Hadrons As Helmholtzian Factorizations

The Helmholtzian operator factorization is:

$$\mathbf{J} \equiv D_B D_A \mathbf{f} = ((\Box - |m|^2))\mathbf{f}$$

where:

$$D_B \equiv \begin{pmatrix} D_0 & D_3^{\Leftrightarrow} & -D_2^{\Leftrightarrow} & -D_1 \\ -D_3^{\Leftrightarrow} & D_0 & D_1^{\Leftrightarrow} & -D_2 \\ D_2^{\Leftrightarrow} & -D_1^{\Leftrightarrow} & D_0 & -D_3 \\ -D_1^{\Updownarrow} & -D_2^{\Updownarrow} & -D_3^{\Updownarrow} & D_0^{\Updownarrow} \end{pmatrix} \quad \& \quad D_A \equiv \begin{pmatrix} D_0^{\Updownarrow} & -D_3^{\Leftrightarrow} & D_2^{\Leftrightarrow} & -D_1 \\ D_3^{\Leftrightarrow} & D_0^{\Updownarrow} & -D_1^{\Leftrightarrow} & -D_2 \\ -D_2^{\Leftrightarrow} & D_1^{\Leftrightarrow} & D_0^{\Updownarrow} & -D_3 \\ -D_1^{\Updownarrow} & -D_2^{\Updownarrow} & -D_3^{\Updownarrow} & D_0 \end{pmatrix}$$

and:

$$D_i^+ \equiv (\partial_i + m_i) \quad , \quad D_i^- \equiv (\partial_i - m_i)$$

$$D_i \equiv \begin{pmatrix} D_i^+ & 0 \\ 0 & D_i^- \end{pmatrix} \; , \; D_i^{\Updownarrow} \equiv \begin{pmatrix} D_j^- & 0 \\ 0 & D_i^+ \end{pmatrix} \; , \; D_i^{\Leftrightarrow} \equiv \begin{pmatrix} 0 & D_i^- \\ D_i^+ & 0 \end{pmatrix} \; , \; D_i^{\Leftrightarrow\Updownarrow} \equiv \begin{pmatrix} 0 & D_i^+ \\ D_j^- & 0 \end{pmatrix}$$

and:

$$|m|^2 \equiv \sum_{j=0}^{4-1} m_j^2$$

and:

$$\mathbf{f} \equiv \begin{pmatrix} f^1 \\ f^2 \\ f^3 \\ f^0 \end{pmatrix} \quad , \qquad f^j \equiv \begin{pmatrix} f_+^j \\ f_-^j \end{pmatrix}$$

$$f_+ \equiv \begin{pmatrix} \begin{pmatrix} f_+^1 \\ 0 \end{pmatrix} \\ \begin{pmatrix} f_+^2 \\ 0 \end{pmatrix} \\ \begin{pmatrix} f_+^3 \\ 0 \end{pmatrix} \\ \begin{pmatrix} f_+^0 \\ 0 \end{pmatrix} \end{pmatrix} \; , \; f_- \equiv \begin{pmatrix} \begin{pmatrix} 0 \\ f_-^1 \end{pmatrix} \\ \begin{pmatrix} 0 \\ f_-^2 \end{pmatrix} \\ \begin{pmatrix} 0 \\ f_-^3 \end{pmatrix} \\ \begin{pmatrix} 0 \\ f_-^0 \end{pmatrix} \end{pmatrix} \; , \; f \equiv \begin{pmatrix} \begin{pmatrix} f_+^1 \\ f_-^1 \end{pmatrix} \\ \begin{pmatrix} f_+^2 \\ f_-^2 \end{pmatrix} \\ \begin{pmatrix} f_+^3 \\ f_-^3 \end{pmatrix} \\ \begin{pmatrix} f_+^0 \\ f_-^0 \end{pmatrix} \end{pmatrix} = f_+ + f_-$$

$$\Rightarrow \begin{pmatrix} -D_0 & D_3^{\Leftrightarrow} & -D_2^{\Leftrightarrow} & -D_1 \\ -D_3^{\Leftrightarrow} & -D_0 & D_1^{\Leftrightarrow} & -D_2 \\ D_2^{\Leftrightarrow} & -D_1^{\Leftrightarrow} & -D_0 & -D_3 \\ -D_1^{\Updownarrow} & -D_2^{\Updownarrow} & -D_3^{\Updownarrow} & D_0^{\Updownarrow} \end{pmatrix} \begin{pmatrix} -D_0^{\Updownarrow} & -D_3^{\Leftrightarrow} & D_2^{\Leftrightarrow} & -D_1 \\ D_3^{\Leftrightarrow} & -D_0^{\Updownarrow} & -D_1^{\Leftrightarrow} & -D_2 \\ -D_2^{\Leftrightarrow} & D_1^{\Leftrightarrow} & -D_0^{\Updownarrow} & -D_3 \\ -D_1^{\Updownarrow} & -D_2^{\Updownarrow} & -D_3^{\Updownarrow} & D_0 \end{pmatrix} \begin{pmatrix} f^1 \\ f^2 \\ f^3 \\ f^0 \end{pmatrix} = D_B D_A \mathbf{f}$$

$$= \begin{pmatrix} -D_0 & D_3^{\Leftrightarrow} & -D_2^{\Leftrightarrow} & -D_1 \\ -D_3^{\Leftrightarrow} & -D_0 & D_1^{\Leftrightarrow} & -D_2 \\ D_2^{\Leftrightarrow} & -D_1^{\Leftrightarrow} & -D_0 & -D_3 \\ -D_1^{\Updownarrow} & -D_2^{\Updownarrow} & -D_3^{\Updownarrow} & D_0^{\Updownarrow} \end{pmatrix} \begin{pmatrix} B_{\Updownarrow}^1 + E^1 \\ B_{\Updownarrow}^2 + E^2 \\ B_{\Updownarrow}^3 + E^3 \\ -\overset{m*}{\nabla}_{\Updownarrow} \cdot \mathbf{f} \end{pmatrix} = D_B(\mathbf{B}_{\Updownarrow} + \mathbf{E})$$

Note:

$$D_A \mathbf{f} = (\mathbf{B}_{\Updownarrow} + \mathbf{E}) \quad \& \quad D_B \mathbf{f} = (\mathbf{B}_{\Updownarrow} + \mathbf{E}_{\Updownarrow})$$

Further:

$$\widetilde{D_B} \equiv \begin{pmatrix} -D_0^{\Updownarrow} & D_3^{\Leftrightarrow} & -D_2^{\Leftrightarrow} & -D_1 \\ -D_3^{\Leftrightarrow} & -D_0^{\Updownarrow} & D_1^{\Leftrightarrow} & -D_2 \\ D_2^{\Leftrightarrow} & -D_1^{\Leftrightarrow} & -D_0^{\Updownarrow} & -D_3 \\ -D_1^{\Updownarrow} & -D_2^{\Updownarrow} & -D_3^{\Updownarrow} & D_0 \end{pmatrix} \quad \& \quad \widetilde{D_A} \equiv \begin{pmatrix} -D_0 & -D_3^{\Leftrightarrow} & D_2^{\Leftrightarrow} & -D_1 \\ D_3^{\Leftrightarrow} & -D_0 & -D_1^{\Leftrightarrow} & -D_2 \\ -D_2^{\Leftrightarrow} & D_1^{\Leftrightarrow} & -D_0 & -D_3 \\ -D_1^{\Updownarrow} & -D_2^{\Updownarrow} & -D_3^{\Updownarrow} & D_0^{\Updownarrow} \end{pmatrix}$$

(ie.: $\widetilde{D_B}$ & $\widetilde{D_A}$ are transformations of D_B & D_A where D_0 & $D_0^{\Updownarrow}$ are exchanged)

$$\Rightarrow \widetilde{D_B}\widetilde{D_A} = \widetilde{D_A}\widetilde{D_B} = [\Box - |m|^2] \begin{pmatrix} \mathbf{I}_2 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_2 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}_2 \end{pmatrix} = D_B D_A = D_A D_B$$

and:

$$\widetilde{D_B}\mathbf{f} = \begin{pmatrix} -D_0^{\Updownarrow} & D_3^{\Leftrightarrow} & -D_2^{\Leftrightarrow} & -D_1 \\ -D_3^{\Leftrightarrow} & -D_0^{\Updownarrow} & D_1^{\Leftrightarrow} & -D_2 \\ D_2^{\Leftrightarrow} & -D_1^{\Leftrightarrow} & -D_0^{\Updownarrow} & -D_3 \\ -D_1^{\Updownarrow} & -D_2^{\Updownarrow} & -D_3^{\Updownarrow} & D_0 \end{pmatrix} \begin{pmatrix} f^1 \\ f^2 \\ f^3 \\ f^0 \end{pmatrix} = \begin{pmatrix} -D_0^{\Updownarrow}f^1 + D_3^{\Leftrightarrow}f^2 - D_2^{\Leftrightarrow}f^3 - D_1f^0 \\ -D_3^{\Leftrightarrow}f^1 - D_0^{\Updownarrow}f^2 + D_1^{\Leftrightarrow}f^3 - D_2f^0 \\ D_2^{\Leftrightarrow}f^1 - D_1^{\Leftrightarrow}f^2 - D_0^{\Updownarrow}f^3 - D_3f^0 \\ -D_1^{\Updownarrow}f^1 - D_2^{\Updownarrow}f^2 - D_3^{\Updownarrow}f^3 + D_0f^0 \end{pmatrix}$$

$$= \begin{pmatrix} E^1 - B^1_{\Updownarrow} \\ E^2 - B^2_{\Updownarrow} \\ E^3 - B^3_{\Updownarrow} \\ -D_1^{\Updownarrow}f^1 - D_2^{\Updownarrow}f^2 - D_3^{\Updownarrow}f^3 + D_0f^0 \end{pmatrix} = \mathbf{E} - \mathbf{B}_{\Updownarrow}$$

Concerning the fundamental particles:

As shown in [2], a fundamental object is a vector components of which are mass-generalzed electromagnetic field components, classified as follows:

$$e(i) \equiv \overline{\alpha_i} = \overline{(E^1,E^2,E^3)}_i \quad , \quad \nu(i) \equiv \beta_i \equiv (B^1,B^2,B^3)_i$$

$$u_j(i) \equiv \phi_{ji} = (\eta_{j-1}(E^1),\eta_{j-2}(E^2),\eta_{j-3}(E^3))_i \ , \ d_j(i) \equiv \overline{\psi_{ji}} = \overline{(\eta_{j-1}(B^1),\eta_{j-2}(B^1),\eta_{j-3}(B^1))}_i$$

where: :

$$\eta_j(R^h_k) \equiv \begin{cases} R^h_k & , \ j \neq 0 \\ E^h_k & , \ j = 0 \ , \ \mathbf{R} = \mathbf{B} \\ B^h_k & , \ j = 0 \ , \ \mathbf{R} = \mathbf{E} \end{cases} \quad , \ \sigma_j(\mathbf{R}_k) \equiv \begin{pmatrix} \eta_{j-1}(R^1_k) \\ \eta_{j-2}(R^2_k) \\ \eta_{j-3}(R^3_k) \end{pmatrix}$$

