# Sum of Geometric Series with Negative Exponents

Chinnaraji Annamalai
School of Management, Indian Institute of Technology, Kharagpur, India
Email: anna@iitkgp.ac.in
https://orcid.org/0000-0002-0992-2584

**Abstract**: This paper presents the summations and sums of Single terms and successive terms of geometric series with negative exponents (negative powers). This will be useful for the researchers who are involving to solve the scientific problems.

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**Sum of one term of geometric series with negative exponents:**

$$x^{-1}=\frac{x^{-1+1}-x^{-1}}{x-1}=\frac{1-x^{-1}}{x-1},\qquad x^{-2}=\frac{x^{-1}-x^{-2}}{x-1},\qquad \cdots,\qquad x^{-r}=\frac{x^{-r+1}-x^{-r}}{x-1}.$$

**Sum of two successive terms of geometric series with negative exponents:**

$$x^{-2}+x^{-1}=\frac{1-x^{-2}}{x-1},\qquad x^{-3}+x^{-2}=\frac{x^{-1}-x^{-3}}{x-1},\cdots,\qquad x^{-r-2}+x^{-r-1}=\frac{x^{-r}-x^{-r-2}}{x-1}.$$

**Sum of three successive terms of geometric series with negative exponents:**

$$x^{-3}+x^{-2}+x^{-1}=\frac{1-x^{-3}}{x-1},\qquad x^{-4}+x^{-3}+x^{-2}=\frac{x^{-1}-x^{-4}}{x-1},\qquad \cdots,$$

$$x^{-r-3}+\ x^{-r-2}+x^{-r-1}=\frac{x^{-r}-x^{-r-3}}{x-1}$$

Similarly, this process continues up to multiple successive terms of geometric series with negative exponents.

**Sum of multiple successive terms of geometric series with negative exponents:**

$$\sum_{i=-n}^{-1}x^{i}=x^{-n}+x^{-n+1}+x^{-n+2}+x^{-n+3}\cdots+x^{-2}+x^{-1}=\frac{1-x^{-n}}{x-1}.$$

$$\sum_{i=-n}^{-k}x^{i}=x^{-n}+x^{-n+1}+x^{-n+2}+x^{-n+3}\cdots+x^{-k-1}+x^{-k}=\frac{x^{-k+1}-x^{-n}}{x-1}.$$