

# The Differential Geometry of the Consciousness

By

*Dr. Moninder Singh Modgil*

Email: [mmodgil@gmail.com](mailto:mmodgil@gmail.com)

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## Abstract

Our visual perception places us at the origin of a 3-dimensional cartesian coordinate system. On a cosmological scale, the consciousness (as manifested in the brain) can be approximated as point like. It is proposed that perceptual experiences can be regarded as occurring on a tangent space at the point where the consciousness is located. A differential geometric framework is developed, for consciousness propagating on a curved space-time manifold. Interaction between consciousnesses is discussed at classical and quantum mechanical level. Quantum mechanical experiments such as those of Schrodinger's cat in Many World Interpretation, are examined in this framework.

## 1. Introduction

Differential Geometry [1] has been used successfully in modeling objects such as black holes [2], and universes with various metrics and matter distributions [3]. Here in, we develop a differential geometric frame work for modelling the consciousness. Our visual perception places us at the origin of a 3-dimensional cartesian coordinate system. On a cosmological scale, we can indeed be considered as points. The consciousness, denoted as  $C$  is considered to be localized on to a point, which we will denote as  $p$ . In a certain sense, the consciousness can be regarded as a Dirac  $\delta$  function – with certain additional properties, which we will discuss later. Due to our psychological experience of time asymmetry [4], we have the ansatz that,  $C$  is traversing irreversibly along the future time-like direction – on the (3+1) dimensional space-time manifold  $M_{3,1}$  with the metric  $g_{\mu\nu}$ .

For irreversible future evolution of  $C$  we can specify its “time velocity (so to say)” as  $N$  Planck Time units per second.

In general, the space-time manifold could have a non-trivial topology. So, the space of perception can be regarded as existing on a tangent space located at the point  $p$ . We will denote

this Tangent space as  $T_p^C$ , i.e., - the ‘Perceptual Tangent Space of the Consciousness  $C$ , located at  $p$ . In general, multiple consciousness’ –

$$C_1, C_2, \dots, C_k$$

can be regarded as located on a series of points –

$$p_1, p_2, \dots, p_k$$

on  $M_{3,1}$ .

Our approach of flat perceptual tangent space consciousness, has some similarity with dualistic view of Sir Eccles [5] – the dual entities being –

- (1) matter existing on  $M_{3,1}$ , and
- (2) The  $T_{p_i}^{C_i}$ , the perceptual tangent spaces of consciousnesses.

We can define a Collective Consciousness -

$$\mathfrak{R}^{\text{Collective Consciousness}} = \bigcup_{k=1}^{N_C} C_k$$

where,  $N_C$  is number of consciousnesses propagating in  $M_{3,1}$ .

## 2. The Perceptual Tangent Space.

We define a map  $f_i$ , which takes points on  $M_{3,1}$  to points on  $T_{p_i}^{C_i}$ . Under this mapping, a neighborhood  $N_{p_i}$  of  $p_i$ , on  $M_{3,1}$  can be mapped on to a neighborhood of  $p_i$  on  $T_{p_i}^{C_i}$  –

$$f_i: N_{p_i} \rightarrow N_{p_i}^{T_{p_i}^{C_i}}$$

The physical body associated with  $C_i$  on  $M_{3,1}$  will be referred to as  $\mathfrak{S}_3^{C_i}$  – where the subscript 3 indicates the dimensions. Its geometrical (spatial) extent of  $\mathfrak{S}_3^{C_i}$  would be referred to as  $G_3^{C_i}$ , whose 2 dimensional surface is  $\partial G_3^{C_i}$ . Their mapping onto the Tangent space is  $f(G_3^{C_i})$  and  $f(\partial G_3^{C_i})$ . Due to - (1) Our motor abilities, and (2) Irreversible passage of time; these objects are time dependent and may be written as  $\mathfrak{S}_3^{C_i}(t)$ ,  $G_3^{C_i}(t)$ ,  $B_3^{C_i}(t)$ ,  $f_i(G_3^{C_i}(t))$ ,  $\partial G_3^{C_i}(t)$ ; where  $t$  is the time. As consciousness  $C_i$  moves around in the space-time continuum  $M_{3,1}$ ,  $G_3^{C_i}$ , the tangent space of consciousness  $T_{p_i}^{C_i}(t)$  also moves accordingly. We can associate a world line  $l^{C_i}$ , parameterized by  $t$ , for temporal evolution of consciousness  $C_i$ . Thus, we have the fiber bundle [6] –

$$\mathcal{F}_i = T_{p_i}^{C_i} \times l^{C_i}$$

On  $T_p^C$  we define a special class of 3 dimensional cartesian coordinate systems, whose origin is the point of tangency, i.e. – the origin is touching  $p$  . In general, we have five senses, namely – sight, sound, taste, smell and touch. Each of them differs in –

- (1) The physics of the interface between external world and the body, and
- (2) Processing of the input by the brain, and its extension – the nervous system.

