Cosmological Special Theory of Relativity and Cosmological Quantum Physics with Uncertainty Principles, Cosmological General Theory of Relativity

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ABSTRACT
In cosmological special relativity theory, we study Maxwell equations, electromagnetic wave equations and functions. In the cosmological special theory of relativity, we study energy-momentum relations, the Klein-Gordon equation and wave functions. We study Yukawa's potential dependence on time in a cosmological inertial frame. If we solve the Klein-Gordon equation, we obtain the Yukawa potential dependent on time in the cosmological inertial frame. The Schrodinger equation is a wave equation. The wave function used as a probability amplitude in quantum mechanics. We make the Schrodinger equation from Klein-Gordon free particle's wave function in the cosmological special theory of relativity. The Dirac equation is a one order wave equation. The wave function used as a probability amplitude in quantum mechanics. We make the Dirac equation from the wave function, Type A in the cosmological inertial frame. The Dirac equation satisfies the Klein-Gordon equation in the cosmological inertial frame. We found equations of complex scalar fields and electromagnetic fields on the interaction of complex scalar fields and electromagnetic fields in Klein-Gordon-Maxwell theory from Type A of the wave function and Type B of the expanded distance in the cosmological inertial frame. In the cosmological special theory of relativity, we quantized the Klein-Gordon scalar field. We treat Lagrangian density and Hamiltonian in a quantized Klein-Gordon scalar field. We address the particle's force and kinetic energy in the cosmological special theory of relativity. In expanded universe, we found gravity field equation and solution. We found Schwarzschild solution, Kerr-Newman solution in expanded universe. Hence, We found new general relativity theory-Cosmological General Theory of Relativity(CGTR).

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1. Introduction

Special relativity theory is the relativity theory in empty space in weak gravity field because the universe's space is expanded beyond the gravity zone. Special relativity theory is not concerned about the universe's time (The cosmological time).Cosmological special relativity theory is a special relativity theory that treats in a non-gravity local empty space. In a non-gravity local empty space, we are able to use universe's space as static space. But, in global empty space, we have to treat the expanded universe by the variable cosmological time in the cosmology of general relativity theory.

In cosmological special relativity theory, we can treat quantum mechanics and electromagnetism of the expanded universe. Of course, cosmological special relativity theory should be dealt with the cosmological time that universe had a flat space after the epoch of inflation

Our article's aim is that we make the cosmological special theory of relativity and apply it to quantum mechanics. We have to make Cosmological General theory of Relativity (CGRT).

First, the Robertson-Walker metric is

$$d\tau^2 = dT^2 - \frac{1}{c^2}\Omega^2(T)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right] \tag{1}$$

Our universe's k is 0, 1,-1. If k is 0 for flat space and if t_0 is cosmological time[6],

$$k = 0\ ,\ T = t_0 + \Delta t \approx t_0, t_0 >> \Delta t\ ,\ t = T - t_0 = \Delta t\ , dt = dT = d(\Delta t)$$

t is the time of an inertial frame in cosmological special relativity ,

Δt is the period of matter's motion (2)

Hence, the proper time, Eq(1) is in cosmological time t_0 ,[18]

$$\begin{aligned} d\tau^2 &= dt^2 - \frac{1}{c^2}\Omega^2(t_0)[dr^2 + r^2 d\Omega^2] \\ &= dt^2 - \frac{1}{c^2}\Omega^2(t_0)[dx^2 + dy^2 + dz^2] \\ &= dt^2(1 - \frac{1}{c^2}\Omega^2(t_0)V^2)\ ,\quad V^2 = \frac{dx^2 + dy^2 + dz^2}{dt^2} \end{aligned} \tag{3}$$

At this time,

$$d\bar{t} = dt, d\bar{x} = \Omega(t_0)dx, d\bar{y} = \Omega(t_0)dy, d\bar{z} = \Omega(t_0)dz \tag{4}$$

The cosmological special theory of relativity's coordinate transformations are

$$\begin{aligned} c\bar{t} &= ct = \gamma(c\bar{t}' + \frac{v_0}{c}\Omega(t_0)\bar{x}') = \gamma(ct + \frac{v_0}{c}\Omega(t_0)x'\Omega(t_0)) \\ \bar{x} &= x\Omega(t_0) = \gamma(\bar{x}' + v_0\Omega(t_0)\bar{t}') = \gamma(\Omega(t_0)x' + v_0\Omega(t_0)t') \\ \bar{y} &= \Omega(t_0)y = \bar{y}' = \Omega(t_0)y', \\ \bar{z} &= \Omega(t_0)z = \bar{z}' = \Omega(t_0)z' \quad ,\ \gamma = 1/\sqrt{1 - \frac{v_0^2}{c^2}\Omega^2(t_0)} \end{aligned} \tag{5}$$

Therefore, the proper time is

$$\begin{aligned} d\tau^2 &= d\bar{t}^2 - \frac{1}{c^2}[d\bar{x}^2 + d\bar{y}^2 + d\bar{z}^2] \\ &= dt^2 - \frac{1}{c^2}\Omega^2(t_0)[dx^2 + dy^2 + dz^2] \\ &= dt'^2 - \frac{1}{c^2}\Omega^2(t_0)[dx'^2 + dy'^2 + dz'^2] \\ &= d\bar{t}'^2 - \frac{1}{c^2}[d\bar{x}'^2 + d\bar{y}'^2 + d\bar{z}'^2] \end{aligned} \tag{6}$$

Hence, velocities are

$$\frac{dx}{dt}=V_x=\frac{V_x'+v_0}{1+\frac{\Omega^2(t_0)}{c^2}V_x'\cdot v_0},V_x'=\frac{dx'}{dt'}$$

$$\frac{dy}{dt}=V_y=\frac{V_y'}{\gamma(1+\frac{\Omega^2(t_0)}{c^2}V_x'\cdot v_0)},V_y'=\frac{dy'}{dt'}$$

$$\frac{dz}{dt}=V_z=\frac{V_z'}{\gamma(1+\frac{\Omega^2(t_0)}{c^2}V_x'\cdot v_0)},V_z'=\frac{dz'}{dt'} \quad (7)$$

In the Cosmological Special Theory of Relativity (CSTR)'s the differential operators are

$$\frac{1}{c}\frac{\partial}{\partial\overline{t}}=\frac{1}{c}\frac{\partial}{\partial t}=\gamma(\frac{1}{c}\frac{\partial}{\partial\overline{t}'}-\frac{v_0}{c}\Omega(t_0)\frac{\partial}{\partial\overline{x}'})$$

$$=\gamma(\frac{1}{c}\frac{\partial}{\partial t'}-\frac{v_0}{c}\frac{\partial}{\partial x'})$$

$$\frac{\partial}{\partial\overline{x}}=\frac{\partial}{\partial x}\frac{1}{\Omega(t_0)}=\gamma(\frac{\partial}{\partial\overline{x}'}-\frac{v_0}{c}\Omega(t_0)\frac{1}{c}\frac{\partial}{\partial\overline{t}'})$$

$$=\gamma(\frac{\partial}{\partial x'}\frac{1}{\Omega(t_0)}-\frac{v_0}{c}\Omega(t_0)\frac{1}{c}\frac{\partial}{\partial t'})$$

$$\frac{\partial}{\partial\overline{y}}=\frac{\partial}{\partial y}\frac{1}{\Omega(t_0)}=\frac{\partial}{\partial\overline{y}'}=\frac{\partial}{\partial y'}\frac{1}{\Omega(t_0)}$$

$$\frac{\partial}{\partial\overline{z}}=\frac{\partial}{\partial z}\frac{1}{\Omega(t_0)}=\frac{\partial}{\partial\overline{z}'}=\frac{\partial}{\partial z'}\frac{1}{\Omega(t_0)},\gamma=1/\sqrt{1-\frac{v_0^2}{c^2}\Omega^2(t_0)} \quad (8)$$

Hence,

$$\frac{1}{c^2}\frac{\partial^2}{\partial\overline{t}^2}-\overline{\nabla}^2=\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\frac{1}{\Omega^2(t_0)}\{(\frac{\partial}{\partial x})^2+(\frac{\partial}{\partial y})^2+(\frac{\partial}{\partial z})^2\}$$

$$=\frac{1}{c^2}\frac{\partial^2}{\partial t'^2}-\frac{1}{\Omega^2(t_0)}\{(\frac{\partial}{\partial x'})^2+(\frac{\partial}{\partial y'})^2+(\frac{\partial}{\partial z'})^2\} \quad (9)$$

