The Twin Prime Conjecture is True

Jim Rock

Abstract: Let P_n be the ***n-th*** prime. For twin primes $P_n – P_{n-1} = 2$. Let X be the number of $(6j –1, 6j+1)$ pairs in the open interval $[P_n, P_n^2]$. The actual number of twin primes TPA_n in $[P_n, P_n^2]$ is $((P_n - a_n)/P_n)((P_{n-1} - a_{n-1})/P_{n-1})((P_{n-2} - a_{n-2})/P_{n-2})\ldots((5 - a_3)/5)(X)$. $P_3=5$, $1.7 < a_n, a_{n-1},\ldots,a_3 < 2.3$.
We exhibit a formula showing as P_n increases, the actual number of twin primes TPA_n in the interval $[P_n, P_n^2]$ also increases. Let $P_n – P_{n-1} = c$. For $n \geq 4$, $(TPA_{n-1})(1+(2c –2)/2P_{n-1}+(c^2–2c)/2P_{n-1}^2) < TPA_n$

Determining the actual number of twin primes TPA_n in the interval $[P_n, P_n^2]$.

TPA_n is all the $(6j –1, 6j+1)$ pairs with no factor less than P_{n+1} in the interval $[P_n, P_n^2]$. Let X be the number of $(6j –1, 6j+1)$ pairs in the interval $[P_n, P_n^2]$. X_m is the number of $(6j –1, 6j+1)$ pairs in the interval $[P_n, P_n^2]$ with no factor less than P_m. X_{m+1} is the number of $(6j –1, 6j+1)$ pairs in $[P_n, P_n^2]$ with no factor less than P_{m+1}.

$$X_{m+1} = (X_m)(P_m – a_m) / P_m.$$

Using this formula repeatedly starting with $m = n$ decreasing incrementally by one to $m = 3$ gives

$TPA_n = ((P_n - a_n)/P_n)((P_{n-1} - a_{n-1})/P_{n-1})((P_{n-2} - a_{n-2})/P_{n-2})\ldots((5 - a_3)/5)(X)$ for $n \geq m \geq 3$, $P_3=5$, $1.7 < a_m < 2.3$.

The graph below (P_m in descending order) plots a_m **values for** ***44*** **equally spaced** P_m in the intervals **[*5, 743*]**, **[*5, 3011*]**, **[*5, 10007*]** and **[*5, 19993*]** to illustrate the above formula for $[743, 743^2]$, $[3011, 3011^2]$, $[10007, 10007^2]$ and $[19993, 19993^2]$. For selected P_m they show the values of a_m of $(P_m – a_m)$. Since the primes less than P_{m+1} are removed in a linear fashion like the sieve of Eratosthenes, similar a_m patterns to the four plots on the graph are found in all $[P_n, P_n^2]$ intervals for $P_n > 500$.

a_m values for *44* selected P_m in descending order in the intervals [*5, 743*], [*5, 3011*], [*5, 10007*] and [*5, 19993*]

