Mass Spectrum of the Charmoniums with Insights into the Nature of Exotic Hadrons

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Abstract

Which charmoniums are diquarks, and which are exotics (tetraquarks, pentaquarks, hexaquarks, heptaquarks, etc.) is revealed in this paper. A possible addition to our understanding of the nature of the X(3872) is also elucidated. It is very likely a 'd' tetraquark, but with a factoring quirk. A prime number figures prominantly in its factoring.

Contents

1. Introduction

Hypersphere surface volume factoring is based on the theory that quarks are not point particles, but volumes of mass/energy that take up simply defined multiples of hypersphere surface volumes. Two of these volumes combine to form mesons, three combine to form baryons, etc. The mass of the combination is a constant times the two (or more) volumes multiplied together along with Planck's constant's coefficient (h=6.62607015 Mev/c^2). The fact that hypersphere surface volume factoring works is strong evidence that subatomic particles are made of *higher dimensional matter*. Factoring results can add precision to experimental mass determinations - up to eight digits of accuracy. Hypersphere surface volume factoring relies heavily on experimental results for guidance, but conversely, it can make predictions, thus can give guidance to the experimenters.

2. How to Read the Mass Spectrums

Col 1: **n** The first column of the mass spectrum holds '**n**', the *multiplier* of the *unit-of-factorization*.

Col 2: **nS8h** The second column shows the *theoretical mass*, which resuls from multiplying '**n**' with the *unit-of-factorization (S8h in this case)*.

Col 3: **ExpMass** The third column holds 'ExpMass', the *experimental mass*. It is placed next to the theoretical mass in the mass spectrum that most closely matches it.

Col 4: **Error** The fourth column holds the *error of the experimental mass*. It is plus or minus.

Col 5: **dm** This is the difference between the experimental and theoretical masses.

Col 6: **dm /Error** This is **dm** divided by the **Error,** and the result is shown as a percentage. It shows the difference between theoretical and experimental mass as a percentage of error size.

3. How Factoring Is Done

First, take a cue from the experimentalists about the quark content of the particle you want to factor. About 25% of the time (roughly) they will be correct, so start there. If the experimentalists quark content doesn't factor to anything significant, you just have to try all the possibilities. First try dividing the experimental mass of interest by hypersphere surface volume units of factorization from S4h to S17h.

If the particle factors significantly with **S4h** or **S5h** it could be a 'du', or 'dd' meson.

If the particle factors significantly with **S6h** or **S7h** it could be a 'cd', 'sd', or 'cu' meson.

If the particle factors significantly with **S8h** or **S9h** it could be a 'cs' or 'cc' meson, or it could be a 'dddu' or 'dddd' tetraquark.

If the particle factors significantly with **S10h** or **S11h** it could be a pentaquark (or high dimension meson). Test for a pentaquark by dividing into the native pentaquark unit of factorization, d^5h.

If the particle factors significantly with **S12h** or **S13h** it could be a hexaquark (or high dimension meson). Test for a hexaquark by dividing into d^6h.

If the particle factors significantly with **S14h** or **S15h** it could be a heptaquark (or high dimension meson). Test for a heptaquark by dividing into d^7h.

If the particle factors significantly with **S16h** or **S17h** it could be an octaquark (or high dimension meson). Test for an octaquark by dividing into d^8h.

If none of those work, the particle may be a 'c' tetraqaurk, so try dividing its mass by **TQ1** to **TQ7**.

The only particles that factor with **S18h** are a few of the heaviest 'bb' mesons. S18h factoring means the particle could be a 'd' nonaquark (or a high dimension meson).

4. Quark Content and Factoring of Each Charmonium

Charmonium	Approx. Key Mass	Quark Content	Factoring Unit	Factoring Details
1. nc(1S)	2982.212	6d	S12h	1580(16)/900 S12h
2. J/Y(1S)	3096.914	2c	S8h	2591/180 S8h
3. Xco(1P)	3415.496	7d	S14h	61.4400 S14h
4. Xc1(1P)	3510.705	6d	S12h	33.06666 S12h
5. hc(1P)	3525.484	8d	S17h	222.0000 S17h
6. Xc2(1P)	3557.808	7d	S14h	64.00000 S14h
7. nc(2S)	3633.472	4c	**TQ1**	**8.000000 TQ1**
8. Y(2S)	3686.097	8d	S17h	51(8102)/1800 S17h
9. Y(3770)	3774.952	6d	S12h	35.55555 S12h
10. Y2(3823)	3822.139	6d	S12h	36.00000 S12h
11. Y3(3842)	3842.71	6d	S13h	48.98888 S13h
12. Xco(3860)	3862	--	----	-------------
13. Xc1(3872)	3871.69	4d	S8h	4049/225 S8h
14. Zc(3900)	3887.2	4c	**TQ2**	**60.00000 TQ2**
15. X(3915)	3918.4	6d	S13h	50.00000 S13h
16. Xc2(3930)	3922.028	6d	S13h	50.00000 S13h
17. X(3940)	3942	6d	S13h	50.00000 S13h
18. X(4020)	4024.1	4c	**TQ2**	**62.00000 TQ2**
19. Y(4040)	4039	4c	**TQ2**	**62.25000 TQ2**
20. X(4050)	4051	5d	S11h	29.50000 S11h
21. X(4055)	4054	5d	S11h	29 50/90 S11h
22. X(4100)	4096	4c	**TQ2**	**63.12500 TQ2**
23. Xc1(4140)	4146.8	4c	**TQ2**	**64.00000 TQ2**
24. Y(4160)	4191	4d	d4h	261 d4h/10290
25. X(4160)	4156	4d	d4h	33 d4h/1312
26. Zc(4200)	4196	5d	S11h	30.55555 S11h
27. Y(4230)	4218	4c	**TQ2**	**65.00000 TQ2**
28. Rco(4240)	4239	4c	**TQ1**	**9.333333 TQ1**
29. X(4250)	4248	6d	S12h	40.01111 S12h
30. Y(4260)	4230	--	----	-------------
31. Xc1(4274)	4274.4	5d	S11h	31.1250 S11h
32. X(4350)	4350.6	5d	S11h	31.6800 S11h
33. Y(4360)	4368	4c	**TQ2**	**67.00000 TQ2**
34. Y(4390)	4391.5	6d	S13h	56.00000 S13h
35. Y(4415)	4412	4c	**TQ2**	**68.00000 TQ2**
36. Zc(4430)	4478	4c	**TQ2**	**69.00000 TQ2**
37. Xco(4500)	4506	5d	S10h	26.66666 S10h
38. Y(4660)	4643	5d	S11h	34.00000 S11h
39. Xco(4700)	4704	4c	**TQ2**	**72.50000 TQ2**

Note: These two charmoniums were not factored

Charmonium		Reason
12. Xco(3860)	3862	No satisfactory factoring was found
30. Y(4260)	4230	Omitted from PDG's 2021 listings

1. ηc(1S)

S12h Factoring / 'd' Hexaquark

S12h = 106.1705373 MeV/c^2

Res = (1/900) **S12h**

	n	(28+n/900) S12h	ExpMass	Error	dm	dm/Error
	10	2973.9547				
	12	2974.1907				
	14	2974.4266	2974.4	1.9	0.0266	0.1%
1576(16)/900 S12h	**16**	**2974.6625**				
	18	2974.8985				
	20	2975.1344				
	22	2975.3703				
	24	2975.6063				
	26	2975.8422	2975.8	3.9/1.2	0.0422	1.1%
	28	2976.0781	2976	8	0.0781	1.0%
	30	2976.3141	2976.3	2.3/1.3	0.0141	0.6%
1577(16)/900 S12h	**32**	**2976.5500**	**2976.6**	2.9/1.3	0.0500	1.7%
	34	2976.7859				
	36	2977.0219				
	38	2977.2578				
	40	2977.4937	2977.5	1.0/1.2	0.0063	0.6%
	42	2977.7297				
	44	2977.9656				
	46	2978.2015				
1578(16)/900 S12h	**48**	**2978.4375**				
	50	2978.6734				
	52	2978.9093				
	54	2979.1453				
	56	2979.3812				
	58	2979.6171	2979.6	2.9/1.3	0.0171	0.6%
	60	2979.8531	2979.8	0.8/3.5	0.0531	6.6%
	62	2980.0890				
	63	2980.2069	2980.2	1.6	0.0069	0.4%
1579(16)/900 S12h	**64**	**2980.3249**	**2980.4**	2.3/0.6	0.0751	3.3%
	66	2980.5609				
	68	2980.7968				
	70	2981.0328				
	72	2981.2687				
	74	2981.5046				
	76	2981.7406	2981.8	1.3/1.5	0.0594	4.6%
	78	2981.9765	2982	5	0.0235	0.5%
1580(16)/900 S12h	**80**	**2982.2124**	**2982.2**	1.5/0.1	0.0124	0.8%
	82.5	2982.5073	2982.5	0.4/1.4	0.0073	1.8%
	83	2982.5663	2982.6	2.7/2.3	0.0337	1.2%
	84	2982.6843	2982.7	1.8/2.2	0.0157	0.9%
	85	2982.8022	2982.8	1.0/0.5	0.0022	0.2%
	86	2982.9202				
	88	2983.1562				
	90	2983.3921				
	91	2983.5100	2983.5	1.4/1.6	0.0100	0.7%
	92	2983.6280				
	94	2983.8640				
	95	2983.9819	2984	2.3/4.0	0.0181	0.8%
1581(16)/900 S12h	**96**	**2984.0999**	**2984.1**	1.1/2.1	0.0001	0.0%
	98	2984.3358	2984.3	0.6/0.6	0.0358	6.0%
	99	2984.4538	2984.49	1.16/.52	0.0362	3.1%
	100	2984.5718	2984.6	0.7/2.2	0.0282	4.0%
	102	2984.8077				
	104	2985.0436				
	106	2985.2796				
	107	2985.3975	2985.4	1.5	0.0025	0.2%
	108	2985.5155				
	110	2985.7514	2985.8	1.5/3.1	0.0486	3.2%
	111	2985.8694	2985.9	0.7/2.1	0.0306	4.4%
1582(16)/900 S12h	**112**	**2985.9874**				
	113	2986.1053	2986.1	1.0/2.5	0.0053	0.5%
	114	2986.2233				
	116	2986.4592				
	118	2986.6952	2986.7	0.5/0.9	0.0048	1.0%

Source of ExpMass and Error data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

2. J/Ψ(1S)

S8h Factoring / cc Diquark

S8h = 215.1464901 MeV/c^2

Res = 1/(256*180)**S8h**

	a	b	(a + b/256) S8h / 180	ExpMass	Error	dm	dm/Error
	2590	101	3096.6621	**3096.66**	0.19/0.02	0.0021	1.1%
	2590	236	3096.8208				
	2590	237	3096.8255				
	2590	238	3096.8302				
	2590	239	3096.8348				
	2590	240	3096.8395				
	2590	241	3096.8442				
	2590	242	3096.8488				
	2590	243	3096.8535				
	2590	244	3096.8582				
	2590	245	3096.8628				
	2590	246	3096.8675				
	2590	247	3096.8722				
	2590	248	3096.8768				
	2590	249	3096.8815				
	2590	250	3096.8862				
	2590	251	3096.8909	**3096.89**	0.09	0.0009	1.0%
	2590	252	3096.8955				
	2590	253	3096.9002	**3096.900**	0.002/0.006	0.0002	9.6%
	2590	254	3096.9049				
	2590	255	3096.9095	**3096.91**	0.03/0.01	0.0005	1.7%
(2591/180) S8h	**2591**	**0**	**3096.9142**	**Key Mass**			
	2591	0.6	3096.9170	**3096.917**	0.010/0.007	0.0000	0.0%
	2591	1	3096.9189				
	2591	2	3096.9235				
	2591	3	3096.9282				
	2591	4	3096.9329	**3096.93**	0.09	0.0029	3.2%
	2591	5	3096.9375				
	2591	6	3096.9422				
	2591	7	3096.9469				
	2591	8	3096.9516	**3096.95**	0.1/0.3	0.0016	1.6%
	2591	9	3096.9562				
	2591	10	3096.9609				
	2591	11	3096.9656				
	2591	12	3096.9702				
	2591	13	3096.9749				
	2591	14	3096.9796				
	2591	15	3096.9842				
	2591	16	3096.9889				
	2591	17	3096.9936				
	2591	18	3096.9982				
	2591	19	3097.0029				
	2591	20	3097.0076				
	2591	128	3097.5118	**3097.5**	0.3	0.0118	3.9%
	2592	64	3098.4083	**3098.4**	2.0	0.0083	0.4%

Source of ExpMass and Error data except [n]: P.A. Zylaet al.(Particle Data Group), Prog. Theor. Exp. Phys.2020, 083C01 (2020) and 2021 update

2. J/Ψ(1S)
Commentary

This might be the correct factoring of J/Ψ (S_8 is the surface volume of an 8-sphere, 'h' is Planck's constant's coefficient, and 2591 is a prime number):

$$\frac{2591}{180} S_8h = 3096.9142 \text{ MeV}/c^2$$

But S_8h factoring implies a 'cs' meson, because c= S_5 and s = S_4, and when they are multiplied together you get S_8, as shown below.

$$cs = S_5S_4$$
$$S_4 = 2\ \pi^2\ r^3$$
$$S_5 = 8/3\ \pi^2\ r^4$$
$$S_5S_4 = 16/3\ \pi^4\ r^7$$

And: $S_8 = 1/3\ \pi^4\ r^7$

Therefore: $16\ S_8 = S_5S_4$

The two expressions, the one for S_8 and the one for S_5S_4, have the same powers of 'π' and 'r' in them, 4 and 7. They differ only in their multipliers, 1/3 versus 16/3. So S_8 and S_5S_4 represent the same expression except for their constants of multiplication. The only difference between the two is that S_5S_4 is sixteen times bigger than S_8.

In the literature, however, J/Ψ is commonly referred to as a 'cc' meson - a charm anti-charm meson. That would imply that it factors with S_9 rather than S_8, because 'cc' = S_5S_5, which equals $70/3\ S_9$, as shown below:

$$cc = S_5S_5$$
$$S_5 = 8/3\ \pi^2\ r^4$$
$$S_5 = 8/3\ \pi^2\ r^4$$
$$S_5S_5 = 64/9\ \pi^4\ r^8$$

And: $S_9 = 32/105\ \pi^4\ r^8$

Therefore: $70/3\ S_9 = S_5S_5$

But heuristically speaking, J/Ψ factors better with S_8 than with S_9:

With S_8: $\frac{2591}{180} S_8h = 3096.9142$

With S_9: $\frac{7*2591}{1152} S_9h = 3096.9142$

(2591/180) is a slightly less complex fraction than (7*2591)/1152). J/Ψ could also have quark content 'dddu' or 'dddd', instead of 'cs' or 'cc', because they have the same 'π' and 'r' powers as S_8 and S_9. So, simplicity of factoring slightly favors the 'cs' or 'dddu' quark content interpretation. However, the simplest factoring does not always lead to the correct quark content. Factoring by itself cannot unequivicably determine the correct quark content of a given particle. It can only narrow down the possibilities.