The electric charge density ρ and the electric current density $\vec{j}$ are

$$j^\mu=\rho_0\frac{dx^\mu}{d\tau},j^0=c\rho=c\gamma\rho_0,j^i=\vec{j}=\rho\vec{u},\ i=1,2,3 \quad (10)$$

In the CSTR, transformations of the electric charge density and the electric current density are likely as follows coordinate transformations are

$$c\overline{\rho}=c\rho=\gamma(c\overline{\rho}'+\frac{v_0}{c}\Omega(t_0)\overline{j}_x')=\gamma(c\rho'+\frac{v_0}{c}\Omega(t_0)j_x'\Omega(t_0))$$

$$\overline{j}_x=j_x\Omega(t_0)=\gamma(\overline{j}_x'+v_0\Omega(t_0)\overline{\rho}')=\gamma(\Omega(t_0)j_x'+v_0\Omega(t_0)\rho')$$

$$\overline{j}_y=\Omega(t_0)j_y=\overline{j}_y'=\Omega(t_0)j_y',$$

$$\overline{j}_z=\Omega(t_0)j_z=\overline{j}_z'=\Omega(t_0)j_z' \quad ,\ \gamma=1/\sqrt{1-\frac{v_0^2}{c^2}\Omega^2(t_0)} \quad (11)$$

2. Electrodynamics in CSTR

The electromagnetic potential A^μ is 4-vector potential. Hence, transformations of A^μ are[18]

$$\bar{\phi} = \phi = \gamma(\bar{\phi}' + \frac{V_0}{c}\Omega(t_0)\bar{A}_x') = \gamma(\phi' + \frac{V_0}{c}\Omega(t_0)A_x'\Omega(t_0))$$

$$\bar{A}_x = A_x\Omega(t_0) = \gamma(\bar{A}_x' + \frac{V_0}{c}\Omega(t_0)\bar{\phi}') = \gamma(\Omega(t_0)A_x' + \frac{V_0}{c}\Omega(t_0)\phi')$$

$$\bar{A}_y = \Omega(t_0)A_y = \bar{A}_y' = \Omega(t_0)A_y', \quad , \ \gamma = 1/\sqrt{1 - \frac{V_0^{\ 2}}{c^2}\Omega^2(t_0)} \qquad (12)$$

$$\bar{A}_z = \Omega(t_0)A_z = \bar{A}_z' = \Omega(t_0)A_z'$$

In the CSTR, the electric field $\bar{\vec{E}}$ and magnetic field $\bar{\vec{B}}$ must satisfy Maxwell equations of special relativity theory. Hence, in CSRT, Maxwell equations are likely as a special theory of relativity,

$$\bar{\vec{\nabla}} \cdot \bar{\vec{E}} = 4\pi\bar{\rho} \qquad (13\text{-i})$$

$$\bar{\vec{\nabla}} \cdot \bar{\vec{B}} = 0 \qquad (13\text{-ii})$$

$$\bar{\vec{\nabla}} \times \bar{\vec{E}} = -\frac{1}{c}\frac{\partial \bar{\vec{B}}}{\partial \bar{t}} \qquad (13\text{-iii})$$

$$\bar{\vec{\nabla}} \times \bar{\vec{B}} = \frac{1}{c}\frac{\partial \bar{\vec{E}}}{\partial \bar{t}} + \frac{4\pi}{c}\bar{\vec{j}} \qquad (13\text{-iv})$$

At this time, Eq(13-i) is

$$\bar{\vec{\nabla}} \cdot \bar{\vec{E}} = \frac{1}{\Omega(t_0)}\vec{\nabla} \cdot \bar{\vec{E}} = 4\pi\bar{\rho} = 4\pi\rho \qquad (14)$$

Hence, $\bar{\vec{E}} = \vec{E}\Omega(t_0)$ .According to special relativity, $\bar{\vec{B}} = \vec{B}\Omega(t_0)$

Eq(13-ii) is

$$\bar{\vec{\nabla}} \cdot \bar{\vec{B}} = \frac{1}{\Omega(t_0)}\vec{\nabla} \cdot \vec{B}\Omega(t_0) = \vec{\nabla} \cdot \vec{B} = 0 \qquad (15)$$

Eq(13-iii) is

$$\bar{\vec{\nabla}} \times \bar{\vec{E}} = \frac{1}{\Omega(t_0)}\vec{\nabla} \times \vec{E}\Omega(t_0) = \vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \bar{\vec{B}}}{\partial \bar{t}} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}\Omega(t_0) \qquad (16)$$

Eq(13-iv) is

$$\bar{\vec{\nabla}} \times \bar{\vec{B}} = \frac{1}{\Omega(t_0)}\vec{\nabla} \times \vec{B}\Omega(t_0) = \vec{\nabla} \times \vec{B}$$

$$= \frac{1}{c}\frac{\partial \bar{\vec{E}}}{\partial \bar{t}} + \frac{4\pi}{c}\bar{\vec{j}} = \Omega(t_0)(\frac{1}{c}\frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c}\vec{j}) \qquad (17)$$

Hence, in the CSTR, Maxwell equations are

$$\vec{\nabla} \cdot \vec{E} = 4\pi\rho \qquad (18\text{-i})$$

$$\vec{\nabla} \cdot \vec{B} = 0 \qquad (18\text{-ii})$$

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}\Omega(t_0) \qquad (18\text{-iii})$$

$$\vec{\nabla} \times \vec{B} = \Omega(t_0)(\frac{1}{c}\frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c}\vec{j}) \qquad (18\text{-iv})$$

Therefore, in the CSTR, the electric field $\vec{E}$ and the magnetic field $\vec{B}$ are

$$\bar{\vec{E}} = \vec{E}\Omega(t_0) = \Omega(t_0)(-\vec{\nabla}\phi - \frac{1}{c}\frac{\partial\vec{A}}{\partial t})$$

$$= -\Omega(t_0)\vec{\nabla}\phi - \Omega(t_0)\frac{1}{c}\frac{\partial\vec{A}}{\partial t} = -\bar{\vec{\nabla}}(\bar{\phi}\Omega^2(t_0)) - \frac{1}{c}\frac{\partial\bar{\vec{A}}}{\partial\bar{t}} \quad (19)$$

$$\bar{\vec{B}} = \vec{B}\Omega(t_0) = \Omega(t_0)\vec{\nabla}\times\vec{A} = \Omega(t_0)\bar{\vec{\nabla}}\times\bar{\vec{A}} \quad (20)$$

3. Electromagnetic Wave in CSTR

The electromagnetic wave equation is in CSTR,[18]

$$\frac{1}{c}\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{E}) = -\Omega(t_0)\frac{1}{c^2}\frac{\partial^2\vec{B}}{\partial t^2}$$

$$= \vec{\nabla}\times(\frac{1}{c}\frac{\partial\vec{E}}{\partial t}) = \vec{\nabla}\times(\frac{1}{\Omega(t_0)}\vec{\nabla}\times\vec{B}), \vec{\nabla}\times\vec{j} = \vec{0}$$

$$= \frac{1}{\Omega(t_0)}\{-\nabla^2\vec{B} + \vec{\nabla}(\vec{\nabla}\cdot\vec{B})\} = -\frac{1}{\Omega(t_0)}\nabla^2\vec{B} \quad (21)$$

Hence, the electromagnetic wave equation is

$$\Omega(t_0)\frac{1}{c^2}\frac{\partial^2\vec{B}}{\partial t^2} - \frac{1}{\Omega(t_0)}\nabla^2\vec{B} = \vec{0} \quad (22)$$

And,

$$\frac{1}{c}\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B}) = \Omega(t_0)\frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}, \frac{1}{c}\frac{\partial\vec{j}}{\partial t} = \vec{0}$$

$$= \vec{\nabla}\times(\frac{1}{c}\frac{\partial\vec{B}}{\partial t}) = \vec{\nabla}\times(-\frac{1}{\Omega(t_0)}\vec{\nabla}\times\vec{E})$$

$$= -\frac{1}{\Omega(t_0)}\{-\nabla^2\vec{E} + \vec{\nabla}(\vec{\nabla}\cdot\vec{E})\} = \frac{1}{\Omega(t_0)}\nabla^2\vec{E}, \vec{\nabla}(4\pi\rho) = \vec{0} \quad (23)$$

Hence, the electromagnetic wave equation is

$$\Omega(t_0)\frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} - \frac{1}{\Omega(t_0)}\nabla^2\vec{E} = \vec{0} \quad (24)$$

