Are gravimeters sensitive enough to measure gravitational waves?

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Calculations show that the sensitivity of common gravimeters is sufficient to measure GW in wide frequency range around 0.1 Hz. Initial evaluations have confirmed that it is possible to extract the coordinates and frequency drift of known binary star systems with good accuracy from multi-year data records of gravimeters distributed around the world. This opens the possibility of an Earth-based search for continuous GW several years before LISA.

1 Introduction

The detection of continuous gravitational waves opens another window for astronomy to observe distant celestial bodies. GW cause extraordinarily small changes in the length of scales, which is why suitable detectors cannot be large enough. As a way out, one can use the Earth itself as the largest possible test body. Gravimeters have been recording the smallest changes in gravity for decades and provide an excellent database for searching for gravitational waves. A hint for their existence was discovered by geophysicists decades ago: The Earth gives off a relentless hum of countless notes, but the origin of this sound remains a mystery [2]. The researchers found that the hum's amplitude does not vary over time and doesn't depend on the seasons. Do continuous GWs cause the never ending hum of the Earth?

In the records of gravimeters, peculiar oscillations are noticed which are not self-resonances of the earth and are also not excited by earthquakes. The green curve in Figure 1 shows the measurements of the SG Strasbourg after the noise was reduced with a band-pass filter (28 μHz $< f <$ 900 μHz). This oscillation resembles the highly stretched end of the GW150914 chirp signal and tempts comparison with the predictions given there. Equation (8) given in [3] leads to

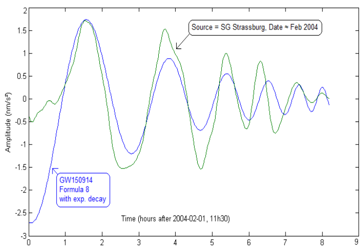

Figure 1): *A striking damped oscillation measured by a gravimeter (green) and the theoretical prediction.*

$$f_{chirp} = (k \cdot (t - t_0))^{-3/8} \qquad (1)$$

where k depends on the mass of the two stars and t is the time. Shortly before the final merger of two supermassive stars, friction will reduce the amplitude of the emitted GW. Supplementing equation (1) with a suitable exponential function, one obtains the blue curve in figure 1. A certain similarity cannot be overlooked. Did the gravimeter really observe a supermassive merger? We will not discuss that here.

Here we are only concerned with the questions: What are the characteristics of continuous GW? Does the communication technology know how to measure signals with comparable properties? Is the intrinsic noise of gravimeters weak enough to detect the periodic changes of the Earth's diameter caused by the GW of distant celestial bodies?

2 The signatures of a continuous GW

Below we discuss properties of GW in the range 10^{-4} Hz $< f_{GW} < 1$ Hz, such as those produced by binary star systems. Examples concern a narrow range around 5 mHz, where the intrinsic noise of gravimeters is very low. The drift of typical GW is about 5 nHz per year, exceeding the target bandwidth (BW) of ~ 2 nHz necessary to ensure a good signal-to-noise (S/N) ratio. This point will be discussed in more detail in section 5.

The frequency of a GW increases slowly and proportionally to the time. Non-linear curves are not to be expected because the observation period is far shorter than the remaining lifetime of the GW source.

The amplitude of the GW is constant during the entire measurement period. This property can be difficult to verify in the presence of poor S/N. The *selective integration* method provides reliable results [11].

The received GW is dual phase modulated because the gravimeters move in the wave field of the GW:

- The high orbital velocity of 30 km/s of the Earth's orbit (frequency $f_{year} = 1/356.256$ days) generates a frequency deviation of about 500 nHz, which exceeds the targeted bandwidth by orders of magnitude (section 6).
- The PM as a result of the Earth's proper rotation ($f_{day} = 1/24$ h) generates two sidebands spaced $\Delta f = 11.574$ μHz to the left and right of the central frequency f_{GW}. In section 7 it is shown that the corresponding spectral lines cannot be detected.

Further modulations of GW are not known.

3 The sensitivity of gravimeters

Gravimeters are sensors near the surface of the Earth to determine the value of the acceleration due to gravity (about 9.81 $\frac{m}{s^2}$). Some types measure the absolute value, others only the relative change. Both types can provide years of uninterrupted data every second.

Up to now gravimeters were considered as instruments of geophysics, were mounted directly on the ground and therefore react very strongly to earthquakes and the natural frequencies of the earth excited by them. Also celestial bodies like the moon or the planets strongly influence the gravimeters with precisely defined frequencies around 11 μHz (tides) [1]. All these influences can be distinguished from GW by the simplest means because they are modulated differently from GW. Irregularly occurring earthquakes produce a very broadband spectrum, which interferes the less, the lower the bandwidth of the signal processing is chosen.

