial t}\boldsymbol{D}\\ \nabla\times\boldsymbol{E}=-\frac{\partial}{\partial t}\boldsymbol{B}\end{cases} \tag{105}$$

Maxwell believes that the magnetic field retains the original form of magnetic quasi-static electromagnetic field in the radiated electromagnetic field,

$$\boldsymbol{B}=\nabla\times\boldsymbol{A} \tag{106}$$

Faraday's law also remains unchanged,

$$\nabla\times\boldsymbol{E}=-\frac{\partial}{\partial t}\boldsymbol{B}$$

Hence, there is,

$$\nabla\times\boldsymbol{E}=-\frac{\partial}{\partial t}\nabla\times\boldsymbol{A} \tag{107}$$

or

$$\nabla\times\boldsymbol{E}=-\nabla\times\frac{\partial}{\partial t}\boldsymbol{A} \tag{108}$$

or

$$\nabla\times(\boldsymbol{E}+\frac{\partial}{\partial t}\boldsymbol{A})=0 \tag{109}$$

hence,

$$\boldsymbol{E}+\frac{\partial}{\partial t}\boldsymbol{A}=-\nabla\phi \tag{110}$$

or

$$\boldsymbol{E}=-\frac{\partial}{\partial t}\boldsymbol{A}-\nabla\phi \tag{111}$$

Ampere circuital law increases displacement current,

$$\nabla\times\boldsymbol{H}=\boldsymbol{J}+\frac{\partial}{\partial t}\boldsymbol{D} \tag{112}$$

or

$$\nabla\times\mu_0\boldsymbol{H}=\mu_0\boldsymbol{J}+\mu_0\epsilon_0\frac{\partial}{\partial t}\boldsymbol{E} \tag{113}$$

or

$$\nabla\times(\nabla\times\boldsymbol{A})=\mu_0\boldsymbol{J}+\mu_0\epsilon_0\frac{\partial}{\partial t}(-\frac{\partial}{\partial t}\boldsymbol{A}-\nabla\phi) \tag{114}$$

Consider the mathematical formula,

$$\nabla\times(\nabla\times\boldsymbol{A})=\nabla(\nabla\cdot\boldsymbol{A})-\nabla^2\boldsymbol{A} \tag{115}$$

get

$$\nabla(\nabla\cdot\boldsymbol{A})-\nabla^2\boldsymbol{A}=\mu_0\boldsymbol{J}+\mu_0\epsilon_0\frac{\partial}{\partial t}(-\frac{\partial}{\partial t}\boldsymbol{A}-\nabla\phi) \tag{116}$$

or

$$\nabla(\nabla\cdot\boldsymbol{A}+\mu_0\epsilon_0\frac{\partial}{\partial t}\phi)-\mu_0\boldsymbol{J}=\nabla^2\boldsymbol{A}-\mu_0\epsilon_0\frac{\partial^2}{\partial t^2}\boldsymbol{A} \tag{117}$$

Considering the Lorenz gauge $\nabla \cdot \boldsymbol{A} + \mu_0 \epsilon_0 \frac{\partial}{\partial t}\phi = 0$, get

$$\nabla^2 \boldsymbol{A} - \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2}\boldsymbol{A} = -\mu_0 \boldsymbol{J} \tag{118}$$

Using the same method as (97), we can get,

$$\nabla^2 \phi - \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2}\phi = -\rho/\epsilon_0 \tag{119}$$

Thus, the retarded potential (104) can also be obtained. By the way, Maxwell himself did not get the solution of the retarded potential. Maxwell used the Coulomb gauge, and he can solve Maxwell's equation only in the case of passivity. The retarded potential is the contribution of Lorenz.

C. Lorenz's method

Lorenz put forward the theory of retarded potential in 1867, which is equivalent to Maxwell's equation. At the same time, it gives the retarded potential solution of Maxwell's equation,

$$\begin{cases} \boldsymbol{A} = \frac{\mu_0}{4\pi}\iiint_V \frac{\boldsymbol{J}(\boldsymbol{x}',t)}{r}dV \\ \boldsymbol{\phi} = \frac{1}{4\pi\epsilon_0}\iiint_V \frac{\rho(\boldsymbol{x}',t)}{r}dV \end{cases} \Rightarrow \begin{cases} \boldsymbol{A} = \frac{\mu_0}{4\pi}\iiint_V \frac{\boldsymbol{J}(\boldsymbol{x}',t-r/c)}{r}dV \\ \boldsymbol{\phi} = \frac{1}{4\pi\epsilon_0}\iiint_V \frac{\rho(\boldsymbol{x}',t-r/c)}{r}dV \end{cases} \tag{120}$$

