An Affluent Prime Reservoir (or Induction Lens)

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ABSTRACT[1]

A simple yet productive primes-generating relationship is proposed that amounts to a 'qualitative recursion' and arises from the [author's] metaphor of a Prime Reservoir: *p=p0*2^k*3^l-2^a*5^b*. The study builds on one the size of 100 which can arbitrarily be rescaled at various fill-in rates to accommodate prime sums (befitting the smaller primes) versus differences (pertaining to the larger ones yet to be reconsidered in terms of sums). The implied kernel X=p0 likewise proves to be prime, *a^+b+k^+l=odd* routinely promising primality.

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Ulterim Corollary

If only in order to spare some reading "pain" for the busy reader, the core result will be presented shortly, followed by a basic rationale. The central finding comes in two parts, 'weak' (the X *kernel* 'naively' restricted to naturals) versus 'strong' (X narrowed down to an even more productive, prime domain while showing interlinkage with the rest of the power parameters).

$$p = p(a, b, k, l) = X * 2^{\hat{k}} 3^{l} - 2^{\hat{a}} 5^{b} \ \in \boldsymbol{P}, X \in \boldsymbol{N}^{+} \quad (1W)$$

Conjecture 1: Most productively, X comes as an [input] *prime* p_0 thus implying a qualitative-like recursion, with *k-hat* and *a-hat* taking on zero values intermittently only (never simultaneously) amid *b* being restricted to 2 effectively (*a*-hat to 1) and *k-hat* possibly assuming higher values (X sticking around 1 most of the time) whenever the latter upper bound is attained. Degenerate parameter vectors (e.g. a=b=l) tend to result in either non-primes or negative values under $p_0=1$, the same holding for monotonously rising parameter-values, say (0,1,1,2m), under p0=3 (zeros disregarded). Same primes can be represented in a variety of ways.

These results are largely captured in the specified version of (1W) below as (1S)-(1aS):

$$p = p_0 * 2^{\hat{k}} 3^{l} - 2^{\hat{a}} 5^{b} \quad (1S)$$

$$\hat{k}\hat{a} = 0, \qquad \hat{k} + \hat{a} > 0 \quad (1aS)$$

To illustrate the prime-generating mechanism's use and throughput, consider plugging in the parameter values in conformity with (1aS). While at it, it should come "clear as noonday" that 2

[1] Against the world's evil powers that [shall no longer] be

or 5 are disqualified as a prime input/prior (same going for any multiples/powers thereof as per the 'naïve' X kernel case) so as to rule out composite/nP outcomes, even though the above values may well *result* as posteriors somehow.

Again, consider primes as garnered from parametric vectors *(p0; a, b, k, l)*. E.g.: p(1; 0,1,1,1)=1, p(1;0,1,1,2)=13, p(1;0,1,2,1)=7, p(1; 0,1,2,2)=31, p(1;1,0,0,1)=1, p(1; 0,2,2,2)=9=3^2 (nP), p(1; 0,2,2,3)=83, p(1; 0,2,3,1)=19, p(1;0,3,3,3)=91=7*13 (nP), p(1;0,4,4,4)=671=11*61 (nP), p(1; 2,2,0,2)=9=3^2; p(3; 0,1,1,1)=13, p(3; 0,1,1,2)=49=7^2 (nP), p(3;0,1,1,3)=157, p(3; 0,1,1,4)=481=13*37 (nP), p(3; 1,0,0,1)=7, p(3;1,1,0,2)=17, p(3; 1,2,0,2)=29, p(3;1,1,0,3)=71, p(3; 1,2,0,3)=56=2^3*7 (nP), p(3;0,2,4,0)=23, p(3;0,1,5,0)=91=7*13 (nP), p(3;0,1,3,1)=67; p(7; 0,1,1,0)=9=3^2 (nP), p(7;0,1,1,1)=37, p(7;1,1,0,1)=11, p(7;1,1,0,2)=53, p(7;0,2,2,1)=79, p(7;1,2,0,2)=13, p(7;1,2,0,3)=139, p(7;0,1,2,0)=23, p(7;0,1,3,0)=51=3*17 (nP), p(7;0,2,4,0)=87=3*29 (nP), p(7;0,2,2,0)=3 (to name just a few success hits without hiding any loose ends).

