Mesons As Helmholtzian Factorizations

Abstract: Mesons may be written as simple Helmholtzian factorizations.

The Helmholtzian operator factorization is:

$$\mathbf{J} \equiv D_B D_A \mathbf{f} = ((\Box - |m|^2))\mathbf{f}$$

where:

$$D_B \equiv \begin{pmatrix} D_0 & D_3^{\Leftrightarrow} & -D_2^{\Leftrightarrow} & -D_1 \\ -D_3^{\Leftrightarrow} & D_0 & D_1^{\Leftrightarrow} & -D_2 \\ D_2^{\Leftrightarrow} & -D_1^{\Leftrightarrow} & D_0 & -D_3 \\ -D_1^{\Updownarrow} & -D_2^{\Updownarrow} & -D_3^{\Updownarrow} & D_0^{\Updownarrow} \end{pmatrix} \quad \& \quad D_A \equiv \begin{pmatrix} D_0^{\Updownarrow} & -D_3^{\Leftrightarrow} & D_2^{\Leftrightarrow} & -D_1 \\ D_3^{\Leftrightarrow} & D_0^{\Updownarrow} & -D_1^{\Leftrightarrow} & -D_2 \\ -D_2^{\Leftrightarrow} & D_1^{\Leftrightarrow} & D_0^{\Updownarrow} & -D_3 \\ -D_1^{\Updownarrow} & -D_2^{\Updownarrow} & -D_3^{\Updownarrow} & D_0 \end{pmatrix}$$

and:

$$D_i^{+} \equiv (\partial_i + m_i) \quad , \quad D_i^{-} \equiv (\partial_i - m_i)$$

$$D_i \equiv \begin{pmatrix} D_i^{+} & 0 \\ 0 & D_i^{-} \end{pmatrix} , \quad D_i^{\Updownarrow} \equiv \begin{pmatrix} D_j^{-} & 0 \\ 0 & D_i^{+} \end{pmatrix} , \quad D_i^{\Leftrightarrow} \equiv \begin{pmatrix} 0 & D_i^{-} \\ D_i^{+} & 0 \end{pmatrix} , \quad D_i^{\Leftrightarrow\Updownarrow} \equiv \begin{pmatrix} 0 & D_i^{+} \\ D_j^{-} & 0 \end{pmatrix}$$

and:

$$\mathbf{f} \equiv \begin{pmatrix} f^1 \\ f^2 \\ f^3 \\ f^0 \end{pmatrix} , \qquad f^j \equiv \begin{pmatrix} f_+^j \\ f_-^j \end{pmatrix}$$

$$f_+ \equiv \begin{pmatrix} \begin{pmatrix} f_+^1 \\ 0 \end{pmatrix} \\ \begin{pmatrix} f_+^2 \\ 0 \end{pmatrix} \\ \begin{pmatrix} f_+^3 \\ 0 \end{pmatrix} \\ \begin{pmatrix} f_+^0 \\ 0 \end{pmatrix} \end{pmatrix} , \quad f_- \equiv \begin{pmatrix} \begin{pmatrix} 0 \\ f_-^1 \end{pmatrix} \\ \begin{pmatrix} 0 \\ f_-^2 \end{pmatrix} \\ \begin{pmatrix} 0 \\ f_-^3 \end{pmatrix} \\ \begin{pmatrix} 0 \\ f_-^0 \end{pmatrix} \end{pmatrix} , \quad f \equiv \begin{pmatrix} \begin{pmatrix} f_+^1 \\ f_-^1 \end{pmatrix} \\ \begin{pmatrix} f_+^2 \\ f_-^2 \end{pmatrix} \\ \begin{pmatrix} f_+^3 \\ f_-^3 \end{pmatrix} \\ \begin{pmatrix} f_+^0 \\ f_-^0 \end{pmatrix} \end{pmatrix} = f_+ + f_-$$

