New Approach to the Hilbert's Hotel Paradox.

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0- Abstract:

In this short article I wanted to expand the Hilbert's Grand Hotel Paradox. Building on the first two statements, I add a third, possibly necessary, statement.

1- Introduction, Classic version:

Hilbert's Hotel paradox is a paradox illuminating the nature of infinite sets[1], it has two main statements:

(a) If a hotel with infinitely many rooms is full and another guest arrives, that guest can be accommodated by each existing guest moving from their current room to the room with the next higher number, leaving Room 1 free for the new arrival.

(b) Indeed, if an infinite number of extra guests arrived, they could be accommodated efficiently by each existing guests moving to the room whose number is twice their existing room number, leaving an infinite number of odd-numbered rooms available for the new arrivals.

2- My own expansion of the paradox.

I can also add a generalization in the infinite numbers (linear infinite cardinals):

(c) Moreover, if a function of number $n\infty$ of guests arrived, they could be accommodated efficiently by each existing guest moving to the room whose number is the previous function number plus one, leaving $n\infty$ -numbered rooms available for the new arrivals.

3- Schemes of the three cases and examples:

In the first case, case a, we have an infinite number of rooms full, and a new one guest arrives, being f(n) the new arrivals:

(a) $f(1) \rightarrow \infty+1$ Will be the final number of guests. (With 1 more guest)

In the second case, still in the Classic version:

(b) $f(\infty) \rightarrow 2\infty$ Will be the final number of guests. (With ∞ more guests).

And in the third case, my expansion:

(c) $f(n\infty) \rightarrow n\infty+\infty=(n+1)\infty$ Will be the final number of guests. (With $n\infty$ more guests).

Example 1: If we have a number of 3∞ new arrivals, every number of the rooms multiplied by four times, leaving space 3∞ new arrivals, and our final number of guests will be 4∞ :

(c.1) $f(3\infty) \rightarrow 3\infty + \infty = 4\infty$

Example 2: If we have a number of 7∞ new arrivals, every number of the rooms multiplied by eight times, leaving space 7∞ new arrivals, and our final number of guests will be 8∞ :

(c.2) $f(7\infty) \rightarrow 7\infty + \infty = 8\infty$

4- References:

[(1)] The Concise Oxford Dictionary of Mathematics. (Oxford University Press. ed. 6)