(i denoting generation, j denoting color)

where:

$$\mathbf{E} = \left(\left(-D_0^{\Updownarrow}f^1 - D_1f^0\right),\left(-D_0^{\Updownarrow}f^2 - D_2f^0\right),\left(-D_0^{\Updownarrow}f^3 - D_3f^0\right),*\right)$$

$$\mathbf{B} = ((D_2f^3 - D_3f^2),(-D_1f^3 + D_3f^1),(D_1f^2 - D_2f^1),*)$$

$$\mathbf{E}_{\Updownarrow} = \left(\left(-D_0^{\Leftrightarrow\Updownarrow}f^1 - D_1^{\Leftrightarrow}f^0\right),\left(-D_0^{\Leftrightarrow\Updownarrow}f^2 - D_2^{\Leftrightarrow}f^0\right),\left(-D_0^{\Leftrightarrow\Updownarrow}f^3 - D_3^{\Leftrightarrow}f^0\right),*\right)$$

$$\mathbf{B}_{\Updownarrow} = ((D_2^{\Leftrightarrow}f^3 - D_3^{\Leftrightarrow}f^2),(-D_1^{\Leftrightarrow}f^3 + D_3^{\Leftrightarrow}f^1),(D_1^{\Leftrightarrow}f^2 - D_2^{\Leftrightarrow}f^1),*)$$

(where $*$ denotes a guage component)

$$\Rightarrow \mathbf{E} = \begin{pmatrix} -D_0^{\Updownarrow} & 0 & 0 & -D_1 \\ 0 & -D_0^{\Updownarrow} & 0 & -D_2 \\ 0 & 0 & -D_0^{\Updownarrow} & -D_3 \\ -D_1^{\Updownarrow} & -D_2^{\Updownarrow} & -D_3^{\Updownarrow} & D_0 \end{pmatrix} \begin{pmatrix} f^1 \\ f^2 \\ f^3 \\ f^0 \end{pmatrix} \ , \ \mathbf{B}_{\Updownarrow} = \begin{pmatrix} 0 & -D_3^{\Leftrightarrow} & D_2^{\Leftrightarrow} & 0 \\ D_3^{\Leftrightarrow} & 0 & -D_1^{\Leftrightarrow} & \\ -D_2^{\Leftrightarrow} & D_1^{\Leftrightarrow} & 0 & 0 \\ -D_1^{\Updownarrow} & -D_2^{\Updownarrow} & -D_3^{\Updownarrow} & D_0 \end{pmatrix} \begin{pmatrix} f^1 \\ f^2 \\ f^3 \\ f^0 \end{pmatrix}$$

so: $\mathbf{E} = D_A\mathbf{f}$ & $\mathbf{B}_{\Updownarrow} = D_A\mathbf{f}$

So, in particular (written horizontally as vectors for brevity):

$e^- = e(1) = \overline{(E^1,E^2,E^3)}_1$	$\mu^- = e(2) = \overline{(E^1,E^2,E^3)}_2$	$\tau^- = e(3) = \overline{(E^1,E^2,E^3)}_3$
$\nu_e = \nu(1) = \left(B^1_{\Updownarrow},B^2_{\Updownarrow},B^3_{\Updownarrow}\right)_1$	$\nu_\mu = \nu(2) = \left(B^1_{\Updownarrow},B^2_{\Updownarrow},B^3_{\Updownarrow}\right)_2$	$\nu_\tau = \nu(3) = \left(B^1_{\Updownarrow},B^2_{\Updownarrow},B^3_{\Updownarrow}\right)_3$
$u_R = u_1(1) = \left(B^1_{\Updownarrow},E^2,E^3\right)_1$	$c_R = u_1(2) = \left(B^1_{\Updownarrow},E^2,E^3\right)_2$	$t_R = u_1(3) = \left(B^1_{\Updownarrow},E^2,E^3\right)_3$
$u_G = u_2(1) = \left(E^1,B^2_{\Updownarrow},E^3\right)_1$	$c_G = u_2(2) = \left(E^1,B^2_{\Updownarrow},E^3\right)_2$	$t_G = u_2(3) = \left(E^1,B^2_{\Updownarrow},E^3\right)_3$
$u_B = u_3(1) = \left(E^1,E^2,B^3_{\Updownarrow}\right)_1$	$c_B = u_3(2) = \left(E^1,E^2,B^3_{\Updownarrow}\right)_2$	$t_B = u_3(3) = \left(E^1,E^2,B^3_{\Updownarrow}\right)_3$
$d_R = d_1(1) = \overline{\left(E^1,B^2_{\Updownarrow},B^3_{\Updownarrow}\right)}_1$	$s_R = d_1(2) = \overline{\left(E^1,B^2_{\Updownarrow},B^3_{\Updownarrow}\right)}_2$	$b_R = d_1(3) = \overline{(E^1,B^2,B^3)}_3$
$d_G = d_2(1) = \overline{\left(B^1_{\Updownarrow},E^2,B^3_{\Updownarrow}\right)}_1$	$s_G = d_2(2) = \overline{\left(B^1_{\Updownarrow},E^2,B^3\right)}_2$	$b_G = d_2(3) = \overline{\left(E^1,B^2_{\Updownarrow},B^3_{\Updownarrow}\right)}_3$
$d_B = d_3(1) = \overline{\left(B^1_{\Updownarrow},B^2_{\Updownarrow},E^3\right)}_1$	$s_B = d_3(2) = \overline{\left(B^1_{\Updownarrow},B^2_{\Updownarrow},E^3\right)}_2$	$b_B = d_3(3) = \overline{\left(B^1_{\Updownarrow},B^2_{\Updownarrow},E^3\right)}_3$

Denoting quark types (u,d) , colors $(1,0,-1)$ & flavours $(1,2,3)$.

(The associated anti-fermion has negaive charge & color of it's counterpart.)

This may be simplified into a purely mathematical data structure
(especially since Left & Right neutrinos have different characteristics):

$\nu_{e_R} = \nu(1) = f\left(0,+1,-1,-1\right)$	$\nu_{\mu_R} = \nu(2) = f\left(0,+1,-1,\ 0\right)$	$\nu_{\tau_R} = \nu(3) = f\left(0,+1,-1,+1\right)$
$e^- = e(1) = f\left(0,-1,\ 0,-1\right)$	$\mu^- = e(2) = f\left(0,-1,\ 0,\ 0\right)$	$\tau^- = e(3) = f\left(0,-1,\ 0,+1\right)$
$\nu_{e_L} = \nu(1) = f\left(0,+1,+1,-1\right)$	$\nu_{\mu_L} = \nu(2) = f\left(0,+1,+1,\ 0\right)$	$\nu_{\tau_L} = \nu(3) = f\left(0,+1,+1,+1\right)$
$u_R = u_1(1) = f(+1,-1,-1,-1)$	$c_R = u_1(2) = f\left(+1,-1,-1,\ 0\right)$	$t_R = u_1(3) = f(+1,-1,-1,+1)$
$u_G = u_0(1) = f\left(+1,-1,\ 0,-1\right)$	$c_G = u_0(2) = f\left(+1,-1,\ 0,\ 0\right)$	$t_G = u_0(3) = f\left(+1,-1,\ 0,+1\right)$
$u_B = u_{-1}(1) = f(+1,-1,+1,-1)$	$c_B = u_{-1}(2) = f\left(+1,-1,+1,\ 0\right)$	$t_B = u_{-1}(3) = f\left(+1,-1,\ 1,+1\right)$
$d_R = d_1(1) = f(+1,+1,-1,-1)$	$s_R = d_1(2) = f\left(+1,+1,-1,\ 0\right)$	$b_R = d_1(3) = f(+1,+1,-1,+1)$
$d_G = d_0(1) = f\left(+1,+1,\ 0,-1\right)$	$s_G = d_0(2) = f\left(+1,+1,\ 0,\ 0\right)$	$b_G = d_0(3) = f\left(+1,+1,\ 0,+1\right)$
$d_B = d_{-1}(1) = f(+1,+1,+1,-1)$	$s_B = d_{-1}(2) = f\left(+1,+1,+1,\ 0\right)$	$b_B = d_{-1}(3) = f(+1,+1,+1,+1)$

For: $f(x_1,x_2,x_3,x_4)$:

$x_1 = \begin{cases} 0: & \text{lepton} \\ 1: & \text{quark} \end{cases}$	$x_2 = \begin{cases} -1: & \text{up} \\ 1: & \text{down} \end{cases}$
$x_3 = color = \begin{cases} -1: & \text{R} \\ 0: & \text{G} \\ 1: & \text{B} \end{cases}$	$x_4 = generation = \begin{cases} -1: \\ 0: \\ 1: \end{cases}$

Now, if a function $c()$ is defined simply by:

$$c((R^1,R^2,R^3)_h) = c(R^1_h) + c(R^2_h) + c(R^3_h),$$
$$c\left(\overline{R^i_h}\right) = -c(R^i_h),$$
$$c(E^i_h) = x,$$
$$c(B^i_h) = y,.$$

then the objects are:

$$c(e(i)) = -3x, c(\nu(i)) = 3y, c(u_j(i)) = 2x + y, c(d_j(i)) = -(x + 2y)$$

Calibrating this with: $-1 = c(e(1)) = -3x, 0 = c(\nu(1)) = 3y \Rightarrow x = \frac{1}{3}, y = 0$

Operating this linear function on the objects, yields:

$$c(e(i)) = -1, c(\nu(i)) = 0$$
$$c(u_j(i)) = \tfrac{2}{3}, c(d_j(i)) = -\tfrac{1}{3}$$
$$\left(E^h \rightrightarrows +\tfrac{1}{3}\ \ \&\ \ B^h_{\Updownarrow} \rightrightarrows 0\right)$$

These correspond to the charge characteristics of all the fermions.

Hadrons are combinations of the fermions.
The possible combinations of fermions are:

- a quark-anti-quark pair of the same color index called mesons, and
- a quark triplet made up of one-and-only-one of each of the color indices.

The reason why only these combinations are allowed is because the component-wise addition of these, and only these combinations result form as ordinary fermions (and as a simple Helmholtzian factorization).