To judge whether a GW of nearly constant frequency may be detected in the noise floor of a gravimeter, one has to know the inherent noise of the gravimeter and compare it with the stress value h_0 predicted by the theory of relativity.

The usual benchmark *Power Spectral Density* (PSD) [4] has the unit $\frac{m^2}{s^4 Hz}$ and provides the first part of the sought answer, if one knows the bandwidth BW of the signal processing. Detailed investigations yield the following values:

- The PSD of superconducting gravimeters in the range around 6 mHz is about 10^{-18} $\frac{m^2}{s^4 Hz}$ [5] and [6], Figure 5.
- The PSD of the widely used gravimeter STS1 is even lower at 10^{-19} $\frac{m^2}{s^4 Hz}$ [7]. It should be possible to improve this value significantly if the gravimeter is not mounted directly on the ground, as has been the case up to now.
- In the successful LISA Pathfinder model experiment, small-scale PSD values around 10^{-29} $\frac{m^2}{s^4 Hz}$ [8] were achieved, which are also hoped for in the future GW observatory LISA. There is one hurdle to overcome: In the model experiment, the spacing of the two test bodies was 0.4 m. For the LISA observatory, three satellites are 2.5×10^9 m apart.

So far gravimeters are instruments of geophysics and therefore mounted directly on the ground. Future *astronomical* gravimeters will be mounted avoiding sound transmission and therefore have little response to earthquakes. This improvement should lower the PSD of gravimeters.

4 The directivity of gravimeters

Gravimeters measure acceleration in the vertical direction. The LIGO observatories measure distances in horizontal direction. If these instruments were operated at the north or south pole of the earth, one could perhaps hope for statements about the polarization of the GW. At all other locations of gravimeters the direction to the center of the earth changes so fast in the course of a day that only mean values can be measured. In the section 5 it is shown that observation durations of days or weeks are not sufficient to detect GW because the S/N and the spectral frequency resolution are too low.

5 Signal bandwidth and the recording period

Not only does the Earth affect gravimeters, but so do the Moon, the Sun, the planets, and certainly distant binary star systems. The result is a signal mixture of oscillations of different frequencies, which can be selectively investigated with the known methods of communications engineering.

Gravimeters receive a very wide frequency range between 10^{-8} Hz and about 100 Hz, which is dominated by very strong tidal signals ($f_{day} \approx 11$ μHz). The amplitudes of the suspected GW in the range around 10 mHz are at least a factor of 10^7 weaker than the tides and can only be detected if the recordings of the gravimeters are processed with the lowest possible bandwidth. The reason is given by communications engineering: Each kind of signal transmission requires a certain frequency range, called channel bandwidth.

Too narrow a bandwidth distorts the signal and generates errors. Too much bandwidth allows unwanted frequencies and unnecessary noise to pass through, degrading the signal-to-noise ratio (S/N).

The choice of bandwidth BW determines the average amplitude A_{noise} of the interfering noise after the filter:

$$A_{noise} = \sqrt{PSD \cdot BW} \tag{2}$$

One cannot narrow the bandwidth of the signal processing arbitrarily in order to eliminate the disturbing noise. Because then the necessary recording period T_{min}, which the filter needs to settle down, increases. This relationship was first equationted by Küpfmüller and is reminiscent of the Heisenberg uncertainty principle.

$$T_{min} \cdot BW \geq 0.5 \tag{3}$$

If one looks for weak GW signals in the recordings of gravimeters, this means: If one wants to reduce the amplitude of the interfering noise to $10^{-14}\ \frac{m}{s^2}$, the bandwidth of the filters must not exceed 1 nHz (equation (2)). Because of (3), gravimeters must be operated for at least 15 years and the frequency of the GW must not vary by more than 0.5 nHz during the entire period to keep the signal within the filter range. Some gravimeters have been recording data for more than 20 years [9].

6 Phase modulation in annual rhythm

In section 2 it was stated that the detection of a PM with $f_{year} = 31.688$ nHz is the primary characteristic of a GW. It would be surprising if any signal generated in our solar system had this signature. Any modulation produces sidebands that occupy what is called the channel bandwidth (Fig 2). If one switches off the modulation, the sidebands also disappear. For PM with a *single* modulation frequency, the spectrum consists of many spectral lines symmetric about the central frequency f_{GW}. The mutual frequency spacing is as large as the modulation frequency f_{year}, the amplitudes are defined by the modulation index η and the Bessel function of the first kind. The definition of the modulation index η is:

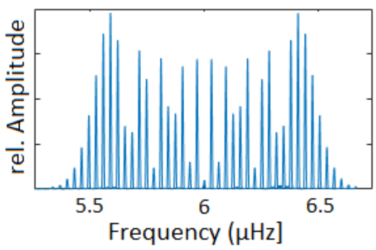

Figure 2): *Das Spektrum einer phasenmodulierten Schwingung mit* $\eta = 10$ *füllt ein breites Band. Die Modulationsfrequenz bestimmt den Abstand der Linien.*

$$\eta = \frac{\textit{max. Frequency deviation from the average frequency}}{\textit{Modulation frequency}} = \frac{\Delta f}{f_{year}} \tag{4}$$

The maximum frequency deviation Δf results from the relativistic Doppler effect due to the orbit around the sun. In the case of HM Cancri, the GW source is nearly in the plane of the ecliptic and the maximum frequency deviation Δf can be calculated from the orbital velocity of the Earth ($v_{orbit} \approx 30$ km/s).

$$\Delta f \approx f_{GW} \cdot \left(\sqrt{\frac{c + v_{orbit}}{c - v_{orbit}}} - 1 \right) = 622 nHz \tag{5}$$

The modulation index η_{year} reaches the surprisingly high value 20 and the corresponding spectrum claims the Carson bandwidth of 1.4 μHz. The entire region is filled with $\frac{1400\ nHz}{31.69\ nHz} \approx 44$ closely spaced spectral lines. It does not seem very promising to search in the noise for a set of 44 spectral lines with unknown amplitude distribution.

7 Phase modulation in daily rhythm

All gravimeters orbit the Earth's axis daily, which is why the receiving frequency is phase modulated with the Earth's rotational frequency (f_{day} =11.574 μHz). Because of the small circumferential velocity at the equator of only 463 $\frac{m}{s}$, the maximum frequency deviation at $f_{GW} = 6220$ μHz is only

$$\Delta f = f_{GW} \cdot \left(\sqrt{\frac{c + v_{equator}}{c - v_{equator}}} - 1 \right) = 9.6\ nHz \tag{6}$$

This value is considerably smaller than f_{day} and provides for the extremely small modulation index

$$\eta_{day} = \frac{\Delta f}{f_{day}} = 8.3 \times 10^{-4} \tag{7}$$

The amplitude of the two sideband frequencies are smaller than the amplitude of the GW by a factor of 2410 and can be seen in the spectrum only at extremely high S/N $> 6 \times 10^6$. No matter how small η becomes, the sideband frequencies remain constant at $f_{GW} \pm f_{day}$. It is a mistake to assume that the position and amplitude of these spectral lines can be changed by other integration time and/or spectral resolution.

8 The reception of an idealized GW

Let us assume that a binary star system generates a GW of constant frequency and the distance to the Earth remains constant. When the GW passes the Earth, the diameter L oscillates in the same rhythm with the maximum amplitude ΔL. The strain h is calculated with the approach

$$h = \Delta L/L = h_0 \cdot sin(\omega t) \tag{8}$$

How large is h_0? Previous estimates give values between 10^{-25} for fast rotating pulsars and 10^{-19} for binary systems in our Galaxy. Very close binary star systems, which have not been noticed by electromagnetic waves so far, could cause even higher strain.

For the following calculations we assume that a close double star system generates a GW of constant frequency $\omega = 0.5\ s^{-1}$ and the strain h_0 here on Earth has the value 10^{-20}. Thus the change of the local gravity $\ddot{L}$ near the Earth surface is

$$\ddot{L} = L \cdot \omega^2 \cdot h_0 = 12.7 \times 10^6\ m \cdot (0.5\ \frac{1}{s})^2 \cdot 10^{-20} \approx 3 \times 10^{-14}\ \frac{m}{s^2} \tag{9}$$

This value exceeds the limit $10^{-14}\ \frac{m}{s^2}$ reached by (from astronomical point of view unfavorably mounted) gravimeters (see section 5). This GW can be detected with gravimeters and appears in the spectrum as a narrow spectral line exceeding the level of background noise (S/N $\approx$ 9). Summing the signal with a selective integrator, the cumulative amplitude increases in proportion to the measurement duration because the amplitude of the GW is constant for years [10].

9 The reception of a real GW

No GW can satisfy these idealized assumptions because, first, the source radiates gravitational energy and therefore increases the orbital frequency. The value of the frequency drift is not known a priori. Secondly, the received frequency oscillates in the annual rhythm (Fig 3) because the Earth moves in the wave field of the GW (Doppler effect).