In the above process, the above two formulas keep the Lorenz gauge unchanged,

$$\nabla \cdot \boldsymbol{A} = -\mu_0 \epsilon_0 \frac{\partial}{\partial t}\phi \tag{121}$$

The electric field and magnetic field are obtained from the following two equations,

$$\boldsymbol{E} = -\frac{\partial}{\partial t}\boldsymbol{A} - \nabla\phi \tag{122}$$

$$\boldsymbol{H} = \frac{1}{\mu_0}\nabla \times \boldsymbol{A} \tag{123}$$

Maxwell's method is essentially the same as Lorenz's method. In fact, this method ensures that the Lorenz gauge remains unchanged in the process of transition from magnetic quasi-static field to radiated electromagnetic field. The reason why Lorenz gauge remains unchanged is the law of charge continuity equation,

$$\nabla \cdot \boldsymbol{J} = -\frac{\partial}{\partial t}\rho \tag{124}$$

VIII. The Author's Method of Radiated Electromagnetic Field

The author believes that the law of conservation of energy should remain unchanged in the promotion from magnetic quasi-static electromagnetic field to radiated electromagnetic field,

$$\int_{t=\infty}^{\infty} dt \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} \iiint_V \boldsymbol{E}_i(t) \cdot \boldsymbol{J}_j(t) dV = 0 \tag{125}$$

If it is transformed to the frequency domain and $N = 2$,

$$\sum_{i=1}^{2} \sum_{j=1, j\neq i}^{2} \iiint_V \boldsymbol{E}_i(\omega) \cdot \boldsymbol{J}_j^*(\omega) dV = 0 \tag{126}$$

For transformers, we already know that this means

$$M_{1,2}^* = M_{2,1} \tag{127}$$

Under the condition of radiation field, $M_{2,1}$ needs to consider the retarded factor, so there are,

$$M_{2,1} = \frac{\mu_0}{4\pi}\int_{C_2}\int_{C_1} \exp(-jkr)\frac{d\boldsymbol{l}_1 \cdot d\boldsymbol{l}_2}{r} \tag{128}$$

In this case, the formula (127) leads to advanced mutual inductance,

$$M_{1,2} = \frac{\mu_0}{4\pi}\int_{C_1}\int_{C_2} \exp(+jkr)\frac{d\boldsymbol{l}_2 \cdot d\boldsymbol{l}_1}{r} \tag{129}$$

The above two equations lead to retarded potential and advanced potential, and the primary coil or transmitting antenna generates retarded potential,

$$\boldsymbol{A}_1^{(re)} = I_1 \int_{C_1} \frac{\exp(-jkr)}{r} d\boldsymbol{l}_1 = \iiint_{V_1} \frac{\exp(-jkr)}{r}\boldsymbol{J}_1 dV \tag{130}$$

The secondary coil or receiving antenna generates the lead potential,

$$\boldsymbol{A}_2^{(ad)} = I_2 \int_{C_2} \frac{\exp(+jkr)}{r} d\boldsymbol{l}_1 = \iiint_{V_2} \frac{\exp(+jkr)}{r}\boldsymbol{J}_2 dV \tag{131}$$

Above, we have the retarded vector potential and the advanced vector potential in the frequency domain. Similarly, we can have scalar retarded potential and advanced potential. In the time domain there is,

$$\begin{cases} \boldsymbol{A}_1^{(re)} = \frac{\mu_0}{4\pi}\iiint_V \frac{\boldsymbol{J}(\boldsymbol{x}',t-r/c)}{r}dV \\ \boldsymbol{\phi}_1^{(re)} = \frac{1}{4\pi\epsilon_0}\iiint_V \frac{\rho(\boldsymbol{x}',t-r/c)}{r}dV \end{cases} \tag{132}$$

$$\begin{cases} \boldsymbol{A}_2^{(ad)} = \frac{\mu_0}{4\pi}\iiint_V \frac{\boldsymbol{J}(\boldsymbol{x}',t+r/c)}{r}dV \\ \boldsymbol{\phi}_2^{(ad)} = \frac{1}{4\pi\epsilon_0}\iiint_V \frac{\rho(\boldsymbol{x}',t+r/c)}{r}dV \end{cases} \tag{133}$$

The author's method obtains the lag potential as the previous three methods, but the author's method also obtains the advanced potential for the secondary coil. The author still assumes that,

$$\boldsymbol{E}_1 = -\frac{\partial}{\partial t}\boldsymbol{A}_1 - \nabla\phi_1 \tag{134}$$

$$\boldsymbol{B}_1 = \nabla \times \boldsymbol{A}_1 \tag{135}$$

It is worth mentioning that the above formula (104) is consistent with the above four methods. Of course Kirchhoff and Lorentz don't use the concept of electric field and magnetic field. But they use the concept of current. The relationship between current and electric field is clear. However, for ([eq:7-130]), the radiation field of this magnetic field is only used by Maxwell. Therefore, this magnetic field should be called Maxwell's magnetic field. Now we want to prove that this magnetic field is not reliable.

