A THREE-DIMENSIONAL BRANE UNIVERSE IN A FOUR-DIMENSIONAL SPACETIME WITH A BIG BANG.

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Abstract. Arguments for the universe being a 3D brane in a 4D bulk are given. Einstein's gravity equations are rewritten as dynamical equations for a 3D metric on a 3D brane travelling in a 4D spacetime with a Big Bang.

1. Introduction.

The idea that our real universe is a 4D structure has deeply penetrated the brains of scientists and ordinary people. However, is it true that this 4D structure as a whole has its material being at each particular moment of time? Let's study this question on the basis of the principles of general relativity and cosmology.

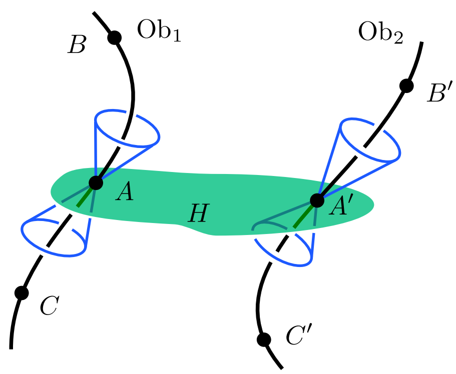

Fig. 1.1

In Fig. 1.1 we see two observers in a 4D spacetime and their trajectories, which are called world lines (see [1] and § 6 in Chapter II of [2]). Let's call them Ob_1 and Ob_2. The observer Ob_1, when passing through the point A of his life, can think as follows: "*I am at the point A and I do exist. Therefore, the point A does exist. The points B and C do not exist right now, since B is my future, while C is my past. I know that I am not a unique observer in the universe and the point A is not a unique point of the universe. There must be another observer, say Ob_2, who coexists with me at some definite point A' of his trajectory. The points B' and C' cannot coexist with me right now, since the point A' does and since B' is the future of A', while C' is its past.*"

The train of thoughts of the observer Ob_1 leads to the conclusion that the 4D spacetime is subdivided into mutually non-overlapping **coexistence classes**. However, these thoughts do not describe the classes completely, though they yield some restrictions for them. The coexistence classes could be

1) smooth structures;
2) partially smooth structures;
3) fractal structures;
4) non-structured formations.

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We choose the option 2 and assume the coexistence classes to form a smooth foliation of 3D hypersurfaces in the 4D spacetime (see [3]). The smoothness of them can be broken at the points corresponding to black holes.

The coexistence classes in the form of 3D hypersurfaces represent successive stages in the evolution of the universe. Only one of them comes to physical being at each particular moment of time. This particular hypersurface, being the present state of the 3D universe, is called the **evolution front**. All observers and all material bodies and fields are enclosed in this evolution front and move together with it along 4D spacetime, though each observer lives with its own pace of time according to laws of Einstein's relativity. As for the 4D spacetime, it is a mathematical abstraction rather than a real physical structure.

The main goal of the present paper is to promote $4 = 3+1$ approach in cosmology as something more than just a mathematical trick for solving equations.

2. Light cone restriction for coexistence classes.

Let A be some point of the $4D$ spacetime and let H be its coexistence class (see Fig. 1.1). Each point A of the spacetime is associated with two light cones. Their walls are formed by light rays passing through the point A. Their interior can be filled with world lines of massive particles passing through the point A. Therefore none of the point from the interior and from the walls of these light cones can belong to the class H. Thus we conclude that each coexistence class H is completely in the exterior of light cones associated with each its point A. This is the light cone restriction for coexistence classes.

According to our choice above, coexistence classes are smooth 3D hypersurfaces at most points of them. At each of those points they have a normal vector $\mathbf{n}$. From the light cone restriction one easily derive that the normal vector $\mathbf{n}$ is in the interior of the light cones. Hence the metric induced to coexistence hypersurfaces is non-degenerate. We take it to be Riemannian (see [4]) and define two matrices

$$g_{ij} = \begin{Vmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{Vmatrix}, \qquad\qquad g^{ij} = \begin{Vmatrix} g^{11} & g^{12} & g^{13} \\ g^{21} & g^{22} & g^{23} \\ g^{31} & g^{32} & g^{33} \end{Vmatrix} \tag{2.1}$$

that correspond to direct and inverse metric tensors.

