Assuming $c < rad^2 abc$, The abc Conjecture Is False

Abdelmajid Ben Hadj Salem[1,2*]

[1*]Residence Bousten 8, Bloc B, Av. Mosquee Raoudha, Soukra, 1181 Soukra Raoudha, Tunisia.

Corresponding author(s). E-mail(s): abenhadjsalem@gmail.com;

Abstract

In this paper, we consider the $\boldsymbol{abc}$ conjecture. Assuming that $\boldsymbol{c < rad^2(abc)}$ is true, we give an elementary proof that the $\boldsymbol{abc}$ conjecture is false using an equivalent statement.

Keywords: Elementary number theory, real functions of one variable.

MSC Classification: 11AXX , 26AXX.

To the memory of my Father who taught me arithmetic,
To my wife Wahida, my daughter Sinda and my son Mohamed Mazen
To the memory of my friend Abdelkader Sellal (1946 - 2017)

1 Introduction and notations

Let a positive integer $a = \prod_i a_i^{\alpha_i}$, a_i prime integers and $\alpha_i \geq 1$ positive integers. We call *radical* of a the integer $\prod_i a_i$ noted by $rad(a)$. Then a is written as :

$$a = \prod_i a_i^{\alpha_i} = rad(a).\prod_i a_i^{\alpha_i - 1} \tag{1}$$

We note:

$$\mu_a = \prod_i a_i^{\alpha_i - 1} \Longrightarrow a = \mu_a.rad(a) \tag{2}$$

The abc conjecture was proposed independently in 1985 by David Masser of the University of Basel and Joseph Œsterlé of Pierre et Marie Curie University (Paris 6) [1].

It describes the distribution of the prime factors of two integers with those of its sum. The definition of the abc conjecture is given below:

Conjecture 1. *(* ***abc Conjecture****): For each $\epsilon > 0$, there exists a constant $K(\epsilon)$ such that, if a, b and c positive integers relatively prime with $c = a + b$, then:*

$$c < K(\epsilon).rad^{1+\epsilon}(abc) \tag{3}$$

$K(\epsilon)$ depending only of ϵ.

The idea to try to write a paper about this conjecture was born after the publication of an article in Quanta magazine about the remarks of professors Peter Scholze of the University of Bonn and Jakob Stix of Goethe University Frankfurt concerning the proof of Shinichi Mochizuki [2]. The difficulty to find a proof of the abc conjecture is due to the incomprehensibility how the prime factors are organized in c giving a, b with $c = a + b$. So, I will give a simple proof, after proposing more then ten proofs since November 2018, assuming $c < rad^2(abc)$ is true, then the abc conjecture is false using an equivalent statement.

2 The Proof of the *abc* Conjecture

We start by giving the equivalent statement of the abc conjecture [1] that we will use:

Equivalence 2. *Let $\epsilon > 0$. Then the set of abc triples for which:*

$$c > rad^{1+\epsilon}(abc) \tag{4}$$

is finite.

Let a, b, c (respectively a, c) positive integers relatively prime with $c = a + b, a > b, b \geq 2$ (respectively $c = a + 1$, $a \geq 2$). We note $R = rad(abc)$ in the case $c = a + b$ or $R = rad(ac)$ in the case $c = a + 1$. We assume in the following that the conjecture $c < R^2$ is true.

We consider $\epsilon \in]0, 1[$. Let one $\epsilon \in]0, 1[$, then it exists p, q two integers $p < q, p \geq 2, q \geq 3$ so that:

$$\frac{p}{q+1} \leq \epsilon < \frac{p}{q} \Longrightarrow \frac{q+p+1}{q+1} \leq 1 + \epsilon \Longrightarrow R^{\frac{q+p+1}{q+1}} \leq R^{1+\epsilon}$$

From the equivalent statement, let (a, b, c) one triple so that $R < c$ and verifying the equivalent statement:

$$R^{1+\epsilon} < c \Longrightarrow R^{\frac{q+p+1}{q+1}} \leq R^{1+\epsilon} < c \Longrightarrow R^{\frac{q+p+1}{q+1}} < c \Longrightarrow \left(\frac{R}{c}\right)^q < \frac{c}{R^{p+1}} < \frac{c}{R^2} < 1 \tag{5}$$

We know [1]:

Proposition 3. *There are infinitely many abc triples verifying $c > R$.*

It follows from the below inequality (5):

$$\left(\frac{R}{c}\right)^q < \frac{c}{R^{p+1}} < \frac{c}{R^2}$$

that it is trivial that the inequality is verified by infinitely many *abc* triples verifying $c > R$. Then the equivalent statement (4) is false and we deduce that the *abc* conjecture is not true.

Q.E.D
Ouf, end of the mystery!

3 Conclusion

Assuming that the conjecture $c < R^2$ holds, we have given an elementary proof so that the *abc* conjecture is false using the equivalent statement cited above of the *abc* conjecture. We can announce the important theorem:

Theorem 4. *(Abdelmajid Ben Hadj Salem, 2023) Assuming that the conjecture $c < rad^2(abc)$ holds, then the abc conjecture is false.*

Acknowledgments. The author is very grateful to Professors Mihailescu Preda and Gérald Tenenbaum for their comments about errors found in previous manuscripts concerning proofs proposed of the *abc* conjecture.

Statements and Declarations:
- The author declares no conflicts of interest.
- No funds, grants, or other support was received.
- The author declares he has no financial interests.
- ORCID - ID:0000-0002-9633-3330.

References

[1] Waldschmidt M. 2013. On the abc Conjecture and some of its consequences presented at The 6th World Conference on 21st Century Mathematics, Abdus Salam School of Mathematical Sciences (ASSMS), Lahore (Pakistan), March 6-9, 2013.

[2] Klaus Kremmerz for Quanta Magazine. 2018. Titans of Mathematics Clash Over Epic Proof of ABC Conjecture. The Quanta Newsletter, 20 September 2018. www.quantamagazine.org.