In CSTR, the electromagnetic wave functions are

$$\vec{E} = \vec{E}_0\sin\Phi, \vec{B} = \vec{B}_0\sin\Phi$$

$$\Phi = \omega\{\frac{t}{\sqrt{\Omega(t_0)}} - \frac{\sqrt{\Omega(t_0)}}{c}(lx + my + nz)\} \quad (25)$$

where,

$$l^2 + m^2 + n^2 = 1 \quad (26)$$

According to Maxwell equations are in CSTR,[1]

$$\Omega(t_0)\{\frac{1}{c}\frac{\partial E_x}{\partial t} + \frac{4\pi}{c}j_x\} = (\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}), \qquad \Omega(t_0)\frac{1}{c}\frac{\partial B_x}{\partial t} = (\frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y})$$

$$\Omega(t_0)\{\frac{1}{c}\frac{\partial E_y}{\partial t} + \frac{4\pi}{c}j_y\} = (\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x}), \qquad \Omega(t_0)\frac{1}{c}\frac{\partial B_y}{\partial t} = (\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z})$$

$$\Omega(t_0)\{\frac{1}{c}\frac{\partial E_z}{\partial t} + \frac{4\pi}{c}j_z) = (\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}), \qquad \Omega(t_0)\frac{1}{c}\frac{\partial B_z}{\partial t} = (\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}) \quad (27)$$

where,

$$\Omega(t_0)\{\frac{1}{c}\frac{\partial E_x}{\partial t'}+\frac{4\pi}{c}j_x'\}=\{\frac{\partial}{\partial y'}\gamma(B_z-\frac{v_0}{c}\Omega(t_0)E_y)-\frac{\partial}{\partial z'}\gamma(B_y+\frac{v_0}{c}\Omega(t_0)E_z)\}$$

$$\Omega(t_0)\{\frac{1}{c}\frac{\partial}{\partial t'}\gamma(E_y-\frac{v_0}{c}\Omega(t_0)B_z)+\frac{4\pi}{c}j_y'\}=\{\frac{\partial B_x}{\partial z'}-\frac{\partial}{\partial x'}\gamma(B_z-\frac{v_0}{c}\Omega(t_0)E_y)\}$$

$$\Omega(t_0)\{\frac{1}{c}\frac{\partial}{\partial t'}\gamma(E_z+\frac{v_0}{c}\Omega(t_0)B_y)+\frac{4\pi}{c}j_z'\}=\{\frac{\partial}{\partial x'}\gamma(B_y+\frac{v_0}{c}\Omega(t_0)E_z)-\frac{\partial B_x}{\partial y'}\} \quad (28)$$

where,[1]

$$\Omega(t_0)\frac{1}{c}\frac{\partial B_x}{\partial t'}=\{\frac{\partial}{\partial z'}\gamma(E_y-\frac{v_0}{c}\Omega(t_0)B_z)-\frac{\partial}{\partial y'}\gamma(E_z+\frac{v_0}{c}\Omega(t_0)B_y)\}$$

$$\Omega(t_0)\frac{1}{c}\frac{\partial}{\partial t'}\gamma(B_y+\frac{v_0}{c}\Omega(t_0)E_z)=\{\frac{\partial}{\partial x'}\gamma(E_z+\frac{v_0}{c}\Omega(t_0)B_y)-\frac{\partial E_x}{\partial z'}\}$$

$$\Omega(t_0)\frac{1}{c}\frac{\partial}{\partial t'}\gamma(B_z-\frac{v_0}{c}\Omega(t_0)E_y)=\{\frac{\partial E_x}{\partial y'}-\frac{\partial}{\partial x'}\gamma(E_y-\frac{v_0}{c}\Omega(t_0)B_z\} \quad (28\text{-i})$$

At this time,

$$\Omega(t_0)\{\frac{1}{c}\frac{\partial E_x'}{\partial t'}+\frac{4\pi}{c}j_x'\}=(\frac{\partial B_z'}{\partial y'}-\frac{\partial B_y'}{\partial z'}), \qquad \Omega(t_0)\frac{1}{c}\frac{\partial B_x'}{\partial t'}=(\frac{\partial E_y'}{\partial z'}-\frac{\partial E_z'}{\partial y'})$$

$$\Omega(t_0)\{\frac{1}{c}\frac{\partial E_y'}{\partial t'}+\frac{4\pi}{c}j_y'\}=(\frac{\partial B_x'}{\partial z'}-\frac{\partial B_z'}{\partial x'}), \qquad \Omega(t_0)\frac{1}{c}\frac{\partial B_y'}{\partial t'}=(\frac{\partial E_z'}{\partial x'}-\frac{\partial E_x'}{\partial z'})$$

$$\Omega(t_0)\{\frac{1}{c}\frac{\partial E_z'}{\partial t'}+\frac{4\pi}{c}j_z')=(\frac{\partial B_y'}{\partial x'}-\frac{\partial B_x'}{\partial y'}), \qquad \Omega(t_0)\frac{1}{c}\frac{\partial B_z'}{\partial t'}=(\frac{\partial E_x'}{\partial y'}-\frac{\partial E_y'}{\partial x'}) \quad (29)$$

Hence, if Eq(28)-(28-i) compare Eq(29) in CSTR, transformations of the electromagnetic field are

$$E_x'=E_x, E_y'=\gamma(E_y-\frac{v_0}{c}\Omega(t_0)B_z), E_z'=\gamma(E_z+\frac{v_0}{c}\Omega(t_0)B_y) \quad (30)$$

$$B_x'=B_x, B_y'=\gamma(B_y+\frac{v_0}{c}\Omega(t_0)E_z), B_z'=\gamma(B_z-\frac{v_0}{c}\Omega(t_0)E_y) \quad (31)$$

In CSTR, electromagnetic wave functions are

$$E_x'=E_{x0}\sin\Phi', E_y'=\gamma(E_{y0}-\frac{v_0}{c}\Omega(t_0)B_{z0})\sin\Phi', E_z'=\gamma(E_{z0}+\frac{v_0}{c}\Omega(t_0)B_{y0})\sin\Phi' \quad (32)$$

$$B_x'=B_{x0}\sin\Phi', B_y'=\gamma(B_{y0}+\frac{v_0}{c}\Omega(t_0)E_{z0})\sin\Phi', B_z'=\gamma(B_{z0}-\frac{v_0}{c}\Omega(t_0)E_{y0})\sin\Phi' \quad (33)$$

At this time,

$$\Phi'=\omega'\{\frac{t'}{\sqrt{\Omega(t_0)}}-\frac{\sqrt{\Omega(t_0)}}{c}(l'x'+m'y'+n'z')\} \quad (34)$$

$$\Phi=\omega\{\frac{t}{\sqrt{\Omega(t_0)}}-\frac{\sqrt{\Omega(t_0)}}{c}(lx+my+nz)\} \quad (35)$$

If we compare Eq(34) and Eq(35),

$$\omega'=\omega\gamma(1-l\Omega(t_0)\frac{v_0}{c}), l'=\frac{l-\frac{v_0}{c}\Omega(t_0)}{1-l\frac{v_0}{c}\Omega(t_0)}$$

$$m' = \frac{m}{\gamma(1 - l\Omega(t_0)\frac{v_0}{c})}, n' = \frac{n}{\gamma(1 - l\Omega(t_0)\frac{v_0}{c})} \quad (36)$$

where,

$$l'^2 + m'^2 + n'^2 = 1 \quad (37)$$

4. Klein-Gordon Equation and Wave Fuction in CSTR

We make the Klein-Gordon equation and wave function in the cosmological special theory of relativity.[19] First, space-time relations are included in the Cosmological Special Theory of Relativity (CSTR).

$$ct = \gamma(ct' + \frac{v_0}{c}\Omega(t_0)x') \quad x\Omega(t_0) = \gamma(\Omega(t_0)x' + v_0\Omega(t_0)t')$$

$$\begin{matrix}\Omega(t_0)y = \Omega(t_0)y', \\ \Omega(t_0)z = \Omega(t_0)z'\end{matrix}, \ \gamma = 1/\sqrt{1 - \frac{v_0^{\ 2}}{c^2}\Omega^2(t_0)}, \ t_0 \text{ is cosmological time} \quad (38)$$

Therefore, the proper time is,

$$d\tau^2 = dt^2 - \frac{1}{c^2}\Omega^2(t_0)[dx^2 + dy^2 + dz^2]$$

$$= dt'^2 - \frac{1}{c^2}\Omega^2(t_0)[dx'^2 + dy'^2 + dz'^2], \ t_0 \text{ is cosmological time} \quad (39)$$