Mesons:

All the possible quark-anti-quark pairs are given by:

$u_0(h) : \overline{u}_0(j)$	$u_1(h) : \overline{u}_1(j)$	$u_{-1}(h) : \overline{u}_{-1}(j)$
$d_0(h) : \overline{d}_0(j)$	$d_1(h) : \overline{d}_1(j)$	$d_{-1}(h) : \overline{d}_{-1}(j)$

$$\|$$

$$\boxed{f(1,x_{20},x_3,x_{40}) : \overline{f(1,x_{21},x_3,x_{41})}}$$

For example:

$$u_R = u_1(1) = (B^1,E^2,E^3)_1 \quad ; \quad d_R = d_1(1) = \overline{(E^1,B^2,B^3)_1}$$

$$u_R = u_1(1) = \left(\begin{pmatrix} B^1_{\Updownarrow} \\ E^2 \\ E^3 \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0}\right) \quad ; \quad \overline{d_R} = \overline{d_1(1)} = \left(\begin{pmatrix} E^1 \\ B^2_{\Updownarrow} \\ B^3_{\Updownarrow} \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0}\right)$$

$$\Rightarrow u_R : \overline{d_R} = \left(\begin{pmatrix} B^1_{\Updownarrow} + E^1 \\ B^2_{\Updownarrow} + E^2 \\ B^3_{\Updownarrow} + E^3 \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0}\right) = \left(\begin{pmatrix} (B_{\Updownarrow} + E)^1 \\ (B_{\Updownarrow} + E)^2 \\ (B_{\Updownarrow} + E)^3 \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0}\right) = (\mathbf{B}_{\Updownarrow} + \mathbf{E})_1 = \pi^+ .$$

$$= \begin{pmatrix} -D_0 & -D_3^{\Leftrightarrow} & D_2^{\Leftrightarrow} & -D_1 \\ D_3^{\Leftrightarrow} & -D_0 & -D_1^{\Leftrightarrow} & -D_2 \\ -D_2^{\Leftrightarrow} & D_1^{\Leftrightarrow} & -D_0 & -D_3 \\ -D_1^{\Updownarrow} & -D_2^{\Updownarrow} & -D_3^{\Updownarrow} & D_0 \end{pmatrix} \left(\begin{pmatrix} f^1 \\ f^2 \\ f^3 \\ f^0 \end{pmatrix}, \mathbf{0}, \mathbf{0} \right) = D_A(\mathbf{f}, \mathbf{0}, \mathbf{0})$$

$$\Rightarrow J(u_R : \overline{d_R}) = ((\Box - |m|^2))(\mathbf{f}, \mathbf{0}, \mathbf{0}) = D_B D_A(\mathbf{f}, \mathbf{0}, \mathbf{0}) = D_B\left((\mathbf{B}_{\Updownarrow} + \mathbf{E})_1 \right) = D_B(\pi^+)$$

$u_B = u_3(1) = (E^1, E^2, B^3)_1 \quad ; \quad d_B = d_3(1) = \overline{(B^1, B^2, E^3)}_1$

$$\overline{u_B} = \overline{u_3(1)} = \left(\begin{pmatrix} \overline{E^1} \\ \overline{E^2} \\ \overline{B^3_{\Updownarrow}} \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0} \right) \quad ; \quad d_B = d_3(1) = \left(\begin{pmatrix} \overline{B^1_{\Updownarrow}} \\ \overline{B^2_{\Updownarrow}} \\ \overline{E^3} \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0} \right)$$

$$\Rightarrow \overline{u_B} : d_B = \left(\begin{pmatrix} \overline{E^1} + \overline{B^1_{\Updownarrow}} \\ \overline{E^2} + \overline{B^2_{\Updownarrow}} \\ \overline{B^3_{\Updownarrow}} + \overline{E^3} \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0} \right) = \left(\begin{pmatrix} \overline{(B_{\Updownarrow} + E)}^1 \\ \overline{(B_{\Updownarrow} + E)}^2 \\ \overline{(B_{\Updownarrow} + E)}^3 \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0} \right) = (\overline{\mathbf{B}_{\Updownarrow} + \mathbf{E}})_1 = \pi^- .$$

$$= \left(\begin{pmatrix} -D_0 & -D_3^{\Leftrightarrow} & D_2^{\Leftrightarrow} & -D_1 \\ D_3^{\Leftrightarrow} & -D_0 & -D_1^{\Leftrightarrow} & -D_2 \\ -D_2^{\Leftrightarrow} & D_1^{\Leftrightarrow} & -D_0 & -D_3 \\ -D_1^{\Updownarrow} & -D_2^{\Updownarrow} & -D_3^{\Updownarrow} & D_0 \end{pmatrix} \overline{\begin{pmatrix} f^1 \\ f^2 \\ f^3 \\ f^0 \end{pmatrix}}, \mathbf{0}, \mathbf{0} \right) = D_A \overline{(\mathbf{f}, \mathbf{0}, \mathbf{0})}$$

$$\Rightarrow J(\overline{u_B} : d_B) = ((\Box - |m|^2))\overline{(\mathbf{f}, \mathbf{0}, \mathbf{0})} = D_B D_A \overline{(\mathbf{f}, \mathbf{0}, \mathbf{0})} = D_B\left(\overline{(\mathbf{B}_{\Updownarrow} + \mathbf{E})}_1 \right) = D_B(\pi^-)$$

$d_G = d_2(1) = \overline{\left(B^1_{\Updownarrow}, E^2, B^3_{\Updownarrow} \right)}_1 \quad ; \quad c_G = u_2(2) = \left(E^1, B^2_{\Updownarrow}, E^3 \right)_2$

$$\Rightarrow d_G : \overline{c_G} = \left(\begin{pmatrix} \overline{B^1_{\Updownarrow}} \\ \overline{E^2} \\ \overline{B^3_{\Updownarrow}} \\ * \end{pmatrix}, \begin{pmatrix} \overline{E^1} \\ \overline{B^2_{\Updownarrow}} \\ \overline{E^3} \\ * \end{pmatrix}, \mathbf{0} \right) = \left(\begin{pmatrix} \overline{(B_{\Updownarrow} + E)}^1 \\ \overline{(B_{\Updownarrow} + E)}^2 \\ \overline{(B_{\Updownarrow} + E)}^3 \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0} \right) = \overline{(\mathbf{B}_{\Updownarrow} + \mathbf{E})}_{1/2}$$

$$= \left(\begin{pmatrix} -D_0 & -D_3^{\Leftrightarrow} & D_2^{\Leftrightarrow} & -D_1 \\ D_3^{\Leftrightarrow} & -D_0 & -D_1^{\Leftrightarrow} & -D_2 \\ -D_2^{\Leftrightarrow} & D_1^{\Leftrightarrow} & -D_0 & -D_3 \\ -D_1^{\Updownarrow} & -D_2^{\Updownarrow} & -D_3^{\Updownarrow} & D_0 \end{pmatrix} \overline{\begin{pmatrix} f^1 \\ f^2 \\ f^3 \\ f^0 \end{pmatrix}}, \mathbf{0}, \mathbf{0} \right) = D_A \overline{(\mathbf{f}, \mathbf{0}, \mathbf{0})}$$

$$\Rightarrow J(d_G : \overline{c_G}) = ((\Box - |m|^2))\overline{(\mathbf{f}, \mathbf{0}, \mathbf{0})} = D_B D_A \overline{(\mathbf{f}, \mathbf{0}, \mathbf{0})} = D_B\left(\overline{(\mathbf{B}_{\Updownarrow} + \mathbf{E})}_{1/2} \right)$$

$c_R = u_1(2) = \left(B^1_{\Updownarrow}, E^2, E^3 \right)_2 \quad ; \quad d_R = d_1(1) = \overline{\left(E^1, B^2_{\Updownarrow}, B^3_{\Updownarrow} \right)}_1$

$$\Rightarrow c_R : \overline{d_R} = \left(\begin{pmatrix} E^1 \\ B^2_{\Updownarrow} \\ B^3_{\Updownarrow} \\ * \end{pmatrix}, \begin{pmatrix} B^1_{\Updownarrow} \\ E^2 \\ E^3_{\Updownarrow} \\ * \end{pmatrix}, \mathbf{0} \right) = \left(\mathbf{0}, \begin{pmatrix} (B_{\Updownarrow} + E)^1 \\ (B_{\Updownarrow} + E)^2 \\ (B_{\Updownarrow} + E)^3 \\ * \end{pmatrix}, \mathbf{0} \right) -= (\mathbf{B}_{\Updownarrow} + \mathbf{E})_2 = D^+$$

$$= \left(\mathbf{0}, \begin{pmatrix} -D_0 & -D_3^{\Leftrightarrow} & D_2^{\Leftrightarrow} & -D_1 \\ D_3^{\Leftrightarrow} & -D_0 & -D_1^{\Leftrightarrow} & -D_2 \\ -D_2^{\Leftrightarrow} & D_1^{\Leftrightarrow} & -D_0 & -D_3 \\ -D_1^{\Updownarrow} & -D_2^{\Updownarrow} & -D_3^{\Updownarrow} & D_0 \end{pmatrix} \begin{pmatrix} f^1 \\ f^2 \\ f^3 \\ f^0 \end{pmatrix}, \mathbf{0} \right) = D_A(\mathbf{0}, \mathbf{f}, \mathbf{0})$$

$$\Rightarrow J(c_R : \overline{d_R}) = ((\Box - |m|^2))(\mathbf{0}, \mathbf{f}, \mathbf{0}) = D_B D_A(\mathbf{0}, \mathbf{f}, \mathbf{0}) = D_B\left((\mathbf{B}_{\Updownarrow} + \mathbf{E})_2 \right) = D_B(D^+)$$

$u_R = u_1(1) = (B^1, E^2, E^3)_1 \quad ; \quad u_G = u_2(1) = (E^1, B^2, E^3)$

$$u_R = u_1(1) = \left(\begin{pmatrix} B^1_{\Updownarrow} \\ E^2 \\ E^3 \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0} \right) \quad ; \quad \overline{u_G} = \overline{u_2(1)} = \left(\begin{pmatrix} \overline{E^1} \\ \overline{B^2_{\Updownarrow}} \\ \overline{E^3} \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0} \right)$$

$$\Rightarrow u_R : \overline{u_G} = \left(\begin{pmatrix} B^1_{\Updownarrow} + \overline{E^1} \\ B^2_{\Updownarrow} + \overline{B^2_{\Updownarrow}} \\ B^3_{\Updownarrow} + \overline{E^3} \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0} \right) \rightrightarrows \nexists$$