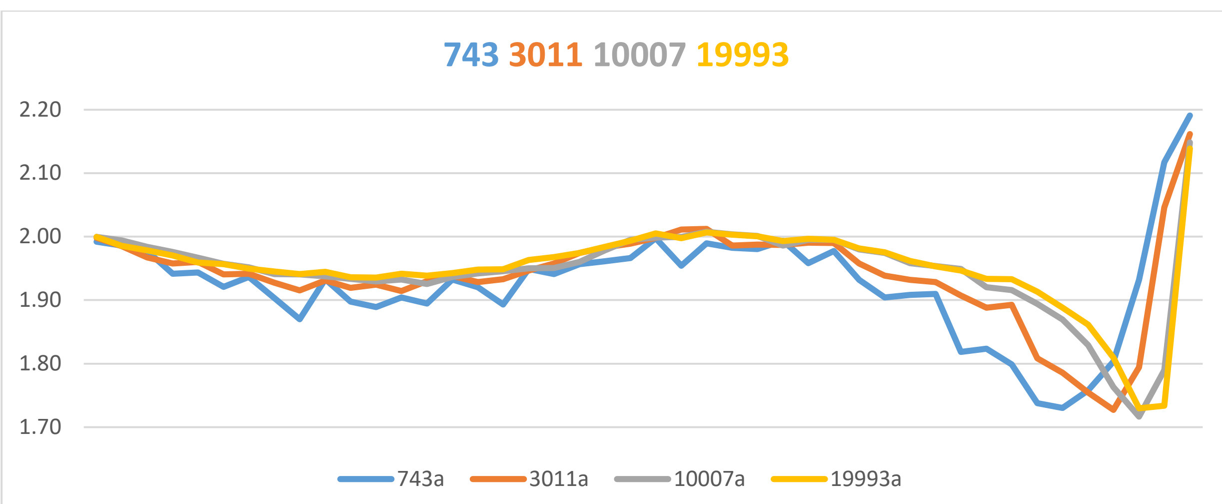


Table 1 shows the number of twin primes calculated ***(TPC_n)*** in [P_n, P_n^2] for $347 \le P_n \le 31153$, when the calculated **average a_m** equals ***2.06,*** Comparing TPC_n with TPA_n shows the average value for $P_m - a_m$

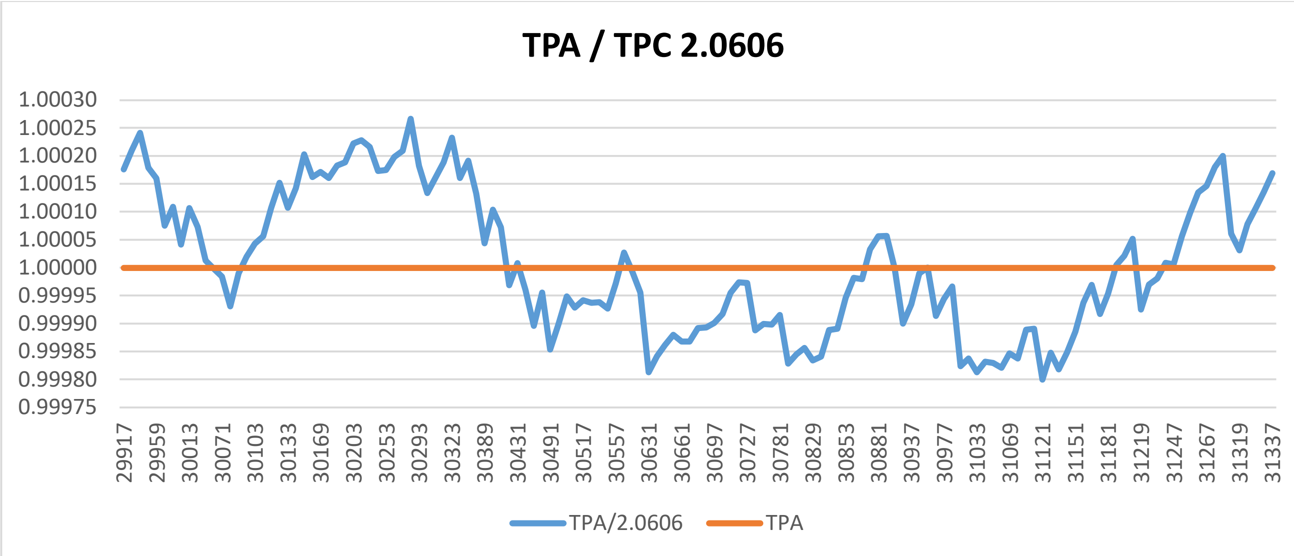
Table 1 – Twin Primes in the interval [P_n, P_n^2] for $347 \le P_n \le 31153$

$TPA_n = ((P_n - a_n)/P_n)((P_{n-1} - a_{n-1})/P_{n-1})((P_{n-2} - a_{n-2})/P_{n-2})\ldots((5 - a_3)/5)(X)$. For $3 \le m \le n$

