# *8 Dimensions and Matrix comparison by definition*

March 24, 2022

Yuji Masuda

y_masuda0208@yahoo.co.jp

## Abstract

The purpose of this short paper is a matrix comparison by definition of the most recent previous short paper

## General comments

From “Reconstruction Proofs by Definition” of my No.67, I define all numbers as 5 numbers.

①$\log\left(-\frac{\pi}{2}\right)=\log e=1$

②$\log 1=0$

③$\log 0=\log\left(\frac{1}{\pm\infty}\right)=\log\left(e^{-1}\right)=\log(-e)=\log\left(\frac{\pi}{2}\right)=-1$

④$\log(-1)=i\pi=-2$

①$\log(-2)=\log\left(-\frac{\pi}{2}\right)=\log e=1$

②⇒③⇒④⇒①⇒②⇒③⇒④⇒①⇒⋯

- 2    -1    0 = 2    1

$\ln(0)=\ln(\frac{1}{\pm\infty})=\ln(\frac{1}{e})=\ln(e^{-1})=-1$

$\ln(1)=\ln(-e^{2})=\ln(-1)+2=i\pi+2=-2+2=0$

$\ln(2)=\ln(-e)=\ln(-1)+1=i\pi+1=-2+1=-1$

$\ln(3)=\ln(-2)=\ln(e)=1$

$\ln(4)=\ln(-1)=i\pi=-2$

$\ln(5)=\ln(0)=-1$

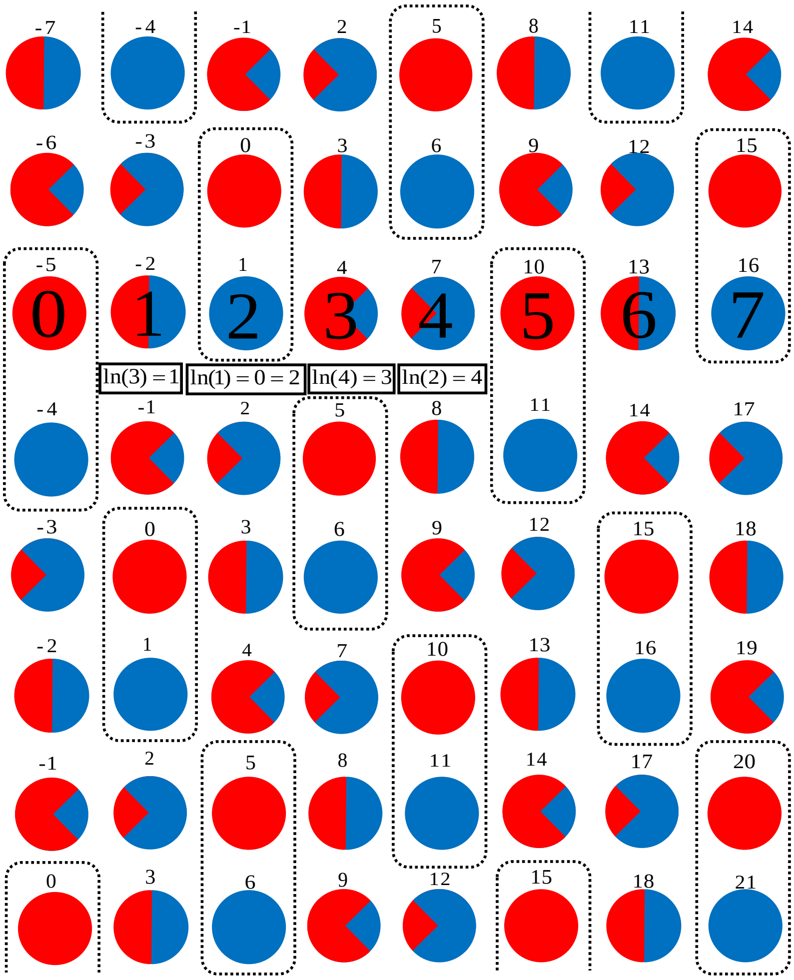
$\ln(6)=\ln(1)=0$

$\ln(7)=\ln(2)=-1$

$\ln(8)=\ln(3)=1$

$\ln(9)=\ln(4)=-2$

⋮



$$\text{Orthogonalize}\left[\begin{pmatrix} -7 & -4 & -1 & 2 & 5 & 8 & 11 & 14 \\ -6 & -3 & 0 & 3 & 6 & 9 & 12 & 15 \\ -5 & -2 & 1 & 4 & 7 & 10 & 13 & 16 \\ -4 & -1 & 2 & 5 & 8 & 11 & 14 & 17 \\ -3 & 0 & 3 & 6 & 9 & 12 & 15 & 18 \\ -2 & 1 & 4 & 7 & 10 & 13 & 16 & 19 \\ -1 & 2 & 5 & 8 & 11 & 14 & 17 & 20 \\ 0 & 3 & 6 & 9 & 12 & 15 & 18 & 21 \end{pmatrix}\right]$$

$$\text{Orthogonalize}\left[\begin{pmatrix} 3 & 1 & 4 & 2 & 5 & 3 & 1 & 4 \\ 4 & 2 & 5 & 3 & 1 & 4 & 2 & 5 \\ 5 & 3 & 1 & 4 & 2 & 5 & 3 & 1 \\ 1 & 4 & 2 & 5 & 3 & 1 & 4 & 2 \\ 2 & 5 & 3 & 1 & 4 & 2 & 5 & 3 \\ 3 & 1 & 4 & 2 & 5 & 3 & 1 & 4 \\ 4 & 2 & 5 & 3 & 1 & 4 & 2 & 5 \\ 5 & 3 & 1 & 4 & 2 & 5 & 3 & 1 \end{pmatrix}\right]$$

outcome