$$\Rightarrow \begin{pmatrix} -D_0 & D_3^{\Leftrightarrow} & -D_2^{\Leftrightarrow} & -D_1 \\ -D_3^{\Leftrightarrow} & -D_0 & D_1^{\Leftrightarrow} & -D_2 \\ D_2^{\Leftrightarrow} & -D_1^{\Leftrightarrow} & -D_0 & -D_3 \\ -D_1^{\Updownarrow} & -D_2^{\Updownarrow} & -D_3^{\Updownarrow} & D_0^{\Updownarrow} \end{pmatrix} \begin{pmatrix} -D_0^{\Updownarrow} & -D_3^{\Leftrightarrow} & D_2^{\Leftrightarrow} & -D_1 \\ D_3^{\Leftrightarrow} & -D_0^{\Updownarrow} & -D_1^{\Leftrightarrow} & -D_2 \\ -D_2^{\Leftrightarrow} & D_1^{\Leftrightarrow} & -D_0^{\Updownarrow} & -D_3 \\ -D_1^{\Updownarrow} & -D_2^{\Updownarrow} & -D_3^{\Updownarrow} & D_0 \end{pmatrix} \begin{pmatrix} f^1 \\ f^2 \\ f^3 \\ f^0 \end{pmatrix}$$

$$= \begin{pmatrix} -D_0 & D_3^{\Leftrightarrow} & -D_2^{\Leftrightarrow} & -D_1 \\ -D_3^{\Leftrightarrow} & -D_0 & D_1^{\Leftrightarrow} & -D_2 \\ D_2^{\Leftrightarrow} & -D_1^{\Leftrightarrow} & -D_0 & -D_3 \\ -D_1^{\Updownarrow} & -D_2^{\Updownarrow} & -D_3^{\Updownarrow} & D_0^{\Updownarrow} \end{pmatrix} \begin{pmatrix} B_{\Updownarrow}^1 + E^1 \\ B_{\Updownarrow}^2 + E^2 \\ B_{\Updownarrow}^3 + E^3 \\ -\overset{m*}{\nabla}_{\Updownarrow} \cdot \mathbf{f} \end{pmatrix}$$

Concerning the fundamental particles:

As shown in [2], a fundamental object is a vector components of which are mass-generalzed electromagnetic field components, classified as follows:

$$e(i) \equiv \overline{\alpha_i} = \overline{(E^1,E^2,E^3)}_i \quad , \quad \nu(i) \equiv \beta_i \equiv (B^1,B^2,B^3)_i$$

$$u_j(i) \equiv \phi_{ji} = (\eta_{j-1}(E^1),\eta_{j-2}(E^2),\eta_{j-3}(E^3))_i \;,\; d_j(i) \equiv \overline{\psi_{ji}} = \overline{(\eta_{j-1}(B^1),\eta_{j-2}(B^1),\eta_{j-3}(B^1))}_i$$

where: :

$$\eta_j(R_k^h) \equiv \begin{cases} R_k^h & , \; j \neq 0 \\ E_k^h & , \; j = 0 \;,\; \mathbf{R} = \mathbf{B} \\ B_k^h & , \; j = 0 \;,\; \mathbf{R} = \mathbf{E} \end{cases} \quad , \; \sigma_j(\mathbf{R}_k) \equiv \begin{pmatrix} \eta_{j-1}(R_k^1) \\ \eta_{j-2}(R_k^2) \\ \eta_{j-3}(R_k^3) \end{pmatrix}$$

(i denoting generation, j denoting color)

where:

$$\mathbf{E} = \left(\left(-D_0^{\Updownarrow} f^1 - D_1 f^0\right),\left(-D_0^{\Updownarrow} f^2 - D_2 f^0\right),\left(-D_0^{\Updownarrow} f^3 - D_3 f^0\right),*\right)$$

$$\mathbf{B} = ((D_2 f^3 - D_3 f^2),(-D_1 f^3 + D_3 f^1),(D_1 f^2 - D_2 f^1),*)$$

$$\mathbf{E}_{\Updownarrow} = \left(\left(-D_0^{\Leftrightarrow\Updownarrow} f^1 - D_1^{\Leftrightarrow} f^0\right),\left(-D_0^{\Leftrightarrow\Updownarrow} f^2 - D_2^{\Leftrightarrow} f^0\right),\left(-D_0^{\Leftrightarrow\Updownarrow} f^3 - D_3^{\Leftrightarrow} f^0\right),*\right)$$

$$\mathbf{B}_{\Updownarrow} = ((D_2^{\Leftrightarrow} f^3 - D_3^{\Leftrightarrow} f^2),(-D_1^{\Leftrightarrow} f^3 + D_3^{\Leftrightarrow} f^1),(D_1^{\Leftrightarrow} f^2 - D_2^{\Leftrightarrow} f^1),*)$$

(where $*$ denotes a guage component)

So, in particular (written horizontally as vectors for brevity):

