

Planck Length, Planck Time and Speed of Gravity When Taking into Account Relativistic Mass with No Knowledge of G , \hbar or c

Espen Gaarder Haug

orcid.org/0000-0001-5712-6091

Norwegian University of Life Sciences, Norway

e-mail espenhaug@mac.com and espen.haug@nmbu.no

May 16, 2022

Abstract

In this paper, we take into account Lorentz's relativistic mass and then derive formulas for the Planck length and the Planck time that are not dependent on any other constants. Thus we can find the Planck length, the Planck time, and also the speed of gravity, from gravitational observations without any knowledge of any physical constants. This is in strong contrast to what has been, and currently is, thought to be the case. Since we take into account relativistic mass, our formulas are also fully accurate for a strong gravitational field. We will claim general relativity theory cannot be fully precise in respect to strong gravitational fields. For example, general relativity theory leads to an imaginary time dilation factor when at the Planck length distance from a Planck mass, but when taking into account Lorentz's relativistic mass, the time dilation works properly all the way down to, and including, the Planck length distance.

Key Words: Planck time, Planck length, speed of gravity, Lorentz's relativistic mass, general relativity theory.

1 Planck length and Planck time from general relativity theory

Gravitational time dilation is given by:

$$T_h = T_L \frac{\sqrt{1 - \frac{v_{e,h}^2}{c^2}}}{\sqrt{1 - \frac{v_{e,L}^2}{c^2}}} \quad (1)$$

where $v_{e,L}$ and $v_{e,h}$ are the escape velocities at radius R_L and R_h , and where $R_h > R_L$. In Einstein's [1] general relativity theory, one is ignoring Lorentz's [2] relativistic mass; see also [3, 4]. When ignoring relativistic mass, then the escape velocity v_e is given by $v_e = \sqrt{\frac{2GM}{R}}$, which is identical to the escape velocity one gets from standard Newton gravity [5]. So general relativity theory and Newton theory here give the same result, despite Newton's theory not taking into account any relativistic effects. Next, replacing the escape velocity formula back into formula 1 above, we get:

$$T_h = T_L \frac{\sqrt{1 - \frac{2GM}{R_h c^2}}}{\sqrt{1 - \frac{2GM}{R_L c^2}}} \quad (2)$$

which is the well-known gravitational time dilation. Since the Schwarzschild radius is $R_s = \frac{2GM}{c^2}$ this can also be re-written as:

$$T_h = T_L \frac{\sqrt{1 - \frac{R_s}{R_h}}}{\sqrt{1 - \frac{R_s}{R_L}}} \quad (3)$$

The general relativity time dilation has several strange properties for a very strong gravitational field. A very strong gravitational field is what we have for a Planck mass when we are at, or close to, the reduced Compton wavelength of the mass, which is the Planck length for the Planck mass. In the special case where we stand at the Planck length, we have $\frac{2Gm_p}{l_p} = 2$, so this would lead to a time dilation factor of:

$$\sqrt{1 - \frac{2GM}{Rc^2}} = \sqrt{1 - \frac{2Gm_p}{l_p c^2}} = \sqrt{1 - 2} = \sqrt{-1} = i \quad (4)$$

In other words, this gives an imaginary time dilation at the Planck length for the Planck mass, as likely first mentioned by Haug [6] in 2018. However, one can easily argue that this is irrelevant as a radius equal to the Planck length for a Planck mass is actually inside the Schwarzschild radius and so this is where general relativity theory in general breaks down. So let's move to the Schwarzschild radius of the Planck mass, which is $R_s = \frac{2Gm_p}{c^2} = 2l_p$. This gives $\frac{2Gm_p}{2l_p} = 1$, so the time dilation factor would be $\sqrt{1 - 1} = \sqrt{0} = 0$, and it could be interpreted as time not existing at the Schwarzschild radius. Time here stands still, which we think is reasonable. Still the question is why the time dilation factor does not work all the way down to the Planck length if the Planck length is the shortest possible length even hypothetical observable. Bear in mind that the Planck length is the reduced Compton wavelength of the Planck mass. To understand the Planck scale more profoundly could be of great importance, so this is one of the questions we will explore in this paper.