3. Normal shift and comoving coordinates. The concept of cosmic time.

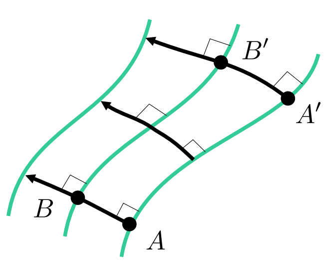

Fig. 3.1

Let's consider our foliation of 3D hypersurfaces (coexistence classes). They are rendered as green lines in Fig. 3.1. Actually they form a dense family filling the whole spacetime. Let's mark normal vectors of the unit length at each point of each hypersurface. As a result we get a smooth vector field $\mathbf{n}$ of unit vectors. Integral curves of this vector field are rendered in black in Fig. 3.1. These integral curves are perpendicular to hypersurfaces. Therefore we see the picture of a normal shift. This is the normal shift along integral curves of a first order dynamical system. Other sorts of normal shift are considered in [5].

Integral curves of a vector field are parametric curves with some numeric parameter (see [6]). Let τ be such a numeric parameter for integral curves in Fig. 3.1. If we choose some curvilinear coordinates x^1, x^2, x^3 on some initial hypersurface and complement them with one more coordinate $x^0 = \tau$, then we get a curvilinear coordinate system in some neighborhood of the initial hypersurface in the 4D spacetime. Coordinates constructed in such a way are called **comoving coordinates**. An observer whose world line coincides with one of the integral curves of the normal shift in Fig. 3.1 is called a **comoving observer** (see [7]).

Two points A and B of two hypersurfaces in Fig. 3.1 are connected by an integral curve of the vector field $\mathbf{n}$. The arc length of the segment $[AB]$ of this curve is called the **orthogonal distance** between two hypersurfaces at the point A (or at the point B). In comoving coordinates we have

$$|AB| = x^0\Big|_B - x^0\Big|_A. \tag{3.1}$$

A similar formula can be written for the points A' and B':

$$|A'B'| = x^0\Big|_{B'} - x^0\Big|_{A'}. \tag{3.2}$$

The normal shift in Fig. 3.1 is called an **equidistant normal shift** if for any two hypersurfaces the orthogonal distances (3.1) and (3.2) are equal to each other for any two points A and A' of one of them. Mathematically, not any normal shift determined by a foliation of hypersurfaces is equidistant. However, here we introduce a new physical postulate.

Postulate 3.1. *Watches of any two comoving observers can be synchronized.*

The normal shift in Fig. 3.1 occurs in the time direction. Indeed, normal vectors to hypersurfaces, which are tangent to integral curves, are in the interior of light cones. Therefore we can divide the distances (3.1) and (3.2) by the speed of light and thus get time intervals for two comoving observers A and A':

$$\triangle t_A = \frac{|AB|}{c} \qquad\qquad \triangle t_{A'} = \frac{|A'B'|}{c}. \tag{3.3}$$

Postulate 3.1 means that the time intervals (3.3) should be equal: $\triangle t_A = \triangle t_{A'}$. Hence we get $|AB| = |A'B'|$ thus proving the following theorem.

Theorem 3.1. *The normal shift associated with coexistence classes in cosmology is always equidistant.*

Using comoving coordinates x^0, x^1, x^2, x^3, one can introduce the time variable

$$t = \frac{x^0}{c}. \tag{3.4}$$

In the case of an equidistant normal shift the time variable (3.4) characterizes each hypersurface as a whole. It is known as the **cosmic time** (see [8]).