3. Xc0(1P)

S14h Factoring / 'd' Heptaquark

S14h = 55.59076334 MeV/c^2

Res = (1/700) **S14h**

43008 + n	n	(n / 700) S14h	ExpMass	Error	dm	dm/Error
	42962	3411.8434				
	42964	3412.0022				
	42966	3412.1611				
	42968	3412.3199				
	42970	3412.4787				
	42972	3412.6375				
	42974	3412.7964				
	42976	3412.9552	**3413.0**	1.9/0.6	0.0448	2.4%
-30	42978	3413.1140				
	42980	3413.2729				
	42982	3413.4317				
-24	42984	3413.5905				
	42986	3413.7494				
	42988	3413.9082				
-18	42990	3414.0670	**3414.1**	0.6/0.8	0.0330	5.5%
	42992	3414.2259	**3414.21**	.39/.27	0.0159	4.1%
	42994	3414.3847				
-12	42996	3414.5435	**3414.6**	1.1	0.0565	5.1%
	42998	3414.7023	**3414.7**	0.7/0.6	0.0023	0.3%
	43000	3414.8612				
-6	43002	3415.0200	**3415**	9	0.0200	0.2%
	43004	3415.1788				
	43006	3415.3377				
Key (4096+2048)/100 S14h	**43008**	**3415.49649**	**3415.5**	0.4/0.4	0.0035	0.9%
	43010	3415.6553				
	43012	3415.8142				
+6	43014	3415.9730	**3416**	3/4	0.0270	0.9%
	43016	3416.1318				
	43018	3416.2907				
+12	43020	3416.4495	**3416.5**	3.0	0.0505	1.7%
	43022	3416.6083				
	43024	3416.7671				
+18	43026	3416.9260				
	43028	3417.0848				
	43030	3417.2436				
+24	43032	3417.4025	**3417.4**	1.8/1.9	0.0025	0.1%
	43034	3417.5613				
	43036	3417.7201				
	43037	3417.7995	**3417.8**	0.4/4	0.0005	0.1%
+30	43038	3417.8790				
	43040	3418.0378				
	43042	3418.1966				
	43044	3418.3555				
	43046	3418.5143				
	43048	3418.6731				
	43050	3418.8319				
	43052	3418.9908				

Source of ExpMass and Error data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

3.0 Xc0(1P)
Possible 'd' Heptaquark

If you take the *Key Mass* of the Xc0(1P), which is 3415.496499 MeV, and divide that into the theoretical mass of the 'd' heptaquark (derived below) you get 96000 exactly. This could mean the Xc0(1P) is a 'd' heptaquark.

To get the theoretical mass of the 'd' heptaquark, multiply the theoretical mass of the 'd' quark, which is ($4 \pi r^2$), together seven times, then multiply that by h = 6.62607015 $MeVc^2$.

<u>Derivation of the Theoretical Mass of the 'd' Heptaquark</u>

d = S3 >>The 'd' quark corresponds to S3,
$S3 = 4 \pi r^2$ >> the surface volume of a 3-sphere.
$(S3)^7 = (4 \pi r^2)^7 = 16384 (\pi^7 r^{14})$

Multiply by 'h' to get the unit of factorization.

$d^7h = 16384 (\pi^7 r^{14}) h$
$d^7h = 327887663.9$ MeV/c^2 >>The theoretical 3d mass of the 'd' heptaquark

Dividing that by the *Key Mass* above you get the integer 96,000.

x = d^7h / *Key Mass*

x = 327887663.9 / 3415.496499 >>Substitute numbers for names

x = 96,000

Being exactly 1/ 96,000 of the *theoretical mass* of the 'd' heptaquark is highly suggestive that the Xc0(1P) is in fact a 'd' heptaquark.

d^7h / **96000** = **3415.4964**
61.44 S14h = **3415.4964**

4. Xc1(1P)

S12h Factoring / 'd' Hexaquark

S12h = 106.1705373 MeV/c^2

Res = (1/900) **S12h**

	n	$(33 + \frac{n}{900})$ S12h	ExpMass	Error	dm	dm/Error	
	11	3504.925					
	12	3505.043	**3505**	4	0.043	1.1%	
	13	3505.161					
33.0333 S12h	**30**	**3507.167**	**3507**	4/4	0.167	4.2%	
	31	3507.285					
	32	3507.403	**3507.4**	1.7	0.003	0.2%	
	33	3507.521					
	34	3507.639					
	35	3507.757					
	36	3507.875					
	37	3507.993					
	38	3508.110					
	39	3508.228					
33.0444 S12h	**40**	**3508.346**	**3508.4**	1.9/0.7	0.054	2.8%	
	41	3508.464					
	42	3508.582					
	43	3508.700					
	44	3508.818					
	45	3508.936	**3509**	11	0.054	0.6%	
	46	3509.054					
	47	3509.172					
	48	3509.290					
	49	3509.408	**3509.4**	0.9	0.008	0.9%	
33.0555 S12h	**50**	**3509.526**					
	51	3509.644					
	52	3509.762					
	53	3509.880					
	54	3509.998					
	55	3510.116	**3510.1**	1.1	0.016	1.5%	
	56	3510.234					
	56.555	3510.2994	**3510.30**	0.14/.16	0.0006	0.4%	
	57	3510.352					
	57.444	3510.4042	**3510.4**	0.6	0.0042	0.7%	
	58	3510.470					
	59	3510.588					
	59.111	3510.6009	**3510.60**	.087/.019	0.0009	1.0%	|
33.0666 S12h	**60**	**3510.7057**	**Key Mass**				| Close Up
	60.307	3510.7101	**3510.71**	.04/.09	0.0001	0.3%	| On
	60.111	3610.7188	**3510.719**	.051/.019	0.0002	0.4%	| Next Page
	61	3510.824					|
	62	3510.942					
	63	3511.060					
	64	3511.178					
	65	**3511.296**	**3511.3**	0.4/0.4	0.004	1.0%	
	66	3511.414					
	67	3511.532					
	68	3511.650					
	69	3511.767					
33.0777 S12h	**70**	3511.885					
	71	3512.003					
	72	3512.121					
	73	3512.239					
	73.555	3512.3048	**3512.3**	0.3/4.0	0.0048	1.6%	
	74	3512.357					
	75	3512.475					
	76	3512.593					
	77	3512.711					
	78	3512.829					
	79	3512.947					
33.0888 S12h	**80**	**3513.065**	**3513**	7	0.065	0.9%	

Source of ExpMass and Error data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

4. Xc1(1P)
Commentary

Five of Xc1(1P)'s experimental mass values near it's *Key Mass* demand a finer resolution than S12h/900 to resolve them, because they are the result of extremely accurate measurements, i.e. - they have extremely small errors. A resolution of S12h/8100, which is 9 times smaller, is needed to resolve them. Actually, even that resolution is too big as you can see in the mass spectrum below, where two higher resolution values, 540.333 and 541.500, had to be inserted. A mass spectrum with the required resolution (almost) is shown below.

Working like a magnifying glass, or microscope, the spectrum below magnifies the interesting area around 33.0666 S12h by 9 times. Five of the high accuracy experimental mass values, which cluster around 33.0666 S12h, are shown resolved. Interestingly, as can be seen from the values in the *dm/Error* column, the errors assigned to the experimental masses shown in the mass spectrum below, as small as they are, are still too big - by a factor of 100 to 300! Obviously, this shows that the experimentalists are very conservative when it comes to assigning error values to their measurements.

Xc1(1P)

S12h Factoring / 'd' Hexaquark

S12h = **106.1705373** MeV/c^2

Res = (1/8100) **S12h**

	n	$(33 + \frac{n}{8100})$ S12h	ExpMass	Error	dm	dm/Error	Source
59(9) =	531	3510.5878	**3510.59**	0.10	0.0022	2.2%	[5]
	532	3510.6009	**3510.60**	.087/.019	0.0009	1.0%	[1]
	533	3510.6140	**3510.61**	.10/.02	0.0040	4.0%	[5]
	534	3510.6271					
	535	3510.6402					
	536	3510.6533					
	537	3510.6664					
	538	3510.6795					
	539	3510.6926					
33.0666 S12h	540	3510.7057	**Key Mass**				
	540.333	3510.7101	**3510.71**	.04/.09	0.0001	0.3%	[1]
	541	3610.7188	**3510.719**	.051/.019	0.0002	0.4%	[1]
	541.500	3510.7254	**3510.725**	.065/.018	0.0004	0.6%	[5]
	542	3510.7319					
	543	3510.7450					
	544	3510.7581					
	545	3510.7713					
	546	3510.7844					
	547	3510.7975					
	548	3510.8106					
61(9) =	549	3510.8237					

The defining mass for this meson that the experimentalists seem to be zeroing in on is 33.0666 S12h. Why isn't it 33.0000 S12h rather than 33.0666 S12h? At this point, without the basic higher dimensional wave equation mathematics to consult for an answer one can only speculate, but 33.0666 can be expressed as the fraction 496/15. What is the significance of that? Well, the fraction's numerator, 496, is a perfect number. What's a perfect number? It's a number, the divisors of which add up to itself. There are very few of them. They were known to the Pythagoreans, who thought they had mystical powers. The first few of them are shown in the table below. Why one of them appears in the expression for a subatomic particle's mass and whether or not it means anything special is unknown.

Perfect Numbers

Decimal	Binary	Prime Factorization
6	110	2^1 (3)
28	11100	2^2 (7)
496	111110000	2^4 (31)
8128	1111111000000	2^6 (127)
33550336	1111111111111000000000000	2^{12} (8191)

5. hc(1P)

S17h Factoring / 'd' Octaquark? / 'c' Tetraquark?

S17h = 15.88056197 MeV/c^2

Res = (1/900)**S17h**

	a	b	(a+b/900) S17h	ExpMass	Error	dm	dm/Error
	222	-62	3524.3907	**3524.4**	0.6/0.4	0.0093	1.5%
	222	-24	3525.0613				
	222	-23	3525.0789				
	222	-22	3525.0966				
	222	-21	3525.1142				
	222	-20	3525.1319				
	222	-19	3525.1495				
	222	-18	3525.1671				
	222	-17	3525.1848				
	222	-16	3525.2024	**3525.20**	0.18/0.12	0.0024	1.3%
	222	-15	3525.2201				
	222	-14	3525.2377				
	222	-13	3525.2554				
	222	-12	3525.2730				
	222	-11	3525.2907				
	222	-10	3525.3083	**3525.31**	0.11/0.14	0.0017	1.5%
	222	-9	3525.3260				
	222	-8	3525.3436				
	222	-7	3525.3612				
	222	-6	3525.3789				
	222	-5	3525.3965	**3525.40**	0.13/0.18	0.0035	2.7%
	222	-4	3525.4142				
	222	-3	3525.4318				
	222	-2	3525.4495				
	222	-1	3525.4671				
Key Mass 222.00 S17h	222	0	3525.4848	**Key Mass**			
	222	1	3525.5024				
	222	2	3525.5200				
	222	3	3525.5377				
	222	4	3525.5553				
	222	5	3525.5730				
	222	6	3525.5906				
	222	7	3525.6083	**3525.6**	0.5	0.0063	1.3%
	222	8	3525.6259				
	222	9	3525.6436				
	222	10	3525.6612				
	222	11	3525.6789				
	222	12	3525.6965				
	222	13	3525.7141				
	222	14	3525.7318				
	222	15	3525.7494				
	222	16	3525.7671				
	222	17	3525.7847				
	222	**18**	3525.8024	**3525.8**	0.2/0.2	0.0024	1.2%
	222	19	3525.8200				
	222	20	3525.8377				
	222	21	3525.8553				
	222	22	3525.8729				
	222	23	3525.8906				
	222	24	3525.9082				
	222	**45**	3526.2788	**3526.28**	0.18/0.19	0.0012	0.7%

Source of ExpMass and Error data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

6. Xc2(1P)

S14h Factoring / 'd' Heptaquark

S14h = 55.59076334 MeV/c^2

Res = (1/700) **S14h**

	n	$(64 + \frac{n}{700})$ S14h	ExpMass	Error	dm	dm/Error
	-62	3552.8851				
	-61	3552.9645				
	-60	3553.0439	**3553**	4	0.0439	1.1%
	-59	3553.1233				
	-58	3553.2028				
	-57	3553.2822				
	-56	3553.3616	**3553.4**	2.2	0.0384	1.7%
	-55	3553.4410				
	-54	3553.5204				
	-34	3555.1087				
	-33	3555.1881				
	-32	**3555.2676**	**3555.3**	0.6/2.2	0.0324	5.4%
	-31	3555.3470				
	-30	3555.4264	**3555.4**			
	-29	3555.5058				
	-28	3555.5852				
	-27	3555.6646				
	-26.5	3555.7043	**3555.70**	0.59/0.39	0.0043	0.7%
	-26	3555.7441				
	-25	3555.8235				
	-24	**3555.9029**	**3555.9**			
	-23	3555.9823				
	-22	3556.0617				
	-21.5	3556.1014	**3556.10**	0.06/0.11	0.0014	2.3%
	-21	3556.1411				
	-20.6	3556.1728	**3556.173**	0.123/0.020	0.0002	0.2%
	-20	3556.2205	**3556.22**	0.131/0.020	0.0005	0.4%
	-19	3556.3000				
	-18	3556.3794	**3556.4**	0.7	0.0206	2.9%
	-17	3556.4588				
	-16	3556.5382				
	-15	3556.6176				
	-14	3556.6970				
	-13	3556.7765				
	-12	3556.8559				
	-11.5	3556.8955	**3556.9**	0.4/0.5	0.0044	1.1%
	-11	3556.9353				
	-10	3557.0147				
	-9	3557.0941	**3557**	1.5	0.0941	6.3%
	-8	3557.1735				
	-7	3557.2529				
	-6	3557.3324				
	-5	3557.4118				
	-4	3557.4912				
	-3	3557.5706				
	-2	3557.6500				
	-1	3557.7294				
Key Factrng **64.000 S14h**	0	3557.8089	**3557.8**	0.2/4	0.0089	4.5%
	1	3557.8883				
	2	3557.9677				
	3	3558.0471				
	4	3558.1265				
	5	3558.2059				
	6	3558.2853				
	7	3558.3648				
	8	3558.4442				
	62	3562.7326				
	63	3562.8120				
	64	3562.8914	**3563**	7	0.1086	1.6%

Source of ExpMass and Error data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

6. Xc2(1P)
Possible 'd' Heptaquark

If you take the *Key Mass* of the Xc2(1P), which is 3557.808853 MeV, and divide that into the theoretical mass of the 'd' heptaquark you get 92,160 exactly. This could mean the Xc2(1P) is a 'd' heptaquark.

Dividing the theoretical mass of the 'd' heptaquark (d^7h) by the Xc2(1P)'s *Key Mass* you get the integer 92,160.

x = d^7h / *Key Mass*

x = 327887663.9 / 3557.808853 >>Substitute numbers for names

x = 92,160

Being exactly 1/ 92,160 of the *theoretical mass* of the 'd' heptaquark is highly suggestive that the Xc2(1P) is in fact a 'd' heptaquark.

Remember, the Xc0(1P)'s Key Mass divided the theoretical mass of the 'd' heptaquark 96,000 times. The difference between the two divisors is 3840:

$$\begin{array}{r} 96{,}000 \\ \underline{-\ 92{,}120} \\ 3840 \end{array}$$

And 3840 parses to a sum of consecutive powers of two.

$$3840 = 2048 + 1024 + 512 + 256$$

The uniqueness of the difference between the two divisors (3840), the fact that it parses to a sum of consecutive powers of two, tends to affirm the correctness of the two factorings.