If we solve the Planck [7, 8] length formula $l_p = \sqrt{\frac{G\hbar}{c^3}}$ with respect to G , we get $G = \frac{l_p^2 c^3}{\hbar}$, and further we solve the reduced Compton [9] wavelength formula $\bar{\lambda}_M = \frac{\hbar}{Mc}$ with respect to M and we get $M = \frac{\hbar}{\lambda_M c}$. Some will likely protest here and say we cannot do that for macroscopic masses, only for such things as electrons and perhaps protons [10, 11], as only these have a Compton wavelength. This would be a mistake, as we have clearly demonstrated one can represent any mass of any size with this formula, because the Compton wavelength of a macroscopic mass represents the composite "sum"¹ of the Compton wavelength in all the elementary particles in that mass; see [12, 13].

Next, if we replace this back into formula 2, we get:

$$\begin{aligned} T_h &= T_L \frac{\sqrt{1 - \frac{2GM}{R_h c^2}}}{\sqrt{1 - \frac{2GM}{R_L c^2}}} \\ T_h &= T_L \frac{\sqrt{1 - \frac{2c^2 l_p^2}{\lambda_M R_h}}}{\sqrt{1 - \frac{2c^2 l_p^2}{\lambda_M R_L}}} \\ T_h &= T_L \frac{\sqrt{1 - \frac{2l_p^2}{\lambda_M R_h}}}{\sqrt{1 - \frac{2l_p^2}{\lambda_M R_L}}} \end{aligned} \quad (5)$$

Next we solve this with respect to l_p and we get:

$$l_p = \frac{\sqrt{\lambda_M R_h R_L (T_h^2 - T_L^2)}}{\sqrt{2R_h T_h^2 - 2R_L T_L^2}} \quad (6)$$

This formula was derived and published by Haug [14, 15] in 2021/2022. It is an exact result under general relativity theory but, as we soon will claim, general relativity theory is incomplete. One of the reasons for this is that it ignores Lorentz relativistic mass.

Further, under general relativity theory, the weak field approximation for the Planck length is given by:

$$l_p \approx \frac{\sqrt{\lambda_M R_h R_L (T_h - T_L)}}{\sqrt{R_h T_h - R_L T_L}} \quad (7)$$

This result was also given in the same papers referred to above. The gravitational field on, for example, the surface of the Earth is weak.

¹The summation is given by $\bar{\lambda} = \sum_i^n = \frac{1}{\lambda_i}$ where λ_i is the reduced Compton wavelength of the elementary particles.

2 Planck length and Planck time when taking into account relativistic mass

We will now incorporate relativistic mass. This gives a different escape velocity than in general relativity theory. We basically assume the small mass m that is acted upon by the gravitational field of M is moving relative to M and is observed from M . The mass M is therefore stationary, while m is moving and should be adjusted for Lorentz's relativistic effects. We have to solve the following equation with respect to v to find the escape velocity:

$$mc^2\gamma - mc^2 - G\frac{Mm\gamma}{R^2} = 0 \quad (8)$$

where γ is the Lorentz factor, $= 1/\sqrt{1 - v^2/c^2}$. This gives

$$v_e = v = \sqrt{\frac{2GM}{R} - \frac{G^2M^2}{R^2c^2}} \quad (9)$$

which is different from the escape velocity due to the last term. The last term inside the square root can be seen as a correction for relativistic mass. This escape velocity leads to a series of implications; see [16, 17]. Such an escape velocity was probably first described in 2009 by Rybczyk [18] and later independently re-discovered by Haug in 2019 [19]. What is interesting is that such an escape velocity has never been fully investigated by the physics' community. Lorentz's relativistic mass was rejected before investigating what implications it would lead to, which is not really the way good science should be performed.