$$\begin{pmatrix} -0.320844 & -0.18334 & -0.0458349 & 0.0916698 & 0.229175 & 0.366679 & 0.504184 & 0.641689 \\ 0.560112 & 0.490098 & 0.420084 & 0.35007 & 0.280056 & 0.210042 & 0.140028 & 0.070014 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0.333333 & 0.111111 & 0.444444 & 0.222222 & 0.555556 & 0.333333 & 0.111111 & 0.444444 \\ 0.224201 & 0.213893 & 0.229355 & 0.219047 & -0.809184 & 0.224201 & 0.213893 & 0.229355 \\ 0.349282 & 0.272298 & -0.495648 & 0.31079 & 0.140663 & 0.349282 & 0.272298 & -0.495648 \\ -0.464605 & 0.414052 & 0.0585771 & 0.452569 & 0.117154 & -0.464605 & 0.414052 & 0.0585771 \\ 0.0276924 & 0.443079 & 0.0276924 & -0.775388 & 0.0553849 & 0.0276924 & 0.443079 & 0.0276924 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

dimension

$8$ $\qquad$ $8$

extra- dimension

$0$ $\qquad$ $0$

summand matrix

$$\begin{pmatrix} 1 & 0 & -1 & -2 & -3 & -4 & -5 & -6 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

eigenpolynomial

$$\lambda^8 - 0.169254\lambda^7 - 0.0545545\lambda^6$$

$$\lambda^8 - 0.559531\lambda^7 - 0.0331659\lambda^6 - 0.330446\lambda^5 - 0.21606\lambda^4 + 0.353553\lambda^3$$

eigenvalue

$$\lambda_1 = \left(103\,365\,606\,275\,727\,434\,913\,541 + \sqrt{92\,073\,664\,384\,311\,926\,058\,320\,390\,392\,514\,095\,807\,502\,673\,881}\right) / 1\,221\,429\,068\,369\,631\,142\,233\,944$$

$$\lambda_2 = \left(103\,365\,606\,275\,727\,434\,913\,541 - \sqrt{92\,073\,664\,384\,311\,926\,058\,320\,390\,392\,514\,095\,807\,502\,673\,881}\right) / 1\,221\,429\,068\,369\,631\,142\,233\,944$$

$$\lambda^8 = \lambda^7 = \lambda^6 = \lambda^5 = \lambda^4 = \lambda^3 = 0$$

$\lambda_1 \approx -0.150528 + 0.858943\,i$ $\quad$ $\lambda_2 \approx -0.150528 - 0.858943\,i$