$u_R = u_1(1) = (B^1, E^2, E^3)_1$

$$u_R = u_1(1) = \left(\begin{pmatrix} B^1_{\Updownarrow} \\ E^2 \\ E^3 \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0} \right)$$

$$\Rightarrow u_R : \overline{u_R} = \left(\begin{pmatrix} B^1_{\Updownarrow} + \overline{B^1_{\Updownarrow}} \\ E^2 + \overline{E^2} \\ E^3 + \overline{E^3} \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0}\right) = \left(\begin{pmatrix} 0 \\ 0 \\ 0 \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0}\right) = (\mathbf{0})_1 = (\mathbf{0},\mathbf{0},\mathbf{0}) = \pi^0 .$$

$$= \begin{pmatrix} -D_0 & -D_3^{\Leftrightarrow} & D_2^{\Leftrightarrow} & -D_1 \\ D_3^{\Leftrightarrow} & -D_0 & -D_1^{\Leftrightarrow} & -D_2 \\ -D_2^{\Leftrightarrow} & D_1^{\Leftrightarrow} & -D_0 & -D_3 \\ -D_1^{\Updownarrow} & -D_2^{\Updownarrow} & -D_3^{\Updownarrow} & D_0 \end{pmatrix} \left(\begin{pmatrix} 0 \\ 0 \\ 0 \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0}\right) = D_A(\mathbf{0},\mathbf{0},\mathbf{0})$$

$$\Rightarrow J(u_R : \overline{d_R}) = ((\Box - |m|^2))(\mathbf{0},\mathbf{0},\mathbf{0}) = D_B D_A(\mathbf{0},\mathbf{0},\mathbf{0}) = D_B((\mathbf{0})_1) = D_B(\pi^0)$$

$$u_R = u_1(1) = (B^1, E^2, E^3)_1$$

$$u_R = u_1(1) = \left(\begin{pmatrix} B^1_{\Updownarrow} \\ E^2 \\ E^3 \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0}\right)$$

$$\Rightarrow u_R : u_R = \left(\begin{pmatrix} B^1_{\Updownarrow} + B^1_{\Updownarrow} \\ E^2 + E^2 \\ E^3 + E^3 \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0}\right) = \left(2\begin{pmatrix} B^1_{\Updownarrow} \\ E^2 \\ E^3 \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0}\right) \rightrightarrows \nexists$$

So, clearly, mesons of the same generation may be written simple Helmholtzian factorizations.
(mesons of differing generations are something like 'split-level' Helmholtzian factorizations.

Baryons:
Baryon order-independent triplets may only be: uuu, uud, udd, ddd .
So, the eight unique baryon triplets of differing color indices are ($j \neq h, k \neq h, k \neq j$):

$u_1(h) : u_0(j) : u_{-1}(k)$	$u_1(h) : u_0(j) : d_{-1}(k)$	$u_1(h) : d_0(j) : u_{-1}(k)$	$u_1(h) : d_0(j) : d_{-1}(k)$
$d_1(h) : d_0(j) : d_{-1}(k)$	$d_1(h) : d_0(j) : u_{-1}(k)$	$d_1(h) : u_0(j) : d_{-1}(k)$	$d_1(h) : u_0(j) : u_{-1}(k)$

$\|$

$u_1(h) : u_0(j) : u_{-1}(k)$	$d_1(h) : d_0(j) : d_{-1}(k)$
$u_1(h) : u_0(j) : d_{-1}(k)$	$d_1(h) : d_0(j) : u_{-1}(k)$
$u_1(h) : d_0(j) : u_{-1}(k)$	$d_1(h) : u_0(j) : d_{-1}(k)$
$u_1(h) : d_0(j) : d_{-1}(k)$	$d_1(h) : u_0(j) : u_{-1}(k)$

$\|$

$f(1,-1,1,h) : f(1,-1,0,j) : f\left(1,\ 1,-1,k\right)$	$f\left(1,\ 1,1,h\right) : f\left(1,\ 1,0,j\right) : f\left(1,\ 1,-1,k\right)$
$f(1,-1,1,h) : f(1,-1,0,j) : f\left(1,\ 1,\ 1,k\right)$	$f\left(1,\ 1,1,h\right) : f\left(1,\ 1,0,j\right) : f(1,-1,-1,k)$
$f(1,-1,1,h) : f\left(1,\ 1,0,j\right) : f\left(1,-1,\ 1,k\right)$	$f\left(1,\ 1,1,h\right) : f(1,-1,0,j) : f\left(1,\ 1,-1,k\right)$
$f(1,-1,1,h) : f\left(1,\ 1,0,j\right) : f\left(1,\ 1,\ 1,k\right)$	$f\left(1,\ 1,1,h\right) : f(1,-1,0,j) : f(1,-1,-1,k)$

(and anti's)

For example:

$u_R u_R d_R = \left(B^1_{\Updownarrow} + B^1_{\Updownarrow} + \overline{E^1}, E^2 + E^2 + \overline{B^2_{\Updownarrow}}, E^3 + E^3 + \overline{B^3_{\Updownarrow}}\right) = \nexists$

$u_R u_R d_G = \left(B^1_{\Updownarrow} + B^1_{\Updownarrow} + \overline{B^1_{\Updownarrow}}, E^2 + E^2 + \overline{E^2}, E^3 + E^3 + \overline{B^3_{\Updownarrow}}\right) = \nexists$

$u_R u_R d_B = \left(B^1_{\Updownarrow} + B^1_{\Updownarrow} + \overline{B^1_{\Updownarrow}}, E^2 + E^2 + \overline{B^2_{\Updownarrow}}, E^3 + E^3 + \overline{E^3}\right) = \nexists$

$u_R u_G d_R = \left(B^1_{\Updownarrow} + E^1 + \overline{E^1}, E^2 + B^2_{\Updownarrow} + \overline{B^2_{\Updownarrow}}, E^3 + E^3 + \overline{B^3_{\Updownarrow}}\right) = \nexists$

$u_R u_G d_G = \left(B^1_{\Updownarrow} + E^1 + \overline{B^1_{\Updownarrow}}, E^2 + B^2_{\Updownarrow} + \overline{E^2}, E^3 + E^3 + \overline{B^3_{\Updownarrow}}\right) = \nexists$

$u_R u_G d_B = \left(B^1_{\Updownarrow} + E^1 + \overline{B^1_{\Updownarrow}}, E^2 + B^2_{\Updownarrow} + \overline{B^2_{\Updownarrow}}, E^3 + E^3 + \overline{E^3}\right) = (E^1, E^2, E^3) \approx \overline{e^-} \rightrightarrows +1 \rightrightarrows p^+$

$u_R u_B d_R = \left(B^1_{\Updownarrow} + E^1 + \overline{E^1}, E^2 + E^2 + \overline{B^2_{\Updownarrow}}, E^3 + B^3_{\Updownarrow} + \overline{B^3_{\Updownarrow}}\right) = \nexists$

$u_R u_B d_G = \left(B^1_{\Updownarrow} + E^1 + \overline{B^1_{\Updownarrow}}, E^2 + E^2 + \overline{E^2}, E^3 + B^3_{\Updownarrow} + \overline{B^3_{\Updownarrow}}\right) = (E^1, E^2, E^3) \approx \overline{e^-} \rightrightarrows +1 \rightrightarrows p^+$

$u_R u_B d_B = \left(B^1_{\Updownarrow} + E^1 + \overline{B^1_{\Updownarrow}}, E^2 + E^2 + \overline{B^2_{\Updownarrow}}, E^3 + B^3_{\Updownarrow} + \overline{E^3}\right) = \nexists$

etc.

$u_R u_G u_G = \left(B^1_{\Updownarrow} + E^1 + E^1, E^2 + B^2_{\Updownarrow} + B^2_{\Updownarrow}, E^3 + E^3 + E^3\right) = \nexists$

$u_R u_G u_B = \left(B^1_{\Updownarrow} + E^1 + E^1, E^2 + E^3 + B^2_{\Updownarrow}, E^3 + E^3 + B^3_{\Updownarrow}\right) = \left(B^1_{\Updownarrow}, B^2_{\Updownarrow}, B^3_{\Updownarrow}\right) + 2(E^1, E^2, E^3) \approx \nu_e + 2\overline{e^-} \rightrightarrows +2 \rightrightarrows \Delta^{++}$

$= \left(B^1_{\Updownarrow} + E^1, E^3 + B^2_{\Updownarrow}, E^3 + B^3_{\Updownarrow}\right) + (E^1, E^2, E^3) \approx \pi^+ + p^+$ (common decay of Δ^{++})

etc.

$u_R d_G d_G = \left(B^1_{\Updownarrow} + \overline{B^1_{\Updownarrow}} + \overline{B^1_{\Updownarrow}}, E^2 + \overline{E^2} + \overline{E^2}, E^3 + \overline{B^3_{\Updownarrow}} + \overline{B^3_{\Updownarrow}}\right) = \nexists$

$u_R d_G d_B = \left(B^1_{\Updownarrow} + \overline{B^1_{\Updownarrow}} + \overline{B^1_{\Updownarrow}}, E^2 + \overline{E^2} + \overline{B^2_{\Updownarrow}}, E^3 + \overline{B^3_{\Updownarrow}} + \overline{E^3}\right) = \left(\overline{B^1_{\Updownarrow}}, \overline{B^2_{\Updownarrow}}, \overline{B^3_{\Updownarrow}}\right) \approx \overline{\nu_e} \rightrightarrows 0 \rightrightarrows n^0$

etc.

