Testing the invariance of the speed of light by means of astrometric data

Attilio Colombo

Abstract

A test case aiming to validate the relativistic velocity composition rule on the basis of experimental data is presented. The test exploits the phenomenon of the aberration of the light coming from celestial objects due to the motion of the observer. In particular, it is based on the analysis of the results obtained by applying the relativistic velocity composition rule to calculate the un-aberrated position of the celestial objects from a series of repeated observations of the same object in the course of time.

I. Introduction

The proposed test case aims to investigate the validity of the Relativistic velocity composition formula, that derives from the Lorentz transformations, versus the Galilean vector sum. The test is based on the analysis of the phenomenon of stellar aberration, i.e. on the observed variation of the apparent position of the celestial objects as a function of the motion of the observer and of its velocity, motion that coincide with that of the Earth along its orbit in the case of a terrestrial telescope.

Since the two formulas for the composition of the velocity of the light with the velocity of the observer are different, the calculated un-aberrated position of the celestial object evaluated by means of the relativistic rule is different from that obtained with the classical vector sum, and the amount of difference depends on the value of the ratio of the speed of the observer with respect to the speed of light. Being the orbital velocity of the Earth about 10^4 times smaller than c, such differences are very small and their analysis therefore requires very accurate measurements of the observed position of the celestial objects in order to resolve the differences between the two cases.

The method introduced in the present work relies in particular on the analysis of the differences in the results derived from a series of observations of the same celestial object repeated during the course of one or more years. The criterion to reject one of the two solutions, and the corresponding velocity composition rule, is based on the detection of a specific twice-per-revolution frequency signature into the reconstructed position of the light source being investigated. Being such specific frequency signature associated to the kinematics of the formula and to the motion of the observer, it can be considered as an artifact due to the processing method adopted and not as an actual physical behaviour of the celestial object, thus allowing to determine which of the two velocity addition formulas has to be discarded.

II. Description of the test case

Let us consider the light coming from a very far celestial source, such that the corresponding wavefront can be considered planar over the entire area covered by the orbit of the Earth around the Sun. For an observer at rest into the center of mass of the Solar system the position of this source is fully characterized by two angles which can be expressed as the in-plane azimuth angle and the out-of-plane elevation angle with respect to the plane of the Earth's orbit (ecliptical plane).

Let $\mathbf{V}$ be the velocity vector describing the motion of an observer that is moving into the ecliptical plane. Let $\mathbf{c}$ be the vector defining the velocity of propagation of the light with respect to the stationary frame, and let us consider a moving reference frame having its x axis aligned with the direction of the velocity vector $\mathbf{V}$ of the observer and the y axis lying into the plane formed by the direction of the incoming light and $\mathbf{V}$. The resultant vector $\mathbf{c}'$ that defines the apparent position of the light source for the moving observer, will also lie into the xy plane according both to the Galilean vector-sum rule and to the relativistic velocity-composition rule. However, the observed variation of the angle of incidence, i.e. the amount of *aberration*, is different in the two cases. It can be calculated by applying the two velocity composition rules and focusing the analysis on the x and y components of the vectors.

Let us define, in the stationary reference frame of the Sun, the direction of the light source by the angle θ that the incoming light vector makes with the direction of the velocity of the observer, see Figure 1. Let v by the speed of the observer, which is assumed to be directed along the positive direction of the x axis of the observer's reference frame, and $\beta = v/c$ be the ratio of the observer speed with respect to the speed of light.

Let us indicate with θ' the aberrated direction of the source as seen by the moving observer. The relationship between the angles θ and θ', derived respectively from the classical vector sum and from the relativistic velocity composition rule, is given by the following two exact trigonometric expressions:

$$sin(\theta - \theta'_G) = \beta \frac{sin(\theta)}{\sqrt{1+\beta^2+2\beta cos(\theta)}} \qquad (1)$$

and

$$sin(\theta - \theta'_R) = \beta \frac{sin(\theta)}{1+\beta cos(\theta)} + \beta^2 \frac{sin(2\theta)}{2\big(1+\beta cos(\theta)\big)(1+\sqrt{1-\beta^2})} \qquad (2)$$

For very small values of the observer speed, compared to the speed of light, the difference between the two angles θ and θ' is also very small, therefore it is possible to determine the solution of the above expressions by approximating the sine function with its argument, $sin(\theta-\theta') \simeq (\theta-\theta')$, thus giving:

$$\theta'_G = \theta - \beta \frac{sin(\theta)}{\sqrt{1+\beta^2+2\beta cos(\theta)}} \qquad (3)$$

and

$$\theta'_R = \theta - \beta \frac{sin(\theta)}{1+\beta cos(\theta)} - \beta^2 \frac{sin(2\theta)}{2\big(1+\beta cos(\theta)\big)(1+\sqrt{1-\beta^2})} \qquad (4)$$

The two expressions (3) and (4) allow to calculate the expected apparent position θ' of the light source for the moving observer when the corresponding position θ of the celestial object into the stationary frame is known.