4. The role of the Big Bang.

For defining the normal shift in Fig. 3.1 we used some initial hypersurface. Therefore the time variable (3.4) is relative one. It is defined up to our choice of an initial hypersurface. However, in many cosmological models the evolution of the universe starts not from a hypersurface, but from a point. This point is called the Big Bang (see [9]). If we include the Big Bang into our concept of coexistence classes, then we would have the initial coexistence class in the form of one point of Big Bang. Other classes would arise through the **normal blow-up** of this point (see Fig. 4.1).

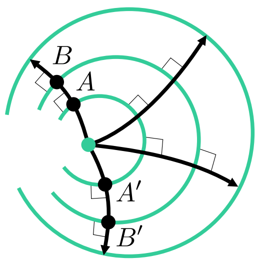

Fig. 4.1

Like the normal shift in Fig. 3.1, the normal blow-up in Fig. 4.1 is a normal shift along integral curves of a first order dynamical system. Other sorts of normal blow-up are considered in [10].

The use of the Big Bang as a reference point in defining the time variable (3.4) makes its choice absolute. The value of this variable for the current epoch is known as the **current age of the universe**. It is approximately 13.8 billion years according to our present knowledge (see [11]).

5. Einstein's equations of gravity in 3 + 1 presentation.

Einstein's equations of gravity are derived for the four-dimensional spacetime (see § 2 in Chapter V of [2]). They are written as follows:

$$r_{ij} - \frac{r}{2}\,G_{ij} - \Lambda\,G_{ij} = \frac{8\,\pi\,\gamma}{c^4}\,T_{ij}. \tag{5.1}$$

Here γ is Newton's gravitational constant (see [12]):

$$\gamma \approx\ 6.674\cdot 10^{-8}\ cm^3\cdot g^{-1}\cdot s^{-2}$$

and c is the speed of light. Now, as a result of redefining other standard units in 2019, the speed of light c is defined as an exact physical constant (see [13]):

$$c = 2.99792458\cdot 10^{10}\ cm\cdot s^{-1}.$$

The constant Λ in (5.1) is the cosmological constant. It is associated with the dark energy (see [14]). Its value is quite uncertain, but very close to zero (see [15]):

$$\Lambda \approx\ 10^{-56}\ cm^{-2}.$$

The term T_{ij} in the right hand side of (5.1) stands for the matter including the dark matter (see [16]) and the regular matter. It represent the components of a symmetric tensor which is called the energy-momentum tensor (see [17]).

The term r_{ij} corresponds to the components of the Ricci tensor and r is the scalar curvature[1] (see § 8 in Chapter IV of [18]). Both of these two terms are expressed

[1] We used lower case letters for r_{ij} and r in order to reserve capital letters for 3D Ricci tensor and for 3D scalar curvature.

through the components of the four-dimensional metric tensor G_{ij}. In order to express the equations (5.1) in 3 + 1 presentation we use comoving coordinates x^0, x^1, x^2, x^3 with the cosmic time (3.4). In these coordinates the components of the metric tensor G_{ij} and the components of the inverse metric tensor G^{ij} are given by the following two matrices:

$$G_{ij} = \begin{Vmatrix} 1 & 0 & 0 & 0 \\ 0 & -g_{11} & -g_{12} & -g_{13} \\ 0 & -g_{21} & -g_{22} & -g_{23} \\ 0 & -g_{31} & -g_{32} & -g_{33} \end{Vmatrix}, \qquad G^{ij} = \begin{Vmatrix} 1 & 0 & 0 & 0 \\ 0 & -g^{11} & -g^{12} & -g^{13} \\ 0 & -g^{21} & -g^{22} & -g^{23} \\ 0 & -g^{31} & -g^{32} & -g^{33} \end{Vmatrix}. \tag{5.2}$$

The quantities g_{ij} and g^{ij} are taken from (2.1). The metric (2.1) is assumed to be Riemannian. Therefore the metric (5.2) is pseudo-Riemannian with the signature $(+\,-\,-\,-)$. This metric produce the metric connection with the components

$$\gamma^k_{ij} = \frac{1}{2}\sum_{s=0}^{3} G^{ks}\left(\frac{\partial G_{sj}}{\partial x^i} + \frac{\partial G_{is}}{\partial x^j} - \frac{\partial G_{ij}}{\partial x^s}\right). \tag{5.3}$$

