Mercury anomaly and Milgrom's constant.

More than 160 years have passed since the discovery of the anomaly of the motion of Mercury. A huge number of measurements have been carried out confirming the presence of this anomaly. Theories have been written to explain this anomaly. But during this period, not a single calculation has been made of the anomalous acceleration of Mercury that is causing this anomalous perihelion shift. At least I have not been able to find such works. Let's try to fill this gap and solve this problem.

A task.

Find the anomalous acceleration of Mercury causing an anomalous shift of its perihelion of 43" over a hundred years?

Solution.

From Newton's second law, the anomalous acceleration of Mercury a_u will have the form:

$$a_u = \frac{F_u}{m} \tag{1}$$

Where Fu is some force acting on the planet of mass m. When the planet moves along its orbit, its speed will continuously decrease due to the braking anomalous acceleration a_u. This movement is described by the equation:

$$V_{avg} = V_0 - a_u t \tag{2}$$

Where $Vavg$ is the average orbital speed. V_0 - initial speed of movement.

Since the speed of the planet's motion decreases due to the deceleration by the anomalous force Fu, the balance between the force of gravity and the centrifugal force of inertia is disturbed, since the centrifugal force of inertia decreases. As a result, under the influence of gravity, the planet will begin to approach the Sun. In this case, the tangential velocity of the planet will begin to increase in accordance with the conservation law of the angular momentum:

$$mV_{avg}r_{avg} = mVr \tag{3}$$

Where V_{avg} is the average speed of movement in the orbit, r_{avg} is the orbital radius for V_{avg}, V is the speed in a new orbit, r is the radius of the new orbit. Then the new speed of the planet will be equal to:

$$V_{avg}r_{avg} = Vr \tag{4}$$

$$V = V_{avg}\frac{r_{avg}}{r} \tag{5}$$

In the equation (5) we substitute the equation (2):

$$V = (V_0 - a_u t)\frac{r_{avg}}{r} \tag{6}$$

The reduction of the planet will occur until the centrifugal force and the force of gravity by the central body are equalized:

$$m\frac{V^2}{r} = G\frac{mM}{r^2} \tag{7}$$

Then:

$$\frac{V^2}{r} = G\frac{M}{r^2} \Rightarrow V^2 = G\frac{M}{r} \tag{8}$$

Substitute (6) into the resulting expression (8):

$$V^2 = (V_0 - a_u t)^2\frac{r_{avg}^2}{r^2} = G\frac{M}{r} \tag{9}$$

$$V^2 r = (V_0 - a_u t)^2\frac{r_{avg}^2}{r} = GM \tag{10}$$

We perform similar (7) transformations for V_0:

$$m\frac{V_0^2}{r_{avg}} = G\frac{mM}{r_{avg}^2} \tag{11}$$

From (7) we express GM:

$$\frac{V_0^2}{r_{avg}} = G\frac{M}{r_{avg}^2} \tag{12}$$

$$GM = V_0^2 r_{avg} \tag{13}$$

Let's combine GM (10) and (13):

$$V^2 r = (V_0 - a_u t)^2\frac{r_{avg}^2}{r} = V_0^2 r_{avg} \tag{14}$$

Find from the resulting expression r:

$$r = \frac{(V_0 - a_u t)^2 r_{avg}}{V_0^2} = (1 - \frac{a_u t}{V_0})^2 r_{avg} \tag{15}$$

r from (15) is substituted into (6):

$$V = (V_0 - a_u t)\frac{r_{avg}}{r} = \frac{(V_0 - a_u t)r_{avg}}{(1-\frac{a_u t}{V_0})^2 r_{avg}} = \frac{V_0 - a_u t}{(1-\frac{a_u t}{V_0})^2} = \frac{V_0 - a_u t}{(\frac{V_0 - a_u t}{V_0})^2} = \frac{V_0^2}{V_0 - a_u t} = \frac{V_0}{1-\frac{a_u t}{V_0}}$$

Thus, we obtain an expression for the new orbital velocity:

$$V = \frac{V_0}{1-\frac{a_u t}{V_0}} \tag{16}$$

We use the formula (16) to calculate the planet's orbital displacement:

$$V = \frac{\mathrm{dS}}{\mathrm{d}t} \tag{17}$$

We integrate over time:

$$S = \int_0^t \frac{V_0}{1-\frac{a_u t}{V_0}} dt = V_0 \int_0^t \frac{1}{1-\frac{a_u t}{V_0}} dt = \frac{V_0^2}{a_u} ln \frac{1}{1-\frac{a_u t}{V_0}} \tag{18}$$