Then, there exists the reverse traffic – actions by a person through the motor system – modifying the external world. For the sensory aspects of consciousness, we can define a sensory tangent space for each of the sense, namely –

$$T_{p_i}^{Vision}, T_{p_i}^{Audio}, T_{p_i}^{Touch}, T_{p_i}^{Taste}, T_{p_i}^{Smell}$$

In addition we define a tangent space for motor actions and propioception –

$$T_{p_i}^{Motor}$$

For completeness we can also add -

$$T_{p_i}^{Dream} \text{ and } T_{p_i}^{ESP}(t)$$

where, superscript *ESP* stands for Extra-Sensory Perception.

Various structures are needed to be defined on these spaces – according to the corresponding functions of brain, nerves and sensors. The perceptual tangent space, can be written as their tensor product, i.e., -

$$T_{p_i}^{C_i}(t) = T_{p_i}^{Vision}(t) \otimes T_{p_i}^{Audio}(t) \otimes T_{p_i}^{Touch}(t) \otimes T_{p_i}^{Taste}(t) \otimes T_{p_i}^{Smell}(t) \\ \otimes T_{p_i}^{Dream}(t) \otimes T_{p_i}^{ESP}(t) \otimes T_{p_i}^{Motor}(t)$$

In the rest of this paper, we will use the following abbreviations -

*Vision* → *V*

*Audio* → *A*

*Touch* → *To*

*Taste* → *Ta*

*Smell* → *S*

*Dream* → *D*

*Motor and Propioception* → *M*

Accordingly, the perceptual tangent space, can be re-written as -

$$T_{p_i}^{C_i}(t) = T_{p_i}^V(t) \otimes T_{p_i}^A(t) \otimes T_{p_i}^{To}(t) \otimes T_{p_i}^{Ta}(t) \otimes T_{p_i}^S(t) \otimes T_{p_i}^D(t) \otimes T_{p_i}^{ESP}(t) \otimes T_{p_i}^M(t)$$

### 3. Properties of the Perceptual Tangent Spaces.

Here, we discuss the visual and audio perceptual tangent spaces.

#### 3.1 The Visual Tangent Space $T_{p_i}^V$

Let  $V_{p_i}^+$  and  $V_{p_i}^-$  be the future and the past light cones at  $p_i$  respectively. Our visual, perceptual experience is limited to absorption of light emitted in the past – and subjected to the processing by the eyes and the brain. Further, it is a subset of the solid angle of  $4\pi$  Sr, an approximately  $60^\circ$  cone. What is coming into eyes is a stream of photons. Below, the superscript  $V$  stands for Vision. Thus, if  $\mathcal{A}^V$  is our visual experience, of an object on  $M_{3,1}$ , we have –

$$\mathcal{A}^V \subset T_{p_i}^V \subset f_i^V(V_{p_i}^-)$$

Let,

$$O_1^V, O_2^V, \dots, O_{N_V}^V$$

be objects lying on  $V_{p_i}^-$ . Here,  $N_V$  stands for number of visual object in field of vision of the consciousness  $C_i$ . Let,  $\mathcal{P}_l^i(O_l^V)$  be the set of photons streaming from  $O_l^V$  into the eyes of the consciousness  $C_i$ . Let  $\mathcal{B}_i^V$ , be the processing by eyes and brain – leading to a sensed object in the Visual Tangent Space. Alternatively,  $\mathcal{B}_i^V$  stands for processing which starts in retina; then information going via optic nerves to Optic Chiasm; splitting into left half and right half of the visual field; then optic radiation going into occipital cortex; visual Brodmann areas; then coming forward and finally coming into the conscious awareness. Then for each  $O_l^V$  we have –

$$\mathcal{B}_i^V(\mathcal{P}_l^i(O_l^V)) \subset T_{p_i}^V$$

And reciprocally the Perceptual Tangent Space is union of all such images of the objects, in the consciousness –

$$T_{p_i}^V = \bigcup_{l=1}^{N_V} \mathcal{B}_i^V(\mathcal{P}_l^i(O_l^V))$$

Note has to be made of the visual perception of the written words – which is processed in association areas of visual and lingual areas.