Hence, energy-momentum relations based on the fact that energy-momentum are 4-vector in CSTR,

$$E = \gamma(E' + v_0\Omega^2(t_0)p_x') \ p_x\Omega(t_0) = \gamma(\Omega(t_0)p_x' + \frac{v_0}{c^2}\Omega(t_0)E')$$

$$\begin{matrix}\Omega(t_0)p_y = \Omega(t_0)p_y', \\ \Omega(t_0)p_z = \Omega(t_0)p_z'\end{matrix}, \gamma = 1/\sqrt{1 - \frac{v_0^{\ 2}}{c^2}\Omega^2(t_0)} \ E = m_0c^2\frac{dt}{d\tau}, \vec{p} = m_0\frac{d\vec{x}}{d\tau} \quad (40)$$

Therefore, the energy-momentum-mass relation is in CSTR,

$$m_0^{\ 2}c^4 = E^2 - \Omega^2(t_0)p^2c^2 \quad (41)$$

The matter wave function is in CSTR,

$$\phi = \phi_0 \exp i\Phi = \phi_0 \exp i[\frac{\omega t}{\sqrt{\Omega(t_0)}} - \vec{k}\cdot\vec{x}\sqrt{\Omega(t_0)}]$$

$$= \phi' = \phi_0 \exp i\Phi' = \phi_0 \exp i[\frac{\omega' t'}{\sqrt{\Omega(t_0)}} - \vec{k}'\cdot\vec{x}'\sqrt{\Omega(t_0)}]$$

ϕ_0 is the amplitude, ω is the angular frequency, $k = |\vec{k}|$ is the wavenumber. (42)

If we use Eq(38) in Eq(42), we obtain angular frequency-wavenumber relation.

$$\omega' = \gamma(\omega - v_0\Omega(t_0)k_1), \ k_1' = \gamma(k_1 - \frac{v_0}{c^2}\Omega(t_0)\omega)$$

$$k_2' = k_2, k_3' = k_3, \gamma = 1/\sqrt{1 - \frac{v_0^{\ 2}}{c^2}\Omega^2(t_0)} \quad (43)$$

At this time, if we define the energy-momentum by the angular frequency-wavenumber,

$$E = \hbar\omega, \vec{p} = \frac{\hbar\vec{k}}{\Omega(t_0)} \quad (44)$$

Hence, we obtain the angular frequency-wave number relation about the energy-momentum-mass relation in CSTR,

$$m_0^{\ 2}c^4 = E^2 - \Omega^2(t_0)p^2c^2 = \hbar^2\omega^2 - \hbar^2k^2c^2 \quad (45)$$

We obtain the next result by the transformation of the angular frequency-wavenumber relation, Eq(43) in

CSTR.

$$m_0^{\ 2}c^4 = \hbar^2\omega^2 - \hbar^2 k^2 c^2 = \hbar^2\omega'^2 - \hbar^2 k'^2 c^2 \qquad (46)$$

If we define the differential operator about energy-momentum in CSTR,

$$E = i\hbar\frac{\partial}{\partial t}, \vec{p} = -i\hbar\frac{1}{\Omega(t_0)}\vec{\nabla} \qquad (47)$$

If we apply Eq(47) to Eq(46),

$$m_0^{\ 2}c^4 = E^2 - \Omega^2(t_0)p^2c^2 = \hbar^2[-(\frac{\partial}{\partial t})^2 + c^2\nabla^2]$$

We finally obtain the Klein-Gordon equation in the CSTR.

$$\frac{m_0^{\ 2}c^2}{\hbar^2}\phi = [-\frac{1}{c^2}(\frac{\partial}{\partial t})^2 + \nabla^2]\phi \qquad (48)$$

The wave function, Eq(42) satisfies the Klein-Gordon equation, Eq(48).

5. Yukawa potential in Klein-Gordon equation in cosmological inertial frame

If we focus on the Klein-Gordon equation, Yukawa potential ϕ is dependent on time,[21]

$$\frac{m_\pi^{\ 2}c^2}{\hbar^2}\phi + \partial_\mu\partial^\mu\phi = \frac{m_\pi^{\ 2}c^2}{\hbar^2}\phi + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\phi - \nabla^2\phi = 0 \qquad (49)$$

At this time, the Yukawa potential ϕ is dependent on time.

$$\phi = -\frac{g^2}{r}\exp(-\frac{m_\pi r c}{\hbar}) + A_0 \sin\omega t$$

Frequency $\omega = \frac{m_\pi c^2}{\hbar}$, m_π is meson's mass (50)

Eq(49)-Klein-Gordon equation is satisfied by Eq(50)-Yukawa potential dependent about time
In the cosmological inertial frame, the Klein-Gordon equation is

$$-\Omega(t_0)\frac{1}{c^2}\frac{\partial^2\phi'}{\partial t^2} + \frac{1}{\Omega(t_0)}\nabla^2\phi' = \frac{m_\pi^{\ 2}c^2}{\hbar^2}\phi' \qquad (51)$$

At this point, in the cosmological inertial frame, space-time transformations in type A of the wave function and the other type B of the expanded distance are

Type A: $r \to r\sqrt{\Omega(t_0)}, t \to \frac{t}{\sqrt{\Omega(t_0)}}$, Type B: $r \to r\Omega(t_0), t \to t$ (52)

The space-time transformation of the Yukawa potential ϕ' depends on Type A

Hence, the Yukawa potential ϕ' dependent on time is

$$\phi' = -\frac{g^2}{r\sqrt{\Omega(t_0)}}\exp[-\frac{m_\pi r\sqrt{\Omega(t_0)}c}{\hbar}] + A_0\sin(\frac{\omega t}{\sqrt{\Omega(t_0)}})$$

Frequency $\omega = \frac{m_\pi c^2}{\hbar}$, m_π is meson's mass (53)

Eq(51)-Klein-Gordon equation is satisfied by Eq(53)-the solution.

6. Schrodinger Equation from Klein-Gordon Free Particle Field in Cosmological Inertial Frame

First, the Klein-Gordon equation is for the free particle field ϕ in the cosmological inertial frame.[22]

$$\frac{m^2c^2}{\hbar^2}\phi + \Omega(t_0)\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} - \frac{1}{\Omega(t_0)}\nabla^2\phi = 0$$

m is the free particle's mass and $\Omega(t_0)$ is the ratio of universe's expansion in cosmological time t_0 (54)

If we write the wave function as a solution of the Klein-Gordon equation for free particles,

$$\phi = A_0 \exp[i(\frac{\omega t}{\sqrt{\Omega(t_0)}} - \vec{k}\cdot\vec{x}\sqrt{\Omega(t_0)})]$$

A_0 is the amplitude, ω is the angular frequency, $k = |\vec{k}|$ is the wavenumber (55)

Energy and momentum are in the cosmological inertial frame,

$$E = \hbar\omega, \vec{p} = \hbar\vec{k} / \Omega(t_0) \quad (56)$$

Hence, the energy-momentum relation is in the cosmological inertial frame

$$E^2 = \hbar^2\omega^2 = \Omega^2(t_0)p^2c^2 + m^2c^4 = \hbar^2k^2c^2 + m^2c^4 \quad (57)$$

Alternatively, the angular frequency- wavenumber relation is

$$\frac{\omega^2}{c^2} = k^2 + \frac{m^2c^2}{\hbar^2} \quad (58)$$

Hence, the wave function is in the cosmological inertial frame,

$$\phi = A_0 \exp[(-\frac{i}{\hbar})(\hbar\frac{\omega t}{\sqrt{\Omega(t_0)}} - \hbar\vec{k}\cdot\vec{x}\sqrt{\Omega(t_0)})]$$

$$= A_0 \exp[(-\frac{i}{\hbar})(\frac{Et}{\sqrt{\Omega(t_0)}} - \vec{p}\cdot\vec{x}\Omega(t_0)\sqrt{\Omega(t_0)}] \quad (59)$$

Because, the Schrodinger equation is made from Klein-Gordon free particle's wave function in the cosmological special theory of relativity,

$$\phi = A_0 \exp[(-\frac{i}{\hbar})(\frac{Et}{\sqrt{\Omega(t_0)}} - \vec{p}\cdot\vec{x}\Omega(t_0)\sqrt{\Omega(t_0)}] \quad (60)$$

