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Concerning the Apparent magnitude

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ABSTRACT

This brief analysis presents observations intended to support the development of a non-standard cosmological model. The project, named "4-Sphere" and currently under development, operates within the framework of Special Relativity.

The Apparent magnitude m, as measure of the brightness of a star, is decisive, together with the Absolute magnitude M, for the correct calculation of the Distance Modulus μ.

The quantity $\mu = m - M$, indeed, is related to the Luminosity distance d by $\mu = 5\,log(d) - 5$ (d in Parsec) from which some verifications of a Cosmological model are then derived.

Contrary to what one might think, the determination of the apparent magnitude depends on the theoretical model adopted. In the absence of galactic recession, its value coincides with the observed one m_o. However, if the existence of a recession is hypothesized, additional conversions are necessary.

This work considers Special Relativity (SR) as the framework for such calculations. Assuming the star is not at rest relative to the observer (an essential condition for the relevance of this analysis), the apparent magnitude calculated in SR assumes a distinct logical weight compared to that obtained based on hypothetical models.

Specifically, the calculation based on SR can be used to verify the validity of a model (the intrinsic validity of SR is not in question). Conversely, to avoid a vicious circle, calculations based on hypotheses can only falsify the very model that generated them or serve to determine its parameters.

In order to make the new K correction practical, an attempt was made to develop a simple corrective factor for the transformation of the Distance Modulus μ. This would have allowed us to exploit the extensive database of existing supernova observations, converting the distance modulus used in the FLRW model into its equivalent in SR. The last paragraph, however, explains the reasons why such a conversion is deemed impractical.

THE K CORRECTION

The calculation of a correction can take place in different ways, but it is in any case necessary to deduce, starting from the observed value m_o, the quantity m [1] to be used in the subsequent procedures.

Now, we will refer to the K_{corr} described in [2], which, here, we will express in a different but equivalent form:

$$m \simeq m_o - K_{corr}$$

(m is given in reverse scale: the brighter the star the lower is m. With $K_{corr} > 0$ the receding star appears further away than it is)

The principal purpose of the K_{corr} is to apply the transformations to be performed between the observed and rest-frame measurements.

In addition to changing the single frequency, redshift can affect the functioning of the photometric equipment for the detection of frequencies within a wavelength band. The correction considers all these aspects. Given the complexity and extreme specificity of the topics involved, it is advisable to rely on articles in literature.

THE K CORRECTION IN SPECIAL RELATIVITY

We will refer here, for simplicity, to a star that behaves like a monochromatic source of light and to a photometric apparatus capable of measuring the intensity of the radiation.

Let us then view the effects of the Galactic Recession on the apparent magnitude *m,* in the Special Relativity context:

An energy δE of radiation, emitted from a source C moving away, is projected through a solid angle $\delta\Omega$ on a surface δS in the time δt towards an observer O at a distance r.

With $\beta = v/c$, for motion in the radial direction then the Lorentz factor is:

$$\gamma = (1 - \beta^2)^{-1/2} \quad \text{with} \quad \beta = ((1+z)^2 - 1)/((1+z)^2 + 1)$$

What the observer will detect will be: (symbol δ stays for infinitesimal quantity)

$\delta E_o = (1+z)^{-1} \delta E_e$ for the redshift of frequency

$\delta\Omega_o = \gamma^2 \delta\Omega_e$ for the Lorentz length contraction only in the direction of motion

$r_o = \gamma^{-1} r_e$ for the Lorentz length contraction only in the direction of motion

$\delta t_o = \gamma \delta t_e$ for the time dilation occurred

HOW THE SOLID ANGLE IS TRANSFORMED

The increase of the solid angle $\delta\Omega$ can be seen more easily starting from 2-dimension: In a circle of radius r and center C (the star) an observer O is placed at the center of an infinitesimal arc δb. An isosceles triangle has vertex in C and base δb tangent to the circle in O.

If now we translate δb moving O along the height $h = r$ of the triangle, squeezing it in the direction of C, the observer O will see the vertex angle increase and the height h shorten.

Expressing h as the Lorentz contraction of the radius toward the observer: $h = r/\gamma$ and returning in 3-dimension we can write:

The solid angle $\delta\Omega_o$ is given by $\delta\Omega_o = \delta S_o/h^2 = \gamma^2 \delta S_o/r^2 = \gamma^2 \delta\Omega_e$ because $\delta S_o = \delta S_e$.