#### 3.2 The Audio Tangent Space $T_{p_i}^A$

Audio and Visual perception have a certain similarity, in the sense that both are based upon incoming physical waves – electromagnetic waves in case of vision, and air pressure waves in case of Audio. Just as vision has association areas for reading and writing (motor areas) similarly, audio has association areas for listening and speaking (motor areas again).

Consider a rock band playing. Sound is being generated by the Drums, Bass Guitar, Rhythm Guitar, Lead Guitar, Keyboards and the Vocalist. The summed air pressure waves are impinging onto the ear of the listener. Then the processing starts at ear drums, then Cochlea, Auditory Nerve, and finally audio cortex, and its association areas. This processing will be represented as  $\mathcal{B}_i^A$ . The hairs in Cochlea perform a Fourier Transform of the incoming air pressure waves. An inverse Fourier Transform is done with in the brain, and an audio-scape of the Rock band is re-created in, i.e. – Consciousness  $\mathcal{C}_i$  is able to identify, the instruments with in the Rock Band.

Analogous to light cones  $V_{p_i}^\pm$ , we define past and future “Audio Cones” –

$${}^A V_{p_i}^-, \text{ and } {}^A V_{p_i}^+$$

We have –

$$\mathcal{A}^A \subset T_{p_i}^A \subset f_i^A({}^A V_{p_i}^-)$$

Let the audio source objects be –

$$O_1^A, O_2^A, \dots, O_{N_A}^A$$

Analogous to Visual Tangent Space, we have –

$$\mathcal{B}^A(W_l^i(O_{p_l}^A)) \subset T_{p_i}^A$$

where,  $W_l^i$  stands for air pressure Waves, originating at point  $p_l$  and reaching the ears of the consciousness located at  $p_i$ . And then,

$$T_{p_i}^A = \bigcup_{l=1}^{N_A} \mathcal{B}^A(\mathcal{P}_l(O_l^A))$$

#### 4. Interaction Between Consciousnesses.

Here, we examine interaction between two consciousnesses  $\mathcal{C}_i$  and  $\mathcal{C}_j$  – both at classical level and Quantum Mechanical level.

##### 4.1. Classical Interaction:

Let  $V_{p_i}^-$  and  $V_{p_j}^-$  be the past light cones of  $C_i$  and  $C_j$  respectively. Let both consciousness lie on the same space-like surface  $\mathcal{F}$  at time instant  $t_o$ . We have –

$$\mathcal{H} = V_{p_i}^- \cap V_{p_j}^- \neq \emptyset$$

#### 4.1.1. Physical Touch.

The condition is –

$$\partial G_i \cap \partial G_j \neq \emptyset$$

#### 4.1.2. Eye Contact.

Clearly, its not possible for  $C_i$  and  $C_j$  to exchange light signals at same time instant. For that to happen, one consciousness has to lie on past light cone of the other. So what is perceived in the eye contact is an image from the past light cone – for both the consciousness.

#### 4.1.3. Speech and Hearing.

As in case of eye contact, the speech heard by  $C_i$ , and uttered by  $C_j$ , comes from the past Audio-cone of the later.

### 4.2. Quantum Mechanical Interaction.

Von Neumann [7], in his seminal work suggested that Consciousness causes the collapse of the wave function in the brain. He pointed out that, while trying to measure a superposed microscopic state, the classical measuring device would also go into a superposed state. Finally, the observing human brain would also go into a superposed state. Since the consciousness sees only a single outcome, and not the superposed state, so the collapse of wave function is caused at the level of the consciousness, with in the brain. Now what would happen, if two consciousnesses  $C_i$  and  $C_j$  are observing the same state, e.g., - the Schrodinger's cat. Clearly  $C_i$  and  $C_j$  should both see that either the cat is dead or alive. We cannot have the situation where for instance,  $C_i$  sees that the cat is alive and  $C_j$  sees that the cat is dead. But if both the consciousnesses are seeing identical state, and also causing collapse of wave function – then the corresponding projection operators with in the  $C_i$  and  $C_j$  should be same. Now, where from this correspondence coming. The possibilities for resolving this paradox are –

- (1) One could say that there is only one consciousness, and it causes universal collapse of wave function every where.
- (2) There is of course the Everett's Many World Interpretation (MWI) [8] – in which  $C_i$  sees the cat alive in Universe 1 (say), and  $C_j$  see the cat dead in Universe 2 – with the two consciousnesses inhabiting different universes.
- (3) The first consciousness which causes the collapse of the wave function, sends a signal (possibly Tachyonic), to all other consciousnesses – and this leads to a universal collapse of the wave function.