$d_R d_G d_G = *\left(\overline{E^1} + \overline{B^1_{\Updownarrow}} + \overline{B^1_{\Updownarrow}}, \overline{B^2_{\Updownarrow}} + \overline{E^2} + \overline{E^2}, \overline{B^3_{\Updownarrow}} + \overline{B^3_{\Updownarrow}} + \overline{B^3_{\Updownarrow}}\right) = \nexists$

$$d_Rd_Gd_B = *\left(\overline{E^1}+\overline{B^1_\Updownarrow}+\overline{B^1_\Updownarrow},\overline{B^2_\Updownarrow}+\overline{E^2}+\overline{B^2_\Updownarrow},\overline{B^3_\Updownarrow}+\overline{B^3_\Updownarrow}+\overline{E^3}\right) = \left(\overline{E^1}+\overline{B^1_\Updownarrow},\overline{E^2}+\overline{B^2_\Updownarrow},\overline{B^3_\Updownarrow}+\overline{E^3}\right)+\left(\overline{B^1_\Updownarrow},\overline{B^2_\Updownarrow},\overline{B^3_\Updownarrow}\right)$$
$$\approx \pi^- + \overline{\nu_e} \rightrightarrows -1 \rightrightarrows \Delta^-$$

So:

$$u_Ru_Gu_B = \left(B^1_\Updownarrow+E^1+E^1,E^2+E^3+B^2_\Updownarrow,E^3+E^3+B^3_\Updownarrow\right) = \left(B^1_\Updownarrow,B^2_\Updownarrow,B^3_\Updownarrow\right)+2(E^1,E^2,E^3) \approx \nu_e + 2\overline{e^-} \rightrightarrows +2 \rightrightarrows \Delta^{++}$$

$$u_R = u_1(1) = \left(\begin{pmatrix}B^1_\Updownarrow\\E^2\\E^3\\*\end{pmatrix},\mathbf{0},\mathbf{0}\right)\quad;\quad u_G = u_2(1) = \left(\begin{pmatrix}E^1\\B^2_\Updownarrow\\E^3\\*\end{pmatrix},\mathbf{0},\mathbf{0}\right)\quad;\quad u_B = u_3(1) = \left(\begin{pmatrix}E^1\\E^2\\B^3_\Updownarrow\\*\end{pmatrix},\mathbf{0},\mathbf{0}\right)$$

$$\Rightarrow u_R : u_G : u_B = \left(\begin{pmatrix}B^1_\Updownarrow+E^1+E^1\\E^2+B^2_\Updownarrow+E^2\\E^3+E^3+B^3_\Updownarrow\\*\end{pmatrix},\mathbf{0},\mathbf{0}\right) = \left(\begin{pmatrix}B^1_\Updownarrow+E^1+E^1+B^1_\Updownarrow-B^1_\Updownarrow\\E^2+B^2_\Updownarrow+E^2+B^2_\Updownarrow-B^2_\Updownarrow\\E^3+B^3_\Updownarrow+E^3+B^3_\Updownarrow-B^3_\Updownarrow\\*\end{pmatrix},\mathbf{0},\mathbf{0}\right)$$

$$= 2\left(\begin{pmatrix}E^1+B^1_\Updownarrow\\E^2+B^2_\Updownarrow\\E^3+B^3_\Updownarrow\\*\end{pmatrix},\mathbf{0},\mathbf{0}\right) - \left(\begin{pmatrix}B^1_\Updownarrow\\B^2_\Updownarrow\\B^3_\Updownarrow\\*\end{pmatrix},\mathbf{0},\mathbf{0}\right)$$

$$= \left(\begin{pmatrix}B^1_\Updownarrow+E^1+E^1-B^1_\Updownarrow+B^1_\Updownarrow\\E^2+B^2_\Updownarrow+E^2-B^2_\Updownarrow+B^2_\Updownarrow\\E^3+B^3_\Updownarrow+E^3-B^3_\Updownarrow+B^3_\Updownarrow\\*\end{pmatrix},\mathbf{0},\mathbf{0}\right)$$

$$= \left(\begin{pmatrix}E^1+B^1_\Updownarrow\\E^2+B^2_\Updownarrow\\E^3+B^3_\Updownarrow\\*\end{pmatrix},\mathbf{0},\mathbf{0}\right) + \left(\begin{pmatrix}E^1-B^1_\Updownarrow\\E^2-B^2_\Updownarrow\\E^3-B^3_\Updownarrow\\*\end{pmatrix},\mathbf{0},\mathbf{0}\right) + \left(\begin{pmatrix}B^1_\Updownarrow\\B^2_\Updownarrow\\B^3_\Updownarrow\\*\end{pmatrix},\mathbf{0},\mathbf{0}\right)$$

$$= \tfrac{1}{2}\left[2\left(\begin{pmatrix}E^1+B^1_\Updownarrow\\E^2+B^2_\Updownarrow\\E^3+B^3_\Updownarrow\\*\end{pmatrix},\mathbf{0},\mathbf{0}\right) - \left(\begin{pmatrix}B^1_\Updownarrow\\B^2_\Updownarrow\\B^3_\Updownarrow\\*\end{pmatrix},\mathbf{0},\mathbf{0}\right) + \left(\begin{pmatrix}E^1+B^1_\Updownarrow\\E^2+B^2_\Updownarrow\\E^3+B^3_\Updownarrow\\*\end{pmatrix},\mathbf{0},\mathbf{0}\right) + \left(\begin{pmatrix}E^1-B^1_\Updownarrow\\E^2-B^2_\Updownarrow\\E^3-B^3_\Updownarrow\\*\end{pmatrix},\mathbf{0},\mathbf{0}\right) + \left(\left(\right.\right.\right.$$

$$= \tfrac{1}{2}\left[3\left(\begin{pmatrix}E^1+B^1_\Updownarrow\\E^2+B^2_\Updownarrow\\E^3+B^3_\Updownarrow\\*\end{pmatrix},\mathbf{0},\mathbf{0}\right) + \left(\begin{pmatrix}E^1-B^1_\Updownarrow\\E^2-B^2_\Updownarrow\\E^3-B^3_\Updownarrow\\*\end{pmatrix},\mathbf{0},\mathbf{0}\right)\right]$$

$$= \tfrac{1}{2}[3((\mathbf{E}+\mathbf{B}_\Updownarrow),\mathbf{0},\mathbf{0}) + ((\mathbf{E}-\mathbf{B}_\Updownarrow),\mathbf{0},\mathbf{0})]$$

and:

$$u_Ru_Gd_B = \left(B^1_\Updownarrow+E^1+\overline{B^1_\Updownarrow},E^2+B^2_\Updownarrow+\overline{B^2_\Updownarrow},E^3+E^3+\overline{E^3}\right) = (E^1,E^2,E^3) \approx \overline{e^-} \rightrightarrows +1 \rightrightarrows p^+$$

$$u_R = u_1(1) = \left(\begin{pmatrix}B^1_\Updownarrow\\E^2\\E^3\\*\end{pmatrix},\mathbf{0},\mathbf{0}\right)\quad;\quad u_G = u_2(1) = \left(\begin{pmatrix}E^1\\B^2_\Updownarrow\\E^3\\*\end{pmatrix},\mathbf{0},\mathbf{0}\right)\quad;\quad d_B = d_3(1) = \left(\overline{\begin{pmatrix}B^1_\Updownarrow\\B^2_\Updownarrow\\E^3\\*\end{pmatrix}},\mathbf{0},\mathbf{0}\right)$$

$$\Rightarrow u_R : u_G : d_B = \left(\begin{pmatrix}B^1_\Updownarrow+E^1+\overline{B^1_\Updownarrow}\\E^2+B^2_\Updownarrow+\overline{B^2_\Updownarrow}\\E^3+E^3+\overline{E^3}\\*\end{pmatrix},\mathbf{0},\mathbf{0}\right)$$

$$= \left(\begin{pmatrix}E^1\\E^2\\E^3\\*\end{pmatrix},\mathbf{0},\mathbf{0}\right) = \left(\begin{pmatrix}E^1+B^1_\Updownarrow-B^1_\Updownarrow\\E^2+B^2_\Updownarrow-B^2_\Updownarrow\\E^3+B^3_\Updownarrow-B^3_\Updownarrow\\*\end{pmatrix},\mathbf{0},\mathbf{0}\right) = \left(\begin{pmatrix}E^1+B^1_\Updownarrow\\E^2+B^2_\Updownarrow\\E^3+B^3_\Updownarrow\\*\end{pmatrix},\mathbf{0},\mathbf{0}\right) - \left(\begin{pmatrix}B^1_\Updownarrow\\B^2_\Updownarrow\\B^3_\Updownarrow\\*\end{pmatrix},\mathbf{0},\mathbf{0}\right)$$

$$= \left(\begin{pmatrix}E^1\\E^2\\E^3\\*\end{pmatrix},\mathbf{0},\mathbf{0}\right) = \left(\begin{pmatrix}E^1-B^1_\Updownarrow+B^1_\Updownarrow\\E^2-B^2_\Updownarrow+B^2_\Updownarrow\\E^3-B^3_\Updownarrow+B^3_\Updownarrow\\*\end{pmatrix},\mathbf{0},\mathbf{0}\right) = \left(\begin{pmatrix}E^1-B^1_\Updownarrow\\E^2-B^2_\Updownarrow\\E^3-B^3_\Updownarrow\\*\end{pmatrix},\mathbf{0},\mathbf{0}\right) + \left(\begin{pmatrix}B^1_\Updownarrow\\B^2_\Updownarrow\\B^3_\Updownarrow\\*\end{pmatrix},\mathbf{0},\mathbf{0}\right)$$

$$= \tfrac{1}{2}\left[\left(\begin{pmatrix} E^1 + B^1_{\Updownarrow} \\ E^2 + B^2_{\Updownarrow} \\ E^3 + B^3_{\Updownarrow} \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0}\right) - \left(\begin{pmatrix} B^1_{\Updownarrow} \\ B^2_{\Updownarrow} \\ B^3_{\Updownarrow} \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0}\right) + \left(\begin{pmatrix} E^1 - B^1_{\Updownarrow} \\ E^2 - B^2_{\Updownarrow} \\ E^3 - B^3_{\Updownarrow} \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0}\right) + \left(\begin{pmatrix} B^1_{\Updownarrow} \\ B^2_{\Updownarrow} \\ B^3_{\Updownarrow} \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0}\right)\right]$$

$$= \tfrac{1}{2}\left[\left(\begin{pmatrix} E^1 + B^1_{\Updownarrow} \\ E^2 + B^2_{\Updownarrow} \\ E^3 + B^3_{\Updownarrow} \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0}\right) + \left(\begin{pmatrix} E^1 - B^1_{\Updownarrow} \\ E^2 - B^2_{\Updownarrow} \\ E^3 - B^3_{\Updownarrow} \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0}\right)\right]$$

$$= \tfrac{1}{2}[3((\mathbf{E} + \mathbf{B}_{\Updownarrow}), \mathbf{0}, \mathbf{0}) + ((\mathbf{E} - \mathbf{B}_{\Updownarrow}), \mathbf{0}, \mathbf{0})]$$

and:

$$u_R : d_G : d_B = \left(B^1_{\Updownarrow} + \overline{B^1_{\Updownarrow}} + \overline{B^1_{\Updownarrow}}, E^2 + \overline{E^2} + \overline{B^2_{\Updownarrow}}, E^3 + \overline{B^3_{\Updownarrow}} + \overline{E^3}\right) = \left(\overline{B^1_{\Updownarrow}}, \overline{B^2_{\Updownarrow}}, \overline{B^3_{\Updownarrow}}\right) \approx \overline{\nu_e} \rightrightarrows 0 \rightrightarrows n^0$$

$$u_R = u_1(1) = \left(\begin{pmatrix} B^1_{\Updownarrow} \\ E^2 \\ E^3 \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0}\right) \quad ; \quad d_G = d_2(1) = \left(\overline{\begin{pmatrix} B^1_{\Updownarrow} \\ E^2 \\ B^3_{\Updownarrow} \\ * \end{pmatrix}}, \mathbf{0}, \mathbf{0}\right) \quad ; \quad d_B = d_3(1) = \left(\overline{\begin{pmatrix} B^1_{\Updownarrow} \\ B^2_{\Updownarrow} \\ E^3 \\ * \end{pmatrix}}, \mathbf{0}, \mathbf{0}\right)$$