Next, we input this escape velocity into the time dilation formula and get:

$$\begin{aligned} T_h &= T_L \frac{\sqrt{1 - \frac{v_{e,h}^2}{c^2}}}{\sqrt{1 - \frac{v_{e,L}^2}{c^2}}} \\ T_h &= T_L \frac{\sqrt{1 - \frac{\frac{2GM}{R_h} - \frac{G^2M^2}{c^2R_h^2}}{c^2}}}{\sqrt{1 - \frac{\frac{2GM}{R_L} - \frac{G^2M^2}{c^2R_L^2}}{c^2}}} \end{aligned} \quad (10)$$

In the case of a Planck mass, if we stand at the Planck length, this leads to:

$$\sqrt{1 - \frac{\frac{2Gm_p}{l_p} - \frac{G^2m_p^2}{c^2l_p^2}}{c^2}} = 0 \quad (11)$$

So here, time stands still at the Planck length, which is the radius where the escape velocity is c when we take into account relativistic mass. That time stands still when $v = c$ makes sense. We are not ending up with the absurdity of imaginary time as in general relativity, even at the shortest possible length, which almost for sure is the Planck length.

Again, if we solve the Planck length formula $l_p = \sqrt{\frac{G\hbar}{c^3}}$ with respect to G we get $G = \frac{l_p^2c^3}{\hbar}$, and in addition we replace M with $M = \frac{\hbar}{\lambda_M} \frac{1}{c}$, then we replace this into the formula 10 and we get:

$$\begin{aligned} T_h &= T_L \frac{\sqrt{1 - \frac{\frac{2c^2l_p^2}{\lambda_M R_h} - \frac{c^4l_p^4}{\lambda^2c^2R_h^2}}{c^2}}}{\sqrt{1 - \frac{\frac{2c^2l_p^2}{\lambda_M R_L} - \frac{c^4l_p^4}{\lambda^2c^2R_L^2}}{c^2}}} \\ T_h &= T_L \frac{\sqrt{1 - \frac{2l_p^2}{\lambda_M R_h} + \frac{l_p^4}{\lambda^2R_h^2}}}{\sqrt{1 - \frac{2l_p^2}{\lambda_M R_L} + \frac{l_p^4}{\lambda^2R_L^2}}} \end{aligned} \quad (12)$$

Next, we solve this with respect to l_p and we get:

$$l_p = \frac{\sqrt{\bar{\lambda}_M R_h R_L (T_h - T_L)}}{\sqrt{R_h T_h - R_L T_L}} \quad (13)$$

Pay attention to the fact that this exact solution, that also holds true under a strong gravitational field, is identical to the weak field approximation we got under general relativity. The reason is that general relativity theory does not take into account relativistic mass. Now in this new theory the weak field approximation for the Planck length is actually general relativities strong field exact solution (Eq. ??). However, it is likely close to impossible to directly test which theory is best, as we only have weak gravitational fields to test in. However, there is more than practical testing that should be used to decide on the best theory; something we have discussed in other papers. Further that general relativity theory not can work all the way down to the Planck length in relation to the Planck mass should make researchers think that perhaps indeed something is missing in general relativity theory, and personally we are convinced that ignoring relativistic mass is one of these things.

Gravitational red-shift is given by:

$$\begin{aligned} z &= \frac{f_h - f_L}{f_L} = \frac{\sqrt{1 - \frac{v_{e,L}^2}{c^2}}}{\sqrt{1 - \frac{v_{e,h}^2}{c^2}}} - 1 \\ z &= \frac{\sqrt{1 - \frac{2l_p^2}{\bar{\lambda}_M R_h} + \frac{l_p^4}{\bar{\lambda}_M^2 R_h^2}}}{\sqrt{1 - \frac{2l_p^2}{\bar{\lambda}_M R_L} + \frac{l_p^4}{\bar{\lambda}_M^2 R_L^2}}} - 1 \end{aligned} \quad (14)$$

Next we solve this with respect to l_p and we get:

$$\begin{aligned} l_p &= \frac{\sqrt{z \bar{\lambda}_M R_h R_L}}{\sqrt{R_h z + R_h - R_L}} \\ l_p &= \frac{\sqrt{\bar{\lambda}_M R_h R_L}}{\sqrt{R_h + \frac{R_h - R_L}{z}}} \end{aligned} \quad (15)$$

That is, we need no knowledge of any other constants to find the Planck length. The reduced Compton wavelength of the gravitational mass $\bar{\lambda}_M$ can be found without knowledge of \hbar and c and G as described by, for example, [12?]. The gravitational red-shift we can find by simply measuring the frequencies of a laser beam at two altitudes $R_h > R_L$.