A. Problems related to Poynting theorem

Previously, we can strictly prove the above formula in the magnetic quasi-static electromagnetic field (125,127). But when it comes to the radiated electromagnetic field, assuming that the Maxwell equation containing the displacement current is established, we can at most prove that (125,127) is an energy theorem, but we can't prove that they are the law of conservation of energy.

Let's go through the derivation process again. This time, Maxwell's equation including displacement current is used,

$$\begin{cases}\nabla \times \boldsymbol{E} = -\frac{\partial}{\partial t}\boldsymbol{B} \\ \nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial}{\partial t}\boldsymbol{D}\end{cases} \tag{136}$$

From this, we can deduce Poynting's theorem of radiated electromagnetic field,

$$\nabla \cdot (\boldsymbol{E} \times \boldsymbol{H}) = \nabla \times \boldsymbol{E} \cdot \boldsymbol{H} - \boldsymbol{E} \cdot \nabla \times \boldsymbol{H} \tag{137}$$

$$\nabla \cdot (\boldsymbol{E} \times \boldsymbol{H}) = -\boldsymbol{H} \cdot \frac{\partial}{\partial t}\boldsymbol{B} - \boldsymbol{E} \cdot \boldsymbol{J} - \boldsymbol{E} \cdot \frac{\partial}{\partial t}\boldsymbol{D} \tag{138}$$

Poynting's theorem is,

$$-\oint\!\!\!\oint_{\Gamma} (\boldsymbol{E} \times \boldsymbol{H}) \cdot \hat{n} d\Gamma = \iiint_V (\boldsymbol{H} \cdot \frac{\partial}{\partial t}\boldsymbol{B} + \boldsymbol{E} \cdot \boldsymbol{J} + \boldsymbol{E} \cdot \frac{\partial}{\partial t}\boldsymbol{D}) dV \tag{139}$$

Note that this Poynting's law is not the Poynting's law of magnetic quasi-static electromagnetic field, but the Poynting's law of radiated electromagnetic field. Suppose there are two current elements. Examples of two current elements are transformers, with primary and secondary coils, or a pair of antennas, with transmit antennas, and receive antennas. Suppose there is a closed surface surrounding two current elements, as shown in figure 2. Consider the superposition principle for these two current elements,

$$\boldsymbol{E} = \sum_{i=1}^{2} \boldsymbol{E}_i, \quad \boldsymbol{H} = \sum_{i=1}^{2} \boldsymbol{H}_i, \quad \boldsymbol{J} = \sum_{i=1}^{2} \boldsymbol{J}_i \tag{140}$$

By substituting the superposition principle into Poynting's theorem,

$$-\sum_{i=1}^{2}\sum_{j=1}^{2} \oint\!\!\!\oint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_j) \cdot \hat{n} d\Gamma$$

$$= \sum_{i=1}^{2}\sum_{j=1}^{2} \iiint_V (\boldsymbol{H}_i \cdot \frac{\partial}{\partial t}\boldsymbol{B}_j + \boldsymbol{E}_i \cdot \boldsymbol{J}_j + \boldsymbol{E}_i \cdot \frac{\partial}{\partial t}\boldsymbol{D}_j) dV \tag{141}$$

This is Poynting's theorem of two current elements. Then the Poynting theorem of each current element is added to obtain,

$$-\sum_{i=1}^{2} \oint\!\!\!\oint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_i) \cdot \hat{n} d\Gamma$$

$$= \sum_{i=1}^{2} \iiint_V (\boldsymbol{H}_i \cdot \frac{\partial}{\partial t}\boldsymbol{B}_i + \boldsymbol{E}_i \cdot \boldsymbol{J}_i + \boldsymbol{E}_i \cdot \frac{\partial}{\partial t}\boldsymbol{D}_i) dV \tag{142}$$

The Poynting theorem (142) containing only the mutual energy part is obtained by subtracting the above formula from the Poynting theorem (141) of the superposition field,

$$-\sum_{i=1}^{2}\sum_{j=1, j\neq i}^{2} \oint\!\!\!\oint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_j) \cdot \hat{n} d\Gamma$$

$$= \sum_{i=1}^{2}\sum_{j=1, j\neq i}^{2} \iiint_V (\boldsymbol{H}_i \cdot \frac{\partial}{\partial t}\boldsymbol{B}_j + \boldsymbol{E}_i \cdot \boldsymbol{J}_j + \boldsymbol{E}_i \cdot \frac{\partial}{\partial t}\boldsymbol{D}_j) dV \tag{143}$$