7. ηc(2S)

56 TQ2

TQ1 Factoring / 'c' Tetraquark

TQ1 = 454.1840724 MeV/c^2

Res = (1/2048) **TQ1**

	n	(8 +n /2048) TQ1	ExpMass	Error	dm	dm/Error
8 - 4/512 TQ1	**-16**	3629.9243				
	-15	3630.1460				
	-14	3630.3678				
	-13	3630.5896				
8 - 3/512 TQ1	**-12**	**3630.8113**	**3630.8**	3.4/1.0	0.0113	0.3%
	-11	3631.0331				
	-10	3631.2549				
	-9	3631.4767				
8 - 2/512 TQ1	**-8**	3631.6984				
	-7	3631.9202				
	-6	3632.1420				
	-5	3632.3637				
	-4	3632.5855				
	-3	3632.8073				
	-2	3633.0290				
	-1	3633.2508				
56.00 TQ2 = 8.0000000 TQ1	**0**	**3633.4726**	**Key Mass**			
	0.5	3633.5839	**3633.6**	1.7/0.6	0.0160	0.9%
	1	3633.6943				
	2	3633.9161				
	3	3634.1379				
	4	3634.3597				
	5	3634.5814				
	6	3634.8032				
	7	3635.0250	**3635.1**	5.8/2.1	0.075	1.3%
8 + 2/512 TQ1	**8**	3635.2467				
	9	3635.4685				
	10	3635.6903				
	11	3635.9120				
8 + 3/512 TQ1	**12**	3636.1338	**3636.1**	3.9/4.2	0.0338	0.9%
	13	3636.3556	**3636.4**	4.1/0.7	0.0444	1.1%
	14	3636.5774				
	15	3636.7991				
8 + 4/512 TQ1	**16**	**3637.0209**	**3637.0**	5.7/3.4	0.0209	0.4%
	17	3637.2427				
	18	3637.4644				
	19	3637.6862	**3637.6**	2.9/1.6	0.0862	3.0%
	20	3637.9080				
	21	3638.1297				
	22	3638.3515				
	23	3638.5733	**3638.5**	1.5/0.8	0.0733	4.9%
8 + 6/512 TQ1	**24**	**3638.7950**				
	25	3639.0168	**3639**	7	0.0168	0.2%
	26	3639.2386				
	27	3639.4604				
	28	3639.6821				
	29	3639.9039				
	30	3640.1257				
	31	3640.3474				
8 + 8/512 TQ1	**32**	**3640.5692**	**3640.5**	3.2/2.5	0.0692	2.2%
	33	3640.7910				
	34	3641.0127				
	35	3641.2345				
	36	3641.4563				
	37	3641.6781				
	38	3641.8998				
	39	3642.1216				
8 +10/512 TQ1	**40**	3642.3434				
	41	3642.5651				
	42	3642.7869	**3642.9**	3.1/1.5	0.1131	3.6%
	43	3643.0087				

Source of ExpMass and Error data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

7. ηc(2S)
Commentary

This 'meson' is a cccc tetraquark. Its key factoring and mass are: 8.00 TQ1 = 3633.472 MeV/c^2. Evidence for this is the experimental mass, 3633.6, which is very close to this mass. It is only (1/4096) TQ1 away from the Key Mass 8.00 TQ1. Also, six other experimental masses fall on (or very near) relatively large factor blocks, which are very close to 8.00 TQ1:

	Factoring	ThrMass	ExpMass	Error	dm	dm/Error
	8 - 4/512 TQ1	3629.9243	UNDISCOVERED			
	8 - 3/512 TQ1	3630.8113	**3630.8**	3.4/1.0	0.0113	0.3%
	8 - 2/512 TQ1	3631.6984	UNDISCOVERED			
Key Mass	**8.0000000 TQ1**	**3633.4726**	**3633.6**	1.7/0.6	0.0160	0.9%
	8 + 2/512 TQ1	3635.2467	**3635.1**	5.8/2.1	0.075	1.3%
	8 + 3/512 TQ1	3636.1338	**3636.1**	3.9/4.2	0.0338	0.9%
	8 + 4/512 TQ1	3637.0209	**3637.0**	5.7/3.4	0.0209	0.4%
	8 + 6/512 TQ1	3638.7950	**3638.75** avg	4	0.0450	1.1%
	8 + 8/512 TQ1	3640.5692	**3640.5**	3.2/2.5	0.0692	2.2%
	8 +10/512 TQ1	3642.3434	UNDISCOVERED			

TQ1 is the abbreviation for a cccc tetraquark with a divisor of 7^1(1000). The 1 refers to the power of the seven in the divisor. Its definition in terms of hypersphere surface volumes is:

$$TQ1 = (S_5)^4 h \;/\; 7000$$

S_5 is the surface volume formula of a 5-sphere.	$S_5 = (8/3)\ \pi^2\ r^4$
The cccc tetraquark unit of factorization equals four S_5's and h = 6.62607015 MeV multiplied together	$TQ = (S_5S_5S_5S_5)\ h = (S_5)^4\ h$
It is divided by 7 then 1000 for ease of factoring.	$TQ1 = (S_5)^4\ h\ /\ 7000$
This is its value.	TQ1 = 454.1840724 MeV/c^2

The values of TQ2, TQ3, TQ4, etc, tetraquark factoring units are found the same way, by successively dividing TQ by higher powers of seven.

$$TQ1 = (S_5)^4\ h\ /\ (7^1{*}1000) = 454.1840724\ MeV/c^2$$
$$TQ2 = (S_5)^4\ h\ /\ (7^2{*}1000) = 64.88343891\ MeV/c^2$$
$$TQ3 = (S_5)^4\ h\ /\ (7^3{*}1000) = 9.269062702\ MeV/c^2$$
$$TQ4 = (S_5)^4\ h\ /\ (7^4{*}100) = 13.24151815\ MeV/c^2$$
$$TQ5 = (S_5)^4\ h\ /\ (7^5{*}10) = 18.9164545\ MeV/c^2$$

One of the 'bb mesons', the Xb2(1P), factors with TQ5.

Factoring	ThrMass	ExpMass	Error	dm	dm/Error	Particle
524.000 TQ5 =	9912.2221	**9912.21**	0.26/0.31	0.0121	4.7%	Xb2(1P)

Several 'light-unflavored mesons' also factor with TQ5. Here's an example.

Factoring	ThrMass	ExpMass	Error	dm	dm/Error	Particle
74.000 TQ5 =	1399.8176	**1399.8**	2.2	0.0176	0.8%	η(1405)

8. Ψ(2S)

S17h Factoring / 'd' Octaquark?/ 'c' Tetraquark?

S17h = 15.88056197 MeV/c^2

Res = (1/1800) **S17h**

	n	n /1800 S17h	ExpMass	Error	dm	dm/Error
	417776	3685.8431				
	417777	3685.8520				
	417778	3685.8608				
	417779	3685.8696				
	417780	3685.8784				
	417781	3685.8873				
	417782	3685.8961				
	417783	3685.9049				
	417784	3685.9137				
	417785	3685.9225				
	417786	3685.9314				
	417787	3685.9402				
	417788	3685.9490	**3685.95**	0.10	0.0010	1.0%
	417789	3685.9578				
	417790	3685.9667				
	417791	3685.9755				
Key **51(8192) S17h/1800**	**417792**	**3685.9843**	**3685.98**	.09/.04	0.0043	4.8%
	417793	3685.9931				
	417794	3686.0019	**3686.00**	0.10	0.0019	1.9%
	417795	3686.0108				
	417796	3686.0196				
	417797	3686.0284				
	417798	3686.0372				
	417799	3686.0461				
	417800	3686.0549				
	417801	3686.0637				
	417802	3686.0725				
	417803	3686.0814				
	417804	3686.0902				
	417805	3686.0990	**3685.099**	.004/.009	0.0000	0.0%
	417806	3686.1078				
	417806.33	3686.1107	**3686.111**	.025/.009	0.0003	1.2%
	417806.70	3686.1139	**3686.114**	.007/.011	0.0000	0.1%
	417807	3686.1166				
	417807.50	3686.1210	**3686.12**	.06/0.10	0.0010	1.7%
(51(8192)+16) S17h/1800	**417808**	**3686.1255**				
	417809	3686.1343				
	417810	3686.1431				
	417811	3686.1519				
	417812	3686.1608				
	417813	3686.1696				
	417814	3686.1784				
	417815	3686.1872				
	417816	3686.1960				
	417817	3686.2049				
	417818	3686.2137				
	417819	3686.2225				
	417820	3686.2313				
	417821	3686.2402				
	417822	3686.2490				
	417823	3686.2578				
	417824	3686.2666				

Source of ExpMass and Error data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

8. Ψ(2S)
Commentary

The experimental masses of this charmonium are of very high accuracy (they have very small errors - all errors are between 0.10 MeV and 0.004 MeV). As can be seen in the mass spectrum, all experimental masses are matched very accurately with S17h factoring, so it definitely factors with S17h. S17h factoring implies that this charmonium has quark content of either 'cccc' or 'dddddddd' - i.e., its either a 'c' tetraquark or a 'd' octaquark.

9. Ψ(3770)

S12h Factoring / 'd' Hexaquark

S12h = 106.1705373 MeV/c^2

Res = (1/900) **S12h**

	n	(n / 900) S12h	ExpMass	Error	dm	dm/Error
-32	31968	3771.1775				
	31970	3771.4134				
	31972	3771.6494				
	31974	3771.8853				
-25	**31975**	**3772.0033**	**3772.0**	1.9	0.0033	0.2%
	31976	3772.1212				
	31978	3772.3572				
	31980	3772.5931				
	31982	3772.8290				
	31984	3773.0650				
	31986	3773.3009				
	31988	3773.5368				
	31990	3773.7728				
	31992	3774.0087				
	31994	3774.2446				
	31996	3774.4806				
	31998	3774.7165				
Key Factrng **35.5555 S12h**	**32000**	**3774.9524**	**Key Mass**			
	32002	3775.1884				
+4	**32004**	3775.4243	**3775.5**	2.4/0.5	0.0757	3.2%
	32005	3775.5423				
	32006	3775.6602				
+8	**32008**	**3775.8962**	**3776**	5/4	0.1038	2.1%
	32010	3776.1321				
	32012	3776.3680				
	32014	3776.6040				
+16	32016	3776.8399				
	32018	3777.0758				
	32020	3777.3118				
	32022	3777.5477				
	32024	3777.7837				
	32025	3777.9016				
	32026	3778.0196				
	32028	3778.2555				
	32029	**3778.3735**	**3778.4**	3.0/1.3	0.0265	0.9%
	32030	3778.4915				
	32031	3778.6094				
+32	**32032**	**3778.7274**	**3778.8**	1.9/0.9	0.0726	3.8%
	32033	3778.8454				
	32034	3778.9633				
	32035	3779.0813				
+36	**32036**	**3779.1993**	**3779.2**	1.8/1.7	0.0007	0.0%
	32037	3779.3172				
	32038	3779.4352				
	32039	3779.5532				
	32040	3779.6711				
+41	**32041**	**3779.7891**	**3779.8**	0.6	0.0109	1.8%
	32042	3779.9071				
	32044	3780.1430				
	32046	3780.3789				
	32048	3780.6149				
	32050	3780.8508				
	32052	3781.0867				
	32054	3781.3227				
	32055	3781.4406				
	32056	3781.5586				
	32058	3781.7945				

Source of ExpMass and Error data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

9. Ψ(3770)
Possible 'd' Hexaquark

The Ψ(3770) factors to **3200 /90 S12h**, which is a fairly significant factoring, because it involves a power of two (32). If that mass is divided into the theoretical mass of the 'd' hexaquark, $\mathbf{d^6h}$, one gets **6912**. Dividing it into three times the theoretical mass of the 'd' hexaquark, $\mathbf{3d^6h}$, one gets **20736**.

$$\frac{3\ d^6h}{35.555\ \text{S12h}} = \mathbf{20736}$$

What am I getting at? Well, if you look at the next charmonium, the Ψ2(3823) on the next page, you'll see that it divides into three times the theoretical mass of the 'd' hexaquark **20480** times.

$$\frac{3\ d^6h}{36.000\ \text{S12h}} = \mathbf{20480}$$

And **20736** is exactly **256** greater than **20480**.

20480 + 256 = 20736

Both these hexaquark factoring numbers parse to large powers of two.

20736 = 16384 + 4096 + 256
20480 = 16384 + 4096

These factorings strongly suggest that both these charmoniums are very likely 'd' hexaquarks.

10. Ψ2(3823)

S12h Factoring / 'd' Hexaquark

S12h = 106.1705373 MeV/c^2

Res = (1/900) **S12h**

	a	b	(a+ b/900) S12h	ExpMass	Error	dm	dm/Error	
	35	884	3820.2519					
	35	885	3820.3698					
	35	886	3820.4878					
	35	887	3820.6058					
	35	888	3820.7237					
	35	889	3820.8417					
	35	890	3820.9597					
	35	891	3821.0776					
	35	892	3821.1956					
	35	893	3821.3136					
	35	894	3821.4315					
	35	895	3821.5495					
	35	896	3821.6675	**3821.7**	1.3/0.7	0.0325	2.5%	
	35	897	3821.7854					
	35	898	3821.9034					
	35	899	3822.0214					
Key Factrng 36.00 S12h	**36**	**0**	**3822.1393**	**3822.2**	1.2	0.0607	5.0%	PDG AVG [8]
	36	1	3822.2573					
	36	2	3822.3753					
	36	3	3822.4932					
	36	4	3822.6112					
	36	5	3822.7292					
	36	6	3822.8471					
	36	7	3822.9651					
	36	8	3823.0831	**3823.1**	1.8/0.7	0.0169	0.9%	
	36	9	3823.2010					
	36	10	3823.3190					
	36	11	3823.4370					
	36	12	3823.5549					
	36	13	3823.6729					
	36	14	3823.7909					
	36	15	3823.9089					
	36	16	3824.0268					
	36	16.50	3824.0858	**3824.08**	0.53/0.14	0.0058	1.1%	
	36	17	3824.1448					
	36	18	3824.2628					
	36	19	3824.3807					
	36	20	3824.4987					
	36	21	3824.6167					
	36	22	3824.7346					
	36	23	3824.8526					
	36	24	3824.9706					

Source of ExpMass and Error Data except [n]: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

10. Ψ2(3823)
Possible 'd' Hexaquark

The charmonium Ψ2(3823) factors as **36.00 S12h,** which is a very significant factoring. If one divides that mass into three times the theoretical mass of the 'd' hexaquark, $3d^6h$, one gets **20480**, which may be considered a factoring of even greater significance.

$$\frac{3\,d^6h}{36.000\text{ S12h}} = \mathbf{20480}$$

In light of the strong significance of this factoring, it can be surmised that the Ψ2(3823), is very likely a 'd' hexaquark.

The previous charmonium Ψ(3770) is also most likely a 'd' hexaquark, as its mass divided into $3d^6h$ yields a special integer. That integer is **20736**, which equals **20480 + 256**.

$$\frac{3\,d^6h}{35.555\text{ S12h}} = \mathbf{20736} \;\mathbf{=}\; 20480 + 256$$

The fact that the hexaquark factorings of these two charmoniums differ by a power of two (256) is also significant, and tends to mutually confirm both factorings.

11. Ψ_3(3842)

S13h Factoring / 'd' Hexaquark

S13h = 78.44057013 MeV/c^2

Res = (1/90) S13h

	a	b	(a+b/900) S13h	ExpMass	Error	dm	dm/Error
	48	74	3829.6429				
	48	75	3830.5145				
	48	76	3831.3861				
	48	77	3832.2576				
	48	78	3833.1292				
	48	79	3834.0008				
	48	80	3834.8723				
	48	81	3835.7439				
	48	82	3836.6154				
	48	83	3837.4870				
	48	84	3838.3586				
	48	85	3839.2301				
	48	86	3840.1017				
	48	87	3840.9733				
	48	88	3841.8448				
	48	89	3842.7164	**3842.71**	0.16/0.12	0.0064	4.0%
Key Factrng 49.0000 S13h	49	0	3843.5879	**Key Mass**			
	49	1	3844.4595				
	49	2	3845.3311				
	49	3	3846.2026				
	49	4	3847.0742				
	49	5	3847.9457				
	49	6	3848.8173				
	49	7	3849.6889				
	49	8	3850.5604				
	49	9	3851.4320				
	49	10	3852.3036				
	49	11	3853.1751				
	49	12	3854.0467				
	49	13	3854.9182				
	49	14	3855.7898				
	49	15	3856.6614				
	49	16	3857.5329				

Source of ExpMass and Error data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

11. $\Psi_3(3842)$
Possible 'd' Hexaquark

When $\Psi_3(3842)$'s theoretical mass of **48.9888 S13h = 3842.7164 is** divided into nine times the theoretical mass of the 'd' hexaquark, the result is the integer 61111.

$$\frac{9\ d^6h}{48.9888\ S13h} = \mathbf{61111}$$

Since 3842.7164 divides $9d^6h$ an integer number of times, it is likely that $\Psi_3(3842)$ is a 'd' hexaquark.