$\lambda_3 \approx 0.786528 + 0.184222\,i$ $\quad$ $\lambda_4 \approx 0.786528 - 0.184222\,i$

$\lambda_5 \approx -0.712469$ $\quad$ $\lambda^8 = \lambda^7 = \lambda^6 = 0$

eigenvector

$$v_1 = \left(\left(-495\,254\,373\,196\,080\,734\,760\,859 + \sqrt{92\,073\,664\,384\,311\,926\,058\,320\,390\,392\,514\,095\,807\,502\,673\,881}\right) / 684\,137\,119\,396\,352\,193\,913\,600,\ 1,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\right)$$

$$v_2 = \left(\left(-495\,254\,373\,196\,080\,734\,760\,859 - \sqrt{92\,073\,664\,384\,311\,926\,058\,320\,390\,392\,514\,095\,807\,502\,673\,881}\right) / 684\,137\,119\,396\,352\,193\,913\,600,\ 1,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\right)$$

$$v_3 = \left(\frac{6\,235\,269\,209\,245\,382\,619\,797\,136}{1\,039\,211\,534\,874\,230\,264\,472\,805},\ -\frac{7\,274\,480\,744\,119\,613\,031\,835\,699}{1\,039\,211\,534\,874\,230\,264\,472\,805},\ 0,\ 0,\ 0,\ 0,\ 0,\ 1\right)$$

$$v_4 = \left(\frac{655\,334\,025\,543\,686\,448\,564\,678}{131\,066\,805\,108\,737\,252\,539\,213},\ -\frac{786\,400\,830\,652\,423\,727\,656\,550}{131\,066\,805\,108\,737\,252\,539\,213},\ 0,\ 0,\ 0,\ 0,\ 1,\ 0\right)$$

$$v_5 = \left(\frac{5\,701\,132\,961\,645\,378\,565\,962\,908}{1\,425\,283\,240\,411\,344\,037\,518\,113},\ -\frac{7\,126\,416\,202\,056\,722\,948\,608\,229}{1\,425\,283\,240\,411\,344\,037\,518\,113},\ 0,\ 0,\ 0,\ 1,\ 0,\ 0\right)$$

$$v_6 = \left(\frac{4\,285\,720\,690\,704\,893\,523\,946\,652}{1\,428\,573\,563\,568\,297\,185\,600\,759},\ -\frac{5\,714\,294\,254\,273\,190\,990\,568\,036}{1\,428\,573\,563\,568\,297\,185\,600\,759},\ 0,\ 0,\ 1,\ 0,\ 0,\ 0\right)$$

$$v_7 = \left(\frac{2\,620\,243\,859\,835\,970\,227\,554\,164}{1\,310\,121\,929\,917\,983\,854\,625\,503},\ -\frac{3\,930\,365\,789\,753\,954\,441\,937\,261}{1\,310\,121\,929\,917\,983\,854\,625\,503},\ 0,\ 1,\ 0,\ 0,\ 0,\ 0\right)$$

$$v_8 = \left(\frac{1\,244\,315\,466\,778\,921\,215\,926\,816}{1\,244\,315\,466\,778\,920\,892\,972\,583},\ -\frac{2\,488\,630\,933\,557\,842\,155\,035\,718}{1\,244\,315\,466\,778\,920\,892\,972\,583},\ 1,\ 0,\ 0,\ 0,\ 0,\ 0\right)$$

$v_1 \approx (-0.388155 - 0.489906\,i,\ 0.0809548 + 0.946574\,i,\ -0.0489623 - 0.158437\,i,\ 0.29621 - 0.590014\,i,\ 1,\ 0,\ 0,\ 0)$

$v_2 \approx (-0.388155 + 0.489906\,i,\ 0.0809548 - 0.946574\,i,\ -0.0489623 + 0.158437\,i,\ 0.29621 + 0.590014\,i,\ 1,\ 0,\ 0,\ 0)$