RADIANT INTENSITY AND INTENSITY

Radiant intensity [3] is the power radiated in a given direction per unit solid angle, it is independent by distance of the source.

From this definition: $I_\Omega = \delta E\ \delta\Omega^{-1} \delta t^{-1}$we can conclude that:

$$I_{\Omega o} = (1+z)^{-1} \gamma^{-3} I_{\Omega \mathrm{e}}$$

As regards Intensity, the light of a star is not uniformly distributed in the solid angle subtended by the entire quasi-spherical surface. Being $I_o/I_e \propto \delta\Omega_e r^2/\delta\Omega_o h^2$ we can conclude that the decrease in the distance from the star is compensated by the increase in the solid angle, so for the Intensity it holds:

$$I_o = (1+z)^{-1} \gamma^{-1} I_e$$

as it had to be from its definition as the power $\delta E\ \delta t^{-1}$ transferred per unit area A, where the area is measured on the plane perpendicular to the direction of propagation of the energy. (From our 2d paradigm $\delta \mathrm{S}, A \propto \delta b^2\ and\ \perp h$)

Note, at last, that term $\delta E/\ \delta t$ changes due to both the redshift of the single photon and the number of photons emitted in the time unit.

Then, for the apparent magnitude relation:

$$I_e/I_o = 2.512^{\Delta m} \quad where\ \Delta m = m_o - m$$

we have:

$$K_{SR\ corr} = 2.5\ log(1+z) + 2.5 \log(\gamma) \quad \text{and} \quad m < m_o.$$

<u>The receding star appears further away than it is</u>.

THE LORENTZ TRANSFORMATION OF THE SOLID ANGLE IN ANALYTIC FORM

About the choice of the coordinate system, in case of contraction of an axis, we notice that, even if we express angles as arctangents of catheti of a right triangle, trigonometry would be of no help. Therefore, the trigonometric functions encountered will be left as they are, even if it is implied that the contraction of an axis can affect the angle.

Hence for the solid angle, the analytical treatment of the Lorenz transformation is important as a verification of previous reasoning:

A star lies at the origin of the $Oxyz$ coordinates in the center of a sphere of radius r. In any point x_o of the x axis an observer moves away from O, with a relative speed $dx/dt = v$ and in a solid reference system $O'x'y'z'$, the axes of which are parallel to those of O. The observer measures, under the Lorentz length contraction, the same radius on the x'axis, obtaining $r' = r/\gamma$. Being this measure independent of the position, x_o can also lie on the surface of the sphere in $x = r$ thus coinciding with the distance from the star.

With θ as the Meridian, φ as the Parallel and a point $P(\theta, \varphi)$ on the surface, we express the infinitesimal surface $\delta S = \delta S\,(r, \theta, \varphi)$ as a square of sides δb and δh centered in *P:* $\delta S \simeq \delta b\ \delta h$. Note that considering two orthogonal great circle ξ, ζ passing for *P:* $\delta b \simeq \delta\xi$ and $\delta h \simeq \delta\zeta$.

A straightforward way to proceed is now to define:

$x_r = r\ \cos\varphi\cos\theta \qquad \delta y_b = \delta b\cos\varphi\cos\theta \qquad \delta z_h = \delta h\cos\varphi\cos\theta$

giving for the Solid Angle:

$\delta S\ = \delta b\ \delta h = \delta y_b \delta z_h (\cos\varphi\cos\theta)^{-2} \qquad\qquad \delta\Omega = \delta S/r^2\ = x_r^{-2}\delta y_b \delta z_h$

Then, from the Length contraction: $\delta\Omega'^2 = \gamma^2\delta\Omega$

because $x_r' = \gamma^{-1}x_r$ while $\delta y_b, \delta z_h$ are orthogonal to the direction of motion: The observed Radiant Intensity I_Ω' is not uniformly distributed.

