Why Particle Ontology is Unavoidable in Quantum Mechanics?

N. Gurappa

Research Center of Physics, Vel Tech Dr.RR & Dr.SR Engineering College,

Avadi, Chennai, Tamil Nadu 600 062, India

Using the quantum formalism, a question - "Why particle ontology is unavoidable in quantum mechanics?" - is analyzed. The frequently outspoken inference, "particle appears to be fuzzy and spread out, i.e., they seem to be at multiple states at once", is shown to be inconsistent with respect to quantum formalism.

In a classroom of quantum mechanics, a frequently encountered inference is, "particle appears to be fuzzy and spread out, i.e., they seem to be at multiple states at once" and a naturally arising immediate question is - how can such a behavior possible? As an answer, the classical particle ontology is shown to be unavoidable in quantum mechanics by considering a free-particle moving in one-dimension (1D). The same analysis does remain valid even for non-free particle in 3D or in $(3 + 1)$D.

The free-particle 1D classical Hamiltonian, H, is given by

$$H = \frac{p^2}{2m} = E_T, \tag{1}$$

where, $p^2/(2m)$, p, x and E_T are the kinetic energy, momentum, position variable and the total energy of a particle of mass m, respectively.

By replacing the commuting physical variables x and p by the corresponding non-commuting operators, $\hat{x}$ and $\hat{p}$, respectively, using Dirac's prescription:

$$\{x\ ,\ p\}_{\text{PB}} = \frac{[\hat{x}\ ,\ \hat{p}]}{i\hbar} \implies [x\ ,\ p] = 0 \longrightarrow [\hat{x}\ ,\ \hat{p}] = i\hbar, \tag{2}$$

where, $\{\ ,\ \}_{\text{PB}}$ stands for the classical Poisson bracket, the time-independent quantum mechanical Hamiltonian can be obtained as,

$$\hat{H} = \frac{\hat{p}^2}{2m} = E\hat{I}, \tag{3}$$

$$\text{such that,} \quad \hat{H}|\psi> = E|\psi> \quad \Longleftrightarrow \quad \frac{d^2\psi(x)}{dx^2} + \frac{2mE}{\hbar^2}\psi(x) = 0\,, \tag{4}$$

where, $i = \sqrt{-1}$, $\hbar = h/(2\pi)$, h is the Planck's constant, E is the energy eigenvalue, $\hat{I}$ is the unit operator and $|\psi >$ is the energy eigenstate; the position basis representation of $|\psi >$ is the Schrödinger's wave function, $< x|\psi > \equiv \psi(x)$.

By comparing Eq. (1) with Eqs. (3) and (4), the mappings given below,

$$H \longrightarrow \hat{H} \ ; \ p \longrightarrow \hat{p} \ ; \ x \longrightarrow \hat{x} \ ; \ E_T \longrightarrow E \text{ and } m \longrightarrow m \tag{5}$$

suggest that *the mass parameter m is intact in both the classical and quantum mechanical situations*. Therefore, the straightforward conclusion is that a particle always remains as a particle from source to detector irrespective of whether its physical situation is classical or quantum mechanical. The same "classical" mass parameter, m, in Eq. (1) enters into the "quantum" situation given in Eq. (3) - which is the actual reason for why the particle ontology of classical mechanics is unavoidable in quantum mechanics.

Dirac's prescription leaves the particle nature untouched. But something new, the energy eigenstate, $|\psi >$, or equivalently the Schrödinger's wave function, $< x|\psi >$, appears in the quantum mechanical case as given in Eq. (4), which is being inferred most of the time as, "particle appears to be fuzzy and spread out, i.e., they seem to be at multiple states at once" - this kind of inference, not supported by the quantum formalism and against the original probabilistic interpretation by Born [1]: "*The wave function determines only the probability that a particle - which brings with itself energy and momentum - takes a path; but no energy and no momentum pertains to the wave*", is one of the major reasons for claiming the quantum mechanics as strange, weird and counter-intuitive. The particle nature, characterized by m (or equivalently by the energy eigenvalue E) in Eq. (3), is not transforming itself into a wave nature described by $< x|\psi >$ - which can be seen easily from Eq. (4) and the same is extremely transparent in Born's probabilistic interpretation. In other words, the wave function itself is not materialistic like the particle, though it represents the moving material particle in accordance with de Broglie's hypothesis. The wave function is needed to compute the probability, but this doesn't mean that it's something like "probability amplitude" as Feynmann famously expounded, "we call it as probability amplitude, because, we don't know what it is" [2]. The 'physical reality' of Schrödinger's wave function along with its relation to the experimentally observed particle behavior is needed to both the derivation of Born's rule and resolution of many quantum paradoxes [3–11].

Let's analyze the following crucial points (CP) present in Born's probabilistic interpretation:

• CP-1: "No energy and momentum pertains to the wave" - this can be straightforwardly seen from the Eq. (4).