Further, we have $g = \frac{GM}{R^2}$, and again by replacing G with $G = \frac{l_p^2 c^3}{\hbar}$, and M with $M = \frac{\hbar}{\lambda c}$, and next solving with respect to c , we get:

$$c = \sqrt{\frac{g R_L^2 \bar{\lambda}_M}{l_p^2}} \quad (16)$$

This speed is linked to both the speed of gravity c_g and the speed of light c ($c_g = c$), as they are the same thing; see also [14]. Next, replacing the Planck length l_p in this formula with the expression above for the Planck length, we get:

$$\begin{aligned} c_g &= \sqrt{\frac{g_L R_L^2 \bar{\lambda}_M}{l_p^2}} \\ c_g &= \sqrt{\frac{g_L R_L^2 \bar{\lambda}_M}{\frac{\bar{\lambda}_M R_h R_L}{R_L + \frac{R_h - R_L}{z}}}} \\ c_g &= \sqrt{\frac{g_L R_L (R_h + \frac{R_h - R_L}{z})}{R_h}} \end{aligned} \quad (17)$$

and the weak field approximation is given by

$$\begin{aligned}
c_g &\approx \sqrt{\frac{g_L R_L^2 \bar{\lambda}_M}{l_p^2}} \\
c_g &\approx \sqrt{\frac{g_L R_L^2 \bar{\lambda}_M}{\frac{\bar{\lambda}_M R_h R_L z(z+2)}{2R_h - 2R_L + 2R_h z^2 + 4R_h z}}} \\
c_g &\approx \sqrt{\frac{g_L R_L}{\frac{R_h z(z+2)}{2R_h - 2R_L + 2R_h z^2 + 4R_h z}}} \\
c_g &\approx \sqrt{\frac{g_L R_L 2(2R_h + \frac{R_h - R_L}{z} + R_h z)}{R_h(z+2)}} \tag{18}
\end{aligned}$$

Thus we can find the speed of gravity from two gravitational observations: the gravitational acceleration and the gravitational red-shift. It would be hard to claim this is anything else than the speed of gravity. No knowledge of the speed of light is needed, nor any expensive equipment to detect gravitational waves. This can be done by a simple drop ball and two time gates or a ball with built in stop-watch [?]. Next, we measure the frequency of a light beam at two different altitudes (similar to the Pound and Rebka [20] experiment), but not its speed. The radiuses R_h and R_L can be easily measured without any knowledge of any constants; further, R_L is simply the radius of the Earth, for example at the ocean level, and R_h is $R_L + H$, where H is the height above ocean level.

Next, we can divide the Planck length by c_g to find the Planck time. The Planck time is therefore given by:

$$\begin{aligned}
t_p &= \frac{l_p}{c_g} = \frac{\sqrt{\bar{\lambda}_M R_h R_L}}{\sqrt{R_h + \frac{R_h - R_L}{z}} \sqrt{\frac{g_L R_L (R_h + \frac{R_h - R_L}{z})}{R_h}}} \\
t_p &= \frac{R_h \sqrt{\bar{\lambda}_M / g_L}}{R_h + \frac{R_h - R_L}{z}} \tag{19}
\end{aligned}$$

To find the Planck time or the Planck length (e.g., 15 and 19) pay attention to needing to find the reduced Compton wavelength of the Earth. However, this can easily be achieved without knowledge of G and \hbar . Whether one needs c to do this is unclear; see [12]. To find the speed of gravity, we do not even need to know the reduced Compton wavelength of the gravitational mass. The weak field approximation when taking into account Lorentz relativistic mass is