13. X(3872)

S8h Factoring / 'd' Tetraquark

S8h = 215.1464901 MeV/c^2

Res = (1/3600) **S8h**

	n	(n / 3600) S8h	ExpMass	Error	dm	dm/Error	Ref
	64774	3871.0830					
	64775	3871.1427					
	64776	3871.2025					
	64777	3871.2623					
	64778	3871.3220	**3871.3**	0.6/0.1	0.0220	7.3%	
	64779	3871.3818	**3871.4**	0.6/0.1	0.0182	3.0%	
	64780	3871.4416					
	64781	3871.5013					
	64782	3871.5611					
	64782.50	3871.5909	**3871.59**	0.06/0.03	0.0009	1.5%	
	64783	3871.6209	**3871.61**	0.16/0.19	0.0109	6.8%	
	64783.33	3871.6407	**3871.64**	0.06/0.01	0.0007	1.2%	
(4*4049)/900 S8h	**64784**	**3871.6806**	**Key Mass 1**				
	64784.25	3871.6955	**3871.695**	.067/.068	0.0005	0.7%	
	64785	3871.7404					
	64786	3871.8001	**3871.8**	3.1/3.0	0.0001	0.0%	
	64787	3871.8599	**3871.85**	0.27/0.19	0.0099	3.4%	
	64788	3871.9197	**3871.9**	0.7/0.2	0.0197	2.8%	
	64788.50	3871.9495	**3871.95**	0.48/0.12	0.0005	0.1%	
	64789	3871.9794					
	64789.50	3872.0093	**3872.0**	0.6/0.5	0.0093	1.5%	
	64790	3872.0392					
	64791	3872.0990					
	64792	3872.1587					
	64793	3872.2185					
	64794	3872.2782					
	64795	3872.3380					
	64796	3872.3978					
	64797	3872.4575					
	64798	3872.5173					
	64799	3872.5771					
Key Mass 18 S8h	64800	3872.6368	**3872.6**	0.5/0.4	0.0368	7.4%	[6]
	64801	3872.6966					
	64802	3872.7563					
	64803	3872.8161					
	64804	3872.8759	**3872.9**	0.6/0.4	0.0241	4.0%	
	64805	3872.9356					
	64806	3872.9954	**3873**	1.8/1.6	0.0046	0.2%	
	64807	3873.0552					
	64808	3873.1149					
	64809	3873.1747					
	64810	3873.2345					
	64811	3873.2942	**3873.3**	1.1/1.0	0.0058	0.5%	
	64812	3873.3540					
	64813	3873.4137	**3873.4**	1.4	0.0137	1.0%	
	64814	3873.4735					
	64815	3873.5333					
(4*4051)/900 S8h	**64816**	**3873.5930**	**Key Mass 2**				
	64817	3873.6528					
	64818	3873.7126					
	64818	3873.7126					
	64819	3873.7723					
	64820	3873.8321					
	64821	3873.8918					
	64822	3873.9516					
	64823	3874.0114					
	64824	3874.0711					
	64839	3874.9676					
	64840	3875.0273					
	64841	3875.0871	**3875.1**	0.7/0.5	0.0129	1.8%	
	64842	3875.1469					
	64843	3875.2066	**3875.2**	0.7	0.0066	0.9%	
	64844	3875.2664					

Source of ExpMass and Error data except [n]: P.A. Zylaet al.(Particle Data Group), Prog. Theor. Exp. Phys.2020, 083C01 (2020) and 2021 update

13. X(3872)
Commentary

There may be two 'tetraquarks' in X(3872)'s mass data. The majority of the experimental mass data for X(3872) reported by PDG clusters around *Key Mass 1* (11 data points), but a few data points (6 data points) cluster around *Key Mass 2*. The 'tetraquark' at *Key Mass 1* has gotten all the attention since it was discovered in 2003, but is there another 'tetraquark' lurking less than 2 MeV away at 3873.5930 MeV? The similarity of their two factorings says there might be.

The first 'tetraquark' - the one at 3871.6806 - factors with 4***4049** in its factoring expression (when a denominatot of 900 is used). The second possible 'tetraquark' - the one at 3873.5930 - also factors with *4 times-a-prime* in its factoring expression (when a denominator of 900 is used). The prime found in the second's factoring expression is **4051**, which is just two larger than the first prime, making it the second prime in the twin prime pair **(4049, 4051)**. The factor of four in the two expressions can be made to disappear depending on the denominator chosen, so maybe it is not significant, but the prime numbers cannot be made to disappear by arithmetic manipulation. The prime number in their factorings might be what make the tetraquark (4049/225) S8h and its sister tetraquark (4051/225) S8h special.

Even when factored with the native dddd tetraquark's *unit-of-factorization*, $d^4 h$, the primes are part of their factorings, as shown below.

13. X(3872)
d^4h Factoring/ dddd Tetraquark

$3\ d^4h\ /128 = 3872.636823$ MeV/c^2

$$\frac{3\ d^4h}{128 + (128/4049)} = (18 - .00444)\ S8h = 17.99555\ S8h = 3871.6806$$

$$\frac{3\ d^4h}{128} = 18\ S8h$$

$$\frac{3\ d^4h}{128 - (128/4051)} = (18 + .00444)\ S8h = 18.00444\ S8h = 3873.5930$$

Another interesting fact about these two factorings is they are the same absolute distance from **18 S8h** along the mass scale. Each is **(4/900) S8h** from 18 S8h, as shown below.

(18 - .00444)	S8h	**=**	4(*4049*) /900 S8h	=	**3871.6806**
18	S8h	=		=	3872.6368
(18 + .00444)	S8h	=	4(*4051*) /900 S8h	=	3873.5930

Are twin prime pairs like **(4049, 4051)** that bracket integers the way that pair does numerous or rare? That's beyond the scope of this paper, but to satisfy my own curiosity I searched and found a pair bracketting the integer 26.

26 - .00444 = 4(*5849*) /900
26 + .00444 = 4(*5851*) /900

It might be worthwhile to do a computer seach of all such prime number factorings and see if any known experimental masses match any of the theoretical results.

14.1 Zc(3900)

TQ3 Factoring / **'c'** Tetraquark

TQ3 = 9.269062702 MeV/c^2

Res = (1/64)TQ3

	a	b	(a+b/64) TQ3	ExpMass	Error	dm	dm/Error	Ref
	418	44	3880.8407					
	418	45	3880.9855					
	418	46	3881.1303					
	418	47	3881.2752	**3881.2**	4.2/52.7	0.0752	1.8%	
418.7500 TQ3	**418**	**48**	3881.4200					
	418	49	3881.5648					
	418	50	3881.7097	**3881.7**	1.6/1.6	0.0097	0.6%	
	418	51	3881.8545					
	418	52	3881.9993					
	418	53	3882.1442					
	418	54	3882.2890					
	418	55	3882.4338					
	418	**56**	3882.5786					
	418	57	3882.7235					
	418	58	3882.8683					
	418	59	3883.0131					
	418	60	3883.1580					
	418	61	3883.3028					
	418	62	3883.4476					
	418	63	3883.5924					
419.0000 TQ3	**419**	**0**	**3883.7373**	**Key Mass**				
419 1/64 TQ3	419	1	3883.8821	**3883.9**	1.5/4.2	0.0179	1.2%	
	419	2	3884.0269					
	419	3	3884.1718					
419.0625 TQ3	419	**4**	3884.3166	**3884.3**	1.2	0.0166	1.4%	[3]
	419	5	3884.4614					
	419	6	3884.6062					
	419	7	3884.7511					
419.1250 TQ3	419	8	3884.8959					
	419	9	3885.0407	**3885**	5/1	0.0407	0.8%	[3]
	419	10	3885.1856					
	419	11	3885.3304					
419.1875 TQ3	419	12	3885.4752					
	419	13	3885.6201					
	419	13.50	3885.6924	**3885.7**	4.3/5.7	0.0076	0.2%	
	419	14	3886.7648					
	419	15	3885.9097					
419.2500 TQ3	**419**	**16**	3886.0545	**3886**	4/2	0.0545	1.4%	
	419	17	3886.1994					
	419	18	3886.3442					
	419	19	3886.4890					
	419	20	3886.6339					
	419	21	3886.7786					
	419	22	3886.9235					
	419	23	3887.0683					
419.3750 TQ3	419	**24**	3887.2131	**3887.2**	2.3	0.0131	0.6%	[4]
	419	25	3887.3580					
	419	26	3887.5028					

Source of ExpMass and Error data except [n]: P.A. Zylaet al.(Particle Data Group), Prog. Theor. Exp. Phys.2020, 083C01 (2020) and 2021 update

14.2 Zc(3900)

60 TQ2

TQ3 Factoring / **'c'** Tetraquark

TQ3 = 9.269062702 MeV/c^2

Res = (1/64)TQ3

	a	b	(a+b/64) TQ3	ExpMass	Error	dm	dm/Error	Ref
	419	44	3890.1098					
	419	45	3890.2546					
	419	46	3890.3994					
	419	47	3890.5442					
-16	**419**	**48**	3890.6891					
	419	49	3890.8339					
	419	50	3890.9787					
	419	51	3891.1236					
	419	52	3891.2684					
	419	53	3891.4132					
	419	54	3891.5580					
	419	55	3891.7029					
	419	**56**	3891.8477					
	419	57	3891.9925					
	419	58	3892.1374					
	419	59	3892.2822					
	419	60	3892.4270					
	419	61	3892.5718					
	419	62	3892.7167					
	419	63	3892.8615					
60 TQ2 = 420.0000 TQ3	**420**	**0**	**3893.0063**	**3893.0**	2.3/19.9	0.0063	0.3%	[2]
420 1/64 TQ3	420	1	3893.1512	**3893.1**	2.2/3.0	0.0512	2.3%	
	420	2	3893.2960					
	420	3	3893.4408					
	420	4	3893.5857					
	420	5	3893.7305					
	420	6	3893.8753					
	420	7	3894.0201					
	420	**8**	3894.1650					
	420	9	3894.3098					
	420	10	3894.4546	**3894.5**	6.6/4.5	0.0454	0.7%	
	420	11	3894.5995					
420.1875 TQ3	420	12	3894.7443	**3894.8**	2.3/3.2	0.0557	2.4%	
	420	13	3894.8891					
	420	14	3895.0339	**3895.0**	5.2	0.0339	0.7%	
	420	15	3895.1788					
+16	**420**	**16**	3895.3236					
	420	17	3895.4684					
	420	18	3895.6133					
	420	19	3895.7581					
	420	20	3895.9029					

14.3 Zc(3900)

TQ3 Factoring / **'c'** Tetraquark

TQ3 = 9.269062702 MeV/c^2

Res = (1/64)TQ3

	a	b	(a+b/64) TQ3	ExpMass	Error	dm	dm/Error	Ref
	420	41	3898.9443					
420.65625 TQ3	420	42	3899.0892	**3899.0**	3.6/4.9	0.0892	2.5%	
	420	43	3899.2340					
	420	44	3899.3788					
	420	45	3899.5236					
	420	46	3899.6685					
	420	47	3899.8133					
-16	420	**48**	3899.9581					
	420	49	3900.1030					
	420	50	3900.2478					
	420	51	3900.3926					
	420	52	3900.5374					
	420	53	3900.6823					
	420	54	3900.8271					
	420	55	3900.9719					
	420	**56**	3901.1168					
	420	57	3901.2616					
	420	58	3901.4064					
	420	59	3901.5513					
	420	60	3901.6961					
	420	61	3901.8409					
	420	62	3901.9857					
	420	63	3902.1306					
421.00000 TQ3	**421**	**0**	**3902.2754**	**Key Mass**				
	421	1	3902.4202					
421.03125 TQ3	421	2	3902.5651	**3902.6**	5.2/5.0	0.0651	1.3%	
	421	3	3902.7099					
	421	4	3902.8547					
	421	5	3902.9995					
	421	6	3903.1444					
	421	7	3903.2892					
	421	**8**	3903.4340					
	421	9	3903.5789					
	421	10	3903.7237					
	421	11	3903.8685					
421.1875 TQ3	421	12	3904.0133	**3904**	9/5	0.0132	0.1%	
	421	13	3904.1582					
	421	14	3904.3030					
	421	15	3904.4478					
+16	421	**16**	3904.5927					
	421	17	3904.7374					
	421	18	3904.8823					

Source of ExpMass and Error data except [n]: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

14.0 Zc(3900)
Mass Spectrum Commentary

On the previous three pages are three high resolution mass spectrums for the Zc(3900)'s experimental mass data. There are three because the experimental mass data for this particle is spread over a 22.8 MeV range and a single mass spectrum of the appropriate resolution is longer than a single page. Each group of data on a page seems to be associated with an integer multiple of TQ3 (TQ3 is the cccc tetraquark's unit-of-factorization divided by 7^3. See the appendix for an exact expression for TQ3.) The first group of data seems to be associated with 419 TQ3, the second group with 420 TQ3, and the third with 421 TQ3. These are the *Key Factorings* of the experimental mass data for this charmonium. They are shown in the abbreviated mass spectrum below. So, instead of one particle, the Zc(3900) could be three particles (at least), one of mass 419 TQ3, one of mass 420 TQ3, and one of mass 421 TQ3.

The entire cluster of Zc(3900)'s experimental mass data is symmetrically distributed around **60 TQ2.** If one had to define the Zc(3900) with a single factoring it would have to be **60 TQ2**. Or **420 TQ3**. The expressions 60 TQ2 and 420 TQ3 both represent the same mass, but could represent particles with different internal structures. Some other property besides mass needs to be examined to determine if two different types of particles of this mass exist, one representing the factoring 60 TQ2 and one representing the factoring 420 TQ3.

Zc(3900)
Abbreviated Mass Spectrum

	a	b	(a+b/64) TQ3	ExpMass	Error	Source
	418	63	3883.5924			
419 TQ3	419	0	3883.7373			
	419	1	3883.8821	**3883.9**	1.5/4.2	
	419	63	3892.8615			
60 TQ2 = 420 TQ3	420	0	3893.0063	**3893.0**	2.3	[2]
	420	1	3893.1512	**3893.1**	2.2	
	420	63	3902.1306			
421 TQ3	421	0	3902.2754			
	421	1	3902.4202			
	421	2	3902.5651	**3902.6**	5.2/5.0	

Source of ExpMass and Error Data except [n]: P.A. Zylaet al.(Particle Data Group), Prog. Theor. Exp. Phys.2020, 083C01 (2020) and 2021 update

15. X(3915)
16. X(3930)
17. X(3940)

S13h Factoring / 'd' Hexaquark

S13h = 78.44057013 MeV/c^2

Res = (1/90) S13h

	a	b	(a + b/90) S13h	ExpMass	Error	dm	dm/Error	Particle or Ref
	49	74	3908.0835					
	49	75	3908.9551					
	49	76	3909.8266					
	49	77	3910.6982					
	49	78	3911.5698					
	49	79	3912.4413					
	49	80	3913.3129					
	49	81	3914.1844					
	49	81.500	3914.6202	**3914.6**	3.8/3.4	0.0202	0.5%	**X(3915)**
50 - 8/90 S13h	**49**	**82**	**3915.0560**	**3915**	3/2	0.0560	1.9%	**X(3915)**
	49	83	3915.9276					
	49	84	3916.7991					
	49	85	3917.6707					
	49	86	3918.5423					
	49	86.666	3919.1233	**3919.1**	3.8/3.4	0.0233	0.6%	**X(3915)**
	49	87	3919.4138	**3919.4**	2.2/1.6	0.0138	0.6%	**X(3915)**
	49	88	3920.2854					
	49	89	3921.1569					
	49	89.850	3921.8977	**3921.9**	0.6/0.2	0.0023	0.4%	**X(3930)**
50.0000 S13h	**50**	**0**	**3922.0285**	**Key Mass**				
	50	0.450	3922.4207	**3922.4**	6.5/2.0	0.0207	0.3%	**[7]**
	50	1	3922.9001					
	50	2	3923.7716	**3923.8**	1.5/0.4	0.0284	1.9%	**X(3915)**
	50	3	3924.6432					
	50	4	3925.5148					
	50	5	3926.3863	**3926.4**	2.2/1.2	0.0137	0.6%	**X(3915)**
	50	5.360	3926.7000	**3926.7**	2.7/1.1	0.0000	0.0%	**X(3930)**
	50	5.500	3926.8220	**3926.8**	2.4/0.8	0.0220	0.9%	**X(3930)**
	50	6	3927.2579					
	50	7	3928.1294					
50 + 8/90 S13h	**50**	**8**	**3929.0010**	**3929**	5/2	0.0010	0.0%	**X(3930)**
	50	9	3929.8726					
	50	10	3930.7441					
	50	11	3931.6157					
	50	12	3932.4872					
	50	13	3933.3588					
	50	14	3934.2304					
	50	15	3935.1019					
50 +16/90 S13h	**50**	**16**	**3935.9735**	**3936**	14	0.0265	0.2%	**X(3940)**
	50	17	3936.8451					
	50	18	3937.7166					
	50	19	3938.5882					
	50	20	3939.4597					
	50	21	3940.3313					
	50	22	3941.2029					
	50	23	3942.0744	**3942**	7/6	0.0744	1.1%	**X(3940)**
50 +24/90 S13h	**50**	**24**	**3942.9460**	**3943**	6/6	0.0540	0.9%	**X(3940)**
	50	25	3943.8176					
	50	26	3944.6891					
	50	27	3945.5607					
	50	28	3946.4322					
	50	29	3947.3038					
	50	30	3948.1754					