Conversely, in order to perform the calculation of the *un-aberrated* position of the source starting from the one observed into the moving frme, it is necessary to use the inverse relationships between θ and θ' that are given by:

$$\theta_G = \theta' + \beta sin(\theta') \qquad (5)$$

and

$$\theta_R = \theta' + \frac{\beta sin(\theta')}{1-\beta cos(\theta')} + \frac{sin(2\theta')}{2} \frac{\sqrt{1-\beta^2}-1}{(1-\beta cos(\theta'))}$$

When $\beta \ll 1$ this last expression can be rewritten as a power series of β truncated to the term of second order, giving:

$$\theta_R \simeq \theta' + \beta sin(\theta') + \frac{1}{4}\beta^2 sin(2\theta') \qquad (6)$$

The comparison of equations (5) and (6) shows that the reconstructed position of the light source calculated using the relativistic formula differs from the the one obtained from the Galilean vector sum by a term which is quadratic into β. For a given value of v, the amplitude of this term depends on the angle between the incident light and the direction of the velocity vector of the observer, being maximum when $|sin(2\theta')| = 1$, therefore when $\theta' = \pi/4 + k\pi$, and being null when the observer velocity forms a right angle with respect to the direction of the incident light and when the source is aligned with the velocity of the observer, i.e. for $\theta' = 0$ and $\theta' = \pi/2$.

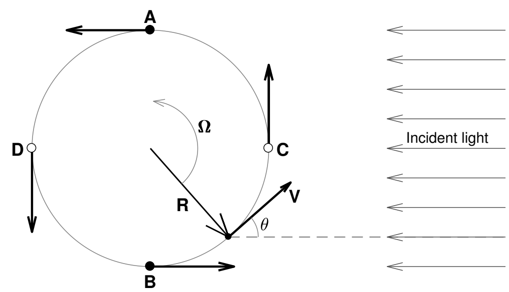

Figure 1: *Orbiting observer with complanar light source*

Let us now consider the case of an observer moving around the Sun with constant angular velocity Ω on a circular orbit having radius R, and of a distant light source located into the same plane of this orbit and stationary with respect to the Sun, as shown in Figure 1. The vector of the observer velocity always lies into the plane of the orbit, therefore in this case the aberration of the incoming light produces, for such a moving observer, an apparent motion of the source which is also always lying into the same plane of the orbit. For this orbiting observer the stationary light source thus shows an apparent oscillation of its position along an horizontal line parallel to the plane of the orbit and characterized by the same time period of the orbit.

It is possible to identify four notable locations along the orbit which are significant because of their peculiar properties with respect to the aberration of the source. In the two positions labeled **A** and **B** the velocity of the observer is parallel to the incident light, therefore when the moving observer is in these two points of the orbit there is no aberration of the incoming light and the observed position of the star coincides with the one observed into the stationary frame of the Sun. The position of the celestial object observed in these two points can therefore be taken as a reference position, since it requires no calculation in order to remove the aberration term.

Conversely, when the moving observer is in the two locations labeled **C** and **D**, its velocity is orthogonal to the direction of the incident light. In these two locations there is the maximum aberration of the apparent position of the star. However, the value of the aberration term is the same for both the classical and the relativistic rule. Therefore, the calculation of the un-aberrated position of the light source, by means of equations (5) or (6) leads to the same result for both the classical and the relativistic rule. In the particular case of a stationary source considered here, the position of the source calculated by the moving observer located in these two points results coincident with the position observed at locations **A**, **B**.

For any other point of the orbit, the un-aberrated position of the source calculated by means of the classical rule will be different from that obtained from the relativistic formula, and the maximum difference between the two results will occur when the moving observer is at the midpoints between **A**,**B** and **C**,**D**, i.e. at an azimuth angle along the orbit equal to $\psi = \pi/4 + k\pi/2$. Assuming a stationary source, since the angle between the direction of the incoming light and the velocity of the observer is $\theta = \Omega t$, one of the two computed results will produce an harmonic oscillation of the horizontal position of the celestial object, having amplitude equal to $\beta^2/4$, and with period equal to one half the period of the observer's orbit. Such peculiar behaviour, characterized by a twice per revolution oscillation that constitutes its specific signature, represents an artifact of the resulting calculated source position, artifact which is due to the inconsistency of the analytical formula used with respect to the actual rule followed by the physical phenomenon.