If we calculate the derivation of the Schrodinger equation,

$$\sum_i (\frac{\partial}{\partial x^i})^2\phi = -\sum_i \frac{(p^i)^2}{\hbar^2}\Omega^3(t_0)\phi = -\frac{p^2}{\hbar^2}\Omega^3(t_0)\phi \quad (61)$$

$$\frac{\partial\phi}{\partial t} = -\frac{i}{\hbar}E\frac{\phi}{\sqrt{\Omega(t_0)}} \quad (62)$$

Energy E is

$$E = \frac{p^2}{2m}\Omega^2(t_0) + V, \quad V \text{ is the potential energy} \quad (63)$$

Hence,

$$E\phi = \frac{p^2}{2m}\Omega^2(t_0)\phi + V\phi, \quad V \text{ is the potential energy} \quad (64)$$

Therefore, by Eq(61),Eq(62)

$$E\phi = i\hbar\frac{\partial\phi}{\partial t}\sqrt{\Omega(t_0)}, \quad \Omega^2(t_0)p^2\phi = -\hbar^2\nabla^2\phi\frac{1}{\Omega(t_0)} \quad (65)$$

Therefore, the Schrodinger equation in the cosmological inertial frame is,

$$E\phi = i\hbar\frac{\partial\phi}{\partial t}\sqrt{\Omega(t_0)} = -\frac{\hbar^2}{2m}\frac{1}{\Omega(t_0)}\nabla^2\phi + V\phi \quad (66)$$

If the energy E is not concerned by time t,

$$\frac{\partial E}{\partial t} = 0 \quad (67)$$

$$\phi = A_0 \exp[-(\frac{i}{\hbar})(\frac{Et}{\sqrt{\Omega(t_0)}} - \vec{p}\cdot\vec{x}\Omega(t_0)\sqrt{\Omega(t_0)})]$$

$$= \varphi \exp[-(\frac{i}{\hbar})(\frac{Et}{\sqrt{\Omega(t_0)}})] \tag{68}$$

Hence, the stationary state of the Schrodinger equation is in cosmological inertial frame,

$$E\varphi \exp[-(\frac{iE}{\hbar})\frac{t}{\sqrt{\Omega(t_0)}}]$$

$$= [-\frac{\hbar^2}{2m}\frac{1}{\Omega(t_0)}\nabla^2\varphi + V\varphi]\exp[-(\frac{iE}{\hbar})\frac{t}{\sqrt{\Omega(t_0)}}] \tag{69}$$

Hence, the stationary state of the Schrodinger equation is

$$E\varphi = -\frac{\hbar^2}{2m}\frac{1}{\Omega(t_0)}\nabla^2\varphi + V\varphi \tag{70}$$

Or,

$$\frac{1}{\Omega(t_0)}\nabla^2\varphi + \frac{2m}{\hbar^2}(E-V)\varphi = 0 \tag{71}$$

Uncertainty Principle is in quantum mechanics in special relativity,

$$x \cdot p \geq \frac{\hbar}{2}, E \cdot t \geq \frac{\hbar}{2} \quad ,$$

If $x = ct, E = pc \rightarrow x \cdot p = ct \cdot E / c = E \cdot t \geq \frac{\hbar}{2}$ (71-i)

According to Eq(59), Cosmological Uncertainty Principle is in cosmological quantum mechanics in cosmological special relativity,

$$x\Omega(t_0) \cdot p\Omega(t_0)\frac{1}{\sqrt{\Omega(t_0)}} = x \cdot p\Omega(t_0)\sqrt{\Omega(t_0)} \geq \frac{\hbar}{2}, \quad ,$$

$$E \cdot t / \sqrt{\Omega(t_0)} \geq \frac{\hbar}{2}$$

If

$$x\Omega(t_0) = ct, E = p\Omega(t_0)c$$

$$\rightarrow x \cdot p\Omega(t_0)\sqrt{\Omega(t_0)} = ct \cdot E / c\sqrt{\Omega(t_0)} = E \cdot t / \sqrt{\Omega(t_0)} \geq \frac{\hbar}{2} \tag{71-ii}$$

7. Dirac Equation from Wave Function-Type A in Cosmological Inertial Frame

The Dirac equation is in special relativity theory,[23]

$$(i\hbar\gamma^{\mu}\partial_{\mu} - mcI)\psi = 0 \quad ,$$

I is the 4×4 unit matrix ,

$$\gamma^0 = \begin{pmatrix} I' & 0 \\ 0 & -I' \end{pmatrix}, \ \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \ \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

I' is the 2×2 unit matrix and σ^i is Pauli's matrix. (72)

The Dirac equation is the wave equation. Therefore, the Dirac equation is in the cosmological inertial frame,

Wave function Type A:

$$r \rightarrow r\sqrt{\Omega(t_0)} \quad , \quad t \rightarrow \frac{t}{\sqrt{\Omega(t_0)}} ,$$

t_0 is the cosmological time. $\Omega(t_0)$ is the expanding ratio of the universe in cosmological time t_0 .

$$(i\hbar\sqrt{\Omega(t_0)}\gamma^0\partial_0 + i\hbar\frac{1}{\sqrt{\Omega(t_0)}}\gamma^i\partial_i - mcI)\phi = 0 \tag{73}$$

If $\overline{\partial}_\mu$ is

$$\overline{\partial}_\mu = (\sqrt{\Omega(t_0)}\partial_0, \frac{1}{\sqrt{\Omega(t_0)}}\partial_i) \tag{74}$$

The Dirac equation is in the cosmological inertial frame,

$$(i\hbar\gamma^\mu\overline{\partial}_\mu - mcI)\phi = 0 \tag{75}$$

Eq(75) multiply $i\hbar\gamma^\upsilon\overline{\partial}_\upsilon$, hence

$$(-\hbar^2(\gamma^\mu\overline{\partial}_\mu)(\gamma^\upsilon\overline{\partial}_\upsilon) - i\hbar(\gamma^\upsilon\overline{\partial}_\upsilon)mcI)\phi = 0 \tag{76}$$

At this time,

$$i\hbar\gamma^\upsilon\overline{\partial}_\upsilon\phi = mcI\phi \tag{77}$$

Hence, Eq(76) is

$$(-\hbar^2\gamma^\mu\gamma^\upsilon\overline{\partial}_\mu\overline{\partial}_\upsilon - m^2c^2I)\phi = 0 \tag{78}$$

In this time, matrix γ^μ is

$$\frac{1}{2}(\gamma^\mu\gamma^\upsilon + \gamma^\upsilon\gamma^\mu) = \frac{1}{2}\{\gamma^\mu, \gamma^\upsilon\} = \eta^{\mu\upsilon}I \tag{79}$$

Therefore,

$$\frac{1}{2}(\gamma^\mu\gamma^\upsilon + \gamma^\upsilon\gamma^\mu)\overline{\partial}_\mu\overline{\partial}_\upsilon\phi + \frac{m^2c^2}{\hbar^2}I\phi$$

$$= (\eta^{\mu\upsilon}\overline{\partial}_\mu\overline{\partial}_\upsilon + \frac{m^2c^2}{\hbar^2})I\phi = 0 \tag{80}$$

Eq(80) is the matrix equation of Klein-Gordon.

The Dirac spinor ϕ is $\phi = (\phi_1, \phi_2, \phi_3, \phi_4)$. ϕ's hermitian conjugate $\phi^+ = (\phi^*{}_1, \phi^*{}_2, \phi^*{}_3, \phi^*{}_4)$.