The rest can be reconstructed more directly: e.g. 7 arises from, say, 7+2^0*5^2=2^5*3^0 (2+5+5^2=2^5=2+5+25 alone carrying some elusive charm to it possibly rendering it special as a basis digit), i.e. as p(1;0,2,5,0) where 2 and 5 remarry. It would appear like any attempts at reconstructing 2 or 5 would result in the trap of *assuming/permitting* these in the input priors, in violation of the restriction albeit still in line with the primes' recurrent nature. One way of bypassing this would be to invoke *ab*normal parameter-values below the lower (above the upper) bound recommended; at this rate, 5 obtains as (13;3,0,0,0). Even more straightforward from a definition of *twin* primes, $p\text{-}p_0=2$, here 2 results from a variety of degenerate vectors with twin-enabling kernels, e.g. (p0;1,0,0,0) for X=p0=5, 7, 13, 19, 31, 43, etc. Otherwise p=2 remains as disputable as does p0=2, this value definitely standing out as part of the basis.

The Origin

To usher you in on how the formula has been induced, consider a "*prime reservoir*" whereby the particular size can be filled in or fitted by partial prime sums based on a particular rate. This, in turn, fits squarely into my *identity-based fitting* paradigm (one alternate way of fancying *residuality*).

$$r * \sum p \equiv S, \qquad e.g.\ S = 100, \qquad r = 2, 5, 10$$

To illustrate the point:
100=5*(7+13)=5*(1+3+5+11)=5*(3+17)=10*(3+7)=2*(19+31)=2*(1+13+17+19)=2*(2+3+5+17+23)=2*(3+47)=2*(7+43), etc.

While *sums* befit the *smaller* primes, *differences* could come in informative when tackling regularities likely characteristic of the *larger* prime values, in particular as confined to a domain comparable with the prime reservoir size. E.g. 100=2(97-47)=2(53-3)=2(89-29-7-3)=2(79-29)=2(61-11)=2(73-23)=2(67-17)=5(43-23)=5(43-19-3-1)=5(73-53)=5(67-47)=5(79-59)=10(23-13)=10(53-43)=10(47-37)=10(29-19), etc. If we now recover the sums of the difference constituencies, these will prove multiples of 2^k3^l (which routinely holds for *two*-term differences yet not necessarily larger subsets). Based on this, a source relationship (A) could have been recovered around the smaller prime giving rise to (1W) and working as an "*induction lens*":

$$2p_{low} + 2^a5^b \equiv X * 2^k3^l \leftrightarrow p = X * 2^{k-1}3^l - 2^{a-1}5^b \quad (A)$$

Apparently, *k* and *l* could contribute excessively, especially under X=1, which routinely holds for (a,b)=(1,2) with (53+3)=7*8 suggesting one exception, 53=p(7;1,1,0,2) building on X=p0=7 as shown from the outset. Interestingly enough, most such 'irregularities' (resulting in composites in the first section) seem to be featuring primes that have 7 or its *generalizations* (see Shevenyonov 2022), notably 17, 71, 37, 61, etc. Other than that, it would appear that nP-"irregularities" are coupled with parametric conflations (sequences) that build on the multiples/powers or repetitions/singularities of the basis digits: 1,2,3,5 (net-of-zeros).

Somewhat cautiously, another proposition can be set forth.

Conjecture 2: For input primes (i.e. X=p0) greater than 1, a parameter vector adding up to an *even trace* points to *composite* [reconstruction] potential most of the time (p=3=p(7;0,2,2,0) suggesting a dual degeneracy making an exception beyond p0=7), with *odd* parameter-sums showing promise of *primality*.

We now check the hypothesis for input/priors outside the sample studied above. As per X=p0=13: p(13;0,1,1,1)=73, p(13;0,1,1,0)=21=3*7 (nP, trace=even), p(13;0,2,1,4)=2081, p(13;1,1,0,5)=3149=47*67 (nP, trace=odd=1+1+5=7). The latter is one further instance of 7 emerging a ubiquitous irritant violating the patterns and appearing in the composition (if any) digits. What is more, it is in a sense implied in the very basis (e.g. 2+5=2^2+3=7) whose powered sub-sums produce primes in their own right: 2^0+3^0=2, 2^1+3^1=5, 2^2+3^1=7=2^1+5^1, 2^3+3^1=11=2^1+3^2, 2^3+3^2=17, 2^4+3^1=19, 2^3+5^1=13, etc. That said, p(13; 0,1,3,3)=2803 does live up to the expectation, even as the trace amounts to 7, save for the apparent reduplication (doubly odd, so to speak).

References

Shevenyonov, Arthur V. (2022). Primality's Ultra-Natural Nature: An Inquiry into Composites. *viXra: 2203.0057*