It is easy to derive that

$$\gamma^k_{ij} = \Gamma^k_{ij} \quad \text{for} \quad 1 \leqslant i, j, k \leqslant 3, \tag{5.4}$$

where Γ^k_{ij} are the components of the metric connection for the metric (2.1):

$$\Gamma^k_{ij} = \frac{1}{2}\sum_{s=1}^{3} g^{ks}\left(\frac{\partial g_{sj}}{\partial x^i} + \frac{\partial g_{is}}{\partial x^j} - \frac{\partial g_{ij}}{\partial x^s}\right). \tag{5.5}$$

The rest of the components (5.3) are distributed as follows:

$$\gamma^0_{ij} = \frac{1}{2}\,\frac{\partial g_{ij}}{\partial x^0} = \frac{1}{2\,c}\,\frac{\partial g_{ij}}{\partial t} = \frac{\dot g_{ij}}{2\,c} \quad \text{for} \quad 1 \leqslant i, j \leqslant 3, \tag{5.6}$$

$$\gamma^k_{0j} = \gamma^k_{j0} = \frac{1}{2}\sum_{s=1}^{3} g^{ks}\,\frac{\partial g_{sj}}{\partial x^0} = \sum_{s=1}^{3} g^{ks}\,\gamma^0_{sj} \quad \text{for} \quad 1 \leqslant k, j \leqslant 3, \tag{5.7}$$

$$\gamma^q_{00} = \gamma^0_{q0} = \gamma^0_{0q} = 0 \quad \text{for} \quad 0 \leqslant q \leqslant 3. \tag{5.8}$$

The formulas (5.4), (5.6), (5.7), and (5.8) are easily derived from (5.3) with the use of the formulas (5.2) and (5.5).

Note that the quantities (5.6), when restricted to a 3D hypersurface representing some coexistence class, constitute the components of a symmetric tensor field. This tensor field is usually denoted through $\mathbf{b}$ and is called the **second quadratic form** of a hypersurface (see § 5 in Chapter IV of [18]). Its components are denoted through b_{ij}. Raising one of two indices of the second quadratic form, we get

$$b^k_j = \sum_{s=1}^{3} g^{ks}\,b_{sj}. \tag{5.9}$$

The quantities (5.9) are also the components of a tensor field. This tensor field is denoted through the same letter $\mathbf{b}$ as the second quadratic form. Using the quantities b_{ij} and b^k_j, we can rewrite the formulas (5.6) and (5.7) as follows:

$$\gamma^0_{ij} = b_{ij} \quad \text{for} \quad 1 \leqslant i, j \leqslant 3, \tag{5.10}$$

$$\gamma^k_{0j} = \gamma^k_{j\,0} = b^k_j \quad \text{for} \quad 1 \leqslant k, j \leqslant 3. \tag{5.11}$$

The Ricci tensor in Einstein's equations (5.1) is calculated through the curvature tensor by means of the following formula (see § 8 in Chapter IV of [18]):

$$r_{ij} = \sum_{k=0}^{3} r^k_{ikj}, \tag{5.12}$$

where the components of the curvature tensor are

$$r^k_{isj} = \frac{\partial \gamma^k_{ji}}{\partial x^s} - \frac{\partial \gamma^k_{si}}{\partial x^j} + \sum_{q=0}^{3} \gamma^k_{sq}\,\gamma^q_{ji} - \sum_{q=0}^{3} \gamma^k_{jq}\,\gamma^q_{si}. \tag{5.13}$$

