# Along the side of the Onsager's solution, the Ekagi language–Part Three

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(Dated: May 28, 2022)

## Abstract

We continue to consult the Ekagi-Dutch-English-Indonesian Dictionary by J. Steltenpool. In this short note, we remove all the multiple countings of an entry in a letter's section which have gone in in the companion paper "Along the side of the Onsager's solution, the Ekagi language; viXra: 2205.0065[Condensed Matter]". We draw the natural logarithm of the number of entries, denoted as f, normalised, starting with a letter vs the natural logarithm of the rank of the letter, denoted as k. We find that $\frac{lnf}{lnf_{max}}$ vs $\frac{lnk}{lnk_{lim}}$ is matched by the graph of the reduced magnetisation vs the reduced temperature of the exact Onsager solution of the two dimensional Ising model in the absence of the external magnetic field

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| letter | A | B | D | E | G | I | J | K | M | N | O | P | T | U | W |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| number | 909 | 510 | 567 | 901 | 462 | 472 | 511 | 978 | 877 | 296 | 430 | 693 | 717 | 420 | 421 |
| splitting | 523+386 | 378+132 | 421+146 | 576+325 | 373+89 | 287+185 | 350+161 | 734+244 | 532+345 | 178+118 | 242+188 | 519+174 | 600+117 | 276+144 | 350+71 |

TABLE I. The Ekagi language entries: the first row represents letters of English alphabet in the serial order, the second row is the respective number of entries, the third row describes the splitting of entries.

## I. INTRODUCTION AND RESULTS

We continue to consult the Ekagi-Dutch-English-Indonesian Dictionary by J. Steltenpool, [1]. In this short note, we remove all the multiple countings of an entry in a letter's section which have gone in in the companion paper "Along the side of the Onsager's solution, the Ekagi language", [2] The result is the table, I.
Highest number of entries, nine hundred seventy eight, starts with the letter K followed by entries numbering nine hundred nine beginning with A, nine hundred one with the letter E etc. To visualise the pattern of change of number of entries along the the letters initiating with, we draw the number of entries vs. sequence number of the respective letters in the fig.1.
For the purpose of exploring graphical law, we assort the letters according to the number of entries, in the descending order, denoted by $f$ and the respective rank, denoted by $k$, [3]. Moreover, we attach a limiting rank, $k_{lim}$, and a limiting number of entries. The limiting rank is maximum rank plus one, denoted as $k_{lim}$ or, $k_d$. Here it is sixteen and the limiting number of entries is one. As a result, $k$ is a positive integer starting from one and both $\frac{lnf}{lnf_{max}}$ and $\frac{lnk}{lnk_{lim}}$ varies from zero to one. Then we tabulate in the adjoining table, II and plot $\frac{lnf}{lnf_{max}}$ against $\frac{lnk}{lnk_{lim}}$ in the figure fig.2. We then ignore the letter with the highest number of entries, tabulate in the adjoining table, II and redo the plot, normalising the $lnf$s with next-to-maximum $lnf_{nextmax}$, and starting from $k=2$ in the figure fig.3. This program then we repeat up to $k=3$, resulting in figures up to fig.4.