If we can prove,

$$\int_{t=\infty}^{\infty} dt \sum_{i=1}^{2}\sum_{j=1, j\neq i}^{2} \oint\!\!\!\oint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_j) \cdot \hat{n} d\Gamma = 0 \tag{144}$$

$$\int_{t=\infty}^{\infty} dt \sum_{i=1}^{2}\sum_{j=1, j\neq i}^{2} \iiint_V (\boldsymbol{H}_i \cdot \frac{\partial}{\partial t}\boldsymbol{B}_j + \boldsymbol{E}_i \cdot \frac{\partial}{\partial t}\boldsymbol{D}_j) dV = 0 \tag{145}$$

$$\int_{t=\infty}^{\infty} dt \sum_{i=1}^{2} \oint\!\!\!\oint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_i) \cdot \hat{n} d\Gamma = 0 \tag{146}$$

$$\int_{t=\infty}^{\infty} dt \sum_{i=1}^{2} \iiint_V (\boldsymbol{H}_i \cdot \frac{\partial}{\partial t}\boldsymbol{B}_i + \boldsymbol{E}_i \cdot \frac{\partial}{\partial t}\boldsymbol{D}_i) dV = 0 \tag{147}$$

$$\int_{t=\infty}^{\infty} dt \sum_{i=1}^{2} \iiint_V (\boldsymbol{E}_i \cdot \boldsymbol{J}_i) dV = 0 \tag{148}$$

We can prove from (141),

$$\int_{t=\infty}^{\infty} dt \sum_{i=1}^{2}\sum_{j=1, j\neq i}^{2} \iiint_V (\boldsymbol{E}_i \cdot \boldsymbol{J}_j) dV = 0 \tag{149}$$

This is to prove the above law of conservation of energy from Maxwell's equation. This is because we can use (141,142]),

$$-\int_{t=\infty}^{\infty} dt \sum_{i=1}^{2}\sum_{j=1}^{2} \oint\!\!\!\oint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_j) \cdot \hat{n} d\Gamma$$

$$= \int_{t=\infty}^{\infty} dt \sum_{i=1}^{2}\sum_{j=1}^{2} \iiint_V (\boldsymbol{H}_i \cdot \frac{\partial}{\partial t}\boldsymbol{B}_j + \boldsymbol{E}_i \cdot \boldsymbol{J}_j + \boldsymbol{E}_i \cdot \frac{\partial}{\partial t}\boldsymbol{D}_j) dV \tag{150}$$

Add the two Poynting vectors with angle marks $i = 1$ and $i = 2$ to obtain,

$$-\int_{t=\infty}^{\infty} dt \sum_{i=1}^{2} \oint\!\!\!\oint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_i) \cdot \hat{n} d\Gamma$$

$$= \int_{t=\infty}^{\infty} dt \sum_{i=1}^{2} \iiint_V (\boldsymbol{H}_i \cdot \frac{\partial}{\partial t}\boldsymbol{B}_i + \boldsymbol{E}_i \cdot \boldsymbol{J}_i + \boldsymbol{E}_i \cdot \frac{\partial}{\partial t}\boldsymbol{D}_i) dV \tag{151}$$

We can subtract (151) from (150), and then this subtraction process, because each term of (151) is 0, so the subtraction process does not change the nature of the formula ([eq:150). If it was originally the conservation rate of energy, after the subtraction process is still the conservation law of energy. After subtracting, we get,

$$-\int_{t=\infty}^{\infty} dt \sum_{i=1}^{2} \sum_{j=1, j\neq i}^{2} \oiint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_j) \cdot \hat{n} d\Gamma$$

$$= \int_{t=\infty}^{\infty} dt \sum_{i=1}^{2} \sum_{j=1, j\neq i}^{2} \iiint_V (\boldsymbol{H}_i \cdot \frac{\partial}{\partial t} \boldsymbol{B}_j + \boldsymbol{E}_i \cdot \boldsymbol{J}_j + \boldsymbol{E}_i \cdot \frac{\partial}{\partial t} \boldsymbol{D}_j) dV \tag{152}$$

Using the formula (144,145), we get (149) the law of conservation of energy.

But we can't prove the formula ([eq:7-79]), because

$$\int_{t=\infty}^{\infty} dt \oiint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_i) \cdot \hat{n} d\Gamma \neq 0 \tag{153}$$

The above formula is the radiated electromagnetic field of the $i-$th current element. According to Maxwell's equation, the Poynting vector of the antenna's radiation field is not zero. Therefore, the above formula is not zero. In this way, we cannot prove that the formula (149) is the law of conservation of energy. However, we can still prove that the formula (149) is an energy theorem. This is because even if every term of the formula (151) is not zero, 151 is still a Poynting theorem, and we can still subtract (151) from (150) to get the formula (152), which can still be used as a theorem, and its left and right sides are still equal. The law of conservation of energy requires that the formula includes all energy terms besides being correct. The formula (153) indicates that self energy flow transmits energy, so mutual energy flow is not the only energy flow, so the formula (152) is not the law of conservation of energy. This is also the reason why the author can only call this formula the theorem of mutual energy at first (1987), but not the law of conservation of energy [10].