Due to (5.12) in (5.13) we need only those terms where $s = k$:

$$r^k_{ikj} = \frac{\partial \gamma^k_{ji}}{\partial x^k} - \frac{\partial \gamma^k_{ki}}{\partial x^j} + \sum_{q=0}^{3} \gamma^k_{kq}\,\gamma^q_{ji} - \sum_{q=0}^{3} \gamma^k_{jq}\,\gamma^q_{ki}. \tag{5.14}$$

Applying (5.4), (5.10), and (5.11) to (5.14), we derive

$$r^k_{ikj} = R^k_{ikj} + b^k_k\,b_{ij} - b^k_j\,b_{ki} \quad \text{for} \quad 1 \leqslant i, j, k \leqslant 3. \tag{5.15}$$

Here R^k_{ikj} are the components of the 3D curvature tensor. They are given by a formula similar to (5.13) upon setting $s = k$ in it:

$$R^k_{isj} = \frac{\partial \Gamma^k_{ji}}{\partial x^s} - \frac{\partial \Gamma^k_{si}}{\partial x^j} + \sum_{q=1}^{3} \Gamma^k_{sq}\,\Gamma^q_{ji} - \sum_{q=1}^{3} \Gamma^k_{jq}\,\Gamma^q_{si}. \tag{5.16}$$

The 3D Ricci tensor is derived from (5.16) by means of the formula

$$R_{ij} = \sum_{k=1}^{3} R^k_{ikj}, \tag{5.17}$$

which is the 3D version of the formula (5.12).

Now let's consider the case $k = 0$ and $1 \leqslant i, j \leqslant 3$ in (5.14). In this case we have

$$r^0_{i\,0j} = \frac{\partial \gamma^0_{ji}}{\partial x^0} - \frac{\partial \gamma^0_{0i}}{\partial x^j} + \sum_{q=0}^{3} \gamma^0_{0q}\,\gamma^q_{ji} - \sum_{q=0}^{3} \gamma^0_{jq}\,\gamma^q_{0i}. \tag{5.18}$$

Applying (5.8), (5.10), and (5.11) to (5.18), we reduce this formula to

$$r^0_{i\,0j} = \frac{\partial b_{ji}}{\partial x^0} - \sum_{q=1}^{3} b_{jq}\,b^q_i \quad \text{for} \quad 1 \leqslant i, j \leqslant 3. \tag{5.19}$$

The next step is $i=0$ and $1\leqslant j,k\leqslant 3$. in (5.14). In this case we have

$$r^k_{0kj}=\frac{\partial b^k_j}{\partial x^k}-\frac{\partial b^k_k}{\partial x^j}+\sum^3_{q=1}\Gamma^k_{kq}\,b^q_j-\sum^3_{q=1}\Gamma^k_{jq}\,b^q_k. \tag{5.20}$$

We add two terms to (5.20) and rearrange the terms in it:

$$r^k_{0kj}=\frac{\partial b^k_j}{\partial x^k}+\sum^3_{q=1}\Gamma^k_{kq}\,b^q_j-\sum^3_{q=1}\Gamma^q_{kj}\,b^k_q-\frac{\partial b^k_k}{\partial x^j}-\sum^3_{q=1}\Gamma^k_{jq}\,b^q_k+\sum^3_{q=1}\Gamma^q_{jk}\,b^k_q.$$

Due to the symmetry $\Gamma^q_{kj}=\Gamma^k_{jq}$ two extra terms that was added do cancel each other. But they let us apply the concept of covariant derivatives to the above formula (see § 6 in Chapter IV of [18]). As a result we obtain

$$r^k_{0kj}=\nabla_k\,b^k_j-\nabla_j\,b^k_k\ \text{ for }\ 1\leqslant k,j\leqslant 3. \tag{5.21}$$

The case $j=0$ and $1\leqslant i,k\leqslant 3$. in (5.14) is treated similarly:

$$r^k_{ik0}=\nabla_k\,b^k_i-\nabla_i\,b^k_k\ \text{ for }\ 1\leqslant k,j\leqslant 3. \tag{5.22}$$