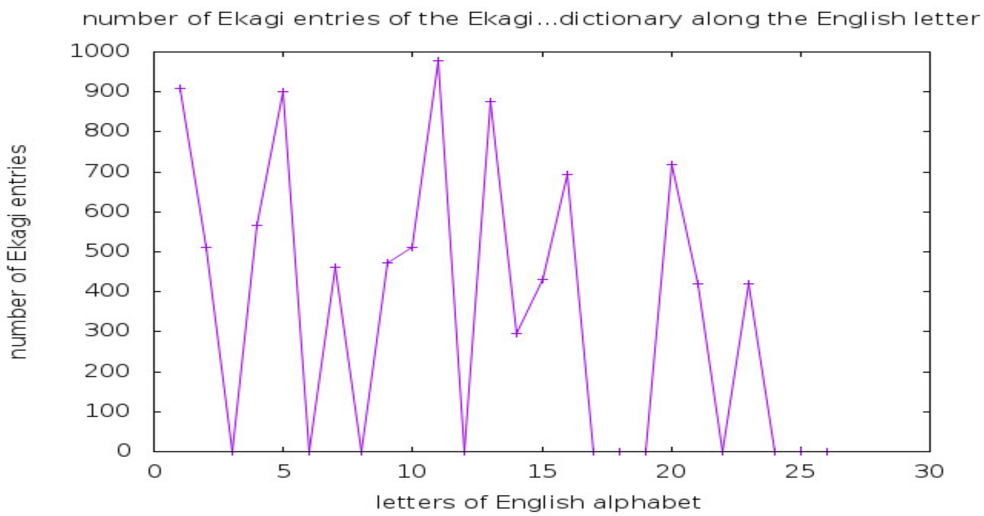


FIG. 1. The vertical axis is number of entries of the Ekagi language and the the horizontal axis is the respective letters of the Ekagi alphabet. Letters are represented by the sequence number in the English alphabet beginning with A.

| k | lnk | lnk/$lnk_{lim}$ | f | lnf | lnf/$lnf_{max}$ | lnf/$lnf_{n-max}$ | lnf/$lnf_{2n-max}$ |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 978 | 6.886 | 1 | Blank | Blank |
| 2 | 0.69 | 0.249 | 909 | 6.812 | 0.989 | 1 | Blank |
| 3 | 1.10 | 0.397 | 901 | 6.804. | 0.988 | 0.999 | 1 |
| 4 | 1.39 | 0.502 | 877 | 6.777 | 0.984 | 0.995 | 0.996 |
| 5 | 1.61 | 0.581 | 717 | 6.575 | 0.955 | 0.965 | 0.966 |
| 6 | 1.79 | 0.646 | 693 | 6.541 | 0.950 | 0.960 | 0.961 |
| 7 | 1.95 | 0.704 | 567 | 6.340 | 0.921 | 0.931 | 0.932 |
| 8 | 2.08 | 0.751 | 511 | 6.236 | 0.906 | 0.915 | 0.917 |
| 9 | 2.20 | 0.794 | 510 | 6.234 | 0.905 | 0.915 | 0.916 |
| 10 | 2.30 | 0.830 | 472 | 6.157 | 0.894 | 0.904 | 0.905 |
| 11 | 2.40 | 0.866 | 462 | 6.136 | 0.891 | 0.901 | 0.902 |
| 12 | 2.48 | 0.895 | 430 | 6.064 | 0.881 | 0.890 | 0.891 |
| 13 | 2.56 | 0.924 | 421 | 6.043 | 0.878 | 0.887 | 0.888 |
| 14 | 2.64 | 0.953 | 420 | 6.040 | 0.877 | 0.887 | 0.888 |
| 15 | 2.71 | 0.978 | 296 | 5.690 | 0.826 | 0.835 | 0.836 |
| 16 | 2.77 | 1 | 1 | 0 | 0 | 0 | 0 |

TABLE II. The Ekagi language entries: ranking, natural logarithm, normalisations

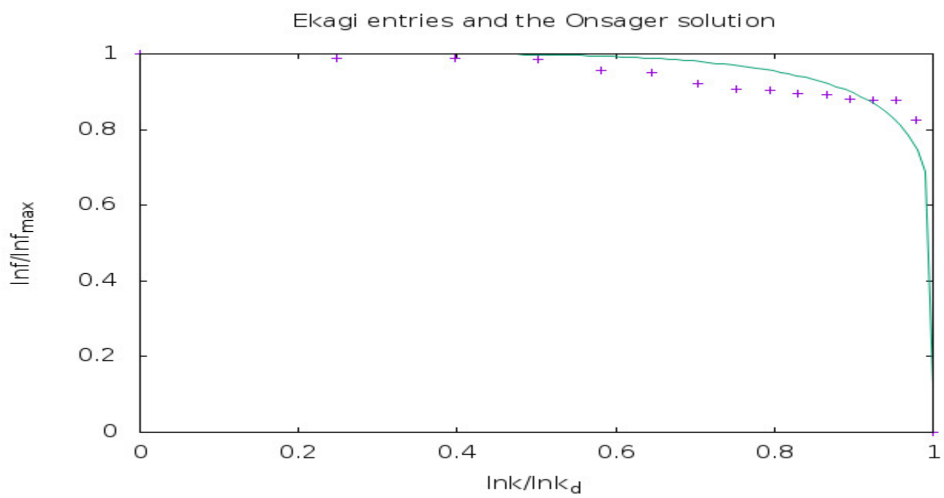


FIG. 2. The vertical axis is $\frac{lnf}{lnf_{max}}$ and the horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the entries of the Ekagi language The reference curve is the Onsager solution.