Source of ExpMass and Error data: P.A. Zylaet al.(Particle Data Group), Prog. Theor. Exp. Phys.2020, 083C01 (2020) and 2021 update

15. X(3915)
16. X(3930)
17. X(3940)

Possible 'd' Hexaquark

The *significant factoring* associated with the experimental mass data for these three charmoniums is:

$$50.0000\ S_{13}h = 3922.0285\ \text{MeV}$$

The formula for $S_{13}h$ is:

$$S_{13}h = \frac{128}{10395}\ \pi^6\ r^{12}\ h$$

The theoretical mass formula for the 'd' hexaquark is:

$$d^6 h = (4\ \pi\ r^2)^6\ h$$

$$d^6 h = 4096\ \pi^6\ r^{12}\ h$$

This is the theoretical mass of the 'd' hexaquark (to 10 digits of accuracy):

$$d^6 h = 26{,}092{,}471.17\ \text{MeV}$$

The particle represented by the factoring 50.00 $S_{13}h$, might be a 'd' hexaquark, because it divides ten times the theoretical mass of the 'd' hexaquark an integer number of times.

$$\frac{10\ d^6 h}{50\ S_{13}h} = \mathbf{66528}$$

18. X(4020)

62 TQ2

TQ3 Factoring / **'c'** Tetraquark

TQ3 = 9.269062702 MeV/c^2

Res = (1/64) TQ3

	a	b	(a + b /64) TQ3	ExpMass	Error	dm	dm/Error
	433	48	4020.4559				
	433	49	4020.6008				
	433	50	4020.7456				
	433	51	4020.8904				
	433	52	4021.0353				
	433	53	4021.1801				
	433	54	4021.3249				
	433	55	4021.4698				
434.875 TQ3	433	56	4021.6146				
	433	57	4021.7594				
	433	58	4021.9042				
	433	59	4022.0491				
	433	60	4022.1939				
	433	61	4022.3387				
	433	62	4022.4836				
	433	63	4022.6284				
62 TQ2 = **434.000 TQ3**	**434**	**0**	**4022.7732**	Key Mass			
	434	**1**	4022.9180	**4022.9**	0.8/2.7	0.0180	2.3%
	434	2	4023.0629				
	434	3	4023.2077				
	434	4	4023.3525				
	434	5	4023.4974				
	434	6	4023.6422				
	434	7	4023.7870				
434.125 TQ3	434	**8**	4023.9318	**4023.9**	2.2/3.8	0.0318	1.4%
	434	9	4024.0767				
	434	10	4024.2215				
	434	11	4024.3663				
	434	12	4024.5112				
	434	13	4024.6560				
	434	14	4024.8008				
	434	15	4024.9456				
434.250 TQ3	434	16	4025.0905				
	434	17	4025.2353				
	434	18	4025.3801				
	434	19	4025.5250	**4025.5**	2.0/4.7	0.0250	1.2%
	434	20	4025.6698				
	434	21	4025.8146				
	434	22	4025.9595				
	434	23	4026.1043				
434.375 TQ3	434	**24**	4026.2491	**4026.3**	2.6/3.7	0.0509	2.0%
	434	25	4026.3939				
	434	26	4026.5388				
	434	27	4026.6836				
	434	28	4026.8284				
	434	29	4026.9733				
	434	30	4027.1181				
	434	31	4027.2629				
	434	32	4027.4077				
	434	33	4027.5526				
	434	34	4027.6974				
	434	35	4027.8422				
	434	36	4027.9871				

Source of ExpMass and Error data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

18. X(4020)
Mass Spectrum Commentary

The Key Factoring for this charmonium is **62 TQ2,** or **434 TQ3**. The two units are related by a factor of 7. The bigger unit, TQ2, is seven times bigger than TQ3: **TQ2** = **7 TQ3**. Two other experimental masses associated with this charmonium factor to relatively large fractions of **TQ3**.

This 'meson' is a **cccc** tetraquark with a divisor of 7^2(1000) , which is given the abbreviation TQ2. The two in the TQ2 means the seven in its divisor is raised to the power of two. (A higher power of two than 2 could be used, but not a lower power.) The key factoring and mass of this 'meson' are:

62.00 **TQ2** = 4022.7732
or
434.00 **TQ3** = 4022.7732

The closest experimental mass to **434 TQ3** is (1 /64) **TQ3** away, as shown in the table below.

Factoring			ThrMass	ExpMass	Error
434	**TQ3**	=	4022.7732	**Key Mass**	
434 + 1/64	**TQ3**	=	4022.9180	4022.9	0.8/2.7

Below is an abbrviated mass spectrum of X(4020)'s experimental mass data. It shows all the salient features of the mass spectrum table above.

18. X(4020)
TQ3 Factoring / **cccc** Tetraquark

		Factoring		ThrMass	ExpMass	Error	dm	dm/Error
62 TQ2	=	**434.0000 TQ3**	**=**	4022.7732	**Key Mass**			
		434 1/64 TQ3	=	4022.9180	**4022.9**	0.8/2.7	0.0180	2.3%
		434.1250 TQ3	=	4023.9318	**4023.9**	2.2/3.8	0.0318	1.4%
		434.2500 TQ3	=	4025.0904	UNDISCOVERED			
		434.3750 TQ3	**=**	4026.2491	**4026.3**	2.6/3.7	0.0509	2.0%

19. Ψ(4040)

TQ2 Factoring / '**c**' Tetraquark

TQ2 = 64.88343891 MeV/c^2

Res = (1/128) TQ2

	a	b	(a+b /128) TQ2	ExpMass	Error	dm	dm/Error	
62.125 TQ2	62	16	4030.8836					
	62	17	4031.3905					
	62	18	4031.8974					
	62	19	4032.4043					
	62	20	4032.9112					
	62	21	4033.4182					
	62	22	4033.9251					
	62	23	4034.4320					
	62	24	4034.9389					
	62	25	4035.4458					
	62	26	4035.9527					
	62	27	4036.4596					
	62	28	4036.9665	**4037**	2	0.0335	1.7%	
	62	29	4037.4734					
	62	30	4037.9803					
	62	31	4038.4872					
Key Mass 62.250 TQ2	62	32	4038.9941	**4039**	1	0.0059	0.6%	PDG ESTIMATE
	62	33	4039.5010	**4039.6**	4.3	0.0990	2.3%	
	62	34	4040.0079	**4040**	1	0.0079	0.8%	
	62	35	4040.5148					
	62	36	4041.0217					
	62	37	4041.5286					
	62	38	4042.0355					
	62	39	4042.5424					
	62	40	4043.0493					
	62	41	4043.5562					
	62	42	4044.0631					
	62	43	4044.5700					
	62	44	4045.0769					
	62	45	4045.5838					
	62	46	4046.0907					
	62	47	4046.5976					
62.375 TQ2	62	48	4047.1045					

Source of ExpMass and Error data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

19. Ψ(4040)
20. X(4050)
21. X(4055)

S11h Factoring / 'd' Pentaquarks

S11h = 137.3262492 MeV/c^2

Res = (1/32) **S11h**

	n	n S11h	ExpMass	Error	dm	dm/Error	Particle	Ref
29.000 S11h	**29.00000**	**3982.4612**	**3982.5**	2.8/3.0	0.0388	1.4%	**Zcs(3982)**	[4]
	29.03125	3986.7527						
	29.06250	3991.0441						
	29.09375	3995.3356						
	29.12500	3999.6270						
	29.15625	4003.9185						
	29.18750	4008.2099						
	29.21875	4012.5013						
	29.25000	4016.7928						
	29.28125	4021.0842						
	29.31250	4025.3757						
	29.34375	4029.6671						
29.375 S11h	**29.37500**	**4033.9586**	**4034**	6	0.0414	0.7%	**Ψ(4040)**	
	29.40625	4038.2500						
	29.43750	4042.5415						
	29.46875	4046.8329						
29.500 S11h	**29.50000**	**4051.1244**	**4051**	14	0.1244	0.9%	**X(4050)**	
	+2/96	4053.9853	**4054**	3/1	0.0147	0.5%	**X(4055)**	
	29.53125	4055.4158						
	29.56250	4059.7072						
	29.59375	4063.9987						
	29.62500	4068.2901						
	29.65625	4072.5816						
	29.68750	4076.8730						
	29.71875	4081.1645						
	29.75000	4085.4559						
	29.78125	4089.7474						
	29.81250	4094.0388						
	29.84375	4098.3302						
	29.87500	4102.6217						
	29.90625	4106.9131						
	29.93750	4111.2046						
	29.96875	4115.4960						
	30.00000	4119.7875						

Source of ExpMass and Error data except [n]: P.A. Zylaet al.(Particle Data Group), Prog. Theor. Exp. Phys.2020, 083C01 (2020) and 2021 update

22. X(4100)

63 TQ2

TQ2 Factoring / 'c' Tetraquark

TQ2 = 64.88343891 MeV/c^2

Res = (1/32) TQ2

	n	n TQ2	ExpMass	Error	dm	dm/Error
	62.50000	4055.2149				
	62.53125	4057.2425				
	62.56250	4059.2701				
	62.59375	4061.2978				
	62.62500	4063.3254				
	62.65625	4065.3530				
	62.68750	4067.3806				
	62.71875	4069.4082				
	62.75000	4071.4358				
	62.78125	4073.4634				
	62.81250	4075.4910				
	62.84375	4077.5186				
	62.87500	4079.5462				
	62.90625	4081.5738				
	62.93750	4083.6014				
	62.96875	4085.6290				
63.000 TQ2	**63.00000**	**4087.6567**	**Key Mass**			
	63.03125	4089.6843				
	63.06250	4091.7119				
	63.09375	4093.7395				
63.125 TQ2	**63.12500**	**4095.7671**	**4096**	20	0.2329	1.2%
	63.15625	4097.7947				
	63.18750	4099.8223				
	63.21875	4101.8499				
	63.25000	4103.8775				
	63.28125	4105.9051				
	63.31250	4107.9327				
	63.34375	4109.9603				
	63.37500	4111.9879				
	63.40625	4114.0155				
	63.43750	4116.0432				
	63.46875	4118.0708				
	63.50000	4120.0984				

Source of ExpMass and Error data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

23. Xc1(4140)

64 TQ2

TQ3 Factoring / **'c'** Tetraquark

TQ3 = 9.269062702 MeV/c^2

Res = (1/32)**TQ3**

	n	n TQ3	ExpMass	Error	dm	dm/Error
	447.75000	4140.9538				
	447.78125	4141.2434				
	447.81250	4141.5331				
	447.84375	4141.8227				
	447.87500	4142.1124				
	447.90625	4142.4021				
	447.93750	4142.6917				
	447.96875	4142.9814	**4143.0**	2.9/1.2	0.0186	0.6%
447.00 TQ3	**447.00000**	**4143.2710**				
	447.015625	4143.4158	**4143.4**	2.9/3.0	0.0158	0.5%
	447.03125	4143.5607				
	447.06250	4143.8503				
	447.09375	4144.1400				
	447.12500	4144.4297				
	447.15625	4144.7193				
	447.18750	4145.0090				
	447.21875	4145.2986				
447.25 TQ3	**447.25000**	**4145.5883**				
	447.28125	4145.8780				
	447.31250	4146.1676				
	447.34375	4146.4573	**4146.5**	4.5	0.0427	0.9%
	447.37500	4146.7469				
	447.40625	4147.0366				
	447.43750	4147.3262				
	447.46875	4147.6159				
447.50 TQ3	**447.50000**	**4147.9056**	**4148.0**	2.1/6.3	0.0944	4.5%
	447.53125	4148.1952				
	447.56250	4148.4849				
	447.59375	4148.7745				
	447.62500	4149.0642				
	447.65625	4149.3539				
	447.68750	4149.6435				
	447.71875	4149.9332				
447.75 TQ3	**447.75000**	**4150.2228**				
	447.78125	4150.5125				
	447.81250	4150.8021				
	447.84375	4151.0918				
	447.87500	4151.3815				
	447.90625	4151.6711				
	447.93750	4151.9608				
	447.96875	4152.2504				
64 TQ2 = 448.00 TQ3	**448.00000**	**4152.5401**	**4152.5**	1.7/6.2	0.0401	2.4%
	448.03125	4152.8297				
	448.06250	4153.1194				
	448.09375	4153.4091				
	448.12500	4153.6987				
	448.15625	4153.9884				
	448.18750	4154.2780				
	448.21875	4154.5677				
	448.25000	4154.8574				
	448.70000	4159.0284	**4159.0**	4.3/6.6	0.0284	0.7%

Source of ExpMass and Error data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

24. Ψ(4160)

'd' Tetraquark

dTQ = $d^4h/(2\cdot3\cdot5\cdot7^3)$ = 16.05758061 MeV/c^2

Res = (1/8)**dTQ**

	a	b	(a + b/8) dTQ / 10290	ExpMass	Error	dm	dm/Error
	258	**0**	**4142.8558**				
	258	1	4144.8630				
	258	2	4146.8702				
	258	3	4148.8774				
	258	4	4150.8846	**4151**	4	0.1154	2.9%
	258	5	4152.8918				
	258	6	4154.8990	**4155**	5	0.1010	2.0%
	258	7	4156.9062				
Key Mass	**259**	**0**	**4158.9134**	**4159**	20	0.0866	0.4%
	259	1	4160.9206				
	259	2	4162.9278				
	259	3	4164.9350				
	259	4	4166.9422				
	259	5	4168.9494				
	259	6	4170.9566				
	259	7	4172.9638				
	260	**0**	**4174.9710**				
	260	1	4176.9782				
	260	2	4178.9854				
	260	3	4180.9925				
	260	4	4182.9997				
	260	5	4185.0069				
	260	6	4187.0141				
	260	7	4189.0213				
Key Mass	**261**	**0**	**4191.0285**	**4191**	5	0.0285	0.6%
	261	0.333	4191.6976	**4191.7**	6.5	0.0024	0.0%
	261	1	4193.0357	**4193**	7	0.0357	0.5%
	261	2	4195.0429				
	261	3	4197.0501				
	261	4	4199.0573				
	261	5	4201.0645				
	261	6	4203.0717				
	261	7	4205.0789				
	262	**0**	**4207.0861**				
	262	1	4209.0933				
	262	2	4211.1005				
	262	3	4213.1077				
	262	4	4215.1149				
	262	5	4217.1221				
	262	6	4219.1293				
	262	7	4221.1365				

Source of ExpMass and Error data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

25. X(4160)

d^4h Factoring / 'd' Tetraquark

d^4h = 165232.5044 MeV/c^2

	n	$33\ d^4h\ /n$	ExpMass	Error	dm	dm/Error
	1332	4093.5981				
	1331	4096.6737				
	1330	4099.7539				
	1329	4102.8387				
	1328	4105.9282				
	1327	4109.0223				
	1326	4112.1212				
	1325	4115.2246				
	1324	4118.3328				
	1323	4121.4457				
	1322	4124.5633				
	1321	4127.6856				
	1320	4130.8126				
	1319	4133.9444				
	1318	4137.0809				
	1317	4140.2222				
	1316	4143.3683				
	1315	4146.5191				
	1314	4149.6748				
	1313	4152.8352				
Key Factoring	**1312**	**4156.0005**	**4156**	25/20	0.0005	0.0%
	1311	4159.1706				
	1310	4162.3455				
	1309	4165.5253				
	1308	4168.7100				
	1307	4171.8995				
	1306	4175.0939				
	1305	4178.2932				
	1304	4181.4974				
	1303	4184.7066				
	1302	4187.9206				
	1301	4191.1396				
	1300	4194.3636				
	1299	4197.5925				
	1298	4200.8264				
	1297	4204.0653				
	1296	4207.3091				
	1295	4210.5580				
	1294	4213.8119				
	1293	4217.0709				
	1292	4220.3349				

Source of ExpMass and Error data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

25. X(4160)
Commentary

Because of the large error size and the fact that there is only one experimental mass for this charmonium, the factoring shown here may not be correct. To increase the confidence in the correctness of this factoring a more accurate mass measurement is needed, and/or, more data points. The one thing lending confidence to the correctness of this factoring is that 1312 is the sum of just three powers of two.

$$1312 = 1024 + 256 + 32$$

Another factoring possibility for this charmonium is:

$$64.05333 \text{ TQ2} = 4156.000540 \text{ MeV}$$

The only thing significant about this factoring is that it is close to the significant factoring 64 TQ2.