$$\Rightarrow u_R : d_G : d_B = \left(\begin{pmatrix} B^1_{\Updownarrow} + \overline{B^1_{\Updownarrow}} + \overline{B^1_{\Updownarrow}} \\ E^2 + \overline{E^2} + \overline{B^2_{\Updownarrow}} \\ E^3 + \overline{B^3_{\Updownarrow}} + \overline{E^3} \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0}\right)$$

$$= \left(\begin{pmatrix} \overline{B^1_{\Updownarrow}} \\ \overline{B^2_{\Updownarrow}} \\ \overline{B^3_{\Updownarrow}} \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0}\right) = \left(\begin{pmatrix} \overline{B^1_{\Updownarrow}} + \overline{E^1} - \overline{E^1} \\ \overline{B^2_{\Updownarrow}} + \overline{E^2} - \overline{E^2} \\ \overline{B^3_{\Updownarrow}} + \overline{E^3} - \overline{E^3} \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0}\right) = \left(\begin{pmatrix} \overline{E^1} + \overline{B^1_{\Updownarrow}} \\ \overline{E^2} + \overline{B^2_{\Updownarrow}} \\ \overline{E^3} + \overline{B^3_{\Updownarrow}} \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0}\right) - \left(\begin{pmatrix} \overline{E^1} \\ \overline{E^2} \\ \overline{E^3} \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0}\right)$$

$$= \left(\begin{pmatrix} \overline{B^1_{\Updownarrow}} \\ \overline{B^2_{\Updownarrow}} \\ \overline{B^3_{\Updownarrow}} \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0}\right) = \left(\begin{pmatrix} \overline{B^1_{\Updownarrow}} - \overline{E^1} + \overline{E^1} \\ \overline{B^2_{\Updownarrow}} - \overline{E^2} + \overline{E^2} \\ \overline{B^3_{\Updownarrow}} - \overline{E^3} + \overline{E^3} \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0}\right) = \left(\begin{pmatrix} E^1 - B^1_{\Updownarrow} \\ E^2 - B^2_{\Updownarrow} \\ E^3 - B^3_{\Updownarrow} \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0}\right) + \left(\begin{pmatrix} \overline{E^1} \\ \overline{E^2} \\ \overline{E^3} \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0}\right)$$

$$= \tfrac{1}{2}\left[\left(\begin{pmatrix} \overline{E^1} + \overline{B^1_{\Updownarrow}} \\ \overline{E^2} + \overline{B^2_{\Updownarrow}} \\ \overline{E^3} + \overline{B^3_{\Updownarrow}} \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0}\right) - \left(\begin{pmatrix} \overline{E^1} \\ \overline{E^2} \\ \overline{E^3} \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0}\right) + \left(\begin{pmatrix} E^1 - B^1_{\Updownarrow} \\ E^2 - B^2_{\Updownarrow} \\ E^3 - B^3_{\Updownarrow} \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0}\right) + \left(\begin{pmatrix} \overline{E^1} \\ \overline{E^2} \\ \overline{E^3} \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0}\right)\right]$$

$$= \tfrac{1}{2}\left[\left(\begin{pmatrix} \overline{E^1} + \overline{B^1_{\Updownarrow}} \\ \overline{E^2} + \overline{B^2_{\Updownarrow}} \\ \overline{E^3} + \overline{B^3_{\Updownarrow}} \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0}\right) + \left(\begin{pmatrix} E^1 - B^1_{\Updownarrow} \\ E^2 - B^2_{\Updownarrow} \\ E^3 - B^3_{\Updownarrow} \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0}\right)\right]$$

$$= \tfrac{1}{2}\left[\overline{((\mathbf{E} + \mathbf{B}_{\Updownarrow}), \mathbf{0}, \mathbf{0})} + ((\mathbf{E} - \mathbf{B}_{\Updownarrow}), \mathbf{0}, \mathbf{0})\right]$$

and:

$$d_R d_G d_B = *\left(\overline{E^1} + \overline{B^1_{\Updownarrow}} + \overline{B^1_{\Updownarrow}}, \overline{B^2_{\Updownarrow}} + \overline{E^2} + \overline{B^2_{\Updownarrow}}, \overline{B^3_{\Updownarrow}} + \overline{B^3_{\Updownarrow}} + \overline{E^3}\right) = \left(\overline{E^1} + \overline{B^1_{\Updownarrow}}, \overline{E^2} + \overline{B^2_{\Updownarrow}}, \overline{B^3_{\Updownarrow}} + \overline{E^3}\right) + \left(\overline{B^1_{\Updownarrow}}, \overline{B^2_{\Updownarrow}}, \overline{B^3_{\Updownarrow}}\right)$$

$$d_R = d_1(1) = \left(\overline{\begin{pmatrix} E^1 \\ B^2_{\Updownarrow} \\ \overline{B^3_{\Updownarrow}} \\ * \end{pmatrix}}, \mathbf{0}, \mathbf{0}\right) \quad ; \quad d_G = d_2(1) = \left(\begin{pmatrix} E^1 \\ B^2_{\Updownarrow} \\ E^3 \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0}\right) \quad ; \quad d_B = d_3(1) = \left(\overline{\begin{pmatrix} B^1_{\Updownarrow} \\ B^2_{\Updownarrow} \\ E^3 \\ * \end{pmatrix}}, \mathbf{0}, \mathbf{0}\right)$$

$$\Rightarrow d_R : d_G : d_B = \left(\begin{pmatrix} \overline{E^1} + \overline{B^1_{\Updownarrow}} + \overline{B^1_{\Updownarrow}} \\ \overline{B^2_{\Updownarrow}} + \overline{E^2} + \overline{B^2_{\Updownarrow}} \\ \overline{B^3_{\Updownarrow}} + \overline{B^3_{\Updownarrow}} + \overline{E^3} \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0}\right) = \left(\begin{pmatrix} \overline{E^1} \\ \overline{E^2} \\ \overline{E^3} \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0}\right)$$

$$= \left(\begin{pmatrix} \overline{E^1} + \overline{B^1_{\Updownarrow}} - \overline{B^1_{\Updownarrow}} \\ \overline{E^2} + \overline{B^2_{\Updownarrow}} - \overline{B^2_{\Updownarrow}} \\ \overline{E^3} + \overline{B^1_{\Updownarrow}} - \overline{B^1_{\Updownarrow}} \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0}\right) = \left(\begin{pmatrix} \overline{E^1} + \overline{B^1_{\Updownarrow}} \\ \overline{E^2} + \overline{B^2_{\Updownarrow}} \\ \overline{E^3} + \overline{B^1_{\Updownarrow}} \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0}\right) - \left(\begin{pmatrix} \overline{B^1_{\Updownarrow}} \\ \overline{B^2_{\Updownarrow}} \\ \overline{B^1_{\Updownarrow}} \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0}\right)$$

$$= \left(\begin{pmatrix} \overline{E^1} - \overline{B^1_\Updownarrow} + \overline{B^1_\Updownarrow} \\ \overline{E^2} - \overline{B^2_\Updownarrow} + \overline{B^2_\Updownarrow} \\ \overline{E^3} - \overline{B^1_\Updownarrow} + \overline{B^1_\Updownarrow} \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0} \right) = \left(\begin{pmatrix} \overline{E^1} - \overline{B^1_\Updownarrow} \\ \overline{E^2} - \overline{B^2_\Updownarrow} \\ \overline{E^3} - \overline{B^1_\Updownarrow} \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0} \right) + \left(\begin{pmatrix} \overline{B^1_\Updownarrow} \\ \overline{B^2_\Updownarrow} \\ \overline{B^1_\Updownarrow} \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0} \right)$$

$$= ½ \left[\left(\begin{pmatrix} \overline{E^1} + \overline{B^1_\Updownarrow} \\ \overline{E^2} + \overline{B^2_\Updownarrow} \\ \overline{E^3} + \overline{B^1_\Updownarrow} \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0} \right) + \left(\begin{pmatrix} \overline{E^1} - \overline{B^1_\Updownarrow} + \overline{B^1_\Updownarrow} \\ \overline{E^2} - \overline{B^2_\Updownarrow} + \overline{B^2_\Updownarrow} \\ \overline{E^3} - \overline{B^1_\Updownarrow} + \overline{B^1_\Updownarrow} \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0} \right) \right]$$

$$= ½ \left[\overline{((\mathbf{E} + \mathbf{B}_\Updownarrow), \mathbf{0}, \mathbf{0})} + ((\mathbf{E} - \mathbf{B}_\Updownarrow), \mathbf{0}, \mathbf{0}) \right]$$

and:

$$u_R c_G b_B = \left(B^1_\Updownarrow + E^1 + \overline{B^1_\Updownarrow}, E^2 + B^2_\Updownarrow + \overline{B^2_\Updownarrow}, E^3 + E^3 + \overline{E^3} \right) = (E^1, E^2, E^3)$$

$$u_R = u_1(1) = \left(\begin{pmatrix} B^1_\Updownarrow \\ E^2 \\ E^3 \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0} \right) \quad ; \quad c_G = u_2(2) = \left(\mathbf{0}, \begin{pmatrix} E^1 \\ B^2_\Updownarrow \\ E^3 \\ * \end{pmatrix}, \mathbf{0} \right) \quad ; \quad b_B = d_3(3) = \left(\mathbf{0}, \mathbf{0}, \overline{\begin{pmatrix} B^1_\Updownarrow \\ B^2_\Updownarrow \\ E^3 \\ * \end{pmatrix}} \right)$$

$$\Rightarrow u_R : c_G : b_B = \left(\begin{pmatrix} B^1_\Updownarrow \\ E^2 \\ E^3 \\ * \end{pmatrix}, \begin{pmatrix} E^1 \\ B^2_\Updownarrow \\ E^3 \\ * \end{pmatrix}, \begin{pmatrix} \overline{B^1_\Updownarrow} \\ \overline{B^2_\Updownarrow} \\ \overline{E^3} \\ * \end{pmatrix} \right)$$