Hence, ϕ's adjoint spinor $\overline{\phi}$ is

$$\overline{\phi} = \phi^+\gamma^0, \ \overline{\phi}(i\gamma^\mu\overleftarrow{\overline{\partial}}_\mu + mcI) = 0 \tag{81}$$

Hence, the positive probability density j^0 is

$$j^0 = \overline{\phi}\gamma^0\phi = \phi^+\phi = |\phi_1|^2 + |\phi_2|^2 + |\phi_3|^2 + |\phi_4|^2 \tag{82}$$

8. Equations of Interaction of Complex Scalar Fields and Electromagnetic Fields in Cosmological Inertial Frame

The Lagrangian L of complex scalar fields ϕ, ϕ^* and electromagnetic fields $F^{\mu\upsilon}, F_{\mu\upsilon}$ is Klein-Gordon-Maxwell theory in special relativity theory,[24]

$$L = (\partial_\mu\phi + ieA_\mu\phi)(\partial^\mu\phi^* - ieA^\mu\phi^*) - \frac{m^2c^2}{\hbar^2}\phi\phi^* - \frac{1}{4}F^{\mu\upsilon}F_{\mu\upsilon}$$

ϕ^* is ϕ's adjoint scalar and m is the mass of scalar fields ϕ, ϕ^*

$$F^{\mu\upsilon} = \partial^\mu A^\upsilon - \partial^\upsilon A^\mu, F_{\mu\upsilon} = \partial_\mu A_\upsilon - \partial_\upsilon A_\mu \tag{83}$$

The Lagrangian L of the interaction of complex scalar fields and electromagnetic fields is Klein-Gordon-Maxwell theory in the cosmological inertial frame,

$$L = (\overline{\partial}_\mu\phi + ie\overline{A}_\mu{}'\phi)(\overline{\partial}^\mu\phi^* - ie\overline{A}^\mu{}'\phi^*) - \frac{m^2c^2}{\hbar^2}\phi\phi^* - \frac{1}{4}F^{\mu\upsilon}{}'F_{\mu\upsilon}{}' \tag{84}$$

We consider Type A of the wave function and Type B of the expanded distance,

Type A of wave function: $r \to r\sqrt{\Omega(t_0)}$, $t \to \frac{t}{\sqrt{\Omega(t_0)}}$,

Type B of expanded distance: $r \to r\Omega(t_0), t \to t$

$$\bar{\partial}_\mu = (\sqrt{\Omega(t_0)}\frac{\partial}{c\partial t}, \frac{1}{\sqrt{\Omega(t_0)}}\vec{\nabla}), \bar{\partial}^\mu = (\sqrt{\Omega(t_0)}\frac{\partial}{c\partial t}, -\frac{1}{\sqrt{\Omega(t_0)}}\vec{\nabla})$$

$$\bar{A}_\mu{}' = (\phi, \vec{A}\Omega(t_0)), \bar{A}^\mu{}' = (\phi, -\vec{A}\Omega(t_0)), \bar{F}_{\mu\upsilon}{}' = F_{\mu\upsilon}\Omega(t_0), \bar{F}^{\mu\upsilon}{}' = F^{\mu\upsilon}\Omega(t_0)$$

t_0 is the cosmological time. $\Omega(t_0)$ is the expanding ratio of the universe in cosmological time t_0.

(85)

Complex scalar field equations are in Klein-Gordon-Maxwell theory in the cosmological inertial frame,

$$\bar{\partial}_\mu(\frac{\partial L}{\partial(\bar{\partial}_\mu\phi)}) - \frac{\partial L}{\partial\phi} = (\bar{\partial}_\mu - ie\bar{A}_\mu{}')(\bar{\partial}^\mu\phi^* - ie\bar{A}^\mu{}'\phi^*) + \frac{m^2c^2}{\hbar^2}\phi^* = 0 \tag{86}$$

The other equation is in Klein-Gordon-Maxwell theory in the cosmological inertial frame,

$$\bar{\partial}_\mu(\frac{\partial L}{\partial(\bar{\partial}_\mu\phi^*)}) - \frac{\partial L}{\partial\phi^*} = (\bar{\partial}^\mu + ie\bar{A}^\mu{}')(\bar{\partial}_\mu\phi + ie\bar{A}_\mu{}'\phi) + \frac{m^2c^2}{\hbar^2}\phi = 0 \tag{87}$$

If operator $\bar{\partial}_\mu{}', \bar{\partial}^\mu{}'$ is in the cosmological inertial frame,

$$\bar{\partial}_\mu{}' = (\frac{\partial}{c\partial t}, \frac{1}{\Omega(t_0)}\vec{\nabla}), \bar{\partial}^\mu{}' = (\frac{\partial}{c\partial t}, -\frac{1}{\Omega(t_0)}\vec{\nabla})$$

$$\bar{F}^{\mu\upsilon}{}' = \bar{\partial}^\mu{}'\bar{A}^\upsilon{}' - \bar{\partial}^\upsilon{}'\bar{A}^\mu{}', \bar{F}_{\mu\upsilon}{}' = \bar{\partial}_\mu{}'\bar{A}_\upsilon{}' - \bar{\partial}_\upsilon{}'\bar{A}_\mu{}' \tag{88}$$

The electromagnetic field equations are in Klein-Gordon-Maxwell theory in the cosmological inertial frame,

$$\bar{\partial}_\upsilon{}'(\frac{\partial L}{\partial(\bar{\partial}_\upsilon{}'\bar{A}_\mu{}')}) - \frac{\partial L}{\partial\bar{A}_\mu{}'} = \frac{1}{4}\bar{\partial}_\upsilon{}'(\bar{\partial}^\mu{}'\bar{A}^\upsilon{}' - \bar{\partial}^\upsilon{}'\bar{A}^\mu{}') - ie\phi(\bar{\partial}^\mu\phi^* - ie\bar{A}^\mu{}'\phi^*) + ie\phi^*(\bar{\partial}^\mu\phi + ie\bar{A}^\mu{}'\phi)$$

$$= \frac{1}{4}\bar{\partial}_\upsilon{}'\bar{F}^{\mu\upsilon}{}' - ie\phi(\bar{\partial}^\mu\phi^* - ie\bar{A}^\mu{}'\phi^*) + ie\phi^*(\bar{\partial}^\mu\phi + ie\bar{A}^\mu{}'\phi) = 0 \tag{89}$$

Hence,

$$\bar{\partial}_\upsilon{}'\bar{F}^{\mu\upsilon}{}' = -4\pi e\bar{J}^\mu{}' = 4ie[\phi(\bar{\partial}^\mu\phi^* - ie\bar{A}^\mu{}'\phi^*) - \phi^*(\bar{\partial}^\mu\phi + ie\bar{A}^\mu{}'\phi)]$$

$$\bar{J}^\mu{}' = -\frac{1}{\pi}i[\phi(\bar{\partial}^\mu\phi^* - ie\bar{A}^\mu{}'\phi^*) - \phi^*(\bar{\partial}^\mu\phi + ie\bar{A}^\mu{}'\phi)]$$

$$= \frac{1}{\pi}i[\phi^*(\bar{\partial}^\mu\phi + ie\bar{A}^\mu{}'\phi) - \phi(\bar{\partial}^\mu\phi^* - ie\bar{A}^\mu{}'\phi^* \tag{90}$$

The other equation is in Klein-Gordon-Maxwell theory in the cosmological inertial frame,

$$\bar{\partial}^\upsilon{}'(\frac{\partial L}{\partial(\bar{\partial}^\upsilon{}'\bar{A}^\mu{}')}) - \frac{\partial L}{\partial\bar{A}^\mu{}'}$$

$$= \frac{1}{4}\bar{\partial}^\upsilon{}'(\bar{\partial}_\mu{}'\bar{A}_\upsilon{}' - \bar{\partial}_\upsilon{}'\bar{A}_\mu{}') + ie\phi^*(\bar{\partial}_\mu\phi + ie\bar{A}_\mu{}'\phi) - ie\phi(\bar{\partial}_\mu\phi^* - ie\bar{A}_\mu{}'\phi^*)$$

$$= \frac{1}{4}\bar{\partial}^\upsilon{}'\bar{F}_{\mu\upsilon}{}' + ie\phi^*(\bar{\partial}_\mu\phi + ie\bar{A}_\mu{}'\phi) - ie\phi(\bar{\partial}_\mu\phi^* - ie\bar{A}_\mu{}'\phi^*) = 0 \tag{91}$$

Hence,

$$\bar{\partial}^\upsilon{}'\bar{F}_{\mu\upsilon}{}' = -4\pi e\bar{J}_\mu{}' = -4ie[\phi^*(\bar{\partial}_\mu\phi + ie\bar{A}_\mu{}'\phi) - \phi(\bar{\partial}_\mu\phi^* - ie\bar{A}_\mu{}'\phi^*)]$$

$$\bar{J}_\mu{}' = i\frac{1}{\pi}[\phi^*(\bar{\partial}_\mu\phi + ie\bar{A}_\mu{}'\phi) - \phi(\bar{\partial}_\mu\phi^* - ie\bar{A}_\mu{}'\phi^*)] \tag{92}$$

9. Quantization of Klein-Gordon Scalar Field in CSTR

We quantify the Klein-Gordon scalar field in the Cosmological Special Theory of Relativity (CSTR).[20] First, space-time relations are included in the Cosmological Special Theory of Relativity (CSTR).