26. Zc(4200)

S11h Factoring / 'd' Pentaquark

S11h = 137.3262492 MeV/c^2

Res = (1/90) S11h

	a	b	(a+b / 90) S11h	ExpMass	Error	dm	dm/Error
	30	32	4168.6146				
	30	33	4170.1404				
	30	34	4171.6663				
	30	35	4173.1921				
	30	36	4174.7180				
	30	37	4176.2438				
	30	38	4177.7697				
	30	39	4179.2955				
	30	40	4180.8214				
	30	41	4182.3472				
	30	42	4183.8731				
	30	43	4185.3989				
	30	44	4186.9248				
	30	45	4188.4506				
	30	46	4189.9764				
	30	47	4191.5023				
	30	48	4193.0281				
	30	49	4194.5540				
Key Factrng 30.5555 S11h	**30**	**50**	**4196.0798**	**4196**	31/29	0.0798	0.3%
	30	51	4197.6057				
	30	52	4199.1315				
	30	53	4200.6574				
	30	54	4202.1832				
	30	55	4203.7091				
	30	56	4205.2349				
	30	57	4206.7608				
	30	58	4208.2866				
	30	59	4209.8125				
	30	60	4211.3383				
	30	61	4212.8642				
	30	62	4214.3900				
	30	63	4215.9159				
	30	64	4217.4417				
	30	65	4218.9675				
	30	66	4220.4934				
	30	67	4222.0192				
	30	68	4223.5451				

Source of ExpMass and Error data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

27. Ψ(4230)

65 TQ2

TQ2 Factoring / **'c'** Tetraquark

TQ2 = 64.88343891 MeV/c2

Res = (10/900) TQ2

		n	(65 + n/90) TQ2	ExpMass	Error	dm	dm/Error
		-240	4200.1213				
		-233.333	4200.6018	**4200.6**	7.9/13.3	0.0018	0.0%
		-230	4200.8422				
		-220	4201.5631				
		-210	4202.2840				
		-200	4203.0049				
		-190	4203.7259				
		-180	4204.4468				
		-170	4205.1678				
		-160	4205.8887				
		-150	4206.6096				
		-140	4207.3305				
		-130	4208.0515				
		-120	4208.7724				
		-110	4209.4933	**4209.5**	7.4/1.4	0.0067	0.1%
		-100	4210.2143				
		-90	4210.9352				
		-80	4211.6561				
		-70	4212.3770				
		-60	4213.0980				
		-50	4213.8189				
		-40	4214.5398				
		-30	4215.2607				
		-20	4215.9817				
		-10	4216.7026	**4216.7**	8.9/4.1	0.0026	0.0%
Key Factrng	**65.0000 TQ2**	**0**	**4217.4235**	**Key Mass**			
		8	4218.0002	**4218**	5.5/4.5	0.0002	0.0%
		10	4218.1445				
		15	4218.5049	**4218.5**	1.6/4.0		
		16	4218.5770	**4218.6**	3.8/2.5		
		20	4218.8654				
		30	4219.5863				
		40	4220.3072	**4220.4**	2.4/2.3	0.0938	3.7%
		50	4221.0282				
		60	4221.7491				
	455.500 TQ3	64 2/7	4222.0580	**4222.0**	3.1/1.4	0.0580	1.9%
		70	4222.4700				
		80	4223.1909	**4223.3**	1.6/2.5	0.1091	6.8%
		90	4223.9119				
		100	4224.6328				
		110	4225.3537				
		120	4226.0747				
		130	4226.7956				
		140	4227.5165				
		150	4228.2374				
		155	4228.5978	**4228.6**	4.1/6.3	0.0022	0.1%
		160	4228.9584				
		170	4229.6793				
		174.444	4229.9997	**4230**	8/6	0.0003	0.0%
		180	4230.4002				
		190	4231.1211				
	65.2222 TQ2	200	4231.8421	**4231.9**	5.3/4.9	0.0579	1.1%
		210	4232.5630				
		220	4233.2839				
		230	4234.0049				
		240	4234.7258				
		250	4235.4467				
		260	4236.1676				

Source of ExpMass and Error data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

28. Rco(4240)

TQ2 Factoring / **'c'** Tetraquark

TQ2 = 64.88343891 MeV/c^2

Res = (5/90) TQ2

		a	b	(a + b/90) TQ2	ExpMass	Error	dm	dm/Error	Particle
		64	0	4152.5401					
		64	5	4156.1447					
		64	10	4159.7494					
		64	15	4163.3540					
		64	20	4166.9586					
		64	25	4170.5633					
		64	30	4174.1679					
		64	35	4177.7725					
		64	40	4181.3772					
		64	45	4184.9818					
		64	50	4188.5864					
		64	55	4192.1911					
		64	**60**	**4195.7957**					
		64	65	4199.4004					
		64	70	4203.0050					
		64	75	4206.6096					
		64	80	4210.2143					
		64	85	4213.8189					
	65.0000 TQ2	**65**	**0**	**4217.4235**					
		65	5	4221.0282					
		65	10	4224.6328					
		65	15	4228.2374					
		65	20	4231.8421					
		65	25	4235.4467					
Key Factrng	**65.3333 TQ2**	**65**	**30**	**4239.0513**	**4239**	18	0.0513	0.3%	**Rco(4240)**
		65	35	4242.6560					
		65	40	4246.2606					
		65	45	4249.8652					
		65	50	4253.4699					
		65	55	4257.0745					
		65	60	4260.6792					
		65	65	4264.2838					
		65	70	4267.8884					
		65	75	4271.4931					
		65	80	4275.0977					
		65	85	4278.7023					

Source of ExpMass and Error data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

28. Rco(4240)
Commentary

The Key Mass of Rco(4240) can be factored down from a TQ2 expression to a TQ1 expression of even greater factoring significance.

```
65.3333 TQ2 = 4239.0513
 9.3333 TQ1 = 4239.0513   >> greater significance
```

The TQ1 expression can be further reduced to a TQ expression, arguably, of yet greater factoring significance.

```
     9.3333 TQ1 = 4239.0513
     (28/3) TQ1 = 4239.0513   >> TQ1 = TQ/7000, so
(28/3) TQ/7000 = 4239.0513   >> sub TQ/7000 for TQ1
28/(3*7000) TQ = 4239.0513
        TQ/750 = 4239.0513   >> greater significance
```

This factoring shows that the unit-of-factorization of the cccc tetraquark, TQ = $(S_5)^4$ h, divided by 750, gives the mass of Rco(4240).

29. Ψ(4250)

S12h Factoring / 'd' Hexaquark

S12h = 106.1705373 MeV/c^2

Res = (1/90) **S12h**

	a	b	(a+b/90)S12h	ExpMass	Error	dm	dm/Error
	39	70	4223.2280				
	39	71	4224.4077				
	39	72	4225.5874				
	39	73	4226.7671				
	39	74	4227.9467				
	39	75	4229.1264				
	39	76	4230.3061				
	39	77	4231.4857				
	39	78	4232.6654				
	39	79	4233.8451				
	39	80	4235.0248				
	39	81	4236.2044				
	39	82	4237.3841				
	39	83	4238.5638				
	39	84	4239.7435				
	39	85	4240.9231				
	39	86	4242.1028				
	39	87	4243.2825				
	39	88	4244.4621				
	39	89	4245.6418				
40.0000 S12h	**40**	**0**	**4246.8215**	**Key Mass**			
	40	1	4248.0012	**4248**	44/29	0.0012	0.0%
	40	2	4249.1808				
	40	3	4250.3605				
	40	4	4251.5402				
	40	5	4252.7199				
	40	6	4253.8995				
	40	7	4255.0792				
	40	8	4256.2589				
	40	9	4257.4385				
	40	10	4258.6182				
	40	11	4259.7979				
	40	12	4260.9776				
	40	13	4262.1572				
	40	14	4263.3369				
	40	15	4264.5166				
	40	16	4265.6963				
	40	17	4266.8759				
	40	18	4268.0556				
	40	19	4269.2353				
	40	20	4270.4149				

Source of ExpMass and Error data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

29. Ψ(4250)

Possible 'd' Hexaquark

This charmonium is another that might be a 'd' hexaquark. When the Key Mass **40 S12h** is divided into the theoretical mass of the 'd' hexaquark, the result is an integer.

$$\frac{d^6h}{40.0000\ S12h} = 6144$$

And that integer is special. It is the sum of two consecutive powers of two.

$$6144 = 4096 + 2048$$

31. X(4274)

S11h Factoring / 'd' Pentaquark

S11h = 137.32624921 MeV/c^2

Res = (1/32) **S11h**

	a	b	(a + b/32) S11h	ExpMass	Error	dm	dm/Error
	31	0	4257.1137				
	31	1	4261.4052				
	31	2	4265.6966				
	31	3	4269.9881				
	31	3.75	4273.2066	**4273.3**	8.3	0.0934	1.1%
31.125 S11h	31	4	4274.2795	**4274.4**	8.4/1.9	0.1205	1.4%
	31	5	4278.5710				
	31	6	4282.8624				
	31	7	4287.1538				
	31	8	4291.4453				
	31	9	4295.7367				
	31	10	4300.0282				
	31	11	4304.3196				
	31	12	4308.6111				
	31	13	4312.9025				
	31	14	4317.1940				
	31	15	4321.4854				
	31	16	4325.7768				
	31	17	4330.0683				
	31	18	4334.3597				
	31	19	4338.6512				
	31	20	4342.9426				
	31	21	4347.2341				
	31	22	4351.5255				
	31	23	4355.8170				
	31	24	4360.1084				
	31	25	4364.3999				
	31	26	4368.6913				
	31	27	4372.9827				
	31	28	4377.2742				
	31	29	4381.5656				
	31	30	4385.8571				
	31	31	4390.1485				

Source of ExpMass and Error data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

32. X(4350)

S11h Factoring / 'd' Pentaquark

S11h = 137.3262492 MeV/c^2

Res = (1/100) S11h

		n	nS11h	ExpMass	Error	dm	dm/Error
		31.48	4323.0303				
		31.49	4324.4036				
		31.50	4325.7768				
		31.51	4327.1501				
		31.52	4328.5234				
		31.53	4329.8966				
		31.54	4331.2699				
		31.55	4332.6432				
		31.56	4334.0164				
		31.57	4335.3897				
		31.58	4336.7629				
		31.59	4338.1362				
		31.60	4339.5095				
		31.61	4340.8827				
		31.62	4342.2560				
		31.63	4343.6293				
		31.64	4345.0025				
		31.65	4346.3758				
		31.66	4347.7490				
		31.67	4349.1223				
Key Factrng	**31.68 S11h**	**31.68**	**4350.4956**	**4350.6**	4.6/5.1	0.1044	2.3%
		31.69	4351.8688				
		31.70	4353.2421				
		31.71	4354.6154				
		31.72	4355.9886				
		31.73	4357.3619				
		31.74	4358.7351				
		31.75	4360.1084				
		31.76	4361.4817				
		31.77	4362.8549				
		31.78	4364.2282				
		31.79	4365.6015				
		31.80	4366.9747				
		31.81	4368.3480				
		31.82	4369.7212				
		31.83	4371.0945				
		31.84	4372.4678				
		31.85	4373.8410				
		31.86	4375.2143				
		31.87	4376.5876				
		31.88	4377.9608				

Source of ExpMass and Error data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

32. X(4350)
Probable 'd' Pentaquark

The constant of multiplication of the *Key Factoring* for this charmonium (3168) equals a sum of powers of two.

$$\frac{3168}{100}\ S11h = 4350.4955\ MeV$$

$$3168 = 2048 + 1024 + 64 + 32$$

The smallest power of two in that sum, 32, when divided by 100 and multiplied by S11h yields 43.9443 MeV, which is a large increment, therefore, this is a significant factoring, meaning it is likely correct.

Division into eleven times the theoretical mass of the 'd' pentaquark yields an integer.

$$\frac{11d^5h}{31.68\ S11h} = 5250$$

This corroborates the S11h factoring and also indicates that the X(4350) is likely a 'd' pentaquark.

What made finding this factoring possible was the high accuracy of X(4350)'s mass measurement. Experimentaers assigned a statistical error of 4.6 MeV to their measurement, but in fact, assuming the 31.68 S11h theoretical mass is X(4350)'s correct mass, their actual error was only 0.1044 MeV. That means that the error given by the experimenters was 44 times bigger than warranted, in other words, the measurement is more accurate than the experimenters thought it was, and high precision measurements help to find correct high confidence factorings.

33. Ψ(4360)

67 TQ2

TQ2 Factoring / 'c' Tetraquark

TQ2 = 64.88343891 MeV/c^2

Res = (1/32) TQ2

		a	b	(a + b/32) TQ2	ExpMass	Error	dm	dm/Error
		66	16	4314.7487				
		66	17	4316.7763				
		66	18	4318.8039				
		66	18.50	4319.8177	**4320.0**	10.4/7.0	0.1823	1.8%
		66	19	4320.8315				
		66	20	4322.8591				
		66	20.50	4323.8729	**4324**	24	0.1271	0.5%
		66	21	4324.8867				
		66	22	4326.9143				
		66	23	4328.9419				
		66	24	4330.9695				
		66	25	4332.9972				
		66	26	4335.0248				
		66	27	4337.0524				
		66	28	4339.0800				
		66	28.50	4340.0937	**4340**	16/9	0.0937	0.6%
		66	29	4341.1076				
		66	30	4343.1352				
		66	31	4345.1628				
Key Mass	**67.0000 TQ2**	**67**	**0**	**4347.1904**	**4347**	6/3	0.1904	3.2%
		67	1	4349.2180				
		67	2	4351.2456				
		67	3	4353.2732				
		67	4	4355.3008	**4355**	9/10	0.3008	3.3%
		67	5	4357.3284				
		67	6	4359.3561				
		67	7	4361.3837	**4361**	9/9	0.3837	3.8%
		67	8	4363.4113				
		67	9	4365.4389				
		67	10	4367.4665				
		67	11	4369.4941				
		67	12	4371.5217				
		67	13	4373.5493				
		67	14	4375.5769				
		67	15	4377.6045				
		67	16	4379.6321				
		67	17	4381.6597				
		67	18	4383.6873	**4383.7**	2.9/6.2	0.0127	0.4%
		67	18.05	4383.7999	**4383.8**	4.2/0.8	0.0001	0.0%
		67	19	4385.7149				
		67	20	4387.7426				
		67	21	4389.7702				
		67	22	4391.7978				
		67	23	4393.8254				
		67	24	4395.8530				
		67	25	4397.8806				
		67	26	4399.9082				
		67	27	4401.9358				
		67	28	4403.9634				
		67	29	4405.9910				
		67	30	4408.0186				
		67	31	4410.0462				

Source of ExpMass and Error data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

34. Ψ(4390)