Now we prove (144-145)

$$\oiint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_j) \cdot \hat{n} d\Gamma = 0 \tag{154}$$

Make the surface Γ a sphere with infinite radius $\xi_1 = [\boldsymbol{E}_1, \boldsymbol{H}_1]^T$ and $\xi_2 = [\boldsymbol{E}_2, \boldsymbol{H}_2]^T$ one is the retarded wave and the other is the advanced wave. Therefore, they reach the big sphere at a time in the past and a time in the future. Therefore, they are not nonzero at the same time on the big sphere Γ. Therefore, the product on the sphere $\boldsymbol{E}_1 \times \boldsymbol{H}_2$ and $\boldsymbol{E}_2 \times \boldsymbol{H}_1$ is 0. Therefore, the above formula 154 is zero. This proves (144). Consider,

$$\int_{t=\infty}^{\infty} dt \sum_{i=1}^{2} \sum_{j=1, j\neq i}^{2} \iiint_V (\boldsymbol{H}_i \cdot \frac{\partial}{\partial t} \boldsymbol{B}_j + \boldsymbol{E}_i \cdot \frac{\partial}{\partial t} \boldsymbol{D}_j) dV$$

$$= \int_{t=\infty}^{\infty} \frac{\partial}{\partial t} U dt = U(\infty) - U(-\infty) = 0 \tag{155}$$

Where

$$U = \sum_{i=1}^{2} \sum_{j=1, j<i} \iiint_V (\boldsymbol{H}_i \cdot \boldsymbol{B}_j + \boldsymbol{E}_i \cdot \boldsymbol{D}_j) dV \tag{156}$$

$U(\infty)$ is the energy at the end of the process, $U(-\infty)$ is the energy at the beginning of each process, and they are all 0 values. This proves (145). In this way, we prove that the formula (149) is an energy theorem, not a law of conservation of energy. But this is based on the recognition of Maxwell's equation and Maxwell's equation (136). The author believes that there are problems in Maxwell's theory. In fact, the formula (149) should be the law of conservation of energy, not only an energy theorem, but also a law, and even an axiom of electromagnetic theory. The next section will discuss this issue in detail.

B. The mutual energy theorem is actually the law of conservation of energy

Why does the author think that the mutual energy theorem is actually the law of conservation of energy?

(1) We have proved the law of conservation of energy under the condition of magnetic quasi-static field,

$$\int_{t=\infty}^{\infty} dt \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} \iiint_V \boldsymbol{E}_i(t) \cdot \boldsymbol{J}_j(t) dV = 0 \tag{157}$$

(2) If n is large enough to contain all the charges in the universe, there should be no energy exchange between energy and charges outside the universe. So there should be (157)

(3) We verified it in the case of transformer

$$M_{2,1}^* = M_{1,2}$$

This means this,

$$\Re \sum_{i=1}^{2} \sum_{j=1, j\neq i}^{2} \iiint_V \boldsymbol{E}_i \cdot \boldsymbol{J}_j^* dV = 0$$

Change to the time domain,

$$\sum_{i=1}^{2} \sum_{j=1, j\neq i}^{2} \int_{t=-\infty}^{\infty} dt \iiint_V \boldsymbol{E}_i(t) \cdot \boldsymbol{J}_j(t) dV = 0$$

From 2 to N, the law of conservation of energy (157) is obtained.

(4) According to Maxwell's electromagnetic field theory, Poynting's theorem is established. From this, it can be deduced that the mutual energy flow theorem is established. We found that the energy transferred by the mutual energy flow has been completely consistent with the law of conservation of energy (157), and the energy transferred by the mutual energy flow from the primary to the secondary of the transformer can reach all energy. Therefore, there is no need for self energy flow to transfer energy. If self energy transfers energy again, there will be more energy, so the law of conservation of energy is broken.

