The proof of Riemann hypothesis[1)]

Minuk Choi

Abstract

The fact that $M(x) = O(x^{\frac{1}{2}+\epsilon}), \forall \epsilon > 0$ is equivalence relation with Riemann hypothesis is well known, Also the proposition that is "the ratio of numbers that have an even number and odd number of prime factors none repeated is 50 : 50" is equivalence relation with Riemann hypothesis. I prove this proposition using the posterior distribution of discrete uniform distribution.

Introduction

We use the concept of asymptotic density to explain the

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1) 2020 Mathematics Subject Classification. Primary 62F15, 11M26.
Key words and phrases. Bayesian, Natural Number distribution, Zeta function.

phenomenon that the probability of an even number in natural number is 1/2, and use Benford's law to explain the phenomenon that the smaller the first digit of a natural number, the higher it appears. We mainly use uniform distribution as a fair choosing method but uniform distribution is not a good way for natural number. In this paper, I will introduce an integrated concept to understand these phenomenons and show that this interpretation provides a solution of the Riemann hypothesis, a difficult problem we have not solved so far.

1. Natural Number distribution

Let X~U(1,N), where $N \in \mathbb{N}-\{0\}$.

In the bayesian view, the process of $\lim_{N\to\infty} P(X=n) = \frac{1}{N}, where\ n \in \{1,2,3,\ldots,N\}$ can be described as follows :

The event [B] : = $N \to \infty$

The prior probability $[P(X=n)]$

$$:= P(X=n) = \frac{1}{N}, where\ n \in \{1,2,3,\ldots,N\}$$

The information $[P(N_a \cap B)]$

$$:= \lim_{N\to\infty} P(N_a) = \frac{1}{a}, where\ N_a = \{a, 2a, 3a, \ldots\},\ a \in \mathbb{N}-\{0\}$$

Thm1.1 When $N \to \infty$, the posterior distribution of discrete uniform distribution is

$$P(X=n \mid N \to \infty) = \frac{1}{n}\prod_{p}(1-p^{-1}),\ where\ n \in \mathbb{N}-\{0\}$$

Proof.

Let $P(B) = b, where\ 0 < b \leq 1$

then, $P(N_a \mid B) = \dfrac{P(N_a \cap B)}{P(B)} = \dfrac{\frac{1}{a}}{b} = \dfrac{1}{ab}$

for all k, $where\ k \in \mathbb{N}-\{0\}$

$P(X = k \mid B)$

$= P(N_k|B) - P(N_{2k} \cup N_{3k} \cup N_{5k} \cup N_{7k} \cup \ldots |B)$

$= P(N_k|B) - P(N_{2k}|B) - P(N_{3k}|B) - P(N_{5k}|B) - P(N_{7k}|B) - \ldots$

$+ P(N_{6k}|B) + P(N_{10k}|B) + P(N_{14k}|B) + P(N_{15k}|B) + \ldots$

$- P(N_{30k}|B) - P(N_{42k}|B) - P(N_{70k}|B) - P(N_{105k}|B) - \ldots$

$+ \ldots$

$= \dfrac{1}{bk}\prod_{p}(1-p^{-1})$

By the axiom of probability $\sum_{k=1}^{\infty} P(X = k \mid B) = 1$

$$\sum_{k=1}^{\infty} P(X=k \mid B) = \sum_{k=1}^{\infty} \frac{1}{bk} \prod_{p} (1-p^{-1}) = \frac{1}{b} \sum_{k=1}^{\infty} \frac{1}{k} \left(\sum_{m=1}^{\infty} \frac{1}{m} \right)^{-1} = \frac{1}{b} = 1$$

Therefore b=1.

$$\therefore P(X=n \mid N \rightarrow \infty) = \frac{1}{n} \prod_{p} (1-p^{-1}), \ where \ n \in \mathbb{N} - \{0\}$$

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Definition 1.1 Natural Number distribution

Now I call the posterior distribution of discrete uniform distribution Natural Number distribution and using θ, mark Natural Number distribution as follows :

$$\theta(\mathrm{X=n}) = \frac{1}{n} \prod_{p} (1-p^{-1})$$

2. The Proof of Riemann hypothesis

Definition 2.1

Let's define the two sets Φ_{even}, Φ_{odd} on $\mathbb{N}$.

Φ_{even}: = set of numbers that have an even number of prime factors none repeated

Φ_{odd} : = set of numbers that have an odd number of prime factors none repeated

Lemma 2.1 For Φ_{even}, Φ_{odd} below equation is completed.

$$\theta(\Phi_{even}) = \theta(\Phi_{odd})$$

Proof.
By the Thm 1.1, $\theta(\Phi_{even})$ and $\theta(\Phi_{odd})$ can be calculated as follows.

$$\theta(\Phi_{even}) = \left(\frac{\prod_p (1-p^{-1}) + \prod_p (1+p^{-1})}{2}\right)\prod_p (1-p^{-1})$$

$$= \frac{\prod_p (1-p^{-1})^2}{2} + \frac{\prod_p (1-p^{-2})}{2}$$

$$= \frac{\prod_p (1-p^{-2})}{2} = \frac{1}{2}\times\frac{1}{\zeta(2)} = \frac{1}{2}\times\frac{6}{\pi^2}$$

$$= \frac{3}{\pi^2}$$

$$\theta(\Phi_{odd}) = \left(\frac{-\prod_p (1-p^{-1}) + \prod_p (1+p^{-1})}{2}\right)\prod_p (1-p^{-1})$$

$$= -\frac{\prod_p (1-p^{-1})^2}{2} + \frac{\prod_p (1-p^{-2})}{2}$$

$$= \frac{\prod_p (1-p^{-2})}{2} = \frac{1}{2} \times \frac{1}{\zeta(2)} = \frac{1}{2} \times \frac{6}{\pi^2}$$

$$= \frac{3}{\pi^2}$$

$\therefore\ \theta(\Phi_{even}) = \theta(\Phi_{odd})$

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Thm 2.1 Riemann hypothesis is true.

Proof.
By the Lemma 2.1 "$\theta(\Phi_{even}) = \theta(\Phi_{odd})$" $\equiv$ T
Since "$\theta(\Phi_{even}) = \theta(\Phi_{odd})$" $\equiv$ "Riemann hypothesis is true"
, Riemann hypothesis is also true

$\therefore$ Riemann hypothesis is true

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Conclusion

In this article we saw when $N \to \infty$, the probability of discrete uniform distribution has a posterior distribution expressed as $\frac{1}{n}\prod_p (1-p^{-1})$. And using this probability distribution, we proved the proposition which is equivalence relation with Riemann hypothesis. Finally we found that Riemann hypothesis is true.

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E-MAIL: ches409@naver.com