#### 4.2.2 Consciousness and the Many World Interpretation (MWI).

Let's say that at time  $t = t_0$  the Universe  $U_0$  splits into two branches  $U_1$  and  $U_2$  with the cat being alive in  $U_1$  and dead in  $U_2$ . Note that this gives a non-Hausdroff topology to the Multiverse [14]. If we assume that consciousness cannot be created or destroyed, then  $C_i$  passes into  $U_1$  and  $C_j$  into  $U_2$ , at the time of splitting. However, in  $U_1$  body  $\mathfrak{S}_3^{C_j}$  corresponding to  $C_j$  is not inhabited by consciousness. Similarly in  $U_2$ , the body  $\mathfrak{S}_3^{C_i}$  corresponding to  $C_i$  is not inhabited by consciousness. However, without the consciousness, the body dies. Thus we run into a contradiction for MWI.

### 5. Consciousness, Mind and Attention.

Attention is the primary tool of Consciousness – which operates over sensed perceptual objects and mind (thoughts and internalized speech). Sri Aurobindo [9] discusses four types of minds. Below, we summarize their definitions and relate them to the Perceptual Tangent Spaces.

#### (1) The Physical Mind –

This is related to the processing of information from the senses. Analysis of sense data, and arriving at conclusions. In a sense, it is the process of scientific thinking, and exploration of the external world. The Physical Mind corresponds to the Perceptual Tangent Spaces of the 5 senses i.e., -

$$T_{p_i}^{Physical\ Mind}(t) = T_{p_i}^V(t) \otimes T_{p_i}^A(t) \otimes T_{p_i}^{To}(t) \otimes T_{p_i}^{Ta}(t) \otimes T_{p_i}^S(t)$$

#### (2) The Surface Mind –

In our surface mind, we have no direct means of knowing even other men, who are part of our own kind and have a similar mentality and are vitally and physically built on the same model. It takes input from the 5 senses but does not ascribe consciousness or identity to the other humans. We define a new perceptual tangent space for this category –

$$T_{C_i}^{Surface\ Mind}$$

#### (3) The Sense Mind –

Mind (the sense mind), life and body are three powers. However, they cannot be taken quite separately because the life acts as a link and gives its character to body and to a great extent our mentality. We have to define a new Perceptual Tangent Space for this category of mind, namely –

$$T_i^{Intuition}$$

(4) The Vital Mind –

The function of vital mind, is not to think and reason, to perceive, consider and find out or value things, for that is the function of thinking mind proper – but to plan or dream or imagine what can be done. Vital mind corresponds to  $T_i^D$ ,  $T_i^{ESP}$ , and a new Perceptual Tangent Space for imagination, namely –

$$T_i^{Vital\ Mind} = T_i^D \otimes T_i^{ESP} \otimes T_i^{Imagination}$$

## 6. Consciousness as a Dirac $\delta$ Function.

Let  $\mathcal{R}$  be a space-like region in  $M_{3,1}$  containing  $N_C$  number of consciousnesses – which are modelled as Dirac  $\delta$  functions located at a set of points  $P = \{p_1, p_2, \dots, p_{N_C}\}$ , all lying within  $\mathcal{R}$ . We define a function  $g$  on  $\mathcal{R}$ , which is infinite (i.e., Dirac  $\delta$ ) on the elements of  $P$ . We have –

$$\oint_{\mathcal{R}} g dx = N_C$$

The infinite number of Fourier modes, allow an infinite amount of information to be stored on each  $\mathcal{C}_i$ .

## 7. Conclusions.

We have introduced the ansatz that consciousness  $\mathcal{C}$  can be considered as a point-like object propagating on a space-time manifold. Its perceptual spaces are the Tangent Spaces at the point  $p$ , of its location on the manifold. Classical and Quantum Mechanical interaction between two consciousness is considered in this Differential Geometric frame work.

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