$e^- = e(1) = \overline{(E^1,E^2,E^3)_1}$	$\mu^- = e(2) = \overline{(E^1,E^2,E^3)_2}$	$\tau^- = e(3) = \overline{(E^1,E^2,E^3)_3}$
$\nu_e = \nu(1) = (B^1,B^2,B^3)_1$	$\nu_\mu = \nu(2) = (B^1,B^2,B^3)_2$	$\nu_\tau = \nu(3) = (B^1,B^2,B^3)_3$
$u_R = u_1(1) = (B^1,E^2,E^3)_1$	$c_R = u_1(2) = (B^1,E^2,E^3)_2$	$t_R = u_1(3) = (B^1,E^2,E^3)_3$
$u_G = u_2(1) = (E^1,B^2,E^3)_1$	$c_G = u_2(2) = (E^1,B^2,E^3)_2$	$t_G = u_2(3) = (E^1,B^2,E^3)_3$
$u_B = u_3(1) = (E^1,E^2,B^3)_1$	$c_B = u_3(2) = (E^1,E^2,B^3)_2$	$t_B = u_3(3) = (E^1,E^2,B^3)_3$
$d_R = d_1(1) = \overline{(E^1,B^2,B^3)_1}$	$s_R = d_1(2) = \overline{(E^1,B^2,B^3)_2}$	$b_R = d_1(3) = \overline{(E^1,B^2,B^3)_3}$
$d_G = d_2(1) = \overline{(B^1,E^2,B^3)_1}$	$s_G = d_2(2) = \overline{(B^1,E^2,B^3)_2}$	$b_G = d_2(3) = \overline{(B^1,E^2,B^3)_3}$
$d_B = d_3(1) = \overline{(B^1,B^2,E^3)_1}$	$s_B = d_3(2) = \overline{(B^1,B^2,E^3)_2}$	$b_B = d_3(3) = \overline{(B^1,B^2,E^3)_3}$

Mesons may be written as a simple Helmholtzian factorization.
For example:

$u_R = u_1(1) = (B^1,E^2,E^3)_1 = \quad ; \quad d_R = d_1(1) = \overline{(E^1,B^2,B^3)_2}$

$$u_R = u_1(1) = \left(\begin{pmatrix} B^1_{\Updownarrow} \\ E^2 \\ E^3 \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0}\right) \quad ; \quad \overline{d_R} = \overline{d_1(1)} = \left(\begin{pmatrix} E^1 \\ B^2_{\Updownarrow} \\ B^3_{\Updownarrow} \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0}\right)$$

$$\Rightarrow u_R : \overline{d_R} = \left(\begin{pmatrix} B^1_{\Updownarrow} + E^1 \\ B^2_{\Updownarrow} + E^2 \\ B^3_{\Updownarrow} + E^3 \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0}\right) = \left(\begin{pmatrix} (B_{\Updownarrow} + E)^1 \\ (B_{\Updownarrow} + E)^2 \\ (B_{\Updownarrow} + E)^3 \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0}\right) = (B_{\Updownarrow} + E)_1 = \pi^+ .$$

$$= \left(\begin{pmatrix} -D_0 & -D_3^{\Leftrightarrow} & D_2^{\Leftrightarrow} & -D_1 \\ D_3^{\Leftrightarrow} & -D_0 & -D_1^{\Leftrightarrow} & -D_2 \\ -D_2^{\Leftrightarrow} & D_1^{\Leftrightarrow} & -D_0 & -D_3 \\ -D_1^{\Updownarrow} & -D_2^{\Updownarrow} & -D_3^{\Updownarrow} & D_0 \end{pmatrix}\begin{pmatrix} f^1 \\ f^2 \\ f^3 \\ f^0 \end{pmatrix}, \mathbf{0}, \mathbf{0}\right) = D_A(\mathbf{f}, \mathbf{0}, \mathbf{0})$$

$$\Rightarrow J(u_R : \overline{d_R}) = ((\Box - |m|^2))(\mathbf{f}, \mathbf{0}, \mathbf{0}) = D_B D_A(\mathbf{f}, \mathbf{0}, \mathbf{0}) = D_B\big((B_{\Updownarrow} + E)_1\big) = D_B(\pi^+)$$