$$= \tfrac{1}{3} \left(\begin{pmatrix} B^1_\Updownarrow \\ E^2 \\ E^3 \\ * \end{pmatrix} + \begin{pmatrix} E^1 \\ B^2_\Updownarrow \\ E^3 \\ * \end{pmatrix} + \begin{pmatrix} \overline{B^1_\Updownarrow} \\ \overline{B^2_\Updownarrow} \\ \overline{E^3} \\ * \end{pmatrix}, \begin{pmatrix} B^1_\Updownarrow \\ E^2 \\ E^3 \\ * \end{pmatrix} + \begin{pmatrix} E^1 \\ B^2_\Updownarrow \\ E^3 \\ * \end{pmatrix} + \begin{pmatrix} \overline{B^1_\Updownarrow} \\ \overline{B^2_\Updownarrow} \\ \overline{E^3} \\ * \end{pmatrix}, \begin{pmatrix} B^1_\Updownarrow \\ E^2 \\ E^3 \\ * \end{pmatrix} + \begin{pmatrix} E^1 \\ B^2_\Updownarrow \\ E^3 \\ * \end{pmatrix} + \begin{pmatrix} \overline{B^1_\Updownarrow} \\ \overline{B^2_\Updownarrow} \\ \overline{E^3} \\ * \end{pmatrix} \right)$$

$$= \tfrac{1}{3} \left[\left(\begin{pmatrix} B^1_\Updownarrow + E^1 + \overline{B^1_\Updownarrow} \\ E^2 + B^2_\Updownarrow + \overline{B^2_\Updownarrow} \\ E^3 + E^3 + \overline{E^3} \\ * \end{pmatrix}, \begin{pmatrix} B^1_\Updownarrow + E^1 + \overline{B^1_\Updownarrow} \\ E^2 + B^2_\Updownarrow + \overline{B^2_\Updownarrow} \\ E^3 + E^3 + \overline{E^3} \\ * \end{pmatrix}, \begin{pmatrix} B^1_\Updownarrow + E^1 + \overline{B^1_\Updownarrow} \\ E^2 + B^2_\Updownarrow + \overline{B^2_\Updownarrow} \\ E^3 + E^3 + \overline{E^3} \\ * \end{pmatrix} \right) \right]$$

$$= \tfrac{1}{3} \left[\left(\begin{pmatrix} E^1 \\ E^2 \\ E^3 \\ * \end{pmatrix}, \begin{pmatrix} E^1 \\ E^2 \\ E^3 \\ * \end{pmatrix}, \begin{pmatrix} E^1 \\ E^2 \\ E^3 \\ * \end{pmatrix} \right) \right] = \tfrac{1}{3} \left[\left(\begin{pmatrix} E^1 \\ E^2 \\ E^3 \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0} \right) + \left(\mathbf{0}, \begin{pmatrix} E^1 \\ E^2 \\ E^3 \\ * \end{pmatrix}, \mathbf{0} \right) + \left(\mathbf{0}, \mathbf{0}, \begin{pmatrix} E^1 \\ E^2 \\ E^3 \\ * \end{pmatrix} \right) \right]$$

$$= \tfrac{1}{3} \left[\left(\begin{pmatrix} E^1 + B^1_\Updownarrow - B^1_\Updownarrow \\ E^2 + B^2_\Updownarrow - B^2_\Updownarrow \\ E^3 + B^3_\Updownarrow - B^3_\Updownarrow \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0} \right) + \left(\mathbf{0}, \begin{pmatrix} E^1 + B^1_\Updownarrow - B^1_\Updownarrow \\ E^2 + B^2_\Updownarrow - B^2_\Updownarrow \\ E^3 + B^3_\Updownarrow - B^3_\Updownarrow \\ * \end{pmatrix}, \mathbf{0} \right) + \left(\mathbf{0}, \mathbf{0}, \begin{pmatrix} E^1 + B^1_\Updownarrow - B^1_\Updownarrow \\ E^2 + B^2_\Updownarrow - B^2_\Updownarrow \\ E^3 + B^3_\Updownarrow - B^3_\Updownarrow \\ * \end{pmatrix} \right) \right]$$

$$= \tfrac{1}{3} \left[\left(\begin{pmatrix} E^1 + B^1_\Updownarrow \\ E^2 + B^2_\Updownarrow \\ E^3 + B^3_\Updownarrow \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0} \right) - \left(\begin{pmatrix} B^1_\Updownarrow \\ B^2_\Updownarrow \\ B^3_\Updownarrow \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0} \right) + \left(\mathbf{0}, \begin{pmatrix} E^1 + B^1_\Updownarrow \\ E^2 + B^2_\Updownarrow \\ E^3 + B^3_\Updownarrow \\ * \end{pmatrix}, \mathbf{0} \right) - \left(\mathbf{0}, \begin{pmatrix} B^1_\Updownarrow \\ B^2_\Updownarrow \\ B^3_\Updownarrow \\ * \end{pmatrix}, \mathbf{0} \right) + \left(\mathbf{0}, \mathbf{0}, \begin{pmatrix} E^1 + B^1_\Updownarrow \\ E^2 + B^2_\Updownarrow \\ E^3 + B^3_\Updownarrow \\ * \end{pmatrix} \right) - \left(\mathbf{0}, \right.$$

$$= \tfrac{1}{3} \left[\left(\begin{pmatrix} E^1 + B^1_\Updownarrow \\ E^2 + B^2_\Updownarrow \\ E^3 + B^3_\Updownarrow \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0} \right) + \left(\mathbf{0}, \begin{pmatrix} E^1 + B^1_\Updownarrow \\ E^2 + B^2_\Updownarrow \\ E^3 + B^3_\Updownarrow \\ * \end{pmatrix}, \mathbf{0} \right) + \left(\mathbf{0}, \mathbf{0}, \begin{pmatrix} E^1 + B^1_\Updownarrow \\ E^2 + B^2_\Updownarrow \\ E^3 + B^3_\Updownarrow \\ * \end{pmatrix} \right) \right] +$$

$$- \tfrac{1}{3} \left[\left(\begin{pmatrix} B^1_\Updownarrow \\ B^2_\Updownarrow \\ B^3_\Updownarrow \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0} \right) + \left(\mathbf{0}, \begin{pmatrix} B^1_\Updownarrow \\ B^2_\Updownarrow \\ B^3_\Updownarrow \\ * \end{pmatrix}, \mathbf{0} \right) + \left(\mathbf{0}, \mathbf{0}, \begin{pmatrix} B^1_\Updownarrow \\ B^2_\Updownarrow \\ B^3_\Updownarrow \\ * \end{pmatrix} \right) \right]$$

$$= \tfrac{1}{3} \left[\left(\begin{pmatrix} E^1 + B^1_\Updownarrow \\ E^2 + B^2_\Updownarrow \\ E^3 + B^3_\Updownarrow \\ * \end{pmatrix}, \begin{pmatrix} E^1 + B^1_\Updownarrow \\ E^2 + B^2_\Updownarrow \\ E^3 + B^3_\Updownarrow \\ * \end{pmatrix}, \begin{pmatrix} E^1 + B^1_\Updownarrow \\ E^2 + B^2_\Updownarrow \\ E^3 + B^3_\Updownarrow \\ * \end{pmatrix} \right) - \left(\begin{pmatrix} B^1_\Updownarrow \\ B^2_\Updownarrow \\ B^3_\Updownarrow \\ * \end{pmatrix}, \begin{pmatrix} B^1_\Updownarrow \\ B^2_\Updownarrow \\ B^3_\Updownarrow \\ * \end{pmatrix}, \begin{pmatrix} B^1_\Updownarrow \\ B^2_\Updownarrow \\ B^3_\Updownarrow \\ * \end{pmatrix} \right) \right]$$

$$= \tfrac{1}{3}\left[\left(\begin{pmatrix} E^1 - B^1_{\Updownarrow} + B^1_{\Updownarrow} \\ E^2 - B^2_{\Updownarrow} + B^2_{\Updownarrow} \\ E^3 - B^3_{\Updownarrow} + B^3_{\Updownarrow} \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0}\right) + \left(\mathbf{0}, \begin{pmatrix} E^1 - B^1_{\Updownarrow} + B^1_{\Updownarrow} \\ E^2 - B^2_{\Updownarrow} + B^2_{\Updownarrow} \\ E^3 - B^3_{\Updownarrow} + B^3_{\Updownarrow} \\ * \end{pmatrix}, \mathbf{0}\right) + \left(\mathbf{0}, \mathbf{0}, \begin{pmatrix} E^1 - B^1_{\Updownarrow} + B^1_{\Updownarrow} \\ E^2 - B^2_{\Updownarrow} + B^2_{\Updownarrow} \\ E^3 - B^3_{\Updownarrow} + B^3_{\Updownarrow} \\ * \end{pmatrix}\right)\right]$$

$$= \tfrac{1}{3}\left[\left(\begin{pmatrix} E^1 - B^1_{\Updownarrow} \\ E^2 - B^2_{\Updownarrow} \\ E^3 - B^3_{\Updownarrow} \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0}\right) + \left(\begin{pmatrix} B^1_{\Updownarrow} \\ B^2_{\Updownarrow} \\ B^3_{\Updownarrow} \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0}\right) + \left(\mathbf{0}, \begin{pmatrix} E^1 - B^1_{\Updownarrow} \\ E^2 - B^2_{\Updownarrow} \\ E^3 - B^3_{\Updownarrow} \\ * \end{pmatrix}, \mathbf{0}\right) + \left(\mathbf{0}, \begin{pmatrix} B^1_{\Updownarrow} \\ B^2_{\Updownarrow} \\ B^3_{\Updownarrow} \\ * \end{pmatrix}, \mathbf{0}\right) + \left(\mathbf{0}, \mathbf{0}, \begin{pmatrix} E^1 - B^1_{\Updownarrow} \\ E^2 - B^2_{\Updownarrow} \\ E^3 - B^3_{\Updownarrow} \\ * \end{pmatrix}\right) + \left(\mathbf{0}, \right.\right.$$

$$= \tfrac{1}{3}\left[\left(\begin{pmatrix} E^1 - B^1_{\Updownarrow} \\ E^2 - B^2_{\Updownarrow} \\ E^3 - B^3_{\Updownarrow} \\ * \end{pmatrix}, \begin{pmatrix} E^1 - B^1_{\Updownarrow} \\ E^2 - B^2_{\Updownarrow} \\ E^3 - B^3_{\Updownarrow} \\ * \end{pmatrix}, \begin{pmatrix} E^1 - B^1_{\Updownarrow} \\ E^2 - B^2_{\Updownarrow} \\ E^3 - B^3_{\Updownarrow} \\ * \end{pmatrix}\right) + \left(\begin{pmatrix} B^1_{\Updownarrow} \\ B^2_{\Updownarrow} \\ B^3_{\Updownarrow} \\ * \end{pmatrix}, \begin{pmatrix} B^1_{\Updownarrow} \\ B^2_{\Updownarrow} \\ B^3_{\Updownarrow} \\ * \end{pmatrix}, \begin{pmatrix} B^1_{\Updownarrow} \\ B^2_{\Updownarrow} \\ B^3_{\Updownarrow} \\ * \end{pmatrix}\right)\right]$$