Average a_m is set at 2.06

P_n	TPA_n	TPA_n / TPC_n 2.06	TPC_n 2.06
347	1405	1.0637	1320.8
349	1419	1.0683	1328.2
1151	10387	1.0430	9958.4
1153	10408	1.0434	9975.2
1997	26735	1.0369	25783.3
1999	26777	1.0375	25808.3
2969	52817	1.0302	51267.4
2971	52877	1.0307	51300.9
3851	82712	1.0224	80901.1
3853	82802	1.0230	80941.9
4649	114842	1.0196	112636
4651	114919	1.0198	112683
5849	171367	1.0156	168737
5851	171471	1.0159	168793
6947	231582	1.0123	228770
6949	231708	1.0126	228834
8387	322646	1.0100	319452
8389	322805	1.0103	319526
9677	415267	1.0091	411530
9679	415417	1.0092	411613
10937	515723	1.0078	511751
10939	515884	1.0079	511842
12251	630469	1.0059	626775
12253	630646	1.0060	626874
13997	798218	1.0048	794435
13999	798427	1.0049	794545
15731	982287	1.0039	978483
15733	982497	1.0040	978604
17291	1162662	1.0026	1159636
17293	1162911	1.0027	1159767
18251	1280482	1.0031	1276491
18253	1280728	1.0032	1276627
19991	1506151	1.0015	1503866
19993	1506427	1.0016	1504011
21191	1671686	1.0015	1669161
21193	1671950	1.0016	1669314
22541	1866304	1.0009	1864560
22543	1866615	1.0010	1864721
23831	2061886	1.0010	2059785
23833	2062203	1.0011	2059953
26111	2428375	1.0000	2428472
26113	2428739	1.0000	2428652
27689	2697588	0.9992	2699839
27691	2697935	0.9992	2700028
29207	2968309	0.9994	2970220
29209	2968674	0.9994	2970418
31151	3333028	0.9987	3337515

Twin Primes in the interval [P_n, P_n^2] for $29917 \le P_n \le 31337$ For $P_n > 29000$ the average a_m values cycle around $(2.0601, 2.0612)$. The graph shows the average a_m ÷ base line 2.0606 ratio.



Establishing a bound for the ratio TPA_n / TPA_{n-1}

X equals approximately $(P_n^2 - P_n)/6$. $m = 3$ to $n \prod P_m - 2 = T_n$ $m = 1$ to $n \prod P_m = J_n$

The number of twin prime pairs in [P_n, P_n^2]is approximately $(T_n)(P_n^2) / J_n$

TPA_n is approximately $(TPA_{n-1})((T_n)(P_n^2) /J_n) / ((T_{n-1})(P_{n-1})^2/J_{n-1})$.

TPA_n is greater than $(TPA_{n-1})((((T_n)(P_n^2) /J_n) / ((T_{n-1})(P_{n-1})^2/J_{n-1}))+1)/2$.

Calculating $((((T_n)(P_n^2) /J_n) / ((T_{n-1})(P_{n-1})^2/J_{n-1}))+1)/2$.

Let $P_n - P_{n-1} = c$.

$$((T_n)(P_n^2) /J_n) / ((T_{n-1})(P_{n-1})^2/J_{n-1}) =$$

$$((T_{n-1})(P_{n-1}+c-2)(P_{n-1}+c)^2/((J_{n-1})(P_{n-1}+c))) / ((T_{n-1})(P_{n-1})^2/J_{n-1}) =$$

$$(P_{n-1}+c-2)(P_{n-1}+c) / P_{n-1}^2 = (P_n-2)(P_n) / P_{n-1}^2$$ **for twin primes $P_n / (P_n - 2)$**

$$(P_{n-1}+c-2)(P_{n-1}+c) / P_{n-1}^2 = 1+(2c-2)/P_{n-1}+(c^2-2c)/P_{n-1}^2$$

$$(1+(1+(2c-2)/P_{n-1}+(c^2-2c)/P_{n-1}^2)) / 2 = 1+(2c-2)/2P_{n-1}+(c^2-2c)/2P_{n-1}^2$$