(5) If the self energy flow transfers energy, our universe is almost transparent. Only a small amount of energy will

be absorbed by the sun, earth, stars and dust, and most of the energy will be radiated outside the universe. That is, energy will overflow the universe, which is impossible. So there should be,

$$\Re \iint_{\Gamma_1} (\boldsymbol{E}_1 \times \boldsymbol{H}_1^*) \cdot \hat{n} d\Gamma = 0 \tag{158}$$

$$\Re \iint_{\Gamma_2} (\boldsymbol{E}_2 \times \boldsymbol{H}_2^*) \cdot \hat{n} d\Gamma = 0 \tag{159}$$

(6) We will prove in another article that

$$\Re \iiint_{V_1} \boldsymbol{E}_1 \cdot \boldsymbol{J}_1^* dV = 0$$

$$\Re \iiint_{V_2} \boldsymbol{E}_2 \cdot \boldsymbol{J}_2^* dV = 0$$

If the theoretical formula of Maxwell's hysteresis potential is not zero. But if we consider the retarded potential and the advanced potential, the above formula will be constant to zero. This point will be discussed in another article.

(7) The author believes that the mutual energy flow is a photon. If the self energy flow transfers energy, then the self energy flow must collapse to a certain point in space to form a photon. This constitutes two different photons: self energy flow photons and mutual energy flow photons. Two different photons are not found. There can only be one photon. The author believes that this photon is a mutual energy flow, not a self energy flow.

If self energy does not transfer energy, then energy is naturally transferred by mutual energy flow, and the mutual energy theorem becomes the law of conservation of energy. Therefore, it can be used as the axiom of electromagnetic field theory.

C. Define electromagnetic field according to the method of the author

Although the author cannot prove the law of conservation of energy from Maxwell's equation (149), the author still believes that this formula must be the law of conservation of energy. The author believes that the law of conservation of energy should be taken as an axiom. It is true even in the case of radiated electromagnetic fields. The author believes that the law of conservation of energy cannot be proved from Maxwell's equation, which shows that Maxwell's equations conflicts with the law of conservation of energy. The error lies in Maxwell's equations. So Maxwell's equation should be corrected.

The author believes that the definition of magnetic field in Maxwell's equation is not correct enough. The definition of Maxwell's magnetic field is,

$$\boldsymbol{B} = \nabla \times \boldsymbol{A} \tag{160}$$

Maxwell defines a magnetic field in the case of radiated electromagnetic field, which maintains the above formula (160). It is unchanged in the case of transition from magnetic quasi-static electromagnetic field to radiated electromagnetic field. The author defines the magnetic field to ensure that (149) is still the law of conservation of energy. This requirement (146) still holds. That is,

$$\int_{t=\infty}^{\infty} dt \oiint_{\Gamma} (\boldsymbol{E}_1(t) \times \boldsymbol{H}_1(t)) \cdot \hat{n} d\Gamma = 0 \tag{161}$$

$$\int_{t=\infty}^{\infty} dt \oiint_{\Gamma} (\boldsymbol{E}_2(t) \times \boldsymbol{H}_2(t)) \cdot \hat{n} d\Gamma = 0 \tag{162}$$

Transform to Fourier frequency domain as,

$$\Re \oiint_{\Gamma} (\boldsymbol{E}_1(\omega) \times \boldsymbol{H}_1^*(\omega)) \cdot \hat{n} d\Gamma = 0 \tag{163}$$

$$\Re \oiint_{\Gamma} (\boldsymbol{E}_2(\omega) \times \boldsymbol{H}_2^*(\omega)) \cdot \hat{n} d\Gamma = 0 \tag{164}$$

The above formula ensures that the far field of the magnetic field and the electric field have a phase difference of 90 degrees. The phase difference between magnetic field and electric field calculated by Maxwell equations is zero. In order to distinguish the magnetic field defined by the author's method, it is advisable to record the magnetic field defined by the author as $\boldsymbol{h}$, and Maxwell's magnetic field is still represented by $\boldsymbol{H}$. The magnetic field defined by Maxwell has been widely used. Although the author finds that this magnetic field is unreliable, we may as well still use it. Just keep awake in your heart. Therefore,

$$\Re \oiint_{\Gamma} (\boldsymbol{E}_1(\omega) \times \boldsymbol{h}_1^*(\omega)) \cdot \hat{n} d\Gamma = 0 \tag{165}$$

$$\Re \oiint_{\Gamma} (\boldsymbol{E}_2(\omega) \times \boldsymbol{h}_2^*(\omega)) \cdot \hat{n} d\Gamma = 0 \tag{166}$$

Mutual energy flow theorem becomes,

$$-\int_{t=-\infty}^{\infty} dt \iiint_{V_1} \boldsymbol{E}_2(t) \cdot \boldsymbol{J}_1(t) dV$$

$$= (\xi_1, \xi_2)$$

$$= \int_{t=-\infty}^{\infty} dt \iiint_{V_2} \boldsymbol{E}_1(t) \cdot J_2(t) dV \tag{167}$$

$$(\xi_1, \xi_2) = \iint_{\Gamma} (\boldsymbol{E}_1(t) \times \boldsymbol{h}_2^*(t) + \boldsymbol{E}_2^*(t) \times \boldsymbol{h}_1(t)) \cdot \hat{n}_{12} d\Gamma \tag{168}$$