$$\begin{aligned}
t_p &\approx \frac{l_p}{c_g} = \frac{\sqrt{\bar{\lambda}_M R_h R_L z(z+2)}}{\sqrt{2R_h - 2R_L + 2R_h z^2 + 4R_h z} \sqrt{\frac{g_L R_L (R_h + \frac{R_h - R_L}{z})}{R_h}}} \\
t_p &\approx \frac{l_p}{c_g} = \frac{R_h \sqrt{\bar{\lambda}_M (1 + 2/z)}}{\sqrt{\frac{2}{z} (R_h + \frac{R_h - R_L}{z}) + \frac{4R_h}{z}} \sqrt{g_L (R_h + \frac{R_h - R_L}{z})}} \\
t_p &\approx \frac{l_p}{c_g} = \frac{R_h \sqrt{\bar{\lambda}_M (z/2 + 1) / g_L}}{R_h + \frac{R_h - R_L}{z} + \sqrt{2R_h (R_h + \frac{R_h - R_L}{z})}} \tag{20}
\end{aligned}$$

Table 1 summarise some of our results. As we can see the exact strong field solution in general relativity theory is actually identical to the weak field approximation when we take into account Lorentz relativistic mass. Further the general relativity theory's weak field approximations are actually identical to the exact strong field solution when we take into account Lorentz relativistic mass. This likely mean general relativity will give incorrect predictions in a very strong gravitational field, such as at or close to the Planck scale. In the table "exact" simply mean it is an exact solution, not that we can measure the Planck length or Planck time exact. There is naturally uncertainty in several of the input parameters needed. The uncertainty in measurement of the Planck length will be the same as before, except we now do not need to go through other physical constants to find it.

	From general relativity theory:	Taking into account Lorentz relativistic mass:
Planck length independent on G, h and c		
From time dilation, exact	$l_p = \frac{\sqrt{\lambda_M R_h R_L (T_h^2 - T_L^2)}}{\sqrt{2R_h T_h^2 - 2R_L T_L^2}}$	$l_p = \frac{\sqrt{\lambda_M R_h R_L (T_h - T_L)}}{\sqrt{R_h T_h - R_L T_L}}$
From time dilation, weak field approx	$l_p \approx \frac{\sqrt{\lambda_M R_h R_L (T_h - T_L)}}{\sqrt{R_h T_h - R_L T_L}}$	$l_p \approx \frac{\sqrt{\lambda_M R_h R_L (T_h^2 - T_L^2)}}{\sqrt{2R_h T_h^2 - 2R_L T_L^2}}$
From red-shift, exact	$l_p = \frac{\sqrt{\lambda_M R_h R_L (z+2)}}{\sqrt{2(2R_h + \frac{R_h - R_L}{z} + R_h z)}}$	$l_p = \frac{\sqrt{\lambda_M R_h R_L}}{\sqrt{R_h + \frac{R_h - R_L}{z}}}$
From red-shift, weak field approx	$l_p = \frac{\sqrt{\lambda_M R_h R_L}}{\sqrt{R_h + \frac{R_h - R_L}{z}}}$	$l_p = \frac{\sqrt{\lambda_M R_h R_L (z+2)}}{\sqrt{2(2R_h + \frac{R_h - R_L}{z} + R_h z)}}$
Planck time independent on G, h		
From time dilation, exact	$t_p = \frac{\sqrt{\lambda_M R_h R_L (T_h^2 - T_L^2)}}{c\sqrt{2R_h T_h^2 - 2R_L T_L^2}}$	$t_p = \frac{\sqrt{\lambda_M R_h R_L (T_h - T_L)}}{c\sqrt{R_h T_h - R_L T_L}}$
From time dilation, weak field approx	$t_p \approx \frac{\sqrt{\lambda_M R_h R_L (T_h - T_L)}}{c\sqrt{R_h T_h - R_L T_L}}$	$t_p \approx \frac{\sqrt{\lambda_M R_h R_L (T_h^2 - T_L^2)}}{c\sqrt{2R_h T_h^2 - 2R_L T_L^2}}$
From red-shift, exact	$t_p = \frac{\sqrt{\lambda_M R_h R_L z(z+2)}}{c\sqrt{2R_h - 2R_L + 2R_h z^2 + 4R_h z}}$	$t_p = \frac{\sqrt{\lambda_M R_h R_L}}{c\sqrt{R_h + \frac{R_h - R_L}{z}}}$
From red-shift, weak field approx	$t_p = \frac{\sqrt{\lambda_M R_h R_L}}{c\sqrt{R_h + \frac{R_h - R_L}{z}}}$	$t_p = \frac{\sqrt{\lambda_M R_h R_L z(z+2)}}{c\sqrt{2R_h - 2R_L + 2R_h z^2 + 4R_h z}}$
Planck time independent on G, h and c		
From red-shift, exact	$t_p = \frac{R_h \sqrt{\lambda_M (z/2+1)/g_L}}{R_h + \frac{R_h - R_L}{z} + \sqrt{2\left(R_h + \frac{R_h - R_L}{z}\right)}}$	$t_p = \frac{R_h \sqrt{\lambda_M / g_L}}{R_h + \frac{R_h - R_L}{z}}$
From red-shift, weak field approx.	$t_p = \frac{R_h \sqrt{\lambda_M / g_L}}{R_h + \frac{R_h - R_L}{z}}$	$t_p \approx \frac{R_h \sqrt{\lambda_M (z/2+1)/g_L}}{R_h + \frac{R_h - R_L}{z} + \sqrt{2R_h \left(R_h + \frac{R_h - R_L}{z}\right)}}$
Speed of gravity independent on G, h and c		
c_g exact solution	$c_g \approx \sqrt{\frac{g_L R_L 2(R_h - R_L + R_h z^2 + 2R_h z)}{R_h z(z+2)}}$	$c_g = \sqrt{\frac{g_L R_L \left(R_h + \frac{R_h - R_L}{z}\right)}{R_h}}$
c_g weak field approx.	$c_g \approx \sqrt{\frac{g_L R_L \left(R_h + \frac{R_h - R_L}{z}\right)}{R_h}}$	$c_g \approx \sqrt{\frac{g_L R_L 2(R_h - R_L + R_h z^2 + 2R_h z)}{R_h z(z+2)}}$

