A note on interpreting special relativity

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Abstract

In this note I have discussed Einstein's theory of special relativity, derived the coordinate transformations in a simple manner, and obtained expressions for time dilation, the relativistic Doppler effect and length contraction. In Einstein's original paper, a factor ϕ appears, which he sets equal to unity. However, ϕ can be reinterpreted as a scale factor when coordinate time is defined via a single coordinate clock and the time delay receiving information via light rays is included. Using Einstein's definition of coordinate time, a moving sphere is construed as an ellipsoid of revolution, whereas the practical definition of time intervals used here - implying simultaneity of reception rather than simultaneity of occurrence - shows it is a rotated sphere.

1 Introduction

One of the most influential papers ever written in the history of mathematical physics was Albert Einstein's 1905 paper entitled "Zur Elektrodynamik bewegter Körper" [1]. He may have called it a paper on electrodynamics, but in fact it changed our perception of space and time, and became known as his special theory of relativity (SR). The theory is accepted today by the scientific community, and forms an important part of the current paradigm, but numerous mathematicians and physicists have questioned its validity (e.g. [2], [3], [4], [5], [6]). In view of this level of scepticism, I decided to re-examine SR myself, using as simple an analysis as possible. These issues have been discussed countless times in the scientific literature since SR was first introduced more than a century ago, and I do not claim here to have discovered anything new. My aim is merely to try to pinpoint and clarify some of the aspects that might be a cause of confusion - and perhaps look at the theory in a different light from usual.

2 Transformation equations

Einstein's original paper [1] contains the following transformation of time and space coordinates between two inertial frames of reference, $O(x, y, z, t)$ and $O'(x', y', z', t')$, moving at a relative speed v in the x or x' direction (where I use my own notation):

$$x' = \phi(v)\gamma(v)\left[x - vt\right]; \quad y' = \phi(v)y; \quad z' = \phi(v)z;$$

$$t' = \phi(v)\gamma(v)\left[t - \frac{vx}{c^2}\right] \tag{1}$$

The function $\gamma = (1 - v^2/c^2)^{-1/2}$ is conventionally called the Lorentz factor. Einstein argued that the function $\phi(v)$ that appears as a multiplication factor affecting all terms is necessarily equal to unity, and so it subsequently disappeared from the equations.

The transforms can be obtained by imagining a frame of reference O' passing your own (stationary) frame O with speed v in the x direction (see Figure 1).

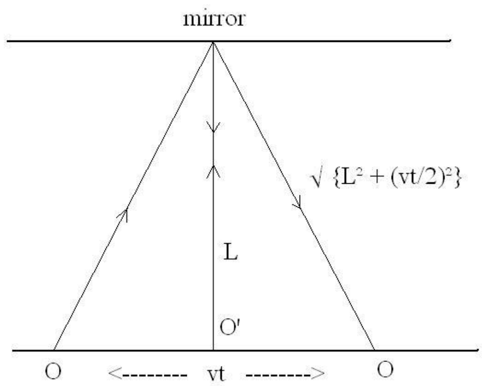

Figure 1: A mirror in space: When the origins of the two frames overlap, a pulse of light is sent to a mirror a distance L away and reflected back to O' and O, but O has moved a distance vt. In SR the coordinate time is larger than the proper time because the light has travelled a larger distance, viz. along the hypotenuse as opposed to the vertical side of the triangle. For a single observer at O to receive the information, the light takes an extra time vt/c (Equation 4) to travel back to the observer O on the left of the diagram.

At the instant the origins overlap, a pulse of light is sent out from O' in all directions. At a distance L from the x' axis a mirror reflects the light signal back to O' in a time $t' = 2L/c$. When O' receives the reflected pulse, an observer in the O frame simultaneously records its arrival. However, this observer in the O frame must be a specific distance, $x = vt$, from the origin, t being the time elapsed between emission and reception of the signal. From the diagram it is clear that the signal has to travel further in the O frame, i.e. along the hypotenuses of the triangles as opposed to along vertical sides in the O' frame. Using Pythagoras's theorem we then have:

$$c^2t^2 = c^2t'^2 + v^2t^2 \quad [x' = 0\,;\, x = vt] \tag{2}$$

Embodied in this equation is Einstein's postulate on the invariance of the speed of light, since we have written it as c in both frames. The other important ingredient is that the observation of simultaneous events in the O frame has required the use of two previously synchronized clocks.