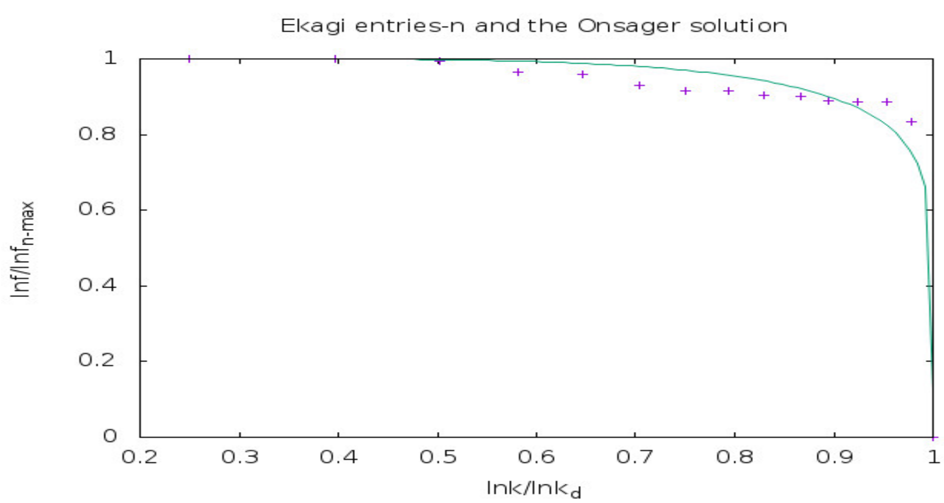


FIG. 3. The vertical axis is $\frac{lnf}{lnf_{next-max}}$ and the horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the entries of the Ekagi language The reference curve is the Onsager solution.

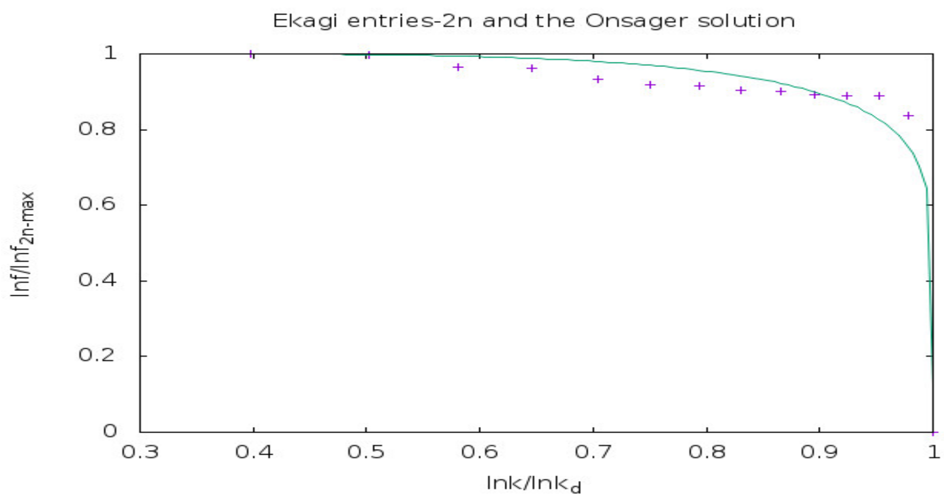


FIG. 4. The vertical axis is $\frac{lnf}{lnf_{nextnext-max}}$ and the horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the entries of the Ekagi language The reference curve is the Onsager solution.

