A Very Intuitive, Constructive [Primality] Test, or Recomposer: Iterations, Algorithm

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ABSTRACT

Based on the ubiquitous 'floor' function, a constructive primality test (tantamount to a dual/residuale de- & re-composition algorithm) is proposed alongside auxiliary conjectures. Implications potentially border on domains as co-distant and versatile as, the #-scores, Shapley value, and ABC conjecture to name but a few.

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Background & Central Result

How would one know the particular number on hand *is* prime? And if the challenge is rather dual, i.e. that of recovering the particular (and unique) *structure*, how does one go about picking the candidate prime-roots along with their eligible exponents, if any? Whilst it would seem like too good a buy to be so low-cost (which it might not end up boasting), please refer to the proposition below.

Proposition. Any candidate prime root enters a particular number as part of [de]composition with a power equivalent to a *floor function differential* (**A**). The maximum possible exponent (times it enters) need not exceed the floor function of the respective fudge (or identity-based, *orduale residuale*). By a dual/residuale token, the subset of candidate roots can effectively be reduced via either 'naive' divisibility tests resulting in further residualization/recurrent procedure or by selecting the candidate power in question (not exceeding the above calibration fudge). Ironically, the higher the power being tested, the lesser the testing cost, or smaller the effective (residuale) subset yet-to-be-scrutinized while bearing in mind the ad-hoc upper bound makes *n^(1/m)* with *(n,m)* referring to the number recomposed and power attempted, respectively.

To begin with, the 'global' upper bound cannot exceed the number itself, which obtains in case of primality. Now, bearing in mind that prior, or 'naïve,' divisibility is straightforward when it comes to 2^k, 3^k, and 5^k (2 and 5 based on the latter digit and 3 versus 9 on whether these divide the respective 'digital root,' *dr(n)* or *#n*), further primes may turn out more problematic than that, save perhaps for symmetric, or 'angelic,' instances (e.g. 2299, 1001, 12321, and the like which are divisible by 11, 111, or more generally 1_n which will be covered in future research as part of "*m-unity*.") That said, as argued elsewhere previously, 7 does stand out even here, as follows.

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[1] To Algirdas Paleckis and his likes/ilk—those who can never prove to be victims as they make Victors, their invincible Victory belonging with convincing rather than tyrannical 'likes'/priorities, shallow 'convictions'/priors

Conjecture. Tentatively, numbers containing 7 at least once in their appearance/flow (as opposed to composition, which does not apply here) *tend* to be prime with material, excessive frequency, unless they meet one of the 'naive' reduction criteria. Of the numbers that actually *are* observed to be prime, about *one-half* may have been accounted for by those marked by the above 7-laden characteristic or alternatively having their #-scores equal 7 or multiples thereof, or both.

One will readily make sure this holds at least for numbers under 4070, more rather than less so for the ones large enough, albeit perhaps variably/heteroskedastically rather than progressively so. That said, this clearly would not go for both [arbitrarily chosen] parts of the number not being coprime. (For instance, not only does 147 reveal #147=12 (or #12=3) divisible by 3, it likewise shows both 14 and 7 having a non-trivial (above 1) gcd.) In either event, one would act safely to choose a lower bound at least above those surefire-reducibility picks. Which is to say, an unaided subset of candidate primes would have the number itself versus 7 as the upper versus lower trials.

Still, as the Proposition holds, somewhat surprisingly, the preselect set could end up all the smaller for the higher powers of potential/candidate roots as chosen arbitrarily. In other words, the upper bound is now compressed all the way down to a low $n^{1/m}$ or its 'floor' core, which is what the PNDT may have to apply to (as the resultant value will likely be so low, the interior of pick it spans proves very compact indeed). For instance, the selfsame 147, checked for roots entering it twice (or as a square) potentially, reduces the upper bound to 11, or indeed to 7 as 11 apparently does not divide it even once. Incidentally, this does suggest a hit: 147=3*7^2. However, the same could be tested rigorously, as per Proposition and (A):

$$\forall n\ \exists(\tilde{p},\tilde{\alpha}): n \equiv \prod_l p_l^{\alpha_l} = \prod_l p_l^{\Delta_l},\ \Delta_l \equiv \sum_k [\frac{n}{p_l^k}] - \sum_k [\frac{n-1}{p_l^k}] \leq\ maxk_l = \left[\frac{logn}{logp_l}\right] \quad (A)$$

Intuitively, I infer this from the long-established, now-standard result pertaining to the number of times (or the effective power) a particular root enters a given *factorial*. Bearing in mind that any number could be expressed as a ratio/quotient of the adjacent factorials, i.e. *n=n!/(n-1)!*, while approaching contributions in the denominator as negative [powers], (A) obtains. And, the simple rationale behind the standard result would be that, per any power, it is the floor function that parsimoniously ensures a maximum attainable multiple/residuale for the number in case of divisibility thereby and less than that for any value below that, which ensures a unity status-differential per applicable levels/powers and an effective overall count (needless to say, if the filter works for higher power, so it does for any lower ones). In contrast, *ir*relevant prime roots or *in*effective powers thereof would garner *zero* differentials, as neither floor hits the potential high/hurdle (i.e. no fixed point secured).

Whilst at it, having assumed (RH) that 1 enters any composition once rather than indefinitely, is supported/formalized by k=0=fudge, delta=1.

Retapping Swathes Remote

The floor-filter could resemble the behavior/nature of the so-called *Shapley values* in cooperative-game theory, which account for actual [marginal, qualitative/effective, posterior] contributions—and bargaining powers exercised—within coalitions, possibly at odds with bare-bones [prior] quantitative weights. Formally, this is seconded by the presence of the Euler beta-like kernel (a factorials ratio/resolvent).

For that matter, the peculiar incidence of *higher* [candidate] power-checks ushering in *greater* efficacy (while allowing for smaller *root-value* picks on top of smaller set powers/*sizes* of any) could come in handy when addressing applications such as the *ABC conjecture*.

Not least, the very nature of the *#-scores* (or digital roots) draws implicitly upon the 'floor' structure which, special-case as it seems (p^k collapsed to base 9), could be re-adjusted towards a generalization/compatibility. Finally, should one find this particular (any such) algorithm (procedures, filters) of questionable merit based on the facility of trying divisibility at *random* or *wholesale* (which really is questionable, let alone least efficient/controlled), the very multitude of domains being bundled and bridged by a shared formal nature looks rewarding (or promising).