Novel Binomial Series and its Summations

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Abstract: This paper presents new binomial series and its summations of the optimized combinations relating to the combinatorics. These series and summations will be useful for the researchers who are involving to solve the scientific problems.

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I. Introduction to Optimized Combination

The optimized combination [1, 2] of combinatorics is expressed as follows:

$$V_r^n = \frac{(r+1)(r+2)\cdots\cdots(r+n)}{n!} = \frac{(n+1)(n+2))\cdots\cdots(n+r)}{r!} = V_n^r,$$

$$i.e., \qquad V_r^n = \prod_{i=1}^{n} \frac{r+i}{n!} = \prod_{i=1}^{r} \frac{n+i}{r!} = V_n^r \quad (n, r \geq 1 \;\&\; n, r \in N),$$

where $N = \{0, 1, 2, 3, \cdots\cdots\}, V_r^n$ is a binomial coefficient, and $n!$ is the factorial of n.

Some results [1, 2] of the optimized combination are provided below:

i). $V_n^0 = V_0^n = 1 \; (n \geq 1 \quad \& \quad n \in N),$

where V_n^0 alaway implies V_0^n , $i.e., V_n^0 \Rightarrow V_0^n$.

ii). $V_r^n = V_n^r \;\; (n, r \geq 1 \;\&\; n, r \in N) \;\&\; V_n^0 = V_0^n$.

iii): $V_0^n + V_1^n + V_2^n + V_3^n + \cdots + V_r^n = V_r^{n+1} \Rightarrow \sum_{i=0}^{n} V_i^n = V_r^{n+1} \quad (n, r \in N).$

II. Novel Series of Optimized Combination

From the result (iii) in this paper, the following series and its summations are expressed:

$$1 + V_1^n + V_2^n + V_3^n + \cdots + V_r^n = V_r^{n+1} \iff 1 + V_n^1 + V_n^2 + V_n^3 + \cdots + V_n^r = V_{n+1}^r$$

This result was newly constituted by Chinnaraji Annamalai[1, 2], Indian Institute of Technology Kharagpur. By using this result, the new series and summations shown below are derived:

$$(1).\quad \sum_{i=0}^{n}\frac{(i+1)}{1!} = 1+2+3+\cdots+n+(n+1) = \frac{(n+1)(n+2)}{2!}.$$

$$(2).\quad \sum_{i=0}^{n}\frac{(i+1)(i+2)}{2!} = 1+3+6+\cdots+\frac{(n+1)(n+2)}{2!} = \frac{(n+1)(n+2)(n+3)}{3!}.$$

$$(3).\quad \sum_{i=0}^{n}\frac{(i+1)(i+2)(i+3)}{3!} = \frac{(n+1)(n+2)(n+3)(n+4)}{4!}.$$

$$(4).\quad \sum_{i=0}^{n}\frac{(i+1)(i+2)(i+3)(i+4)}{4!} = \frac{(n+1)(n+2)(n+3)(n+4)(n+5)}{5!}.$$

Similarly, the series are continued upto *r* times. The r^{th} series and its summation are:

$$(r).\quad \sum_{i=0}^{n}\frac{(i+1)(i+2)(i+3)\cdots(i+r)}{r!} = \frac{(n+1)(n+2)\cdots(n+r)(n+r+1)}{(r+1)!}$$

$$i.e., \sum_{i=0}^{n}\prod_{j=1}^{r}\frac{i+j}{r!} = \prod_{i=1}^{r+1}\frac{n+i}{(r+1)!}.$$

This general expression is novel binomial series and its summation constituted by Chinnaraji Annamalai[1,2].

III. Conclusion

In this paper, the novel binomial series and summations have been derived using the results [1, 2] of the optimized combination in the field of combinatorics. These series and summations will be useful for the researchers who are involving to solve the scientific problems and meet today's challenges.

IV. References

[1] Annamalai C (2020) "Combinatorial Technique for Optimizing the combination", The Journal of Engineering and Exact Sciences – jCEC, Vol. 6(2), pp 0189-0192.

[2] Annamalai C (2019) "A Model of Iterative Computations for Recursive Summability", Tamsui Oxford Journal of Information and Mathematical Sciences – Airiti Library, Vol. 33(1), pp 75-77.