### A. conclusion

From the figures (fig.2-fig.4), we observe that the entries of the Ekagi language, [1], underlies the Onsager solution.
Moreover, the associated correspondence is,

$$\frac{lnf}{lnf_{max}} \longleftrightarrow \frac{M}{M_{max}},$$

$$lnk \longleftrightarrow T.$$

k corresponds to temperature in an exponential scale, [4].

## II. THE ONSAGER'S SOLUTION

The two dimensional Ising model, [5], in the absence of external magnetic field is prototype of an Ising model. In case of square lattice of planar spins, one spin interacts with four other nearsest neighbour spins i.e. on an average to another one spin. Below a certain ambient temperature, denoted as $T_c$, the two dimensional array of spins reduces to a planar magnet with magnetic moment per site varying as a function of $\frac{T}{T_c}$. This function was inferred, [6], by Lars Onsager way back in 1948 and thoroughly deduced thereafter by C.N.Yang, [7]. This function we are referring to as Onsager solution. Moreover, systems, [8], showing behaviour like Onsager solution is rare to come across. Graphically, the Onsager solution appears as in fig.5.

To have a comprehension, let us imagine an arbitrary lattice, with each up spin assigned a value one and a down spin a value minus one, with an unspecified long-range order parameter defined as above by $L = \frac{1}{N}\Sigma_i\sigma_i$, where $\sigma_i$ is i-th spin, N being total number of spins. L can vary from minus one to one. $N = N_+ + N_-$, where $N_+$ is the number of up spins, $N_-$ is the number of down spins. $L = \frac{1}{N}(N_+ - N_-)$. As a result, $N_+ = \frac{N}{2}(1 + L)$ and $N_- = \frac{N}{2}(1 - L)$. Magnetisation or, net magnetic moment , $M$ is $\mu\Sigma_i\sigma_i$ or, $\mu(N_+ - N_-)$ or, $\mu NL$, $M_{max} = \mu N$. $\frac{M}{M_{max}} = L$. $\frac{M}{M_{max}}$ is referred to as reduced magnetisation. Moreover, the Ising Hamiltonian,[5], for the lattice of spins, setting $\mu$ to one, is $-\epsilon\Sigma_{n.n}\sigma_i\sigma_j - H\Sigma_i\sigma_i$, where n.n refers to nearest neighbour pairs.
At a temperature T, below a certain temperature called phase transition temperature, $T_c$, for the two dimensional Ising model in the absence of external magnetic field i.e. for H

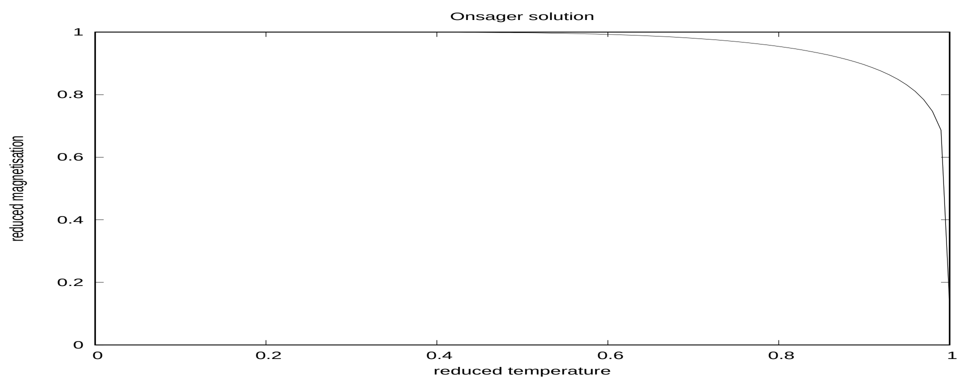


FIG. 5. Reduced magnetisation vs reduced temperature curves for exact solution of two dimensional Ising model, due to Onsager, in the the absence of external magnetic field

equal to zero, the exact, unapproximated, Onsager solution gives reduced magnetisation as a function of reduced temperature as, [6], [9], [7], [10],

$$\frac{M}{M_{max}} = [1 - (sinh\frac{0.8813736}{\frac{T}{T_c}})^{-4}]^{1/8}.$$

and appears as in fig.5.

## III. ACKNOWLEDGMENT

We have used gnuplot for plotting the figures in this paper.

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