To complete the derivation of Equations 1, we now view the events from the O' frame. This enables us to write equivalently:

$$c^2t'^2 = c^2t^2 + v^2t'^2 \;\; [x = 0\,;\, x' = -vt'] \tag{3}$$

The sign of v is reversed. Now we only have to realize there are linear relationships for t' and x' as functions of x and t:

$$t' = Ax + Bt \;\; ; \;\; x' = Cx + Dt \tag{4}$$

where A, B, C, D are functions of v, to be determined. Substituting Equations 2 and 3 into Equations 4 then gives the transform equations in Equations 1 with $\phi = 1$.

Equations with the same mathematical form had already been obtained previously by Lorentz and Larmor, predating Einstein's SR, but the physical meaning of the equations was different. In SR, v is the relative velocity of two inertial frames and c is the invariant speed of light in either of the frames, neither of which is preferred, whereas in Lorentz's theory v represents the speed of a moving frame relative to a stationary aether frame, and c is the speed of light relative to the aether frame. Nevertheless, the transformation continues to be called a Lorentz transformation, even within Einstein's theory.

Einstein's transformations result essentially from a purely abstract thought experiment, in which the frames are equipped everywhere with identical clocks that are all synchronized with each other, so that the time and location of an event can be recorded as it occurs in either frame. In particular, if two events occur in one frame at a single location (as in the above derivation), these events will inevitably occur in different locations in the other frame. Mathematically this is not necessarily a conceptual problem, but from a physics point of view there is an issue with the time measurement, since it takes a finite amount of time for information to travel from one clock to another. This is indeed taken care of by Einstein in his paper, and there is no contradiction in his theory. However, it does cause potential misconceptions, which I shall outline below.

3 Time dilation and single-observer time

From the transforms, we can immediately quantify the effect called time dilation. Rearranging Equation 2 gives:

$$t' = t\sqrt{1 - \frac{v^2}{c^2}} = \frac{t}{\gamma} \tag{5}$$

which is interpreted to mean that a clock in a frame O' moving relative to frame O runs at a slower rate ($t' < t$ for $v > 0$).

From the diagram we see that such a definition of a time interval t requires more than one clock in the O frame. Essentially an infinite number are imagined in the thought experiment, dotted about the universe. However, from a practical point of view, consider that we only have one laboratory clock at our disposal (at the origin of the O frame) plus a telescope, and that events are referred back to this clock from distant locations using light signals. For this purpose I shall define a new time quantity τ by adding the delay time to Einstein's coordinate time t. We then have (see Figure 1):

$$\tau = t + \frac{vt}{c} = \left(1 + \frac{v}{c}\right)t \tag{6}$$

and from Equation 5 one obtains:

$$\tau = \left(1 + \frac{v}{c}\right) \frac{t'}{(1 - v^2/c^2)^{1/2}} = \sqrt{\frac{1 + v/c}{1 - v/c}}\, t' \tag{7}$$

This expression describes the well-known relativistic Doppler effect. Time intervals τ and t' can be construed as the inverse of light frequencies and, since wavelengths are inversely related to frequencies, it describes the relativistic red-shift for a receding source of light. Changing the sign of v inverts the expression, and then the square-root term describes a blue-shift for an approaching source.

Using Einstein's definition of coordinate time t, the quantity ϕ in Equation 1 is indeed unity, since both directions are equivalent. However, with the redefined time quantity τ, the directions ($\pm v$) are not equivalent, and if the coordinate transformations are rewritten in terms of τ, then ϕ is given by

$$\phi = \frac{1}{(1 \pm v/c)} \tag{8}$$

where v is positive for a receding O' frame and negative for an approaching O' frame. (The derivation of this is not immediately straightforward.)

4 Twin age difference

The two ways of defining coordinate time can be compared here by examining the classic twin paradox. As a numerical example, consider twin A taking a journey to a distant planet 3 light years away at a speed $v/c = 3/5$, and immediately returning, while twin B stays at home. For these numbers we have $\gamma = 5/4$. Thus, from the usual time dilation equation, SR predicts twin A takes 4 years to reach the planet plus 4 years to return, i.e. 8 years in total, while twin B ages by 10 years.

Alternatively, according to twin B as the single observer, he/she observes twin A's clock ticking at half rate for a total of 8 years (from Equation 7), because it takes twin A 5 years to reach the destination plus a further 3 years for the light signals to return to B, giving an ageing of $8 \times \frac{1}{2} = 4$ years. This is followed by a sudden change to double rate of ageing for just 2 years, i.e. a further four years. The overall effect is thus the same, as it must be. There is no inconsistency, i.e. the age difference is real, and not dependent on how time quantities are defined. The age or clock difference occurs on both receding and approaching legs of the journey, so it is theoretically not necessary for the traveller to return to prove he/she has aged differently. Ultimately, the overall ageing effect is given by the Lorentz factor alone, and the Doppler shift just gives us some ongoing information during the course of the thought experiment.