Now we consider the cases $i=0$ and $k=0$ with $1\leqslant j\leqslant 3$ and $j=0$ with $k=0$ and $1\leqslant i\leqslant 3$ in (5.14). In these cases we have

$$r^0_{00j}=0, \qquad\qquad r^0_{i00}=0. \tag{5.23}$$

The next case is $i=0$ and $j=0$ with $1\leqslant k\leqslant 3$ in (5.14). In this case we have

$$r^k_{0k0}=-\frac{\partial b^k_k}{\partial x^0}-\sum^3_{q=1}b^k_q\,b^q_k\ \text{ for }\ 1\leqslant k\leqslant 3. \tag{5.24}$$

The last case is the case where $i=0$, $j=0$, and $k=0$ in (5.14):

$$r^0_{000}=0. \tag{5.25}$$

Let's apply (5.15) and (5.19) in order to calculate the components of the Ricci tensor in (5.12). Taking into account (5.17) and the symmetry $b_{ij}=b_{ji}$, we derive

$$r_{ij}=\frac{\partial b_{ij}}{\partial x^0}+R_{ij}+\sum^3_{k=1}b^k_k\,b_{ij}-\sum^3_{k=1}(b_{ki}\,b^k_j+b_{kj}\,b^k_i)\ \text{ for }\ 1\leqslant i,j\leqslant 3. \tag{5.26}$$

Then we apply (5.21), (5.22), and (5.23) to (5.12). This yields

$$\begin{aligned}
&r_{i\,0}=\sum^3_{k=1}\nabla_k\,b^k_i-\sum^3_{k=1}\nabla_i\,b^k_k\ \text{ for }\ 1\leqslant i\leqslant 3,\\
&r_{0j}=\sum^3_{k=1}\nabla_k\,b^k_j-\sum^3_{k=1}\nabla_j\,b^k_k\ \text{ for }\ 1\leqslant j\leqslant 3.
\end{aligned} \tag{5.27}$$

Note that the formulas (5.27) are consistent with the symmetry $r_{ij}=r_{ji}$.

Finally we apply (5.24) and (5.25) to (5.12). As a result we get

$$r_{00}=-\sum_{k=1}^{3}\frac{\partial b^k_k}{\partial x^0}-\sum_{k=1}^{3}\sum_{q=1}^{3}b^k_q\,b^q_k. \tag{5.28}$$

The four-dimensional scalar curvature r is calculated through the Ricci tensor (5.12) by means of the formula

$$r=\sum_{i=0}^{3}\sum_{j=0}^{3}r_{ij}\,G^{ij}, \tag{5.29}$$

see § 8 in Chapter IV of [18]. Applying (5.2), (5.26), and (5.28) to (5.29), we derive

$$r=-2\sum_{k=1}^{3}\frac{\partial b^k_k}{\partial x^0}-R-\sum_{k=1}^{3}\sum_{q=1}^{3}b^k_q\,b^q_k-\sum_{k=1}^{3}\sum_{q=1}^{3}b^k_k\,b^q_q\,. \tag{5.30}$$

Here R is the 3D scalar curvature given by a formula analogous to (5.29):

$$R=\sum_{i=0}^{3}\sum_{j=0}^{3}R_{ij}\,g^{ij}. \tag{5.31}$$

Through R_{ij} in the formula (5.31) we denote the components of the 3D Ricci tensor given by the formula (5.17).

Now we are ready to rewrite Einstein's equations (5.1) in $3+1$ presentation. They are subdivided into three groups. The first group is written as

$$\begin{gathered}\frac{\partial b_{ij}}{\partial x^0}-\sum_{k=1}^{3}\frac{\partial b^k_k}{\partial x^0}\,g_{ij}-\sum_{k=1}^{3}(b_{ki}\,b^k_j+b_{kj}\,b^k_i)-\frac{g_{ij}}{2}\sum_{k=1}^{3}\sum_{q=1}^{3}b^k_q\,b^q_k-\\-\frac{g_{ij}}{2}\sum_{k=1}^{3}\sum_{q=1}^{3}b^k_k\,b^q_q+\sum_{k=1}^{3}b^k_k\,b_{ij}+R_{ij}-\frac{R}{2}\,g_{ij}+\Lambda\,g_{ij}=\frac{8\,\pi\,\gamma}{c^4}\,T_{ij},\end{gathered} \tag{5.32}$$

