Title: Is the Equivalence Principle Schizophrenic? … And a Summary, and a Correction

(Revised Edition)

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Abstract:

The equivalence principle is not always valid. Before trusting it, it is important to verify that there are no inconsistencies in either the General Relativity scenario or in the Special Relativity scenario.

The exponential version of the gravitational time dilation equation is incorrect.

I have derived the correct version of the gravitational time dilation equation, and its Special Relativity version agrees with the CMIF simultaneity method.

The Special Relativity version of the correct gravitational time dilation equation establishes, for an accelerating observer, a "NOW" moment throughout all space, which guarantees that the current age of a distant person, according to that accelerating observer, is fully meaningful and real to the accelerating observer.

Section 1. Introduction

The Equivalence Principle basically says that, if you have a scenario in General Relativity (GR) with no accelerations and no motions, but with a gravitational field "g", and with an equation f(g) that relates the clock rates of various clocks in the scenario, then that scenario is equivalent to a scenario in Special Relativity (SR) with no gravitational fields, but with motions and an acceleration "A" of various clocks whose clock rates are related by the same function "f" as in the GR scenario, with the argument "g" in the function "f" being replaced by the acceleration "A".

For example, in 1907 Einstein defined a scenario in SR, where two clocks separated by a constant distance "L" are initially unaccelerated and stationary, but which then (according to the two clocks) each simultaneously start accelerating (in the direction of their separation) with a constant acceleration "A". Einstein said that the two clocks will tic at different rates.

Specifically, according to an observer (the "AO") located with the trailing clock, the leading clock will run faster than the trailing clock, by the rate ratio

R(A) = exp(A * L),

where the asterisk denotes multiplication.

Then, Einstein said that, by the Equivalence Principle (EP), two stationary clocks, separated by the distance "L" in a uniform (not varying with distance) gravitational field "g" (in the direction of the separation), will tic at different rates. The clock that is farther from the source of the field will run faster than the other clock, by the rate ratio

$$R(g) = \exp(g * L).$$

But the equivalence principle has ALSO been applied by other physicists (e.g., by Wolfgang Rindler) to a somewhat similar pair of scenarios for the case where (in the GR scenario) the gravitational field varies with the distance from the source of the field. In that case,

R(g) = exp(integral from 0 to L of g[x] dx),

with the separation of the two clocks again being the constant L.

But the corresponding SR scenario is nonsense in this case: it would require that the two clocks accelerate at different rates (according to their own accelerometers), yet maintain a constant separation, which is impossible. So, in this case, the equivalence principle is not valid. (This situation is what motivated my characterization of the equivalence principle as “Schizophrenic”: sometimes it is correct, and sometimes it is not. Beware.)

Section 2. The exponential version of the Rate Ratio is Incorrect

Although the Equivalence Principle (EP) is correct in Einstein’s 1907 paper, It’s easy to see that his exponential version of the R(A) equation is incorrect. In his derivation, Einstein assumed a very small value of the acceleration “A”, and his equation was

$$R(A) = 1 + A * L,$$

where the asterisk denotes multiplication. Einstein BELIEVED that the above linear equation was only valid for very small values of “A”, and that the correct equation for arbitrary accelerations was the exponential

$$R(A) = \exp(A * L).$$

But the exponential equation that he and everyone else embraced isn’t correct, and it’s easy to see why. The CHANGE “AC” in the reading of the leading clock during a given change “tau” in the reading of the trailing clock, is just

$$AC = tau * R(A) = tau * \exp(A * L).$$

Now, imagine doing a sequence of experiments, with decreasing values of “tau”, where in all cases you keep the velocity change, due to the acceleration “A” acting for a duration “tau”, constant. So, as you make tau smaller, you make “A” larger, so that their product stays constant. (Their product is just the “rapidity”, theta, resulting from the acceleration, which is related to the final velocity after the acceleration. Rapidity can have arbitrarily large values, but velocity can’t exceed or equal the speed of light.) So, from one experiment to the next, “tau” gets smaller at the same rate that the acceleration gets larger (because their product stays constant). But note that in the equation for AC above, AC is NOT the product of tau and (A * L), it is the product of tau and the EXPONENTIAL of (A * L). So as tau gets smaller, the exponential grows at a MUCH MUCH faster rate than the rate at which tau is getting smaller, and so their product goes to infinity as tau goes to zero. So the reading on the leading clock goes to infinity as tau (the reading on the trailing clock) goes to zero. That contradicts what we

KNOW happens at the reunion of the twins in the twin paradox: the home twin (she) IS older than the traveling twin (he) at their reunion, but she is NOT infinitely older than he is.

I gave a specific numerical example of the above sequence of experiments in my viXra paper, titled “An Inconsistency Between the Gravitational Time Dilation Equation and the Twin Paradox”, https://vixra.org/abs/2109.0076 .

So the exponential version of the gravitational time dilation (GTD) equation is incorrect. Einstein’s linear version of the GDT equation is also incorrect (except for very small values of “A”), but it is not nearly as inaccurate as the exponential version is. His linear equation is at least in the right ballpark, even for arbitrarily large values of “A”.

Section 3. The corrected versions of the Rate Ratio and the Age Change

The corrected version of the Rate Ratio equation is

R(A, t) = [1 + L * A * sech^2 (A * t)],

where the “^” symbol denotes that the sech function is squared. The hyperbolic secant is the reciprocal of the hyperbolic cosine. The hyperbolic secant equals 1.0 when its argument is zero, and then symmetrically curves downward, and approaches zero as the argument approaches +infinity or -infinity.