$$ct = \gamma(ct' + \frac{v_0}{c}\Omega^2(t_0)x'), x\Omega(t_0) = \gamma(\Omega(t_0)x' + v_0\Omega(t_0)t')$$

$$\begin{aligned}\Omega(t_0)y &= \Omega(t_0)y', \\ \Omega(t_0)z &= \Omega(t_0)z'\end{aligned} \quad , \ \gamma = 1/\sqrt{1-\frac{{v_0}^2}{c^2}\Omega^2(t_0)}, \ t_0 \text{ is cosmological time} \tag{93}$$

Proper time is

$$d\tau^2 = dt^2 - \frac{1}{c^2}\Omega^2(t_0)[dx^2 + dy^2 + dz^2]$$
$$= dt'^2 - \frac{1}{c^2}\Omega^2(t_0)[dx'^2 + dy'^2 + dz'^2], \ t_0 \text{ is cosmological time} \tag{94}$$

The angular frequency-wavenumber relation is in the CSTR.

$$\omega' = \gamma(\omega - v_0\Omega(t_0)k_1), \ k_1' = \gamma(k_1 - \frac{v_0}{c^2}\Omega(t_0)\omega)$$
$$k_2' = k_2, k_3' = k_3, \gamma = 1/\sqrt{1-\frac{{v_0}^2}{c^2}\Omega^2(t_0)} \tag{95}$$

Lagrangian density of Klein-Gordon scalar field in CSTR,

$$L = -\frac{1}{2}[-(\frac{1}{c}\frac{\partial\phi}{\partial t})^2\Omega(t_0) + \frac{1}{\Omega(t_0)}\vec{\nabla}\phi\cdot\vec{\nabla}\phi - \frac{{m_0}^2c^2}{\hbar^2}\phi^2] \tag{96}$$

Hence, the Euler-Lagrange equation is in CSTR,

$$\partial_\mu[\frac{\partial L}{\partial(\partial_\mu\phi)}] - \frac{\partial L}{\partial\phi} = [\Omega(t_0)\frac{1}{c^2}(\frac{\partial}{\partial t})^2 - \frac{1}{\Omega(t_0)}\nabla^2 + \frac{{m_0}^2c^2}{\hbar^2}]\phi = 0) \tag{97}$$

Hamiltonian of Klein-Gordon scalar field is in the CSTR,

$$H = \frac{1}{2}[(\frac{1}{c}\frac{\partial\phi}{\partial t})^2\Omega(t_0) + \frac{1}{\Omega(t_0)}\vec{\nabla}\phi\cdot\vec{\nabla}\phi + \frac{{m_0}^2c^2}{\hbar^2}\phi^2] \tag{98}$$

The Klein-Gordon scalar field is divided by the positive frequency mode and negative frequency mode.

$$\phi(x) = \phi^{(+)}(x) + \phi^{(-)}(x) \tag{99}$$

The positive frequency mode is

$$\phi^{(+)}(x) = \int\frac{d^3k}{[(2\pi)^3 2\omega_k]^{\frac{1}{2}}}a(k)f_k(x) \tag{100}$$

The negative frequency mode is

$$\phi^{(-)}(x) = \int\frac{d^3k}{[(2\pi)^3 2\omega_k]^{\frac{1}{2}}}a^{(+)}(k)f_k(x) \tag{101}$$

At this time, $f_k(x)$ is

$$f_k(x) = \frac{1}{[(2\pi)^3 2\omega_k]^{\frac{1}{2}}}\exp[i(\frac{\omega_k t}{\sqrt{\Omega(t_0)}} - \vec{k}\cdot\vec{x}\sqrt{\Omega(t_0)})] \tag{102}$$

In this time,

$$\frac{\omega_k}{c} = (k^2 + \frac{{m_0}^2c^2}{\hbar^2})^{\frac{1}{2}} \tag{103}$$

Quantization of the complex scalar field is in the CSTR,

$$\phi(x) = \int\frac{d^3k}{(2\pi^3) 2\omega_k} [a(k)\ \exp\{i(\frac{\omega_k t}{\sqrt{\Omega(t_0)}} - \vec{k}\cdot\vec{x}\sqrt{\Omega\, t_0} \quad ($$

$$+\int \frac{d^3k}{(2\pi)^2 2\omega_k}[b^+(k)\exp\{-i(\frac{\omega_k t}{\sqrt{\Omega(t_0)}} - \vec{k}\cdot\vec{x}\sqrt{\Omega(t_0)})\}] \qquad (104)$$

$$\phi^+(x) = \int \frac{d^3k}{(2\pi)^3 2\omega_k}[b(k)\exp\{i(\frac{\omega_k t}{\sqrt{\Omega(t_0)}} - \vec{k}\cdot\vec{x}\sqrt{\Omega(t_0)})\}]$$

$$+\int \frac{d^3k}{(2\pi)^2 2\omega_k}[a^+(k)\exp\{-i(\frac{\omega_k t}{\sqrt{\Omega(t_0)}} - \vec{k}\cdot\vec{x}\sqrt{\Omega(t_0)})\}] \qquad (105)$$

Hence, Hamiltonian H is in CSTR,

$$H = \int \frac{d^3k}{(2\pi)^3 2\omega_k}[a^+(k)a(x) + b^+(k)b(k)] \qquad (106)$$

At this time,

$$[a(k), a^+(k')] = (2\pi)^3 2\omega_k \delta^3(\vec{k} - \vec{k}')$$
$$[b(k), b^+(k')] = (2\pi)^3 2\omega_k \delta^3(\vec{k} - \vec{k}') \qquad (107)$$

10. Particle's Force and Kinetic Energy in CSTR

If we are dealing with the particle's force and kinetic energy, the 4-force is in CSTR

$$f^\mu = (f^0, \vec{f}\Omega(t_0))$$

$$f^0 = m_0 \frac{d}{dt}(\frac{cdt}{d\tau}),$$

$$\vec{f}\Omega(t_0) = \frac{d(\vec{p}\Omega(t_0))}{dt} = \frac{d}{dt}(\frac{m_0\vec{u}\Omega(t_0)}{\sqrt{1-\frac{u^2}{c^2}\Omega^2(t_0)}}), \quad \vec{p}\Omega(t_0) = m_0\frac{d\vec{x}\Omega(t_0)}{d\tau} \qquad (108)$$

If the particle is in an electromagnetic field, a 4-Lorentz force will act on the particle in the CSTR.

$$f^0 = m_0\frac{d}{dt}(\frac{cdt}{d\tau}) = q\frac{\vec{u}}{c}\cdot\vec{E}\Omega^2(t_0)$$

$$\vec{f}\Omega(t_0) = \frac{d(\vec{p}\Omega(t_0))}{dt} = \frac{d}{dt}(\frac{m_0\vec{u}\Omega(t_0)}{\sqrt{1-\frac{u^2}{c^2}\Omega^2(t_0)}}) = q\vec{E}\Omega(t_0) + q\frac{\vec{u}}{c}\times\vec{B}\Omega^2(t_0) \qquad (109)$$

If we are dealing with the particle's kinetic energy,

$$KE = \int_0^{V\Omega(t_0)} u\Omega(t_0)d(p\Omega(t_0))$$

$$= \int_0^{V\Omega(t_0)} u\Omega(t_0)d(\frac{m_0u\Omega(t_0)}{\sqrt{1-\frac{u^2\Omega^2(t_0)}{c^2}}})$$

$$= \frac{m_0V^2\Omega^2(t_0)}{\sqrt{1-\frac{V^2\Omega^2(t_0)}{c^2}}} - \int_0^{V\Omega(t_0)}\frac{m_0u\Omega(t_0)}{\sqrt{1-\frac{u^2\Omega^2(t_0)}{c^2}}}d(u\Omega(t_0))$$

$$= \frac{m_0V^2\Omega^2(t_0)}{\sqrt{1-\frac{V^2\Omega^2(t_0)}{c^2}}} + m_0c^2\sqrt{1-\frac{u^2\Omega^2(t_0)}{c^2}}\Big|_0^{V\Omega(t_0)}$$

$$= \frac{m_0 V^2 \Omega^2(t_0)}{\sqrt{1-\frac{V^2\Omega^2(t_0)}{c^2}}} + m_0 c^2 \sqrt{1-\frac{V^2\Omega^2(t_0)}{c^2}} - m_0 c^2 = \frac{m_0 c^2}{\sqrt{1-\frac{V^2\Omega^2(t_0)}{c^2}}} - m_0 c^2 \quad (110)$$

11. Newtonian Gravity in Expanded Universe

Newton's Gravity is built in static universe. Hence, for making our cosmological theory, we modified Newtonian Gravity in expanded universe.