At $f_{GW} = 0.1$ Hz and low ecliptic latitude of the source, the frequency deviation caused by this phase modulation can reach the maximum value of

$$\Delta f = f_{GW} \cdot \left(\sqrt{\frac{c + v_{Earth}}{c - v_{Earth}}} - 1 \right) \approx 10\ \mu Hz \tag{10}$$

To process an FM signal with this frequency deviation without distortion, the filter bandwidth must be at least twice as wide (Carson rule). This value exceeds the desired bandwidth of 1 nHz (see Section 3) by orders of magnitude and makes it unlikely to detect a GW. There are several reasons for this:

- When the bandwidth increases by a factor of 20,000, the noise level increases by a factor of 141 (equation (2)) and the S/N drops dramatically.
- Approximately 20 μHz / 31.69 nHz $\approx$ 630 closely spaced spectral lines with mutual separation f_{year} = 31.69 nHz fill the entire bandwidth (see Fig 2). The defined phase relationships should not change.

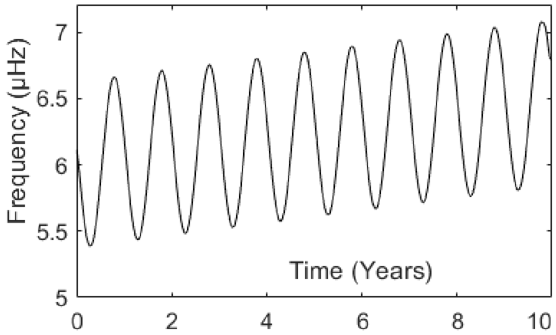

Figure 3): *Example HM Cancri: The image shows the measured frequency wobble and drift of the GW as a function of time. The average value of the GW frequency is 6220 μHz and was reduced to 6 μHz before processing. (Principle of the superhet).*

- The GW transports a certain amount of energy, which is distributed approximately evenly over 630 spectral lines. This reduces the amplitudes of these lines, many of them will disappear in the noise.

- Probably the Milky Way hosts many thousands of GW sources of similar frequency and all of them fill similarly wide Carson bandwidths with their bundle of spectral lines. It is not easy to separate these overlapping spectra. FFT-based methods are hardly suitable for this purpose.

The same problems must be solved for the future LISA observatory; they cannot be eliminated by any modification of the sensors.

Probably the only way out: The signal processing must eliminate both the phase modulation and the frequency drift in order to be able to reduce the receiver bandwidth to $\sim 10^{-9}$ Hz. This measure increases the signal level of the central spectral line and lowers the noise level enough to identify weak GWs. In addition, one can focus on a single spectral line instead of a bundle of many hundreds of lines.

The *modified superhet method* (MSH) described in [11] removes all modulations of the GW signal (drift and phase modulation) and calculates properties of the GW source.

10 Summary

The test body *Earth* is sufficiently large and gravimeters are sensitive enough to measure GW in the frequency range 10^{-5} Hz to 1 Hz despite unfavorable mounting. From equation (9) it follows that the probability of detecting GW in gravimeter readings improves with increasing frequency. The MSH method is a suitable algorithm to remove all interfering modulations of a GW and to reduce the bandwidth to such an extent that the GWs of many sources exceed the critical threshold value S/N $>$ 1. The MSH method also allows to determine the direction in which to look for the GW source.

References

[1] Hartmann and Wenzel, Tidal potential catalogue, (1995)

[2] Nishida, K., Global propagation of body waves revealed by cross-correlation analysis of seismic hum, Geophys. Res. Lett., 40, 1691–1696, doi:10.1002/grl.50269, (2013)

[3] Abbot, B. et.al., The basic physics of the binary black hole merger GW150914, https://onlinelibrary.wiley.com/doi/10.1002/andp.201600209 (2016)

[4] Schaldenbrand, P., What is a Power Spectral Density (PSD)?, https://community.sw.siemens.com/s/article/what-is-a-power-spectral-density-psd (2020)

[5] Banka, D., Crossley, D., Noise levels of superconducting gravimeters at seismic frequencies, Geophys. J. Int. (1999) 139, 87-97

[6] Kimura, M. et.al., Earthquake-induced prompt gravity signals identified in dense array data in Japan, (2019) https://earth-planets-space.springeropen.com/articles/10.1186/s40623-019-1006-x

[7] Bormann, P., Wielandt, E., Seismic Signals and Noise, figure 4.5, DOI: 10.2312/GFZ.NMSOP-2_ch4, (2013)

[8] Bailes, M. et.al., Gravitational-wave physics and astronomy in the 2020s and 2030s, Nature 344, https://www.nature.com/articles/s42254-021-00303-8.pdf (2021)

[9] IGETS Datenbank, GFZ Potsdam, http://isdc.gfz-potsdam.de/igets-data-base/ (2021)

[10] Weidner, H., Measurement of a Continuous Gravitational Wave near 2619.9 µHz, https://vixra.org/abs/2203.0130 (2022)

[11] Weidner, H., Detection of the continuous GW of HM Cancri, https://vixra.org/abs/2201.0172 (2022)