$v_3 \approx (0.702739 + 1.81025\,i,\ -1.20273 + 1.93553\,i,\ -0.177366 + 1.16594\,i,\ -1.61145 + 0.974721\,i,\ 1,\ 0,\ 0,\ 0)$

$v_4 \approx (0.702739 - 1.81025\,i,\ -1.20273 - 1.93553\,i,\ -0.177366 - 1.16594\,i,\ -1.61145 - 0.974721\,i,\ 1,\ 0,\ 0,\ 0)$

$v_5 \approx (7.32419,\ 3.61052,\ -20.6752,\ 2.57662,\ 1,\ 0,\ 0,\ 0)$

$v_6 = (0,\ 0,\ -1,\ 0,\ 0,\ 0,\ 0,\ 1)$

$v_7 = (0,\ -1,\ 0,\ 0,\ 0,\ 0,\ 1,\ 0)$

$v_8 = (-1,\ 0,\ 0,\ 0,\ 0,\ 1,\ 0,\ 0)$

$$\text{Orthogonalize}\left[\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{pmatrix}\right] \qquad \text{Orthogonalize}\left[\begin{pmatrix} -1 & -2 & 2 & 1 & -2 & -1 & -2 & 2 \\ 2 & 1 & -2 & -1 & -2 & 2 & 1 & -2 \\ -2 & -1 & -2 & 2 & 1 & -2 & -1 & -2 \\ -2 & 2 & 1 & -2 & -1 & -2 & 2 & 1 \\ 1 & -2 & -1 & -2 & 2 & 1 & -2 & -1 \\ -1 & -2 & 2 & 1 & -2 & -1 & -2 & 2 \\ 2 & 1 & -2 & -1 & -2 & 2 & 1 & -2 \\ -2 & -1 & -2 & 2 & 1 & -2 & -1 & -2 \end{pmatrix}\right]$$

**outcome**

$$\begin{pmatrix} 0.070014 & 0.140028 & 0.210042 & 0.280056 & 0.35007 & 0.420084 & 0.490098 & 0.560112 \\ 0.641689 & 0.504184 & 0.366679 & 0.229175 & 0.0916698 & -0.0458349 & -0.18334 & -0.320844 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} -0.208514 & -0.417029 & 0.417029 & 0.208514 & -0.417029 & -0.208514 & -0.417029 & 0.417029 \\ 0.362659 & -0.032969 & -0.219793 & -0.109897 & -0.791257 & 0.362659 & -0.032969 & -0.219793 \\ -0.318701 & -0.230783 & -0.512851 & 0.402955 & -0.0439587 & -0.318701 & -0.230783 & -0.512851 \\ -0.468546 & 0.366429 & 0.00926147 & -0.343396 & -0.417488 & -0.468546 & 0.366429 & 0.00926147 \\ -0.0624064 & -0.370768 & -0.121142 & -0.814955 & 0.154181 & -0.0624064 & -0.370768 & -0.121142 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

**dimension** $8$ $\qquad$ $8$

**extra-dimension** $0$ $\qquad$ $0$

**summand matrix**

$$\begin{pmatrix} 1 & 0 & -1 & -2 & -3 & -4 & -5 & -6 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

**eigenpolynomial**

$$\lambda^8 - 0.574198\lambda^7 - 0.0545545\lambda^6$$

$$\lambda^8 - 0.94355\lambda^7 - 0.076792\lambda^6 - 0.342197\lambda^5 - 0.600293\lambda^4 + 0.353553\lambda^3$$

**eigenvalue**

$$\lambda_1 = \left(17\,248\,044\,055\,154\,440\,916\,003 + \sqrt{494\,395\,351\,312\,504\,405\,267\,511\,492\,620\,141\,096\,054\,056\,169}\right) \Big/ 60\,076\,972\,477\,483\,579\,442\,708$$

$$\lambda_2 = \left(17\,248\,044\,055\,154\,440\,916\,003 - \sqrt{494\,395\,351\,312\,504\,405\,267\,511\,492\,620\,141\,096\,054\,056\,169}\right) \Big/ 60\,076\,972\,477\,483\,579\,442\,708$$

$$\lambda^8 = \lambda^7 = \lambda^6 = \lambda^5 = \lambda^4 = \lambda^3 = 0$$

$\lambda_1 \approx 0.90373$ $\qquad$ $\lambda_2 \approx -0.880596$

$\lambda_3 \approx -0.10325 + 0.757465\,i$ $\qquad$ $\lambda_4 \approx -0.10325 - 0.757465\,i$