Note that the above rate ratio varies with time … it is NOT constant during the constant acceleration, in contrast to the exponential equation, and in contrast to Einstein’s linear equation.

The Age Change “AC” after the time tau of the acceleration is

AC(tau) = tau + L * tanh(A * tau) = tau + L * tanh(theta[tau]) = tau + L * v(tau),

where “v(tau)” is the velocity at the end of the acceleration, and where the initial velocity v(0) before the acceleration starts is zero.

In the limit as tau goes to zero, the above equation gives, for an instantaneous change in the velocity,

delta(AC) = L * delta(v),

which gives the same result as the co-moving inertial frames (CMIF) simultaneity method. In fact, the above result is just the delta_CADO equation, which is the easiest way to determine simultaneity in the CMIF method.

The above equations are derived in my viXra paper, titled “A New Gravitational Time Dilation Equation”, https://vixra.org/abs/2201.0015.

Section 4. A “NOW” Moment Throughout All Space for an Accelerating Observer

Einstein showed us how a perpetually inertial observer could set up an array of synchronized clocks, by sending light signals (whose speed is known) between pairs of clocks. That array then defines a “NOW” instant, extending throughout all space, for that perpetually inertial observer (the “PIO”). At a given “NOW” instant, all of the clocks show the same time. Each of those clocks can be accompanied by an inertial “helper friend” (an “HF”). All of the HF’s always have the same age as the IO (according to observers stationary in that inertial frame). Suppose that when the PIO is some age “T”, he would like to know how old some particular distant person (she) is, right then. All he needs to do is find out which HF was co-located with her when the HF was age “T”, and ask that HF what her age was then.

That information given to him must be considered to be completely meaningful to him, because it is based ONLY on the assumption that a light pulse travels at a specific given speed in ALL inertial reference frames: 186,000 miles per second. And if that assumption is false, then there IS no special theory of relativity.

But how about for an accelerating observer? Can a meaningful “NOW” instant throughout all space be defined for him? The answer is “YES”! It is conceptually possible for an accelerating observer (the “AO”) to set up an array of clocks that are always stationary with respect to him (according to him). They aren’t synchronized like in the perpetually inertial case, but the AO can COMPUTE the readings on each of the clocks at any instant in his life, so that does establish a “NOW” moment throughout space for him. The Age Change (AC) equation in the previous section tells the AO what the current ages of each of the “helper friends” (the HF’s) co-located with the clocks are, at any instant in the AO’s life. There are enough HF’s to cover all of space. So, if the AO (at some instant of his life) wants to know the current age of some particular distant person (her) (who may or may not be moving with respect to the AO), he just asks the particular HF, who happens to be momentarily co-located with her at that instant, what her age is when the HF’s age is what the AO computes it to be (using the AC equation) at that instant. It may take him a long time to exchange all that information with that HF, but eventually he can know exactly how old that distant person was at that particular instant in his life. And, that answer MUST be considered to be completely meaningful to him, assuming that the Age Change (AC) equation that I have derived is valid. I’m confident that it IS valid.

The above process could take a very long time to carry out. But that’s not a problem: that procedure’s role is strictly in proving that the“NOW” instant thus determined is real and meaningful for the AO. But the AO doesn’t have to wait to get an answer to how old the distant person is “right now” … he can determine that immediately, because the same answer is also given by the CMIF simultaneity method (most easily using the delta_CADO equation). The value of the array of clocks is strictly that it guarantees that the result the CMIF method gives him is fully meaningful to him, and that the CMIF simultaneity method is the ONLY correct simultaneity method.

Section 5. Correction

I need to correct an error I made in my viXra paper, titled “A New Gravitational Time Dilation Equation”, https://vixra.org/abs/2201.0015. Near the end of Section 1 of that paper, I wrote

“Although the AO and the HF have different ages as the acceleration progresses, they each know, and agree about, what the relationship is between their respective ages. That establishes a “NOW” instant for them, that they both agree about.”

That statement is incorrect. The AO and the HF DON’T agree about the relationship between their ages.

In Section3, I gave the Rate Ratio equation:

$$R(A, t) = [1 + L * A * \text{sech}^2 (A * t)],$$

which gives, according to the AO, the ratio of the HF's rate of ageing to the AO's rate of ageing. The acceleration "A" was taken to be positive, and thus R is greater than 1.0 when the AO is trailing, and the HF is leading. So the HF will be older than the AO, according to the AO.

But if we make "A" negative, that makes the second term in the R equation negative, and R will then be less than 1.0. So for negative "A", the AO says that the HF is the younger one, not the older one. And the AO is no longer the trailing observer, he is now the leading observer. So by making "A" negative, we have effectively switched the roles of the AO and the HF. We can then say that, according to the HF, the AO is ageing less fast than the HF, by the ratio

$$R2(A, t) = [1 - L * |A| * \text{sech}^2 (A * t)].$$

But the quantities R and R2 are NOT reciprocals (because their product is not equal to 1.0), and therefore the AO and the HF do NOT agree about the correspondence of their ages.

That disagreement is of no importance to us though, because it is strictly the AO's view of the "NOW" moment that we want.

The R2 equation IS of some use to us, though, because it also gives the viewpoint of the AO when he is accelerating AWAY FROM the home twin, rather than toward her.

Section 6. Conclusions

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