$d_G = d_2(1) = \overline{\big(B^1_{\Updownarrow}, E^2, B^3_{\Updownarrow}\big)_1} \quad ; \quad c_G = u_2(2) = \big(E^1, B^2_{\Updownarrow}, E^3\big)_2$

$$\Rightarrow d_G : \overline{c_G} = \left(\begin{pmatrix} \overline{B^1_{\Updownarrow}} \\ \overline{E^2} \\ \overline{B^3_{\Updownarrow}} \\ * \end{pmatrix}, \begin{pmatrix} \overline{E^1} \\ \overline{B^2_{\Updownarrow}} \\ \overline{E^3} \\ * \end{pmatrix}, \mathbf{0}\right) = \left(\begin{pmatrix} \overline{(B_{\Updownarrow} + E)}^1 \\ \overline{(B_{\Updownarrow} + E)}^2 \\ \overline{(B_{\Updownarrow} + E)}^3 \\ * \end{pmatrix}, \mathbf{0}, \mathbf{0}\right) = \overline{(B_{\Updownarrow} + E)_{1/2}}$$

$$= \left(\begin{pmatrix} -D_0 & -D_3^{\Leftrightarrow} & D_2^{\Leftrightarrow} & -D_1 \\ D_3^{\Leftrightarrow} & -D_0 & -D_1^{\Leftrightarrow} & -D_2 \\ -D_2^{\Leftrightarrow} & D_1^{\Leftrightarrow} & -D_0 & -D_3 \\ -D_1^{\Updownarrow} & -D_2^{\Updownarrow} & -D_3^{\Updownarrow} & D_0 \end{pmatrix}\overline{\begin{pmatrix} f^1 \\ f^2 \\ f^3 \\ f^0 \end{pmatrix}}, \mathbf{0}, \mathbf{0}\right) = D_A\overline{(\mathbf{f}, \mathbf{0}, \mathbf{0})}$$

$$\Rightarrow J(d_G : \overline{c_G}) = ((\Box - |m|^2))\overline{(\mathbf{f}, \mathbf{0}, \mathbf{0})} = D_B D_A\overline{(\mathbf{f}, \mathbf{0}, \mathbf{0})} = D_B\Big(\overline{(B_{\Updownarrow} + E)_{1/2}}\Big)$$

$c_R = u_1(2) = \big(B^1_{\Updownarrow}, E^2, E^3\big)_2 \quad ; \quad d_R = d_1(1) = \overline{\big(E^1, B^2_{\Updownarrow}, B^3_{\Updownarrow}\big)_1}$

$$\Rightarrow c_R : \overline{d_R} = \left(\begin{pmatrix} E^1 \\ B^2_{\Updownarrow} \\ B^3_{\Updownarrow} \\ * \end{pmatrix}, \begin{pmatrix} B^1_{\Updownarrow} \\ E^2 \\ E^3_{\Updownarrow} \\ * \end{pmatrix}, \mathbf{0}\right) = \left(\mathbf{0}, \begin{pmatrix} (B_{\Updownarrow} + E)^1 \\ (B_{\Updownarrow} + E)^2 \\ (B_{\Updownarrow} + E)^3 \\ * \end{pmatrix}, \mathbf{0}\right) -= (B_{\Updownarrow} + E)_2 = D^+$$

$$= \left(\mathbf{0}, \begin{pmatrix} -D_0 & -D_3^{\Leftrightarrow} & D_2^{\Leftrightarrow} & -D_1 \\ D_3^{\Leftrightarrow} & -D_0 & -D_1^{\Leftrightarrow} & -D_2 \\ -D_2^{\Leftrightarrow} & D_1^{\Leftrightarrow} & -D_0 & -D_3 \\ -D_1^{\Updownarrow} & -D_2^{\Updownarrow} & -D_3^{\Updownarrow} & D_0 \end{pmatrix}\begin{pmatrix} f^1 \\ f^2 \\ f^3 \\ f^0 \end{pmatrix}, \mathbf{0}\right) = D_A(\mathbf{0}, \mathbf{f}, \mathbf{0})$$

$$\Rightarrow J(c_R : \overline{d_R}) = ((\Box - |m|^2))(\mathbf{0}, \mathbf{f}, \mathbf{0}) = D_B D_A(\mathbf{0}, \mathbf{f}, \mathbf{0}) = D_B\big((B_{\Updownarrow} + E)_2\big) = D_B(D^+)$$

So, clearly, mesons of the same generation may be written simple Helmholtzian factorizations.
(mesons of differing generations are something like 'split-level' Helmholtzian factorizations.

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