Experimental results of a simple pendulum and inelastic collisions

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Friday 1st July, 2022

Abstract

In this article, theoretical and experimental values of a simple pendulum and an inelastic collision of two pendulums are calculated, through numerical methods with the python language, and analysis of the experimental data. An accuracy of 98.46% for the collision time and 97.78% for the speed during impact is demonstrated.

1. Introduction

It is common during the study of physics to doubt the effectiveness of developed methods. The purpose of this article is to ensure that they are reliable in a practical manner, even with an experiment full of factors that can influence the result. This experiment simulates two simple pendulums of equal mass colliding, using clackers.

2. Theory

The equation which will be used to determine the time elapsed since the release of a mass in a simple pendulum is:[1]:

$$t_{queda} = \sqrt{\frac{\ell}{2g}} \int_{\theta_f}^{\theta_i} \frac{d\theta}{\sqrt{\cos(\theta) - \cos(\theta_f)}} \tag{1}$$

Where θ_i and θ_f represent the initial and final (impact) angle of the pendulum, respectively. ℓ is the length of the pendulum and g is the acceleration due to gravity.

The following values were experimentally acquired for the pendulum:

$$\ell \overset{m}{=} 14.0\,\text{cm}$$
$$\theta_i \overset{m}{=} 93.3°$$
$$\theta_f \overset{m}{=} 20.7°$$
$$t_{queda} = 0.191\,\text{s}$$

Furthermore, we use $g = 9.81\,\text{ms}^{-2}$.

When the blue ball reaches the peak of its ascent after being bumped, it means that its velocity is zero. We can find the velocity at the moment of impact with the first integral of motion[2]:

$$v = \sqrt{2gl(\cos(\theta_m) - \cos(\theta_p)} \tag{2}$$

Where θ_m is the angle at the moment of impact, and θ_p is the angle during the peak.

The experimental values are:

$$\theta_m \overset{m}{=} 0°$$
$$\theta_p \overset{m}{=} 70.5°$$
$$v \overset{m}{=} 1.32\,\text{ms}^{-1}$$

3. Method and Equipment

During the experiment, a cell phone camera was used, recording at 120 frames per second, and a clacker.

The initial, final (impact) and peak angles were acquired through the GeoGebra website with the geometry tool, with angle calculators between straight lines that were based on images of release, impact and peak.

The fall time was acquired knowing that the time lasted 23 frames ($23/120 \approx 0.191$ s).

To determine the speed, we calculate the ratio between the distance traveled in 4 frames (where the speed variation in each frame is ignoble), approximately 4.4 cm, and the time spent in them ($4/120 = 0.0\bar{3}$s). The result is $1.32\,\text{ms}^{-1}$.

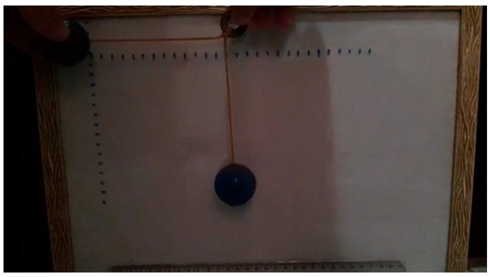

Figure 1: Instant of the release of the green ball

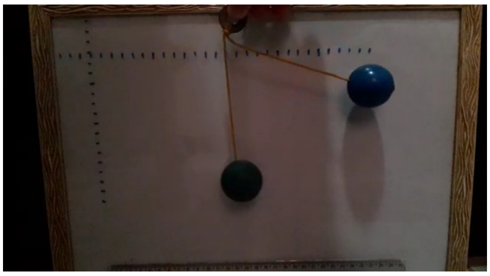

Figure 2: Peak of the blue ball after colliding.

4. Results.

Through the python language and the scipy library, and the formula for the time and the experimental values of the pendulum, we can numerically compute the expected value of the impact time for an ideal pendulum (since (1) is an elliptic integral, preventing it from being computed analytically).

The code used for the computation is here:

```
import numpy as np
from scipy.integrate import quad

l = 0.14
g = 9.81
theta_i = (93.3 / 180) * np.pi
theta_f = (20.7 / 180) * np.pi

t = np.sqrt(l/(2*g)) * quad(lambda
   theta: 1/np.sqrt(np.cos(theta) -
   np.cos(theta_i)), theta_f,
   theta_i)[0]
```

The value acquired is approximately $0.194\,\mathrm{s}$. Comparing with the experimental value ($t = 0.191\,\mathrm{s}$), we get an error of 1.54% (ie, an accuracy of 98.46%).

Speed accuracy is lower but still great. The experimental value is $1.32\,\mathrm{ms^{-1}}$, while the theoretical value, inserting the values of impact angle and peak in equation (2), we obtain $1.35\,\mathrm{ms^{-1}}$. The error is 2.22%, getting an accuracy of 97.78%.

5. Conclusion

Even with precarious tools, the numerical methods developed in mathematical physics have great precision.

References

[1] *Arbitrary-amplitude period formula*, Wikipedia, https://www.wikiwand.com/en/Pendulum_(mathematics)#/Arbitrary-amplitude_period (visited on 07/22/2022).

[2] *First integral of motion*, Wikipedia, https://en.wikipedia.org/w/index.php?title=Pendulum_(mechanics)#math_Eq._2 (visited on 07/22/2022).