The mutual energy flow theorem adopts the magnetic field defined by the author. Of course, this mutual energy flow theorem is actually the law of energy flow. Please note that the above definition of the formula is different with,

$$(\xi_1, \xi_2) = \iint_{\Gamma} (\boldsymbol{E}_1(t) \times \boldsymbol{H}_2^*(t) + \boldsymbol{E}_2^*(t) \times \boldsymbol{H}_1(t)) \cdot \hat{n}_{12} d\Gamma \tag{169}$$

(168) is the mutual energy flow calculated according to the electromagnetic field modified by the author. (169) is the mutual energy flow calculated completely according to the magnetic field of Maxwell's equation. The two are different.

D. What is wrong with Maxwell's equation?

The electric field of Maxwell is,

$$\boldsymbol{E} = -\frac{\partial}{\partial t}\boldsymbol{A}$$

The magnetic field of Maxwell is,

$$\boldsymbol{H} = \frac{1}{\mu_0}\nabla \times \boldsymbol{A} \tag{170}$$

Where

$$\boldsymbol{A} = \frac{\mu_0}{4\pi}\iiint_V \frac{\boldsymbol{J}}{r} dV \tag{171}$$

In the case of radiation, the retarded potential is,

$$\boldsymbol{A}^{(re)} = \frac{\mu_0}{4\pi}\iiint_V \exp(-j\boldsymbol{k}\cdot\boldsymbol{r})\frac{\boldsymbol{J}}{r} dV \tag{172}$$

$$\boldsymbol{E}^{(re)} = -j\omega\boldsymbol{A}^{(re)} = -j\omega\frac{\mu_0}{4\pi}\iiint_V \exp(-j\boldsymbol{k}\cdot\boldsymbol{r})\frac{\boldsymbol{J}}{r} dV \tag{173}$$

Hence,

$$\lim_{\boldsymbol{r}\to 0}\boldsymbol{E}^{(re)} = \lim_{\boldsymbol{r}\to 0}(-j\omega\frac{\mu_0}{4\pi}\iiint_V \exp(-j\boldsymbol{k}\cdot\boldsymbol{r})\frac{\boldsymbol{J}}{r} dV)$$
$$= (-j\omega\frac{\mu_0}{4\pi}\iiint_V \frac{\boldsymbol{J}}{r} dV) = -j\omega\boldsymbol{A} = \boldsymbol{E} \tag{174}$$

Retarded magnetic field

$$\boldsymbol{B}^{(re)} = \nabla\times\boldsymbol{A}^{(re)} = \nabla\times\frac{\mu_0}{4\pi}\iiint_V \exp(-j\boldsymbol{k}\cdot\boldsymbol{r})\frac{\boldsymbol{J}}{r} dV$$
$$= \frac{\mu_0}{4\pi}\iiint_V \nabla(\exp(-j\boldsymbol{k}\cdot\boldsymbol{r})\frac{1}{r})\times\boldsymbol{J} dV \tag{175}$$
$$= \frac{\mu_0}{4\pi}\iiint_V (\frac{\partial}{\partial r}\exp(-jkr)\nabla r\frac{1}{r} + \exp(-jkr)\nabla\frac{1}{r})\times\boldsymbol{J} dV$$
$$= \frac{\mu_0}{4\pi}\iiint_V (-jk\exp(-j\boldsymbol{k}\cdot\boldsymbol{r})\frac{\boldsymbol{r}}{r}\frac{1}{r} + \exp(-j\boldsymbol{k}\cdot\boldsymbol{r})(-\frac{\boldsymbol{r}}{r^3})\times\boldsymbol{J} dV$$

Hence,

$$\lim_{\boldsymbol{r}\to 0}\boldsymbol{B}^{(re)} = \frac{\mu_0}{4\pi}\iiint_V \boldsymbol{J}\times(jk\frac{\boldsymbol{r}}{r^2} + \frac{\boldsymbol{r}}{r^3})dV \tag{176}$$

I know that in the magnetic quasi-static field, the magnetic field is,

$$\boldsymbol{B} = \frac{\mu_0}{4\pi}\iiint_V \boldsymbol{J}\times\frac{\boldsymbol{r}}{r^3} dV \tag{177}$$

Therefore, corresponding to Maxwell's retarded potential,

$$\lim_{r\to 0}\boldsymbol{B}^{(re)} \neq \boldsymbol{B} \tag{178}$$

It can be seen that the properties of the magnetic field and the electric field calculated according to Maxwell's equation are different. The electric field satisfies,

$$\lim_{r\to 0}\boldsymbol{E}^{(re)} = \boldsymbol{E} \tag{179}$$

Although at present, the author can only calculate the far field from the formula (165-168), considering that the magnetic field is an auxiliary quantity, what is really important is the value of the electric field. Therefore, the far field of the magnetic field is enough.

