The equivalence of gravity and gravitational time dilation

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Submitted March 29, 2022

Abstract:

The unsuccessful attempts of quantization of spacetime are corroborating the belief that for the purposes of quantum gravity, the current notion of spacetime must be replaced with a different concept. However, the range of possible alternative approaches is currently restricted by the fact that gravity is considered to be inseparably associated with the concept of curved spacetime.

In order to extend this range of possible alternative approaches, it is shown here on the basis of the Schwarzschild metric how gravity may not only be described by the means of curved spacetime but equivalently also in the form of gravitational time dilation in flat, uncurved space.

Essay written for the Gravity Research Foundation 2022 Awards for Essays on Gravitation.

1. Introduction

Curved spacetime in the form of a pseudo-Riemannian manifold was introduced by Grossmann and Einstein in the year 1913 **[1]**. It was appreciated as an excellent tool when Einstein was looking for a way for the description of gravity and in particular of the principle of equivalence. However, it had one important inconvenient: It was not compatible with quantum mechanics. For several decades, it was tried to make curved spacetime harmonize with quantum mechanics by quantization of spacetime. Today, after all these attempts, we should accept that curved spacetime is only a tool which is not an essential part of general relativity, and which is not indispensable for quantum gravity.

The idea of a universe of gravity without curved spacetime is not new. Already Steven Weinberg wrote in 1972 that it does not matter if we describe gravity as curvature of space and time or as a gravitational field **[2]**, but he felt that leading general relativists rejected this point of view.[1] Today, it seems more important than ever not to exclude any longer this approach, because quantum gravity has become a real *cul-de-sac*.

In this essay, it will be shown on the basis of the Schwarzschild metric that gravity may not only be described by curved spacetime but also by the equivalent concept of gravitational time dilation in flat, uncurved space.

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[1] He wrote: *"The reader should be warned that these views are heterodox and would meet with objections from many general relativists."*

2. The equivalence of gravity and gravitational time dilation

One particular characteristic of the Schwarzschild metric is its amazing simplicity. The equation[2]

$$ds^2 = -c^2\left(1-\frac{2GM}{c^2 r}\right)dt^2 + \frac{dr^2}{1-\frac{2GM}{c^2 r}} + r^2(d\Theta + sin^2\Theta\, \mathrm{d}\phi^2)$$

is simply a combination of the flat Minkowski metric with gravitational time dilation. In order to show this, we denote by **C** (upper case) the gravitational time dilation of the clock of a particle in a gravity field with reference to a potential-free far-away observer:

$$C = \frac{d\tau}{dt} = \sqrt{1-\frac{r_s}{r}} = \sqrt{1-\frac{2GM}{c^2 r}}$$

By inserting **C** into the equation above, we get a modified form of the Schwarzschild metric:

$$ds^2 = -c^2(\boldsymbol{Cdt})^2 + \left(\frac{\boldsymbol{dr}}{\boldsymbol{C}}\right)^2 + r^2(d\Theta + sin^2\Theta\, \mathrm{d}\phi^2)$$

Now we compare this equation with the equation of flat Minkowski metric **[3]**:

$$ds^2 = -c^2\boldsymbol{dt}^2 + \boldsymbol{dr}^2 + r^2(d\Theta + sin^2\Theta\, \mathrm{d}\phi^2)$$

We see that the Schwarzschild metric and the Minkowski metric are very similar, and the gravitational time dilation C is the only difference between curved and uncurved spacetime:

[2] Following the current sign convention (- + + +)

The term dt becomes Cdt and the term dr becomes $\frac{dr}{C}$. By consequence, gravity may be entirely described by gravitational time dilation, and there is no gravitational effect beyond gravitational time dilation.

3. The derivation of the attractive force of gravitation

If gravity and gravitational time dilation are equivalent, the question arises how gravitational time dilation can generate any attractive interaction, as its direct effect is quite the contrary: Gravitational time dilation is slowing down the radial velocity of particles, by some repulsive effect, from the point of view of a potential-free observer **(see below section 4.)**.

There is only one possible answer which is equivalent to the geometry of curved spacetime: the time dilation gradient is acting on the rest energy of mass particles, in other words: mass particles are striving to maximize their respective gravitational time dilation. This concept is shown in fig. 1:

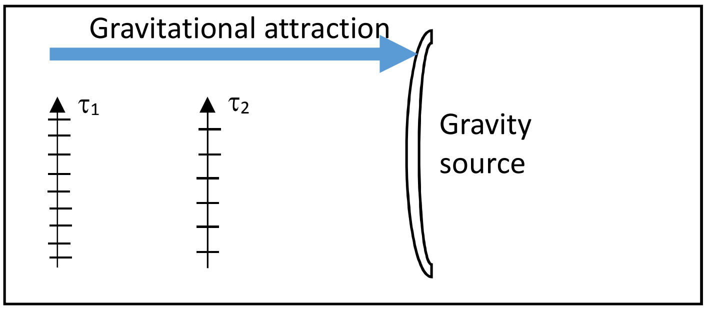

Fig. 1: The effect of the gradient of time dilation: The clock of particle 2 runs slower because it is closer to the gravity source. Particles are attracted because they are striving to maximize their respective time dilation.

The two particle worldlines are not parameterized by some common coordinate time axis but by their respective proper time τ_1 and τ_2. Gravitational time dilation is acting directly by dilating these two proper time parameters (clock frequencies):

The clock τ_2 of the right particle is running at a slower frequency than the clock τ_1 of the left particle because it is closer to the gravity source, with a higher exposition to time dilation. Both particles are striving to slow down their respective clocks, by following the gradient of gravitational time dilation which is oriented towards the gravity source on the right, represented by the blue arrow in the direction of the attractive force.