S13h Factoring / 'd' Hexaquark

S13h = 78.44057013 MeV/c^2

Res = (1/64) S13h

		a	b	(a+b/64) S13h	ExpMass	Error	dm	dm/Error
		55	48	4373.0618				
		55	49	4374.2874				
		55	50	4375.5131				
		55	51	4376.7387				
		55	52	4377.9643				
		55	53	4379.1900				
		55	54	4380.4156				
		55	55	4381.6412				
		55	56	4382.8669				
		55	57	4384.0925				
		55	58	4385.3181				
		55	59	4386.5438				
		55	60	4387.7694				
		55	61	4388.9950				
		55	62	4390.2207				
		55	63	4391.4463	**4391.5**	6.3/6.8	0.0537	0.9%
Key Factrng	**56.00 S13h**	**56**	**0**	**4392.6719**	**Key Mass**			
		56	1	4393.8976				
		56	2	4395.1232				
		56	3	4396.3488				
		56	4	4397.5745				
		56	5	4398.8001				
		56	6	4400.0257				
		56	7	4401.2514				
		56	8	4402.4770				
		56	9	4403.7026				
		56	10	4404.9283				
		56	11	4406.1539				
		56	12	4407.3795				
		56	13	4408.6052				
		56	14	4409.8308				
		56	15	4411.0564				
		56	16	4412.2821				

Source of ExpMass and Error data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

34. Ψ(4390)
Commentary

The *Key Factoring* of this charmonium is a fairly significant factoring, because it's multiplier is an integer (56). However, when converted to S12h factoring the factoring becomes even more significant, because it involves a power of two (4096).

```
56.0000  S13h = 4392.6719
4096/99  S12h = 4392.6719
```

This charmonium is also a possible 'd' hexaquark, because it divides the theoretical mass of the 'd' hexaquark an integer number of times.

$$\frac{d^6 h}{56 \text{ S13h}} = \mathbf{5940}$$

35.1 Ψ(4415)

68 TQ2

TQ2 Factoring / 'c' Tetraquark

TQ2 = 64.88343891 MeV/c^2

Res = (1/64) TQ2

		a	b	(a + b/64) TQ2	ExpMass	Error	dm	dm/Error
		67	48	4395.8530				
		67	49	4396.8668				
		67	50	4397.8806				
		67	51	4398.8944				
		67	52	4399.9082				
		67	53	4400.9220				
		67	54	4401.9358				
		67	55	4402.9496				
		67	56	4403.9634				
		67	57	4404.9772				
		67	58	4405.9910				
		67	59	4407.0048				
		67	60	4408.0186				
		67	61	4409.0324				
		67	62	4410.0462				
		67	63	4411.0600	**4411**	7	0.0600	0.9%
Key Factrng	**68.0000 TQ2**	**68**	**0**	**4412.0738**	**4412**	15	0.0738	0.5%
		68	1	4413.0876				
		68	2	4414.1015	**4414**	7	0.1015	1.5%
		68	3	4415.1153	**4415.1**	7.9	0.0152	0.2%
		68	4	4416.1291				
		68	5	4417.1429	**4417**	10	0.1428	1.4%
		68	6	4418.1567				
		68	7	4419.1705				
		68	8	4420.1843				
		68	9	4421.1981				
		68	10	4422.2119				
		68	11	4423.2257				
		68	12	4424.2395				
		68	13	4425.2533				
		68	14	4426.2671				
		68	15	4427.2809				
		68	16	4428.2947				

Source of ExpMass and Error data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

35.2 Ψ(4415)

S11h Factoring / 'd' Pentaquark

S11h = 137.3262492 MeV/c²

Res = (1/32) S11h

	n	n S11h	ExpMass	Error	dm	dm/Error	Particle
	32.00000	4394.4400					
	32.03125	4398.7314					
	32.06250	4403.0229					
	32.09375	4407.3143					
	32.12500	4411.6058					
	32.15625	4415.8972					
	32.18750	4420.1886					
	32.21875	4424.4801					
32.2222 S11h	32.22222	4424.9569	**4425**	6	0.0431	0.7%	Ψ(4415)
32.2500 S11h	**32.25000**	4428.7715	**4429**	9	0.2285	2.5%	Ψ(4415)
	32.28125	4433.0630	**4433**	4/2	0.0630	1.6%	Zc(4430)
	32.31250	4437.3544					
	32.34375	4441.6459					
	32.37500	4445.9373					
	32.40625	4450.2288					
	32.43750	4454.5202					
	32.46875	4458.8117					
	32.50000	4463.1031					
	32.53125	4467.3945					
	32.56250	4471.6860					
	32.59375	4475.9774					
	32.62500	4480.2689					
	32.65625	4484.5603					
	32.68750	4488.8518					
	32.71875	4493.1432					
	32.75000	4497.4347					
	32.78125	4501.7261					
32.8125 S11h	32.81250	4506.0176	**4506**	11	0.0176	0.2%	Ψ(4500)
	32.84375	4510.3090					
	32.87500	4514.6004					
	32.90625	4518.8919					
	32.93750	4523.1833					
	32.96875	4527.4748					
	33.00000	4531.7662					

Source of ExpMass and Error data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

36. Ψ(4430)

69 TQ2

TQ2 Factoring / 'c' Tetraquark

TQ2 = 64.88343891 MeV/c^2

Res = (1/32)**TQ2**

	n	(69+ n/32) TQ2	ExpMass	Error	dm	dm/Error	Other
	-16	4444.5156					
	-15	4446.5432					
	-14	4448.5708					
	-13	4450.5984					
	-12	4452.6260					
	-11	4454.6536					
	-10	4456.6812					
	-9	4458.7088					
	-8	4460.7364					
	-7	4462.7640					
	-6	4464.7916					
	-5	4466.8192					
	-4	4468.8469					
	-3	4470.8745					
	-2	4472.9021					
	-1	4474.9297	**4475**	7	0.0703	1.0%	
69.00000 TQ2	0	4476.9573	**Key Mass**				
	0.5	4477.9710	**4478**	15/18	0.0290	0.2%	PDG AVERAGE
	1	4478.9849					
	2	4481.0125					
	3	4483.0401					
69.12500 TQ2	4	4485.0677	**4485**	22	0.0677	0.3%	
	5	4487.0953					
	6	4489.1229					
	7	4491.1505					
	8	4493.1781					
	9	4495.2058					
	10	4497.2334					
	11	4499.2610					
	12	4501.2886					
	13	4503.3162					
	14	4505.3438					
	15	4507.3714					
	16	4509.3990					
	17	4511.4266					
	18	4513.4542					

Source of ExpMass and Error data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

37. Ψ(4500)

S11h Factoring / 'd' Pentaquark

S11h=137.3262492 MeV/c^2

Res = (1/32) S11h

	n	n S11h	ExpMass	Error	dm	dm/Error	Particle
	32.00000	4394.4400					
	32.03125	4398.7314					
	32.06250	4403.0229					
	32.09375	4407.3143					
	32.12500	4411.6058					
	32.15625	4415.8972					
	32.18750	4420.1886					
	32.21875	4424.4801					
32.2222 S11h	**32.22222**	4424.9569	**4425**	6	0.0431	0.7%	Ψ(4415)
32.2500 S11h	**32.25000**	4428.7715	**4429**	9	0.2285	2.5%	Ψ(4415)
	32.28125	4433.0630	**4433**	4/2	0.0630	1.6%	Zc(4430)
	32.31250	4437.3544					
	32.34375	4441.6459					
	32.37500	4445.9373					
	32.40625	4450.2288					
	32.43750	4454.5202					
	32.46875	4458.8117					
	32.50000	4463.1031					
	32.53125	4467.3945					
	32.56250	4471.6860					
	32.59375	4475.9774					
	32.62500	4480.2689					
	32.65625	4484.5603					
	32.68750	4488.8518					
	32.71875	4493.1432					
	32.75000	4497.4347					
	32.78125	4501.7261					
32.8125 S11h	32.81250	4506.0176	**4506**	11	0.0176	0.2%	Ψ(4500)
	32.84375	4510.3090					
	32.87500	4514.6004					
	32.90625	4518.8919					
	32.93750	4523.1833					
	32.96875	4527.4748					
	33.00000	4531.7662					

Source of ExpMass and Error data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

38. Ψ2(4660)

S11h Factoring / 'd' Pentaquark

S11h=137.3262492 MeV/c2

Res = (1/64) **S11h**

		n	n S11h	ExpMass	Error	dm	dm/Error
		33.500000	4600.4293				
		33.515625	4602.5751				
		33.531250	4604.7208				
		33.546875	4606.8665				
		33.562500	4609.0122				
		33.578125	4611.1580				
		33.593750	4613.3037				
		33.609375	4615.4494				
		33.625000	4617.5951				
		33.640625	4619.7409				
		33.656250	4621.8866				
		33.671875	4624.0323				
		-1/512	4625.9098	**4625.9**	6.2/6.0	0.0098	0.2%
	33.6875 S11h	**33.687500**	**4626.1780**				
		33.703125	4628.3237				
		33.718750	4630.4695				
		33.734375	4632.6152				
		-3/512	4633.9562	**4634**	8/7	0.0438	0.5%
	33.7500 S11h	**33.750000**	**4634.7609**				
		33.765625	4636.9066				
		33.781250	4639.0524				
		33.796875	4641.1981				
		33.812500	4643.3438				
		33.828125	4645.4895				
		33.843750	4647.6352				
		33.859375	4649.7810				
	33.8750 S11h	**33.875000**	**4651.9267**	**4652**	10/11	0.0733	0.7%
		+2/512	4652.4631	**4652.5**	3.4/1.1	0.0369	1.1%
		33.890625	4654.0724				
		33.906250	4656.2181				
		33.921875	4658.3639				
		33.937500	4660.5096				
		33.953125	4662.6553				
		33.968750	4664.8010				
		33.984375	4666.9468				
Key Factrng	**34.0000 S11h**	**34.000000**	**4669.0925**	**4669**	21/3	0.0925	0.4%
		34.015625	4671.2382				
		34.031250	4673.3839				
		34.046875	4675.5296				
		34.062500	4677.6754				
		34.078125	4679.8211				
		34.093750	4681.9668				
		34.109375	4684.1125				
		34.125000	4686.2583				
		34.140625	4688.4040				
		34.156250	4690.5497				
		34.171875	4692.6954				
		34.187500	4694.8411				
		34.203125	4696.9869				
		34.218750	4699.1326				
		34.234375	4701.2783				
		34.250000	4703.4240				

Source of ExpMass and Error data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

38. Ψ2(4660)
Commentary

Here are three of the pentaquarks from the previous page shown factored with native pentaquark factoring (d^5h factoring) .

S11h Factoring	d^5h Factoring	Power of Two Parsing
34.0000 S11h = 4669.0925	17 d^5h / 7560	7560 = 4096 + 2048 + 1024 + 256 + 128 +8
33.7500 S11h = 4634.7609	d^5h / 448	448 = 256 + 128 + 64
33.6875 S11h = 4626.1780	77 d^5h / 34560	34560 = 32768 + 1024 + 512 + 256

39. Xco(4700)

72.5 TQ2

TQ2 Factoring / 'c' Tetraquark

TQ2 = 64.88343891 MeV/c^2

Res = (1/32) TQ2

		n	n TQ2	ExpMass	Error	dm	dm/Error
		72.00000	4671.6076				
		72.03125	4673.6352				
		72.06250	4675.6628				
		72.09375	4677.6904				
		72.12500	4679.7180				
		72.15625	4681.7456				
		72.18750	4683.7732				
		72.21875	4685.8009				
		72.25000	4687.8285				
		72.28125	4689.8561				
		72.31250	4691.8837				
		72.34375	4693.9113				
		72.37500	4695.9389				
		72.40625	4697.9665				
		72.43750	4699.9941				
		72.46875	4702.0217				
Key Factrng	**72.5000 TQ2**	**72.50000**	**4704.0493**	**4704**	10	0.0493	0.5%
		72.53125	4706.0769				
		72.56250	4708.1045				
		72.59375	4710.1321				
		72.62500	4712.1598				
		72.65625	4714.1874				
		72.68750	4716.2150				
		72.71875	4718.2426				
		72.75000	4720.2702				
		72.78125	4722.2978				
		72.81250	4724.3254				
		72.84375	4726.3530				
		72.87500	4728.3806				
		72.90625	4730.4082				
		72.93750	4732.4358				
		72.96875	4734.4634				
		73.00000	4736.4910				

Source of ExpMass and Error data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

40. ddddu Pentaquarks

S10h Factoring

S10h = 168.9756582 MeV/c^2

Res = (5 / 90) S10h

	a	b	(a+b/90) S10h	ExpMass	Error	dm	dm/Error	Particle	Ref
	25	0	4224.3915						
	25	5	4233.7790						
	25	10	4243.1665						
	25	15	4252.5541						
	25	20	4261.9416						
	25	25	4271.3291						
	25	30	4280.7167						
	25	35	4290.1042						
	25	40	4299.4917						
	25	45	4308.8793						
	25	50	4318.2668						
	25	55	4327.6544						
25.6666 S10h	25	60	4337.0419	**4337**	7/4	0.0419	0.6%	**Pc(4337)**	**[7]**
	25	65	4346.4294						
	25	70	4355.8170						
	25	75	4365.2045						
	25	80	4374.5920						
	25	85	4383.9796						
	26	**0**	**4393.3671**						
	26	5	4402.7546						
	26	10	4412.1422						
	26	15	4421.5297						
	26	20	4430.9173						
26.2777 S10h	26	25	4440.3048	**4440.3**	1.3	0.0048	0.4%	**Pc(4440)**	**[9]**
26.3333 S10h	26	30	4449.6923	**4449.8**	1.7/2.5	0.1077	6.3%	**Pc(4450)**	**[9]**
	26	35	4459.0799						
	26	40	4468.4674						
	26	45	4477.8549						
	26	50	4487.2425						
	26	55	4496.6300						
26.6666 S10h	26	60	4506.0176	**4506**	11	0.0176	0.2%	**Ψ(4500)**	**[1]**
	26	65	4515.4051						
	26	70	4524.7926						
	26	75	4534.1802						
	26	80	4543.5677						
	26	85	4552.9552						

40. ddddu Pentaquarks
Commentary

Here are three pentaquarks from the previous page shown factored with native pentaquark factoring (d^5h factoring).
(To be strictly consistent with S10h factoring, the native factoring used should be d^4uh. But d^5h and d^4uh differ by only a factor of two, so it makes little difference which is used.)

S10h Factoring	d^5h Factoring	Power of Two Parsing
25.666 S10h = **4337.0419**	$\frac{\mathbf{77}\ d^5h}{25.666\ S10h}$ = 36864	36864 = 32768 + 4096
26.333 S10h = **4449.6923**	$\frac{\mathbf{79}\ d^5h}{26.333\ S10h}$ = 36864	36864 = 32768 + 4096
26.666 S10h = **4506.0175**	$\frac{\mathbf{80}\ d^5h}{26.666\ S10h}$ = 36864	36864 = 32768 + 4096

5. Exotics Among the Charmoniums

Below is a list of the different types of particles found among the charmoniums (quantities are approximate).

Type	Qty	
'd' Octaquarks	2	(tentative - they could be 'c' tetraquarks)
'd' Heptaquarks	2	
'd' Hexaquarks	10	
'd' Pentaquarks	7	
'd' Tetraquarks	3	
'c' Tetraquarks	12	(tentative - they could be 'd' octaquarks)
'c' Diquarks	1	

The most numerous types are the 'c' tetraquarks (12), 'd' hexaquarks (10), and the 'd' pentaquarks (7).