Since the acceleration of a_u is of little value, we can simplify the expression of the form:

$$ln(1+\varepsilon) = \varepsilon \tag{19}$$

Considering that:

$$\varepsilon << 1$$

Then, in our case, we have:

$$1+\varepsilon = \frac{1}{1-\frac{a_u t}{V_0}} \Rightarrow \varepsilon = \frac{1}{1-\frac{a_u t}{V_0}} - 1 = \frac{1}{1-\frac{a_u t}{V_0}} - \frac{1-\frac{a_u t}{V_0}}{1-\frac{a_u t}{V_0}} = \frac{1-1+\frac{a_u t}{V_0}}{1-\frac{a_u t}{V_0}} = \frac{\frac{a_u t}{V_0}}{1-\frac{a_u t}{V_0}} = \frac{1}{\frac{V_0}{a_u t} - 1}$$

Thus:

$$\varepsilon = \frac{1}{\frac{V_0}{a_u t} - 1} \tag{20}$$

And accordingly, the movement of S of the planet will be approximately expressed by the ratio:

$$S = \frac{V_0^2}{a_u}\varepsilon = \frac{V_0^2}{a_u} \frac{1}{\frac{V_0}{a_u t} - 1} = \frac{V_0^2}{\frac{V_0}{t} - a_u} = \frac{V_0^2}{\frac{V_0 - a_u t}{t}} = \frac{V_0^2 t}{V_0 - a_u t} \tag{21}$$

The displacement of the planet relative to the undisturbed state will be expressed by the difference:

$$\Delta S = S - S_0 = \frac{V_0^2 t}{V_0 - a_u t} - V_0 t = \frac{V_0^2 t - V_0 t(V_0 - a_u t)}{V_0 - a_u t} = \frac{V_0^2 t - V_0^2 t + V_0 a_u t^2}{V_0 - a_u t} =$$

$$= \frac{V_0 a_u t^2}{V_0 - a_u t} = V_0 t \frac{a_u t}{V_0 - a_u t} = V_0 t \frac{1}{\frac{V_0}{a_u t} - 1} \tag{22}$$

Because the abnormal acceleration $a_u << 1$ is very small, then the expression:

$$\frac{V_0}{a_u t} >> 1 \tag{23}$$

And, accordingly, the unit can be neglected:

$$\Delta S = \frac{V_0 t}{\frac{V_0}{a_u t}} = a_u t^2 \tag{24}$$

The problem of the shift of the planet's perihelion under the action of an external disturbing force is solved analytically and has the final form:

$$\Delta S = a_u t^2 \tag{25}$$

Let's move on to calculating the anomalous acceleration a_u. For an observer on Earth, the perihelion shift can be expressed through the observed angle of the anomalous perihelion shift of Mercury:

$$\Delta S = \Delta\phi \cdot r \tag{26}$$

And taking into account the already found formula (25):

$$a_u = \frac{\Delta S}{t^2} = \frac{\Delta\phi \cdot r}{t^2} \tag{27}$$

We recalculate the angle of the anomalous displacement of the planet from angular seconds per hundred years to the angle of displacement per one year:

$$\Delta\phi = \frac{\frac{\Psi \frac{''}{100years}}{100years} \cdot \pi}{3600\frac{''}{\circ} \cdot 180^\circ} \tag{28}$$

And the final solution for the anomalous acceleration that caused the perihelion shift of Mercury is:

$$a_u = \frac{\Delta\phi \cdot r}{t^2} = \frac{\frac{\frac{\Psi \frac{''}{100years}}{100years} \cdot \pi}{3600\frac{''}{\circ} \cdot 180^\circ} \cdot r}{t^2} = \frac{\frac{\frac{43 \frac{''}{100years}}{100years} \cdot \pi}{3600\frac{''}{\circ} \cdot 180^\circ} \cdot 57909227000m}{\left(365,2564\frac{day}{year} \cdot 24\frac{hour}{day} \cdot 3600\frac{sec}{hour}\right)^2} = 1,21(22) \cdot 10^{-10} \left(\frac{m}{s^2}\right) \tag{29}$$

Where:
- t - sidereal earth year, expressed in seconds,
- r - semi-major axis of the planet's orbit,
- $\Delta\phi$ - ythe angle of the anomalous shift of the planet's perihelion per year,
- ψ - the angle of the anomalous shift of the perihelion of Mercury for a hundred years.

Thus, assuming that Mercury is experiencing an anomalous acceleration causing its anomalous perihelion shift, we have found an analytical solution for this anomalous acceleration and its numerical value. Strange as it may seem, but the numerical value of the anomalous acceleration of Mercury is equal to the fundamental Milgrom constant, the value of which was obtained by calculating and averaging the accelerations of stars in the arms of a huge number of spiral galaxies. This solution was obtained in the process of proving the Hypothesis of Dirac number in 2014. Prepared for separate publication in 2021.

I am deeply grateful to B.I. Makarov, who wrote the book "Laws governing the Universe". Mathematical calculations from Chapter III,§11 "On the displacement of the perihelion of the planets"have been modified and refined by me to solve the problem of finding the anomalous acceleration of Mercury.

Best wishes! Yuri Rabyshko. Minsk. July. 2021 year.