On the notion of quantum ‘indistinguishability’

Alireza Jamali

Independent Researcher

alireza.jamali.mp@gmail.com

July 21, 2022

Abstract

Based on the possibility of ‘indistinguishability’ not being a binary property of quantum particles it is argued that allowing for fractional quanta to occur can provide a means to ‘distinguish’ so far-indistinguishable quantum particles.

The most fundamental assumption of modern statistical mechanics is the notion of *indistinguishability* of particles. I quote Wikipedia:
“There are two methods for distinguishing between particles. The first method relies on differences in the intrinsic physical properties of the particles, such as mass, electric charge, and spin. If differences exist, it is possible to distinguish between the particles by measuring the relevant properties. However, it is an empirical fact that microscopic particles of the same species have completely equivalent physical properties. For instance, every electron in the universe has exactly the same electric charge; this is why it is possible to speak of such a thing as ‘the charge of the electron’.
Even if the particles have equivalent physical properties, there remains a second method for distinguishing between particles, which is to track the trajectory of each particle. As long as the position of each particle can be measured with infinite precision (even when the particles collide), then there would be no ambiguity about which particle is which.
The problem with the second approach is that it contradicts the principles of quantum mechanics. According to quantum theory, the particles do not possess definite positions during the periods between measurements. Instead, they are governed by wavefunctions that give the probability of finding a particle at each position. As time passes, the wavefunctions tend to spread out and overlap. Once this happens, it becomes impossible to determine, in a subsequent measurement, which of the particle positions correspond to those measured earlier. The particles are then said to be indistinguishable.”.

In [1] I showed that the ‘second approach’ mentioned above is no longer tenable: *linearity of quantum mechanics is not a logical necessity.* The current linear quantum mechanics is a special case of a more general nonlinear theory. Once we dispense with linearity, the so-called Uncertainty Principle (non-commutativity) loses ontological importance.

In this light one it is conceivable to look for deviations from quantum indistinguishability, that is to say it is conceivable to expect the possibility of an objective realization of *quantum* ***distinguishability*** of particles. Indeed we have long learned that a simple-minded notion of 'distinguishability' leads to Maxwell-Boltzmann distribution, which is only an approximation. Clearly we cannot retain that old notion of 'distinguishability' anymore. Yet it would not be a contradiction if we had a 'correct' notion of *distinguishability* in statistical physics, to which quantum indistinguishability was an approximation.
I try to clarify this issue in this preliminary note and for simplicity I only consider Fermions in the microcanonical ensemble. I will not approach finding the statistics and postpone that to a full treatment in future.

Following the conventions and notation of [2], the number $w(i)$ of distributing n_i identical indistinguishable particles among g_i energy levels is

$$w_{\text{Fermi-Dirac}}(i) = \binom{g_i}{n_i}, \tag{1}$$

which is to say, $w(i)$ is the number of ways in which the g_i levels can be divided into **two** subgroups— one group consisting of n_i levels (each with one Fermion) and the other group consisting of $g_i - n_i$ levels, which are unoccupied. This is because you only have one kind of Fermion. For a certain level, you cannot allot a 'half-electron' to it; you either allot or not. In other words all electrons are the same to us in that their energy is a complete unit, i.e.

$$n \in \mathbb{N} \text{ in } E = nh\nu. \tag{2}$$

In fact if we use the *Multinomial coefficient*

$$\binom{n}{k_1, k_2, \cdots, k_m} := \frac{n!}{k_1! k_2! k_3! \cdots k_m!}, \tag{3}$$

to re-write (1) as

$$w_{\text{Fermi-Dirac}}(j) = \binom{g_i}{n_i} = \binom{g_i}{n_{i_1}, n_{i_2}}, \tag{4}$$

$$\text{subject to } \; n_{i_1} + n_{i_2} = g_i, \tag{5}$$

we realize that we are *already* 'distinguishing' between Fermions in a very special sense: by (5) we have in fact sub-divided our n_i Fermions into $m = 2$ species: n_{i_1}, which are those *with integer multiples of* $h\nu$, and n_{i_2}, those without.