Let us now consider the case of a terrestrial observer and let's approximate the Earth's orbit with a circle of radius $R = 150x10^6\,km$, and period T equal to one year. In this case the orbital speed is constant and its value is $v \simeq 30\,km/s$, which gives $\beta \simeq 10^{-4}$.

With these approximated values of the or-

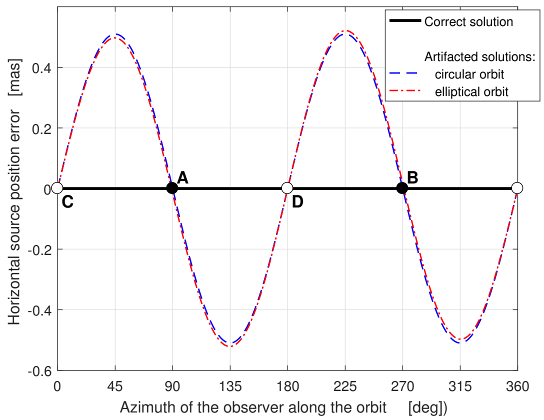

Figure 2: *Comparison of the un-aberrated position of the light source calculated by means of the two different velocity composition rules for an Earth based observer*

bital parameters the two resulting curves of the calculated horizontal position of the source, deriving from the application of equations (5) or (6), are shown in Figure 2. In this figure, also the artifacted solution calculated taking into account the elliptical shape of the Earth's orbit is presented. Due to the low eccentricity ($e \simeq 0.0167$) of the actual orbit of our planet, the deviations of these results from the reference case of a circular trajectory are very small, as shown in the graph that has been calculated considering a celestial object aligned with the major axis of the ecliptic.

The values of the un-aberrated position of the source corresponding to the four notable orbital locations **A**,**B** and **C**,**D** are indicated in the figure with the same markers used in the previous figure. Both the correct and the artifacted curves pass through points **A** and **B**, since for these locations there is no aberration at all and the value of the horizontal position of the celestial object is given directly by the observed position. Both curves also give the same results for locations **C** and **D** where the velocity of the observer is orthogonal to the incoming light direction. [1]

[1] In the general case of a non stationary source, the corresponding computed value of the horizontal position of the object evaluated at **C** and **D** could differ from the one corresponding to the reference locations **A** and **B**.

The above described artifact, characterized by its twice per revolution frequency content, must be present in either the classical or the relativistic computed results, and has the same specific signature characteristics for any observed stationary source lying into the orbital plane, with almost the same amplitude of oscillation and with the same frequency content, independently from the specific celestial object or the specific region of the electromagnetic spectrum being observed.

When the celestial object being analyzed does not lie into the orbital plane there will be also an aberration contribution to the out-of-plane position of the source. Considerations similar to those discussed for an in-plane source apply also to this more general case: the vertical component of the reconstructed position of the source will contain a twice per revolution spurious term in either the classical or the relativistic results. The amplitude of the artifacted vertical component is null when the celestial object is located in the orbital plane, it then increases with the out-of-plane elevation of the source, reaching a maximum for an elevation angle of $\pi/4$, for which the term $\beta^2 sin(2\theta')$ is maximum. For elevations greater than $\pi/4$ the amplitude of the vertical spurious term will then decrease again and will become zero for circumpolar objects, for which also the in-plane component vanishes.

The presence of a twice per revolution frequency term into the computed results of the un-aberrated position of stationary celestial objects is therefore a general characteristics, a specific signature, that allows to identify the incorrect velocity composition rule between the two that have been analyzed.

III. Conclusions

A test case to discriminate between the Galilean and the Relativistic velocity composition rules has been described. The test is based on the analysis of the aberration of the light coming from stationary celestial objects as perceived by an orbiting observer, and on the different results obtained by using the two different velocity composition formulas to remove the aberration term from the observed position of the various light sources of the sky. In order to be applied to measured data, the comparison requires that the observed position of the sources is determined with high accuracy, since the differences that have to be investigated are of the order of milli-arcseconds, a level of accuracy that should be achievable by the most advanced large ground telescopes or space based astrometric instruments like Gaia.

The outcome of the test could constitute a further experimental evidence of the validity of the Lorentz transformations and of the related velocity composition rule applied to a moving observer, thus showing the invariance of the speed of light from the state of motion of the observer. Conversely, should the outcome of the test be in favour of the classical Galilean velocity vector sum, this results could be considered as a supporting element to reconsider physical theories alternative to Special Relativity or as a motivation to go beyond or modify it.