In a gravity field, gravitational time dilation is

$$C = \sqrt{1 - \frac{r_s}{r}}$$

from the point of view of a potential-free observer, and the spatial gradient of C (decreasing spatial distance r) is

$$\frac{dC}{-dr} = -\frac{r_s}{2r^2}\frac{1}{\sqrt{1 - \frac{r_s}{r}}} = -\frac{GM}{r^2}\frac{1}{c^2\sqrt{1 - \frac{r_s}{r}}}$$

Accordingly, the attractive force of the spatial gradient on the rest energy $E_0 = mc^2$ of a mass particle in radial direction is

$$F_{att} = -\left(\frac{dC}{-dr}\right) mc^2 = \frac{GM}{r^2} \frac{mc^2}{c^2\sqrt{1-\frac{r_s}{r}}} = \frac{GMm}{r^2} \frac{1}{\sqrt{1-\frac{r_s}{r}}} = \frac{F_{Newton}}{C}$$

before gravitational time dilation. Due to the effect of gravitational time dilation (**see below section 4.**), we must multiply radial forces with C^2:

$$F'_{att} = C^2 \cdot F_{att} = \frac{C^2 \cdot F_{Newton}}{C} = C \cdot F_{Newton}$$

An infalling observer of a black hole (C = 1) will feel exactly the force of permanent Newtonian attraction and acceleration, whereas from the point of view of the reference frame of an external observer (C is approaching zero), the attraction is approaching zero near the event horizon.

4. The repulsive effect of gravitational time dilation

We saw that gravitational attraction is generated by the spatial gradient of time dilation. But in order to be complete, we must take into account the fact that gravitational time dilation has a direct repulsive effect in radial direction which is reducing the attractive force, from the point of view of a potential-free observer:

Gravitational time dilation C is acting on radial velocity which is v = s/t. Example: If C = 0,5, the object will take twice as much time t for a given distance, and therefore its velocity is divided by 2 (multiplication with C):

$$v' = Cv$$

v = velocity before gravitational time dilation

v' = velocity after time dilation

Accordingly, radial kinetic energy (and, in the same way, radial force) is multiplied with C^2:

$$E'_{kin} = C^2 \cdot E_{kin}$$

So we get for the kinetic energy

$$E'_{kin} = C^2 \cdot E_{kin} = \left(1 - \frac{r_s}{r}\right) \frac{mv^2}{2}$$

Due to time dilation, the particle loses kinetic energy, and the repulsive force is the corresponding spatial gradient of the kinetic energy, that means the loss of kinetic energy with decreasing spatial distance r:

$$F_{rep} = \frac{dE'_{kin}}{-dr} = -\left(\frac{r_s}{r^2}\right) \frac{mv^2}{2} = -\frac{2GM}{c^2 r^2} \frac{mv^2}{2} = -\frac{v^2}{c^2} \frac{GMm}{r^2} = -\frac{v^2}{c^2} \cdot F_{Newton}$$

As a result, the repulsive force corresponds to Newtonian force times squared velocity. Again, the infalling observer does not feel the repulsive force, because it is a direct effect of time dilation C.

5. Attraction and repulsion

In total, we get one attractive and one repulsive radial effect of gravitational time dilation, which are added in order to obtain the total radial force of gravity:

$$F = F_{att} + F_{rep} = C \cdot F_{Newton} - \frac{v^2}{c^2} \cdot F_{Newton} = \left(\sqrt{1 - \frac{r_s}{r}} - \frac{v^2}{c^2} \right) F_{Newton}$$

	generated by:	acts on:
Attraction	gradient of gravitational time dilation	rest energy
Repulsion	gravitational time dilation	kinetic energy

6. Lorentz-invariance

This alternative description of gravity by gravitational time dilation provides gravity with a Lorentz-invariant concept. Gravity is slowing down the frequency of the proper time of worldlines, without acting on any spacetime manifold.

The four-dimensional gravity concept of spacetime and curvature is replaced with the two-dimensional concept of gravitational time dilation along a worldline. The description of gravity as gravitational time dilation requires a reduced number of degrees of freedom because it is independent of space, gravitation "follows" the worldline of the particle.

The concept of gravitational time dilation is based on the three-dimensional manifold of flat space, it is not based on spacetime. Therefore, this concept does not belong to the vain attempts for the integration of gravity within a flat Minkowski spacetime manifold (cf. the summary of Misner-Thorne-Wheeler, p. 177 ff. **[4]**), and it complies with the current results of quantum gravity according to which it seems to be impossible to ally gravity and quantum mechanics within a four-dimensional Lorentzian spacetime manifold.

7. Curved spacetime and gravitational time dilation in flat space: two complementary concepts

The two alternative concepts of gravity adopt two different points of view which are complementary, with two different time concepts: Curved spacetime corresponds to the view of an observer whereas gravitational time dilation refers to the Lorentz-invariant proper time parameter of the observed particle. However, the proper time parameter is not appropriate for observation because each particle worldline has its own proper time parameter, and for observation, the conversion into a common time parameter of some spacetime coordinates is needed (by multiplication with the respective time dilation factors). As a result, curved spacetime will always remain indispensable for observation, but for quantum gravity, we must resort to the underlying fundamental, Lorentz-invariant concept of gravitational time dilation in flat space.

8. References

[1] Albert Einstein, Marcel Grossmann: Entwurf einer verallgemeinerten Relativitätstheorie und einer Theorie der Gravitation, 1913

[2] Steven Weinberg, Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity. Wiley, New York 1972, p. 147

[3] Robert M. Wald: General Relativity, 1984, p.271

[4] Charles Misner, Kip Thorne, John Archibald Wheeler: Gravitation, 1973, p. 177 ff.