Table 1: The table shows how we can find the Planck length without any knowledge off G , \hbar and likely also independent on any knowledge of c based on general relativity theory and when we take into account Lorentz relativistic mass.

One could argue one need to know the speed of light to find the reduced Compton wavelength that is needed to find the Planck length and the Planck time. But we could find the speed of gravity first that do not rely on the Compton wavelength, and then use this speed also for light. That is we find the speed of gravity independent on knowledge of any other physical constants, then we assume the speed of light is the same as this speed, without measuring the speed of light. We can naturally in additional measure the speed of light to check that it holds true, as it do. We will claim this is quite revolutionary as one do not need any of the universal constants that Max Planck combined with dimensional analysis to find the Planck units. In our this shows standard gravity phenomena such as gravitational acceleration and gravitational red-shift are directly linked to the Planck scale. This is no surprise in light of the new unified quantum gravity theory rooted in the Planck scale [21], still this should naturally be investigated by a series of researchers before any conclusion is made.

3 Conclusion

We have shown how to find the Planck length, the Planck time, and the speed of gravity with no prior knowledge of G , c and \hbar , both under general relativity theory, and also when we take into account Lorentz's relativistic mass. This is in strong contrast to what is assumed in the physics' community; namely, that one can only find these indirectly through dimensional analysis. The Planck time and Planck length are linked to the ultimate limit of quantization and directly to gravity.

We have further shown that the gravitational time dilation, when taking into account Lorentz's relativistic mass, is valid all the way down to the Planck length for a Planck mass, while general relativity theory leads to an imaginary time dilation at the Planck length.