It's known that the quantum state vector of a particle can be split into as many components as one wants, which may be recombined later in the case of interference experiments. Consider the splitting of the quantum state vector $|\psi>$ into two orthogonal components, $|\psi_1>$ and $|\psi_2>$; notice that, $|\psi>$ is said to be split into two components only when they are orthogonal to each other:

$$|\psi> = |\psi_1> + |\psi_2>, \tag{6}$$

$$\text{such that,} \quad H|\psi> = H(|\psi_1> + |\psi_2>) = H|\psi_1> + H|\psi_2>, \tag{7}$$

$$\text{where,} \quad H|\psi> = E|\psi> \quad ; \quad H|\psi_1>= E|\psi_1> \quad \text{and} \quad H|\psi_2>= E|\psi_2> . \tag{8}$$

Hence, the fact, "Schrodinger's wave function itself doesn't carry any energy, but merely represents the energy eigenvalue, E" - is clear from Eqs. (6), (7) and (8). If each of $|\psi>$, $|\psi_1>$ and $|\psi_2>$ carry the same E, then the problem of energy conservation pops up, because, the eigenvalue E is a scalar and the state $|\psi>$ is a complex vector and hence, they do not behave the same way while splitting of $|\psi>$ into $|\psi_1>$ and $|\psi_2>$:

$$|\psi>= |\psi_1> + |\psi_2> \Longrightarrow \ |\psi>\sim E; \ |\psi_1>\sim E \text{ and } |\psi_2>\sim E. \tag{9}$$

The energy conservation is not a problem when a classical wave of amplitude, say $u(x,t)$, splits into two orthogonal components, say $u_1(x,t)$ and $u_2(x,t)$, because, the square of wave amplitude, being a scalar, contains the energy unlike the quantum mechanical case; here, t is the time parameter. Therefore,

$$\begin{aligned} &u(x,t) = u_1(x,t) + u_2(x,t); \\ &u^2 \sim E_T; \ u_1^2 \sim E_1 \text{ and } u_2^2 \sim E_2 \ \Longrightarrow \ E_T = E_1 + E_2. \end{aligned} \tag{10}$$

• CP-2: "A particle brings with itself energy and momentum" - implies the fact that the eigenvalues are carried by the particle of mass m.

If a given physical property is a function of the defining properties of a particle, like mass, charge, spin, etc., then the particle can be said to be carrying that physical property.

For example, in classical mechanics, the kinetic energy is given by either $p^2/(2m)$ or $mv^2/2$, where, v is the velocity. Clearly, the kinetic energy is being carried by the particle of mass m. Consider the same case in quantum mechanics for the free-particle; here, $E = k^2\hbar^2/(2m)$, $k = 2\pi/\lambda$ and λ is the de Broglie wavelength. Therefore, akin to the classical case, the particle of mass m carries the energy eigenvalue, E.

Although the Schrödinger wave function, $< x|\psi >$, is a function of $\sqrt{2mE}/\hbar$, it's only an eigenfunction representation for the energy eigenvalue E of the particle of mass m, but it itself does not carry any energy and hence, mass. Notice that, CP-1 and CP-2 present in Born's probabilistic interpretation are crucial to understand Einstein's explanation of photoelectric effect, though Einstein's contribution came much before Born's interpretation.

In conclusion, the eigenvalue E in Eq. (4) represents the particle nature, whereas, the state vector $|\psi >$ (or $< x|\psi >$) represents the wave nature associated with the moving particle. The particle ontology is unavoidable in quantum mechanics, because, the classical mass parameter enters the quantum mechanical description of the same particle. Hence, the inference, "particle appears to be fuzzy and spread out, i.e., they seem to be at multiple states at once" is not supported by the quantum formalism and also by Born's probabilistic interpretation. Such an inference is nothing more than describing a classical wave nature using only the vocabulary of classical particle nature. Also, it's mathematically strange to claim that the eigenvalue itself becomes fuzzy and spread out like its eigenfunction. Moreover, the particle can not be present at multiple states at once, because, such a situation does not respect the conservation laws and also, during the experimental observation, superluminal speeds are required for the instantaneous collapse of the wave function, violating the Cosmic speed limit of the special theory of relativity, as first pointed out by Einstein at the 1927 Solvay Conference. However, if the wave function is non-materialistic unlike a particle, then the postulates of special theory of relativity can't prevent its instantaneous collapse - this doesn't imply that the wave function stands for some abstract non-physical quantity like "probability amplitude". For example, in Young's double-slit experiment, the observed interference pattern is a real physical phenomenon and hence, it must be caused by a real physical wave nature [3–11] - which will be reported in the future update.

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[1] G. Auletta 2001 *Foundations and Interpretation of Quantum Mechanics* (World Scientific) p 104

[2] Richard Feynman - The Character of Physical Law - Part 6 Probability and Uncertainty (full version) https://youtu.be/aAgcqgDc-YM 28:06 - 28:16

[3] N. Gurappa *What's Really Going on in Young's Double-slit Experiment at a Single-quantum Level?* viXra:2108.0127

[4] N. Gurappa *Delayed-Choice Quantum Erasure Experiment: A Causal Explanation Using Wave-Particle Non-Duality* viXra:2107.0150

[5] N. Gurappa *Transmission of a Single-Photon through a Polarizing Filter: An Analysis Using Wave-Particle Non-Duality* viXra:2107.0003

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[11] N. Gurappa *On the Foundations of Quantum Mechanics: Wave-Particle Non-Duality and the Nature of Physical Reality* arXiv:1710.09270 [physics.gen-ph]