For $n \ge 4$ $(TPA_{n-1})(1+(2c-2)/2P_{n-1}+(c^2-2c)/2P_{n-1}^2) < TPA_n$ See table below

(column B)((column D/column C)+1)/2=(column F)

$P_n - P_{n-1} = c. \quad (TPA_{n-1})(1+(2c-2)/2P_{n-1}+(c^2-2c)/2P_{n-1}^2) < TPA_n$

A	B	C	D	E	F	G	H
prime	TPA_{n-1}	$(F_{n-1})(P_{n-1})^2/J_{n-1}$	$(F_n)(P_n)^2/J_n$	*(D/C+1)/2*	*(B)(E)*	TPA_n	*F/G*
71	120	109.0	112.1	1.01408	121.7	123	0.989483
73	123	112.1	127.9	1.07047	131.7	138	0.954117
1019	8420	8935.3	8952.8	1.00098	8428.2	8450	0.997425
1021	8450	8952.8	9111.3	1.00885	8524.8	8586	0.992872
2087	28819	30850.0	30879.6	1.00048	28832.8	28867	0.998816
2089	28867	30879.6	31146.2	1.00432	28991.6	29106	0.996070
3461	68804	74874.0	74917.3	1.00029	68823.9	68872	0.999302
3463	68872	74917.3	75047.1	1.00087	68931.7	69019	0.998735
4637	114316	125244.7	125298.7	1.00022	114340.6	114394	0.999534
4639	114394	125298.7	125460.9	1.00065	114468.0	114580	0.999023
6299	195208	215150.4	215218.7	1.00016	195239.0	195319	0.999590
6301	195319	215218.7	215833.9	1.00143	195598.2	195879	0.998566
8009	297317	329810.8	329893.1	1.00012	297354.1	297454	0.999664
8011	297454	329893.1	330305.0	1.00062	297639.7	297851	0.999291
9857	428957	476792.2	476889.0	1.00010	429000.5	429089	0.999794
9859	429089	476889.0	477953.7	1.00112	429568.0	430004	0.998986
11777	588001	656535.4	656646.9	1.00008	588050.9	588163	0.999809
11779	588163	656646.9	656981.4	1.00025	588312.8	588502	0.999679
13931	791507	885279.3	885406.4	1.00007	791563.8	791704	0.999823
13933	791704	885406.4	889096.0	1.00208	793353.6	794778	0.998208
16187	1033547	1158651.2	1158794.4	1.00006	1033610.9	1033796	0.999821
16189	1033796	1158794.4	1159223.9	1.00019	1033987.6	1034307	0.999691
18041	1254327	1408473.1	1408629.2	1.00006	1254396.5	1254586	0.999849
18043	1254586	1408629.2	1409097.7	1.00017	1254794.6	1255094	0.999761
20147	1527206	1717720.9	1717891.4	1.00005	1527281.8	1527479	0.999871
20149	1527479	1717891.4	1719767.6	1.00055	1528313.1	1529106	0.999481
21839	1763993	1985940.5	1986122.3	1.00005	1764073.7	1764289	0.999878
21841	1764289	1986122.3	1987759.5	1.00041	1765016.2	1765719	0.999602
23741	2047968	2308071.0	2308265.5	1.00004	2048054.3	2048281	0.999889
23743	2048281	2308265.5	2308848.8	1.00013	2048539.8	2048899	0.999825
26861	2555034	2883638.7	2883853.4	1.00004	2555129.1	2555371	0.999905
26863	2555371	2883853.4	2887074.9	1.00056	2556798.3	2558027	0.999520
28619	2861908	3233814.5	3234040.4	1.00003	2862008.0	2862279	0.999905
28621	2862279	3234040.4	3235170.5	1.00017	2862779.1	2863372	0.999793
31319	3365123	3806114.0	3806357.0	1.00003	3365230.4	3365489	0.999923
31321	3365489	3806357.0	3807572.4	1.00016	3366026.3	3366653	0.999814