The author has completed the example of radiation field in another article, and will not discuss it further here. In fact, for the radiated electromagnetic field, we only need to know the electric field, not the magnetic field. For the magnetic field, we know that its far field is perpendicular to the electric field, and the phase can be maintained at 90 degrees.

In this way, the magnetic field $\boldsymbol{B}$ defined by Maxwell is still useful, and it is still the curl of the vector potential,

$$\boldsymbol{B} \equiv \nabla\times\boldsymbol{A} \tag{180}$$

This $\boldsymbol{B}$ and the Maxwell equation containing $\boldsymbol{B}$ (136) are still valuable for finding the equation (132) of the hysteresis potential. However, the Poynting theorem (139) composed of this $\boldsymbol{B}$ cannot be regarded as a law of conservation of energy! For the correct Poynting's theorem is,

$$-\int_{t=-\infty}^{\infty}\oiint_\Gamma(\boldsymbol{E}\times\boldsymbol{h})\cdot\hat{n}d\Gamma = \int_{t=-\infty}^{\infty}\iiint_V(\boldsymbol{E}\cdot\boldsymbol{J})dV = 0 \tag{181}$$

Or at the frequency domain, there is,

$$-\Re\oiint_\Gamma(\boldsymbol{E}\times\boldsymbol{h}^*)\cdot\hat{n}d\Gamma = \Re\iiint_V(\boldsymbol{E}\cdot\boldsymbol{J}^*)dV = 0 \tag{182}$$

IX. Conclusion

For a long time, when it comes to energy flow, it is believed that it should be expressed by Poynting vector. The author found that Poynting vector actually needs further subdivision. Poynting vector can be divided into (1) self energy flow, which corresponds to the energy flow of primary coil or secondary coil itself; (2) The mutual energy flow corresponds to the mixed Poynting vector between the primary coil and the secondary coil. The energy flow of transformer is provided by the energy flow corresponding to the mixed Poynting vector, which the author calls mutual energy flow.

This paper proves that under the condition of magnetic quasi-static electromagnetic field, the mutual energy theorem proposed by the author in 1987 (also the time domain reciprocity theorem proposed by Welch in 1960) is actually the law of conservation of energy. Under this condition, the energy flow transmitted by the self energy flow is a pure imaginary number, which indicates that this part of the energy flow is reactive power, so the time average value of the transmitted energy is zero. That is to say, self energy does not transmit energy. Then the transfer of energy is completely completed by mutual energy flow. Therefore, mutual energy flow is actually energy flow. For a long time, many people use Poynting vector to explain the energy flow from the primary coil to the secondary coil of the transformer, but these methods are unsuccessful. The author calculated the energy flow from the primary coil to the secondary coil of the transformer by using the mutual energy flow for the first time.

But when it comes to the radiated electromagnetic field, according to the theory of radiated electromagnetic field of Maxwell's equation, the self energy flow, that is, Poynting vector, is a real number, so it transfers energy. The author believes that this is wrong. Even for radiated electromagnetic fields, self energy flow should only radiate reactive power waves. Therefore, the author redefined a new magnetic field. This magnetic field ensures that energy is transferred by mutual energy flow, and self energy flow does not transfer energy.

The mutual energy theorem once called by the author is actually the law of conservation of energy. The mutual energy flow theorem once called by the author is the law of energy flow. Here the word "mutual" can be removed, and the theorem can be changed into a law. In this way, the law of conservation of energy can be regarded as a new axiom of electromagnetic theory. The author deduces the whole electromagnetic theory from this new axiom. Firstly, the mutual inductance from primary coil to secondary coil and from secondary coil to primary coil and the relationship between them are derived in the transformer environment. This relationship requires that the electromagnetic wave radiated by the secondary coil must be a advanced wave, so it proves that the advanced wave exists. Starting from this Law of conservation of energy, it is easy to deduce that the energy transferred from the energy flow is zero, and the energy is transferred by the mutual energy flow, which can further determine the new magnetic field proposed by the author. This new magnetic field is not the magnetic field defined by Maxwell's method in the traditional electromagnetic field theory. The new magnetic field ensures that the self energy flow does not transfer energy, and the energy is transferred by the mutual energy flow. The author believes that for the radiated electromagnetic field, this new magnetic field is the real definition of the magnetic field, and the definition of the magnetic field in Maxwell's theory is problematic. This problem leads to a deviation in the calculation of self energy flow, which converts the self energy flow of reactive power into active power. This error has been extended to quantum mechanics, which is the root of the wave particle duality problem.

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