$$= \tfrac{1}{6}\left[\left(\begin{pmatrix} E^1 + B^1_{\Updownarrow} \\ E^2 + B^2_{\Updownarrow} \\ E^3 + B^3_{\Updownarrow} \\ * \end{pmatrix}, \begin{pmatrix} E^1 + B^1_{\Updownarrow} \\ E^2 + B^2_{\Updownarrow} \\ E^3 + B^3_{\Updownarrow} \\ * \end{pmatrix}, \begin{pmatrix} E^1 + B^1_{\Updownarrow} \\ E^2 + B^2_{\Updownarrow} \\ E^3 + B^3_{\Updownarrow} \\ * \end{pmatrix}\right) + \left(\begin{pmatrix} E^1 - B^1_{\Updownarrow} \\ E^2 - B^2_{\Updownarrow} \\ E^3 - B^3_{\Updownarrow} \\ * \end{pmatrix}, \begin{pmatrix} E^1 - B^1_{\Updownarrow} \\ E^2 - B^2_{\Updownarrow} \\ E^3 - B^3_{\Updownarrow} \\ * \end{pmatrix}, \begin{pmatrix} E^1 - B^1_{\Updownarrow} \\ E^2 - B^2_{\Updownarrow} \\ E^3 - B^3_{\Updownarrow} \\ * \end{pmatrix}\right)\right]$$

$$= \tfrac{1}{6}[([(\mathbf{E} + \mathbf{B}_{\Updownarrow}) + (\mathbf{E} - \mathbf{B}_{\Updownarrow})], \mathbf{0}, \mathbf{0}) + (\mathbf{0}, [(\mathbf{E} + \mathbf{B}_{\Updownarrow}) + (\mathbf{E} - \mathbf{B}_{\Updownarrow})], \mathbf{0}) + (\mathbf{0}, \mathbf{0}, [(\mathbf{E} + \mathbf{B}_{\Updownarrow}) + (\mathbf{E} - \mathbf{B}_{\Updownarrow})])]$$

and:

$$u_B c_G s_R = \left(E^1 + E^1 + \overline{E^1}, E^2 + B^2_{\Updownarrow} + \overline{B^2_{\Updownarrow}}, B^3_{\Updownarrow} + E^3 + \overline{B^3_{\Updownarrow}}\right) = (E^1, E^2, E^3)$$

$$u_B = u_3(1) = \left(\begin{pmatrix} E^1 \\ E^2 \\ B^3_{\Updownarrow} \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0}\right) \quad ; \quad c_G = u_2(2) = \left(\mathbf{0}, \begin{pmatrix} E^1 \\ B^2_{\Updownarrow} \\ E^3 \\ * \end{pmatrix}, \mathbf{0}\right) \quad ; \quad s_R = d_1(2) = \overline{\left(E^1, B^2_{\Updownarrow}, B^3_{\Updownarrow}\right)_2} = \left(\mathbf{0}, \overline{\begin{pmatrix} E^1 \\ B^2_{\Updownarrow} \\ B^3_{\Updownarrow} \\ * \end{pmatrix}}, \mathbf{0}\right)$$

$$\Rightarrow u_B c_G s_R = \left(\begin{pmatrix} E^1 \\ E^2 \\ B^3_{\Updownarrow} \\ * \end{pmatrix}, \begin{pmatrix} E^1 \\ B^2_{\Updownarrow} \\ E^3 \\ * \end{pmatrix} + \overline{\begin{pmatrix} E^1 \\ B^2_{\Updownarrow} \\ B^3_{\Updownarrow} \\ * \end{pmatrix}}, \mathbf{0}\right) = \left(\begin{pmatrix} E^1 \\ E^2 \\ B^3_{\Updownarrow} \\ * \end{pmatrix}, \begin{pmatrix} E^1 + \overline{E^1} \\ B^2_{\Updownarrow} + \overline{B^2_{\Updownarrow}} \\ E^3 + \overline{B^3_{\Updownarrow}} \\ * \end{pmatrix}, \mathbf{0}\right)$$

$$= \tfrac{1}{2}\left(\begin{pmatrix} E^1 \\ E^2 \\ B^3_{\Updownarrow} \\ * \end{pmatrix} + \begin{pmatrix} E^1 + \overline{E^1} \\ B^2_{\Updownarrow} + \overline{B^2_{\Updownarrow}} \\ E^3 + \overline{B^3_{\Updownarrow}} \\ * \end{pmatrix}, \begin{pmatrix} E^1 \\ E^2 \\ B^3_{\Updownarrow} \\ * \end{pmatrix} + \begin{pmatrix} E^1 + \overline{E^1} \\ B^2_{\Updownarrow} + \overline{B^2_{\Updownarrow}} \\ E^3 + \overline{B^3_{\Updownarrow}} \\ * \end{pmatrix}, \mathbf{0}\right)$$

$$= \tfrac{1}{2}\left(\begin{pmatrix} E^1 + E^1 + \overline{E^1} \\ E^2 + B^2_{\Updownarrow} + \overline{B^2_{\Updownarrow}} \\ B^3_{\Updownarrow} + E^3 + \overline{B^3_{\Updownarrow}} \\ * \end{pmatrix}, \begin{pmatrix} E^1 + E^1 + \overline{E^1} \\ E^2 + B^2_{\Updownarrow} + \overline{B^2_{\Updownarrow}} \\ B^3_{\Updownarrow} + E^3 + \overline{B^3_{\Updownarrow}} \\ * \end{pmatrix}, \mathbf{0}\right)$$

$$= \tfrac{1}{2}\left(\begin{pmatrix} E^1 + B^1_{\Updownarrow} - B^1_{\Updownarrow} \\ E^2 + B^2_{\Updownarrow} - B^2_{\Updownarrow} \\ E^3 + B^3_{\Updownarrow} - B^3_{\Updownarrow} \\ * \end{pmatrix}, \begin{pmatrix} E^1 + B^1_{\Updownarrow} - B^1_{\Updownarrow} \\ E^2 + B^2_{\Updownarrow} - B^2_{\Updownarrow} \\ E^3 + B^3_{\Updownarrow} - B^3_{\Updownarrow} \\ * \end{pmatrix}, \mathbf{0}\right)$$

$$= \tfrac{1}{2}\left[\left(\begin{pmatrix} E^1 + B^1_{\Updownarrow} \\ E^2 + B^2_{\Updownarrow} \\ E^3 + B^3_{\Updownarrow} \\ * \end{pmatrix}, \begin{pmatrix} E^1 + B^1_{\Updownarrow} \\ E^2 + B^2_{\Updownarrow} \\ E^3 + B^3_{\Updownarrow} \\ * \end{pmatrix}, \mathbf{0}\right) - \left(\begin{pmatrix} B^1_{\Updownarrow} \\ B^2_{\Updownarrow} \\ B^3_{\Updownarrow} \\ * \end{pmatrix}, \begin{pmatrix} B^1_{\Updownarrow} \\ B^2_{\Updownarrow} \\ B^3_{\Updownarrow} \\ * \end{pmatrix}, \mathbf{0}\right)\right]$$

$$= \tfrac{1}{2}\left(\begin{pmatrix} E^1 - B^1_{\Updownarrow} + B^1_{\Updownarrow} \\ E^2 - B^2_{\Updownarrow} + B^2_{\Updownarrow} \\ E^3 - B^3_{\Updownarrow} + B^3_{\Updownarrow} \\ * \end{pmatrix}, \begin{pmatrix} E^1 - B^1_{\Updownarrow} + B^1_{\Updownarrow} \\ E^2 - B^2_{\Updownarrow} + B^2_{\Updownarrow} \\ E^3 - B^3_{\Updownarrow} + B^3_{\Updownarrow} \\ * \end{pmatrix}, \mathbf{0}\right)$$

$$= \tfrac{1}{2}\left[\left(\begin{pmatrix} E^1 - B^1_{\Updownarrow} \\ E^2 - B^2_{\Updownarrow} \\ E^3 - B^3_{\Updownarrow} \\ * \end{pmatrix}, \begin{pmatrix} E^1 - B^1_{\Updownarrow} \\ E^2 - B^2_{\Updownarrow} \\ E^3 - B^3_{\Updownarrow} \\ * \end{pmatrix}, \mathbf{0}\right) + \left(\begin{pmatrix} B^1_{\Updownarrow} \\ B^2_{\Updownarrow} \\ B^3_{\Updownarrow} \\ * \end{pmatrix}, \begin{pmatrix} B^1_{\Updownarrow} \\ B^2_{\Updownarrow} \\ B^3_{\Updownarrow} \\ * \end{pmatrix}, \mathbf{0}\right)\right]$$

$$= ¼\left[\left(\begin{pmatrix} E^1+B^1_{\Updownarrow} \\ E^2+B^2_{\Updownarrow} \\ E^3+B^3_{\Updownarrow} \\ * \end{pmatrix}, \begin{pmatrix} E^1+B^1_{\Updownarrow} \\ E^2+B^2_{\Updownarrow} \\ E^3+B^3_{\Updownarrow} \\ * \end{pmatrix}, \mathbf{0}\right) + \left(\begin{pmatrix} E^1-B^1_{\Updownarrow} \\ E^2-B^2_{\Updownarrow} \\ E^3-B^3_{\Updownarrow} \\ * \end{pmatrix}, \begin{pmatrix} E^1-B^1_{\Updownarrow} \\ E^2-B^2_{\Updownarrow} \\ E^3-B^3_{\Updownarrow} \\ * \end{pmatrix}, \mathbf{0}\right)\right]$$

$$= ¼[([(\mathbf{E}+\mathbf{B}_{\Updownarrow}) + (\mathbf{E}-\mathbf{B}_{\Updownarrow})], \mathbf{0}, \mathbf{0}) + (\mathbf{0}, [(\mathbf{E}+\mathbf{B}_{\Updownarrow}) + (\mathbf{E}-\mathbf{B}_{\Updownarrow})], \mathbf{0})]$$

etc. (all the rest are the variations on the u/d & $R/G/B$ - and the analyses are similar to the above)

REFERENCES

[1] Cassano, Claude.Michael ; "Reality is a Mathematical Model", 2010.
ISBN: 1468120921 ; http://www.amazon.com/dp/1468120921
ASIN: B0049P1P4C ;
http://www.amazon.com/Reality-Mathematical-Modelebook/dp/B0049P1P4C/ref=tmm_kin_swatch_0?_encoding=UTF8&sr=&qid
[2] Cassano, Claude.Michael ; "A Mathematical Preon Foundation for the Standard
Model", 2011.
ISBN:1468117734 ; http://www.amazon.com/dp/1468117734
ASIN: B004IZLHI2 ; http://www.amazon.com/Mathematical-Preon-Foundation-Standardebook/
dp/B004IZLHI2/ref=tmm_kin_swatch_0?_encoding=UTF8&sr=&qid=