The binary nature ($m = 2$) of our counting procedure for Fermions, i.e. the fact that we either *do* give a Fermion one *complete integer unit* or we *do not* at all, is due to the fact that we can only have $E = nh\nu$ and not $E = \frac{n}{3}h\nu$ for example. This suggests that for Fermions, **the quantum hypothesis**(2), i.e. the fact that energy comes in complete *integer* units of $E = h\nu$ **is probably directly tied to indistinguishability** of Fermions. There is another simpler way to see this: among all Bosons which all possess the same amount of unit energy, say $2h\nu$, you cannot distinguish any two.

Suppose now that **fractional quanta are allowed**, and we are *still obeying the Pauli exclusion principle*, i.e.

$$g_i \geq \sum_{j=1}^{m-1} n_{i_j}. \tag{6}$$

except that we now have $m \geq 2$ *species of* n_i *Fermions*, such that

$$\sum_{j=1}^{m} n_{i_j} = g_i. \tag{7}$$

Our counting must now be done by dividing energy levels into m (not necessarily two) groups: those with 0 integer units, those with 1/2, those with 2/3 etc. in other words

$$\{\frac{n}{k}\epsilon\}_{k=1}^{m}. \tag{8}$$

where ϵ can be the unit of energy $h\nu$. The number of those n_i particles which are given one integer unit is n_{i_1}, the number of those which are given 1/3 unit is n_{i_2}, and so on, and the number of those which are given zero unit (no energy) is

$$g_i - \sum_{j=1}^{m-1} n_{i_j}.$$

Therefore the number of arrangements is given by

$$w(i,m) = \frac{g_i!}{n_{i_1}!n_{i_2}!\cdots n_{i_m}!},$$

subject to (7).

Now that fractional quanta are allowed we can 'distinguish' something we could not, when particles had only integer complete units of energy. Quantum mechanics cannot distinguish between all particles with, say, one unit of energy. But if we generalize the distribution from Bionomial to Multinomial we can distinguish among those particles themselves which have one unit of energy. This was not possible in Fermi-Dirac statistics.

Previously we had two groups of particles: those with zero energy, and those with integer units of energy. We can now have other groups as well. For example those with zero energy, and those with integer units of energy *and* those with half-integer units of energy, and so on. It is true that the indistinguishability is now shifted to the new divisions but this shifting of indistinguishability makes the previous division distinguishable: let us say we have three groups of indistinguishable particles, those with zero energy, those with integer units of energy (integer multiples of ϵ) and those with half-integer units of energy. To the eyes of current quantum mechanics, two particles with energies 0 and $\epsilon/2$ are indistinguishable hence they are both counted in the $g_i - n_i$ group: quantum mechanics just sees the integer part.

It is in this sense that quantum particles can be distinguishable. The particles are still indistinguishable nevertheless in the divisions themselves. In this manner the more number of divisions we have the easier it would be to 'distinguish' particles. If we let the 'number' of divisions to become uncountable we would have the notion of *perfect quantum distinguishability.*

If not anything else, this shows that we have not been rigorous enough about the word 'indistinguishability'. *Indistinguishability* seems not to be a binary property, that particles either have or not. There seems to exist **hierarchies of (in)distinguishability** which we have ignored so far. It is highly plausible to devise of a measure that shows 'how much (in)distinguishable a collection of particles are'. This measure should be related to m. For $m = 2$ we retain the current established notion of quantum 'indistinguishability'.
It should be mentioned that even Anyons and Generalized Pauli Principle (Haldane) share this $m = 2$, for[3]

$$w(i) = \frac{[g_i + (n_i - 1)(1 - \alpha)]!}{n_i!\,[g_i - \alpha n_i - (1 - \alpha)]!},$$

is still *bio*nomial; still *one index*.

References

[1] Alireza Jamali. Nonlinear generalisation of quantum mechanics. *10.20944/preprints202108.0525.v3*, 2021.

[2] R. K. Pathria and Paul D. Beale. *Statistical Mechanics*. Academic Press, 2011.

[3] Yong-Shi Wu. Statistical Distribution for Generalized Ideal Gas of Fractional-Statistics Particles. *Physical Review Letters*, 73, 1994.