5 Length contraction

Another prediction of *SR* is length contraction, which is also called Lorentz contraction, since it had already been predicted earlier using Lorentz's aether theory.

Suppose we have a rod of length x' at rest in the primed frame aligned along the x' axis. To find the length of the rod as measured in the O frame, we must make sure to measure the distances to the end points simultaneously in the O frame, which means the measurement is characterized by $t = 0$. Using Equation 1 we then have

$$x = \frac{x'}{\gamma} = x'\sqrt{1 - v^2/c^2} \tag{9}$$

If the rod were aligned instead in the y or z directions, with motion again in the x direction, the same consideration shows that there is no change in length (since the relative velocity in those directions is zero).

In his paper [1], Einstein states that a rigid body at rest, which has the shape of a sphere, has in a state of motion, viewed from the stationary system, the form of an ellipsoid of revolution. Indeed, our perception of special relativity is shaped by this understanding but, as I shall show next, the use of the word "viewed" is not strictly correct. It is only correct in the sense of Einstein's abstract definition of coordinate time.

So, in essence, we come back to the same issue, where the coordinate observer in Einstein's theory is quasi an omnipresent figure with an instantaneous overview of the whole of spacetime, without having to worry about the time it takes for information to reach him or her. This is very different from the lowly physicist in his lab equipped with just one clock, and an optical spectrometer attached to a telescope!

To illustrate this, imagine a cube of side length L in motion along the x axis, with its edges parallel to the coordinate axes x, y, z (Figure 2). According to Einstein's definition of coordinate time t, the front face of the cube (xy plane) is indeed contracted along the x direction by the factor $\sqrt{1 - v^2/c^2}$ (into a rectangle), the y direction remaining unaltered; the other orthogonal faces yz and xz remain square.

Consider yourself now as a single observer viewing the cube by receiving parallel light in a direction perpendicular to the xy plane. The light will take longer to reach you from the back of the cube, than from its front, since it is further away, and the previously obscured yz face will appear rotated towards the x direction by an angle θ, given by $\sin\theta = vL/c$. The contraction of the xy face of the cube in the x direction by the Lorentz factor is also equivalent to a rotation by the same angle, so that the complete picture for an observer viewing the moving cube is that it has rotated about a vertical axis by an angle θ. This effect is explained in more detail by David Appell in his article [7]. The angle increases with velocity v, and for an object approaching the speed of light, it approaches 90 degrees.

For the case of a sphere in motion, then, we would not expect to

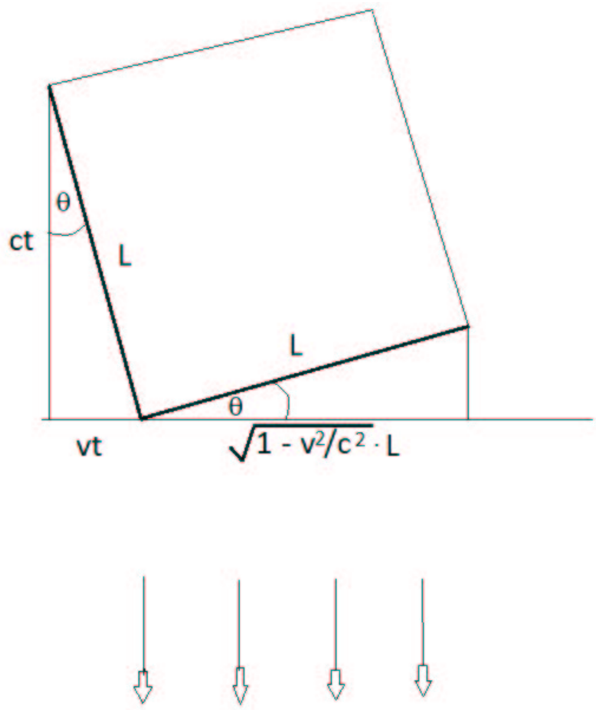

Figure 2: A cube moving with speed v to the right appears rotated (anticlockwise in diagram; The diagram is a plan view of the cube, which is observed from the side)

see a foreshortening of the sphere into an ellipsoid of revolution, as Einstein stated, but we can infer that a simple rotation will occur, with the sphere retaining its circular outline. (This has previously been noted by Roger Penrose, see ref.[7].)

Einstein's coordinate time definition is, therefore, potentially confusing when trying to understand special relativity from a visual or practical point of view, and it is more instructive to take observational time delay into consideration, as I have done throughout this paper, both with regard to the Doppler effect and length contraction.

The bottom line here is that light rays that simultaneously leave a moving object do not necessarily reach the eye or detector at the same time. This leads to most interesting effects that can be the subject of further investigation.

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