References

- [1] A. Einstein. Näherungsweise integration der feldgleichungen der gravitation. *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften Berlin*, 1916.
- [2] H. A. Lorentz. Simplified theory of electrical and optical phenomena in moving systems. *Proc. Acad. Scientific, Amsterdam*, 1, 1899.
- [3] C. G. Adler. Dose mass really depends on velocity dad? *American Journal of Physics*, 55:739, 1987. URL <https://doi.org/10.1119/1.15314>.
- [4] E. Hecht. Einstein never approved the relativistic mass formula. *The Physics Teacher*, 47:336, 2009. URL <https://doi.org/10.1119/1.3204111>.
- [5] A. T. Augousti and A. Radosz. An observation on the congruence of the escape velocity in classical mechanics and general relativity in a schwarzschild metric. *European Journal of Physics*, 376:331–335, 2006. URL <https://doi.org/10.1088/0143-0807/27/2/015>.
- [6] E. G. Haug. Newton and einstein's gravity in a new perspective for planck masses and smaller sized objects. *International Journal of Astronomy and Astrophysics*, 8, 2018. URL <https://doi.org/10.4236/ijaa.2018.81002>.
- [7] M. Planck. *Natuerliche Masseinheiten*. Der Königlich Preussischen Akademie Der Wissenschaften, 1899.
- [8] M. Planck. *Vorlesungen über die Theorie der Wärmestrahlung*. Leipzig: J.A. Barth, p. 163, see also the English translation "The Theory of Radiation" (1959) Dover, 1906.
- [9] A. H. Compton. A quantum theory of the scattering of x-rays by light elements. *Physical Review*, 21(5):483, 1923. URL <https://doi.org/10.1103/PhysRev.21.483>.
- [10] L.S. Levitt. The proton compton wavelength as the 'quantum' of length. *Experientia*, 14:233, 1958. URL <https://doi.org/10.1007/BF02159173>.
- [11] O. L. Trinhammer and H. G. Bohr. On proton charge radius definition. *EPL*, 128:21001, 2019. URL <https://doi.org/10.1209/0295-5075/128/21001>.
- [12] E. G. Haug. Finding the planck length multiplied by the speed of light without any knowledge of g , c , or h , using a newton force spring. *Journal Physics Communication*, 4:075001, 2020. URL <https://doi.org/10.1088/2399-6528/ab9dd7>.
- [13] E. G. Haug. Collision space-time: Unified quantum gravity. *Physics Essays*, 33(1):46, 2020. URL <https://doi.org/10.4006/0836-1398-33.1.46>.
- [14] E. G. Haug. Demonstration that newtonian gravity moves at the speed of light and not instantaneously (infinite speed) as thought! *Journal of Physics Communication.*, 5(2):1, 2021. URL <https://doi.org/10.1088/2399-6528/abe4c8>.
- [15] E. G. Haug. Planck speed: the missing speed of physics? absolute still without breaking lorentz symmetry! *European Journal of Applied Physics*, 4(1):15, 2022. URL <https://www.ej-physics.org/index.php/ejphysics/article/view/144>.

- [16] E. G. Haug. A new full relativistic escape velocity and a new hubble related equation for the universe. *Physics Essays*, 34(4):502, 2021. URL <http://dx.doi.org/10.4006/0836-1398-34.4.502>.
- [17] E. G. Haug. Three dimensional space-time gravitational metric, 3 space + 3 time dimensions. *Journal of High Energy Physics, Gravitation and Cosmology*, 7:1230, 2021. URL <https://doi.org/10.4236/jhepgc.2021.74074>.
- [18] J. A. Rybczyk. Relativistic escape velocity using relativistic forms of potential and kinetic energy. *Working paper made available on the web*, 2009. URL <http://www.mrelativity.net/RelEscapeVelocityPotKinEnergy/Relativistic%20Escape%20Velocity%20using%20Relativistic%20Forms%20of%20Potential%20and%20Kinetic%20Energy.htm>.
- [19] E. G. Haug. Finally a unified quantum gravity theory! collision space-time: the missing piece of matter! gravity is lorentz and heisenberg break down at the planck scale. gravity without g . *ViXra or Research-gate*, 2019. URL <https://vixra.org/abs/1904.0547>.
- [20] R. V. Pound and G. A. Rebka Jr. Gravitational red-shift in nuclear resonance. *Physical Review Letters*, 3(9): 439–441, 1959. URL <https://journals.aps.org/prl/pdf/10.1103/PhysRevLett.3.439>.
- [21] E. G. Haug. Unified quantum gravity field equation describing the universe from the smallest to the cosmological scales. *Physics Essays*, 35:61, 2022. URL <https://doi.org/10.4006/0836-1398-35.1.61>.