At first, Newton's gravity acceleration is

$$\bar{\vec{a}} = \vec{a}\Omega(t_0) = -\bar{\vec{\nabla}}\bar{\phi} = -\frac{1}{\Omega^2(t_0)}\vec{\nabla}\phi \quad (111)$$

$$\bar{\phi} = \phi\frac{1}{\Omega(t_0)} = -\frac{GM}{r\Omega(t_0)} \text{ or } \bar{\phi} = \phi\frac{1}{\Omega(t_0)} = \frac{1}{2}\frac{GM}{R^3\Omega^3(t_0)}r^2\Omega^2(t_0) = \frac{1}{2}\frac{GM}{R^3}r^2\frac{1}{\Omega(t_0)} \quad (112)$$

In this time, if Newton's gravity potential is

$$\bar{\phi} = \phi\frac{1}{\Omega(t_0)} = -\frac{GM}{r\Omega(t_0)} \quad (113)$$

Eq(111) is

$$\bar{\vec{a}} = \vec{a}\Omega(t_0) = -\frac{1}{\Omega^2(t_0)}\vec{\nabla}\phi = -\frac{GM}{r^3}\vec{r}\frac{1}{\Omega^2(t_0)} \quad (114)$$

If Newton's gravity potential is

$$\bar{\phi} = \phi\frac{1}{\Omega(t_0)} = \frac{1}{2}\frac{GM}{R^3}r^2\frac{1}{\Omega(t_0)} \quad (115)$$

Poisson equation is in expanded universe,

$$\bar{\nabla}^2\bar{\phi} = \frac{1}{\Omega^3(t_0)}\nabla^2\phi = 4\pi G\bar{\rho}, \ \bar{\rho} = \frac{\rho}{\Omega^3(t_0)} \quad (116)$$

Newton force is in expanded universe,

$$\bar{\vec{F}} = m_0\bar{\vec{a}} = m_0\vec{a}\Omega(t_0) = \vec{F}\Omega(t_0) \quad (117)$$

12. Cosmological General Theory of Relativity

Einstein's geodesic equation is in expanded universe,

$$\frac{d^2\bar{x}^\mu}{d\tau^2} + \bar{\Gamma}^\mu_{\alpha\beta}\frac{d\bar{x}^\alpha}{d\tau}\frac{d\bar{x}^\beta}{d\tau} = 0 \quad (118)$$

Schwarzschild solution (vacuum solution) is in expanded universe,

$$ds^2 = -c^2(1-\frac{2GM}{\bar{r}c^2})dt^2 + \frac{d\bar{r}^2}{1-\frac{2GM}{\bar{r}c^2}} + \bar{r}^2 d\theta^2 + \bar{r}^2\sin^2\theta d\varphi^2$$

$$= -c^2(1-\frac{2GM}{r\Omega(t_0)c^2})dt^2 + \Omega^2(t_0)[\frac{dr^2}{1-\frac{2GM}{r\Omega(t_0)c^2}} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)] \quad (119)$$

Hence, Newtonian approximation is by Eq(119)

$$\bar{a}_r = \frac{d^2r}{d\tau^2}\Omega(t_0) \approx -\bar{\Gamma}^1_{00}c^2(\frac{dt}{d\tau})^2 \approx \frac{1}{2}c^2\frac{\partial\bar{g}_{00}}{\partial\bar{r}} = \frac{1}{2}\frac{c^2}{\Omega(t_0)}\frac{\partial g_{00}}{\partial r} = -\frac{GM}{r^2}\frac{1}{\Omega^2(t_0)} \quad (120)$$

Hence, the gravity field equation of Einstein in expanded universe,

$$\bar{R}_{\mu\upsilon} - \frac{1}{2}\bar{g}_{\mu\upsilon}\bar{R} = -\frac{8\pi G}{c^4}\bar{T}_{\mu\upsilon} \quad (121)$$

In this time,

$$\bar{T}_{00} = \bar{\rho}c^2 = \frac{\rho}{\Omega^3(t_0)}c^2 = \frac{T_{00}}{\Omega^3(t_0)} \quad (122)$$

Einstein's general solution- Kerr-Newman solution is in expanded universe,

$$ds^2 = \bar{g}_{\mu\upsilon}d\bar{x}^{\mu}d\bar{x}^{\upsilon}$$

$$= -c^2(1 - \frac{2c^2GM\bar{r} - kGQ^2}{c^4\bar{\Sigma}})dt^2 - 2(2c^2MG\bar{r} - kGQ^2)\frac{\bar{a}\sin^2\theta}{c^4\bar{\Sigma}}cdtd\varphi$$

$$+ \frac{c^4\bar{\Sigma}}{\bar{r}^2 - c^{2}2GM\bar{r} + \bar{a}^{2} + kGQ^{2}}d\bar{r}^2 + \bar{\Sigma}d\theta^2$$

$$+\sin^2\theta[\bar{r}^2 + \bar{a}^2 + (2c^2GM\bar{r} - kGQ^2)\frac{\bar{a}^2\sin^2\theta}{c^4\bar{\Sigma}}]d\varphi^2$$

$$\bar{\Sigma} = \bar{r}^2 + \bar{a}^2\cos^2\theta = (r^2 + a^2\Omega^2(t_0)cos^2\theta)\Omega^2(t_0)$$

$$= \Sigma'\Omega^2(t_0) \ , \quad \Sigma' = r^2 + a^2\Omega^2(t_0)cos^2\theta \quad (123)$$

Hence, Kerr-Newman solution is expanded universe,

$$ds^2 = \bar{g}_{\mu\upsilon}d\bar{x}^{\mu}d\bar{x}^{\upsilon}$$

$$= -c^2(1 - \frac{2c^2GMr\Omega(t_0) - kGQ^2}{c^4\Sigma'\Omega^2(t_0)})dt^2$$

$$-2(2c^2MGr\Omega(t_0) - kGQ^2)\frac{a\Omega^2(t_0)\sin^2\theta}{c^4\Sigma'\Omega^2(t_0)}cdtd\varphi$$

$$+\Omega^2(t_0)\{\frac{c^4\Sigma'\Omega^2(t_0)}{r^2\Omega^2(t_0) - c^2 2GMr\Omega(t_0) + a^2\Omega^4(t_0) + kGQ^2}dr^2 + \Sigma'd\theta^2$$

$$+\sin^2\theta[r^2 + a^2\Omega^2(t_0) + (2c^2GMr\Omega(t_0) - kGQ^2)\frac{a^2\Omega^4(t_0)\sin^2\theta}{\Omega^4(t_0)c^4\Sigma'}]d\varphi^2\}$$

$$\bar{\Sigma} = \bar{r}^2 + \bar{a}^2\cos^2\theta = (r^2 + a^2\Omega^2(t_0)cos^2\theta)\Omega^2(t_0)$$

$$= \Sigma'\Omega^2(t_0) \ , \quad \Sigma' = r^2 + a^2\Omega^2(t_0)cos^2\theta \quad (124)$$

Robertson-Walker solution is Minkowski space-time by Einstein gravity field equation in CGTR-Eq(121) in expanded universe.

$$ds^2 = -c^2dt^2 + [d\bar{r}^2 + \bar{r}^2d\Omega^2] = -c^2dt^2 + \Omega^2(t_0)[dr^2 + r^2d\Omega^2] \quad (125)$$

13. Conclusion

We know Maxwell equations, electromagnetic wave equations and functions in the Cosmological Special Theory of Relativity (CSTR). We are able to describe free particles by the Klein-Gordon equation and wave function in the CSTR. We solve the Klein-Gordon equation in the cosmological inertial frame. Hence, we found Yukawa potential dependent time in the cosmological inertial frame. We found Schrodinger equation from Klein-Gordon's free particle equation in the cosmological special theory of relativity. The wave function used as a probability amplitude. We found the Dirac equation from Wave Function-Type A in the cosmological special theory of relativity. The wave function uses as a probability amplitude. We found equations of complex scalar fields and electromagnetic fields on the interaction of complex scalar fields and electromagnetic fields in Klein-Gordon-Maxwell theory from Type A of the wave function and Type B of the expanded distance in cosmological inertial frame. We quantized the Klein-Gordon scalar field in CSTR. We treat the Lagrangian density and Hamiltonian. We address the particle's force and kinetic energy in the cosmological special theory of relativity.We find Cosmological General theory of Relativity. We obtain solution of Einstein gravity field equation in expanded universe..

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