The 'c' Tetraquarks Could Be 'd' Octaquarks

All the charmoniums that factor as 'cccc' tetraquarks can also be factored as 'dddddddd' octaquarks. This is true because the theoretical mass formulae for 'cccc' and 'dddddddd' each have the same power of 'pi' and 'r' in them, $\boldsymbol{\pi^8\ r^{16}}$. The TQ2 equivalent formula for factoring 'd' octaquarks is:

$$(\mathbf{n} / 31{,}752{,}000)\ \mathbf{d^8\,h}$$

Where n is an integer or *power of two* fraction, and, $d = 4\pi r^2$, the theoretical mass of the 'down' quark. It's very interesting that the prime factorization of the number that is key to these factorings, 31,752,000, consists of the first four prime numbers raised to powers.

$$31{,}752{,}000 = 2^6\,3^4\,5^3\,7^2$$

The list below shows that all 'c' tetraquarks that factor using (**n TQ2**), can also be factored using (**n** / 31,752,000) $\mathbf{d^8\,h}$**.**
(The definition of the 'c' tetraquark unit of factorization **TQ2** is: $TQ2 = (S_5)^4 h / 49000$, where S_5 is the surface volume of a 5-sphere: $S_5 = (8/3)\ \pi^2\ r^4$)

'c' Tetraquarks **TQ2** Factoring	'd' Octaquarks $\mathbf{d^8\,h}$ Factoring
56.00 **TQ2** = 3633.472579	(**28.00** / $2^6\,3^4\,5^3\,7^2$) $\mathbf{d^8\,h}$ = 3633.472579
58.00 **TQ2** = 3763.239457	(**29.00** / $2^6\,3^4\,5^3\,7^2$) $\mathbf{d^8\,h}$ = 3763.239457
60.00 **TQ2** = 3893.006335	(**30.00** / $2^6\,3^4\,5^3\,7^2$) $\mathbf{d^8\,h}$ = 3893.006335
62.00 **TQ2** = 4022.773213	(**31.00** / $2^6\,3^4\,5^3\,7^2$) $\mathbf{d^8\,h}$ = 4022.773213
64.00 **TQ2** = 4152.540090	(**32.00** / $2^6\,3^4\,5^3\,7^2$) $\mathbf{d^8\,h}$ = 4152.540090
66.00 **TQ2** = 4282.306968	(**33.00**/ $2^6\,3^4\,5^3\,7^2$) $\mathbf{d^8\,h}$ = 4282.306968
68.00 **TQ2** = 4412.073846	(**34.00**/ $2^6\,3^4\,5^3\,7^2$) $\mathbf{d^8\,h}$ = 4412.073846

Are the 'c' tetraquarks actually 'd' octaquarks? Factoring alone can't make that distinction. Factoring can only say it's one or the other.

7. Summary

The conclusion that should be drawn from the successfull factoring of charmonium masses with hypersphere surface volumes is that matter at the subatomic particle level is very likely higher dimensional.

8. References

[1] P.A. Zyla et al.(Particle Data Group), Prog. Theor. Exp. Phys.2020, 083C01 (2020) and 2021 update

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[3] Exotic charmonium-like states at BESIII, arXiv:1505.04478v1 [hep-ex] 18 May 2015

[4] Explanation of the newly observed Zcs(3985) as a $Ds^{(*)-}D^{(*)0}$ molecular state, arXiv:2011.09404v1 [hep-ex] 18 Nov 2020

[5] Heavy Quarkonium Physics, arXiv: hep-ph/0412158v2 26 Jul 2005

[6] Study on X(3872) from effective field theory with pion exchange interaction, arXiv: 1304.0846v1 [hep-ex] 3 Apr 2013

[7] The Study of $\gamma\gamma \rightarrow \gamma\psi(2S)$ at Belle, arXiv: 2105.06605v3 [hep-ex] 10 Jan 2022

[8] M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018)

[9] Can discovery of hidden charm strange pentaquark states help determine the spins of Pc(4440) and Pc(4457)?, arXiv:2011.07935v1 [hep-ph] 16 Nov 2020

Appendix 1. Hypersphere surface volume formulae

(Dimension 2 - Dimension 21)

Dimension of Sphere	Abbreviation	Surface Formula
2	S2 =	$2\ \pi^{1}\ r^{1}$
3	S3 =	$4\ \pi^{1}\ r^{2}$
4	S4 =	$2\ \pi^{2}\ r^{3}$
5	S5 =	$8/3\ \pi^{2}\ r^{4}$
6	S6 =	$\pi^{3}\ r^{5}$
7	S7 =	$16/15\ \pi^{3}\ r^{6}$
8	S8 =	$1/3\ \pi^{4}\ r^{7}$
9	S9 =	$32/105\ \pi^{4}\ r^{8}$
10	S10 =	$1/12\ \pi^{5}\ r^{9}$
11	S11 =	$64 / 945\ \pi^{5}\ r^{10}$
12	S12 =	$1 / 60\ \pi^{6}\ r^{11}$
13	S13 =	$128 / 10395\ \pi^{6}\ r^{12}$
14	S14 =	$1 / 360\ \pi^{7}\ r^{13}$
15	S15 =	$256 / 135135\ \pi^{7}\ r^{14}$
16	S16 =	$1 / 2520\ \pi^{8}\ r^{15}$
17	S17 =	$512 / 2027025\ \pi^{8}\ r^{16}$
18	S18 =	$1 / 20160\ \pi^{9}\ r^{17}$
19	S19 =	$1024 / 34459425\ \pi^{9}\ r^{18}$
20	S20 =	$1 / 181440\ \pi^{10}\ r^{19}$
21	S21 =	$2048 / 654729075\ \pi^{10}\ r^{20}$

Appendix 2. Quark Assignments
to
Hypersphere Surface Volume Formulae

Dimension	Quark	Corresponding Hypersphere Surface Formula
2	u - up	$2\ \pi^1\ r^1$
3	d - down	$4\ \pi^1\ r^2$
4	s - strange	$2\ \pi^2\ r^3$
5	c - charrm	$8/3\ \pi^2\ r^4$
6	b - bottom	$\pi^3\ r^5$
7	t - top	$16/15\ \pi^3\ r^6$
8	--------	$1/3\ \pi^4\ r^7$
9	--------	$32/105\ \pi^4\ r^8$
10	--------	$1/12\ \pi^5\ r^9$
11	--------	$64 / 945\ \pi^5\ r^{10}$
12	--------	$1 / 60\ \pi^6\ r^{11}$
13	--------	$128 / 10395\ \pi^6\ r^{12}$
14	--------	$1 / 360\ \pi^7\ r^{13}$
15	--------	$256 / 135135\ \pi^7\ r^{14}$
16	--------	$1 / 2520\ \pi^8\ r^{15}$
17	--------	$512 / 2027025\ \pi^8\ r^{16}$
18	--------	$1 / 20160\ \pi^9\ r^{17}$
19	--------	$1024 / 34459425\ \pi^9\ r^{18}$
20	--------	$1 / 181440\ \pi^{10}\ r^{19}$
21	--------	$2048 / 654729075\ \pi^{10}\ r^{20}$

Appendix 3. Values of Hypersphere Surface Volume Units of Factorization

Unit of Factorization		Value
S2 h =	$2\ \pi^1 r^1 h$ =	41.63282661
S3 h =	$4\ \pi^1 r^2 h$ =	83.26565322
S4h =	$2\ \pi^2 r^3 h$ =	130.7933822
S5h =	$8/3\ \pi^2 r^4 h$ =	174.3911763
S6h =	$\pi^3 r^5 h$ =	205.4497644
S7h =	$16/15\ \pi^3 r^6 h$ =	219.1464153
S8h =	$1/3\ \pi^4 r^7 h$ =	215.1464901
S9h =	$32/105\ \pi^4 r^8 h$ =	196.7053624
S10h =	$1/12\ \pi^5 r^9 h$ =	168.9756582
S11h =	$64 / 945\ \pi^5 r^{10} h$ =	137.3262492
S12h =	$1 / 60\ \pi^6 r^{11} h$ =	106.1705373
S13h =	$128 / 10395\ \pi^6 r^{12} h$ =	78.44057013
S14h =	$1 / 360\ \pi^7 r^{13} h$ =	55.59076334
S15h =	$256 / 135135\ \pi^7 r^{14} h$ =	37.91204905
S16h =	$1 / 2520\ \pi^8 r^{15} h$ =	24.94907624
S17h =	$512 / 2027025\ \pi^8 r^{16} h$ =	15.88056197
S18h =	$1 / 20160\ \pi^9 r^{17} h$ =	9.797479330
S19h =	$1024 / 34459425\ \pi^9 r^{18} h$ =	5.869441980
S20h =	$1 / 181440\ \pi^{10} r^{19} h$ =	3.419965454
S21h =	$2048 / 654729075\ \pi^{10} r^{20} h$ =	1.940989032

Note: h = 6.62607015
r = 1.00000000

Appendix 4. Meson Quark Content

If a meson's mass is divisible by S4h then it has quark content S3S2 or du. Thats what the first line of the table below says.
If a meson's mass is divisible by S5h then it has quark content S3S3 or dd. Thats what the second line of the table says, etc.

If meson factors with	It has quark content	Because			
S4h	S3 S2 = du	**S3 S2** = $(4\pi r^2)(2\pi r)$	=	c **S4**	(c = 4)
S5h	S3 S3 = dd	**S3 S3** = $(4\pi r^2)(4\pi r^2)$	=	c **S5**	(c = 6)
S6h	S3 S4 = ds	**S3 S4** = $(4\pi r^2)(2\pi^2 r^3)$	=	c **S6**	(c = 8)
	S2 S5 = uc	**S2 S5** = $(2\pi r)(8/3\ \pi^2 r^4)$	=	c **S6**	(c = 16/3)
S7h	S3 S5 = dc	**S3 S5** = $(4\pi r^2)(8/3\ \pi^2 r^4)$	=	c **S7**	(c = 10)
S8h	S5 S4 = cs	**S5 S4** = $(8/3\ \pi^2 r^4)(2\pi^2 r^3)$	=	c **S8**	(c = 16)
	S6 S3 = bd	**S6 S3** = $(\pi^3 r^5)(4\pi r^2)$	=	c **S8**	(c = 4)
S9h	S5 S5 = cc	**S5 S5** = $(8/3\ \pi^2 r^4)(8/3\ \pi^2 r^4)$	=	c **S9**	(c = 70/3)
	S7 S3 = td	**S7 S3** = $(16/15\ \pi^3 r^6)(4\pi r^2)$	=	c **S9**	(c = 64/15)
S10h	S5 S6 = cb	**S5 S6** = $(8/3\ \pi^2 r^4)(\pi^3 r^5)$	=	c **S10**	(c = 32)
S11h	S5 S7 = ct	**S5 S7** = $(8/3\ \pi^2 r^4)(16/15\ \pi^3 r^6)$	=	c **S11**	(c = 42)
S12h	S6 S7 = bt	**S6 S7** = $(\pi^3 r^5)(16/15\ \pi^3 r^6)$	=	c **S12**	(c = 64)
S13h	S7 S7 = t t	**S7 S7** = $(16/15\ \pi^3 r^6)(16/15\ \pi^3 r^6)$	=	c **S13**	(c = 92.4)
S14h	S7 S8	**S7 S8** = $(16/15\ \pi^3 r^6)(1/3\ \pi^4 r^7)$	=	c **S14**	(c = 128)
	S9 S6	**S9 S6** = $(32/105\ \pi^4 r^8)(\pi^3 r^5)$	=	c **S14**	(c = 768/7)
S15h	S7 S9	**S7 S9** = $(16/15\ \pi^3 r^6)(32/105\ \pi^4 r^8)$	=	c **S15**	(c = 171.6)
S16h	S8 S9	**S8 S9** = $(1/3\ \pi^4 r^7)(32/105\ \pi^4 r^8)$	=	c **S16**	(c = 256)
	S10 S7	**S10 S7** = $(1/12\ \pi^5 r^9)(16/15\ \pi^3 r^6)$	=	c **S16**	(c = 224)
	S11 S6	**S11 S6** = $(64/945\ \pi^5 r^{10})(\pi^3 r^5)$	=	c **S16**	(c = 512/3)
S17h	S9 S9	**S9 S9** = $(32/105\ \pi^4 r^8)(32/105\ \pi^4 r^8)$	=	c **S17**	(c = 2574/7)
S18h	S9 S10	**S9 S10** = $(32/105\ \pi^4 r^8)(1/12\ \pi^5 r^9)$	=	c **S18**	(c = 19305/192)

Appendix 5. d Multiquark Units of Factorization

MQ	Name	Derivation	Expression	Value of Unit (10 digits)
2	d Diquark	$d^2 h = (4\pi^1 r^2)^2 h =$	$16\pi^2 r^4 h =$	1,046.347058 MeVc2
3	d Triquark	$d^3 h = (4\pi^1 r^2)^3 h =$	$64\pi^3 r^6 h =$	13,148.78492 MeVc2
4	d Tetraquark	$d^4 h = (4\pi^1 r^2)^4 h =$	$256\pi^4 r^8 h =$	165,232.5044 MeVc2
5	d Pentaquark	$d^5 h = (4\pi^1 r^2)^5 h =$	$1024\pi^5 r^{10} h =$	2,076,372.888 MeVc2
6	d Hexaquark	$d^6 h = (4\pi^1 r^2)^6 h =$	$4096\pi^6 r^{12} h =$	26,092,471.25 MeVc2
7	d Heptaquark	$d^7 h = (4\pi^1 r^2)^7 h =$	$16384\pi^7 r^{14} h =$	327,887,663.9 MeVc2
8	d Octaquark	$d^8 h = (4\pi^1 r^2)^8 h =$	$65536\pi^8 r^{16} h =$	4,120,357,905 MeVc2

Appendix 6. cccc Tetraquark Units of Factorization

$$TQ = c^4 h = (S_5)^4 h = ((8/3)\pi^2 r^4)^4 h = 3{,}179{,}288.507 \text{ MeVc2}$$

$$TQ1 = \frac{c^4 h}{7000} = \frac{(S_5)^4 h}{7000} = \frac{((8/3)\pi^2 r^4)^4 h}{7000} = 454.1840724 \text{ MeVc2}$$

$$TQ2 = \frac{c^4 h}{7^2(1000)} = \frac{(S_5)^4 h}{7^2(1000)} = \frac{((8/3)\pi^2 r^4)^4 h}{7^2(1000)} = 64.88343891 \text{ MeVc2}$$

$$TQ3 = \frac{c^4 h}{7^3(1000)} = \frac{(S_5)^4 h}{7^3(1000)} = \frac{((8/3)\pi^2 r^4)^4 h}{7^3(1000)} = 9.269062702 \text{ MeVc2}$$

$$TQ4 = \frac{c^4 h}{7^4(100)} = \frac{(S_5)^4 h}{7^4(100)} = \frac{((8/3)\pi^2 r^4)^4 h}{7^4(100)} = 13.24151815 \text{ MeVc2}$$

$$TQ5 = \frac{c^4 h}{7^5(10)} = \frac{(S_5)^4 h}{7^5(10)} = \frac{((8/3)\pi^2 r^4)^4 h}{7^5(10)} = 18.91645449 \text{ MeVc2}$$

Appendix 7. d Multiquarks' Corresponding Surface Volume Formulae

MQ		Corresponding Surface Formula
		$S2 = 2\ \pi^1 r^1$
		$S3 = 4\ \pi^1 r^2$
		$S4 = 2\ \pi^2 r^3$
2d	d Diquark $= 16\ \pi^2 r^4 h$	$S5 = 8/3\ \pi^2 r^4$
		$S6 = \pi^3 r^5$
3d	d Triquark $= 64\ \pi^3 r^6 h$	$S7 = 16/15\ \pi^3 r^6$
		$S8 = 1/3\ \pi^4 r^7$
4d	d Tetraquark $= 256\ \pi^4 r^8 h$	$S9 = 32/105\ \pi^4 r^8$
		$S10 = 1/12\ \pi^5 r^9$
5d	d Pentaquark $= 1024\ \pi^5 r^{10} h$	$S11 = 64 / 945\ \pi^5 r^{10}$
		$S12 = 1 / 60\ \pi^6 r^{11}$
6d	d Hexaquark $= 4096\ \pi^6 r^{12} h$	$S13 = 128 / 10395\ \pi^6 r^{12}$
		$S14 = 1 / 360\ \pi^7 r^{13}$
7d	d Heptaquark $= 16384\ \pi^7 r^{14} h$	$S15 = 256 / 135135\ \pi^7 r^{14}$
		$S16 = 1 / 2520\ \pi^8 r^{15}$
8d	d Octaquark $= 65536\ \pi^8 r^{16} h$	$S17 = 512 / 2027025\ \pi^8 r^{16}$
		$S18 = 1 / 20160\ \pi^9 r^{17}$
		$S19 = 1024 / 34459425\ \pi^9 r^{18}$