where $1\leqslant i,j\leqslant 3$. The second group is written as

$$\sum_{k=1}^{3}\nabla_k\,b^k_j-\sum_{k=1}^{3}\nabla_j\,b^k_k=\frac{8\,\pi\,\gamma}{c^4}\,T_{0j}, \tag{5.33}$$

where $1\leqslant j\leqslant 3$. The third group consists of one equation:

$$-\frac{1}{2}\sum_{k=1}^{3}\sum_{q=1}^{3}b^k_q\,b^q_k+\frac{1}{2}\sum_{k=1}^{3}\sum_{q=1}^{3}b^k_k\,b^q_q+\frac{R}{2}-\Lambda=\frac{8\,\pi\,\gamma}{c^4}\,T_{00}. \tag{5.34}$$

The equations (5.32), (5.33), and (5.34) are derived by substituting (5.26), (5.27), (5.28), and (5.30) into (5.1). Due to the symmetry of the tensors r_{ij}, G_{ij}, and T_{ij} two expressions from (5.27) lead to the same equations (5.33).

If we remember the relationship (3.4), then we can rewrite the equations (5.32) in terms of the time derivatives with respect to the cosmic time:

$$\begin{gathered}\frac{\dot b_{ij}}{c}-\sum_{k=1}^{3}\frac{\dot b^k_k}{c}\,g_{ij}-\sum_{k=1}^{3}(b_{ki}\,b^k_j+b_{kj}\,b^k_i)-\frac{g_{ij}}{2}\sum_{k=1}^{3}\sum_{q=1}^{3}b^k_q\,b^q_k-\\-\frac{g_{ij}}{2}\sum_{k=1}^{3}\sum_{q=1}^{3}b^k_k\,b^q_q+\sum_{k=1}^{3}b^k_k\,b_{ij}+R_{ij}-\frac{R}{2}\,g_{ij}+\Lambda\,g_{ij}=\frac{8\,\pi\,\gamma}{c^4}\,T_{ij},\end{gathered}\tag{5.35}$$

The equations (5.35) should be complemented with the following equations derived from the equations (5.6) and (5.10):

$$\frac{\dot g_{ij}}{2\,c}=b_{ij}.\tag{5.36}$$

The equations (5.35) and (5.36) constitute a system of two evolution equations for two matrices $g_{ij}(x^1,x^2,x^3,t)$ and $b_{ij}(x^1,x^2,x^3,t)$. The equations (5.33) and (5.34) are treated as auxiliary restrictions for solutions of the equations (5.35) and (5.36).

6. Conclusions.

The equations (5.35), (5.36), (5.33), (5.34) are derived in comoving coordinates $x^0=c\,t$, x^1, x^2, x^3. The matter is that they are not new. Similar calculations are performed in § 97 of Chapter XI in [19], though the ultimate result is presented in a form somewhat different from the equations (5.35), (5.36), (5.33), (5.34). In place of comoving coordinates there the term synchronous reference system is used.

The main issue of the present paper is not in calculations, but in their interpretation. The synchronous coordinates in [19] are treated as just coordinates only, ones among many others. In contrast, comoving coordinates in the present paper are associated with a 3D brane representing the state of physical being of the universe. The change of this state is described internally as the evolution of a 3D metric according to the equations (5.35), (5.36), (5.33), (5.34). Externally it can be understood as a motion of a 3D brane in a 4D spacetime. However the spacetime should be considered as a theoretical construction, not a physical entity. Otherwise it would mean fatalism since, being a physical entity, the spacetime would comprise our past, our present, and our future simultaneously.

7. Dedicatory.

This paper is dedicated to my sister Svetlana Abdulovna Sharipova.

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