The Light Intensity is the power transferred per unit area, where the area is measured on the plane perpendicular to the direction of propagation of the energy. The way it is distributed is also straightforward:

$$\delta I' = \frac{\delta E\ \delta t^{-1}}{\delta y_b \delta z_h}(\cos\varphi\cos\theta)^2 = \frac{\delta E\ \delta t^{-1}}{\delta b\ \delta h} = \frac{\delta E\ \delta t^{-1}}{\delta\xi\ \delta\zeta}$$

and the observed Light Intensity I' of the star is uniformly distributed independently of the Lorentz Length contraction.

Thus, precedent results for $I_{\Omega o}$ and I_o are confirmed.

ON THE K CORRECTION IN FRIEDMANN-LEMAITRE-ROBERTSON-WALKER METRIC (FLRW)

Having defined this alternative framework, it becomes crucial to utilize it for validating the cosmological model that incorporates it as a corrective measure. Given the extensive volume of existing observational data, the most pragmatic approach would be to derive the new correction $K_{SR\ corr}$ from the established one.

The great difficulty encountered when trying to compare a model based on *SR* with the standard *FLRW* one is the concept of that correction itself: namely, what to be made to transform the apparent magnitude of a star, with redshift *z,* into the corresponding magnitude it would have if it were at rest.

In the *FLRW* model, redshift is attributed not to motion but to the expansion of space, affecting stars regardless of movement; they are at rest while their distances increase. In contrast, Galactic Recession within Special Relativity allows for a conceptually distinct perspective. In *SR*, one can posit that a star is at rest at a distance *r*. However, this concept is not directly translatable to *FLRW*. Consequently, the challenge lies in converting the *K* correction.

The standard FLRW model employs a workaround: *K* correction does not directly connect the Apparent magnitude with the observed one $m \simeq m_o - K_{corr}$, but appears in the relation of the Distance Modulus between m_o , μ and *M:* the magnitude that the star would have if it were, stationary, at the predetermined distance of *10 Parsec (Pc)*

$$m_o = M + \mu + K_{corr}$$

(in literature [*] the expression is complicated by a further transformation between the observed frequency band *R* and the initial emitting band *Q,* in which we want *M* to be expressed)

But, given *M:*

- in SR, once $K_{SR\ corr}$ is applied to m_o, the star is at rest, and we can deduce the Luminosity distance from μ.
- In FLRW, can we apply the same formula if we cannot separate the movement of the star, still stationary, but at a great distance from us?

More specifically, the goal is to study Supernovae *(SN)* as Standard Candles: Here the procedure in [**] uses a sample of Supernovae near us, whose magnitude *M* is given. From the redshift *z* and the Supernova variations of m_o in time, it selects a value of *M* from the sample and associates it with the *SN* to be studied, getting at the same time the Luminosity distance and the cosmological parameters of *FLRW*. (All that is necessary in the analysis of the Hubble Tension).

These sophisticated methods (and their ancestors) compare the observed variations in the light curve shape with the sample, using a regression analysis as a function of various variables including μ. The sample *SN,* the Distance modulus μ and others chosen are the ones, that as a group, minimizes the χ^2statistics in mean-square estimation.

For us, the direct transformation of K_{corr} in $K_{SR\ corr}$ is not clear, and in any case too complex: The difficulty we are referring to can be understood by reading [***]. The analysis of *FLRW* cosmological parameters has been structured as a regression problem, which, I argue, has diminished the physical interpretation of individual variables, thereby restricting their utility in alternative cosmological models."

Furthermore, many variables, such as extinction (the dimming of the SN) by dust encountered by the light during its travel, are evaluated in the *FLRW* context [****].

To not only propose but also validate an alternative cosmological model, a substantial and challenging undertaking lies ahead, requiring a return to fundamental photometric data, the acquisition of diverse skills, and extensive program code revision.

[*] - [arXiv:astro-ph/0210394] -The K correction

[**] - [arXiv:astro-ph/9904347] -Determination of the Hubble Constant Using a Two-Parameter Luminosity Correction for Type Ia Supernovae

[***] - [arXiv:astro-ph/9608192] -Measurements of the Cosmological Parameters Omega and Lambda from the First 7 Supernovae at z >= 0.35

[****] - [The Astrophysical Journal: Saurabh Jha et al 2007 ApJ 659 122] - Improved Distances to Type Ia Supernovae with Multicolor Light-Curve Shapes: MLCS2k2

References from Wikipedia:

[1] - Apparent magnitude

[2] - K correction

[3] - Radiant intensity

[4] - Intensity