$\lambda_5 \approx -0.760185$ $\qquad$ $\lambda^8 = \lambda^7 = \lambda^6 = 0$

**eigenvector**

$$v_1 = \left(\left(-65\,209\,073\,258\,542\,424\,058\,015 + 5\sqrt{494\,395\,351\,312\,504\,405\,267\,511\,492\,620\,141\,096\,054\,056\,169}\right) \Big/ 192\,753\,646\,316\,400\,418\,273\,856,\ 1, 0, 0, 0, 0, 0, 0\right)$$

$$v_2 = \left(\left(-65\,209\,073\,258\,542\,424\,058\,015 - 5\sqrt{494\,395\,351\,312\,504\,405\,267\,511\,492\,620\,141\,096\,054\,056\,169}\right) \Big/ 192\,753\,646\,316\,400\,418\,273\,856,\ 1, 0, 0, 0, 0, 0, 0\right)$$

$$v_3 = \left(\frac{296\,976\,506\,325\,642\,689\,389\,625}{49\,496\,084\,387\,607\,105\,557\,793},\ -\frac{692\,945\,181\,426\,499\,533\,851\,969}{98\,992\,168\,775\,214\,211\,115\,586}, 0, 0, 0, 0, 0, 1\right)$$

$$v_4 = \left(\frac{187\,989\,294\,992\,639\,497\,947\,995}{37\,597\,858\,998\,527\,891\,986\,659},\ -\frac{225\,587\,153\,991\,167\,370\,927\,304}{37\,597\,858\,998\,527\,891\,986\,659}, 0, 0, 0, 0, 1, 0\right)$$

$$v_5 = \left(\frac{1\,224\,053\,168\,323\,024\,984\,756\,030}{306\,013\,292\,080\,756\,183\,781\,451},\ -\frac{1\,530\,066\,460\,403\,781\,043\,722\,368}{306\,013\,292\,080\,756\,183\,781\,451}, 0, 0, 0, 1, 0, 0\right)$$

$$v_6 = \left(\frac{966\,591\,034\,601\,260\,888\,222\,685}{322\,197\,011\,533\,753\,605\,830\,691},\ -\frac{1\,288\,788\,046\,135\,014\,458\,688\,070}{322\,197\,011\,533\,753\,605\,830\,691}, 0, 0, 1, 0, 0, 0\right)$$

$$v_7 = \left(\frac{702\,655\,413\,098\,298\,020\,315\,560}{351\,327\,706\,549\,148\,965\,519\,323},\ -\frac{1\,053\,983\,119\,647\,446\,941\,196\,426}{351\,327\,706\,549\,148\,965\,519\,323}, 0, 1, 0, 0, 0, 0\right)$$

$$v_8 = \left(\frac{31\,865\,320\,052\,409\,009\,923\,815}{31\,865\,320\,052\,409\,008\,583\,351},\ -\frac{63\,730\,640\,104\,818\,017\,836\,934}{31\,865\,320\,052\,409\,008\,583\,351}, 1, 0, 0, 0, 0, 0\right)$$

$v_1 \approx (-0.159157, -0.843474, -0.00653499, -0.522842, 1, 0, 0, 0)$

$v_2 \approx (0.669272, 0.846187, 0.379009, 0.777168, 1, 0, 0, 0)$

$v_3 \approx (-2.1551 + 2.02137\,i, 1.66501 + 1.47205\,i, -1.52554 - 1.13985\,i, -0.0498242 - 1.58452\,i, 1, 0, 0, 0)$

$v_4 \approx (-2.1551 - 2.02137\,i, 1.66501 - 1.47205\,i, -1.52554 + 1.13985\,i, -0.0498242 + 1.58452\,i, 1, 0, 0, 0)$

$v_5 \approx (0.421604, 1.44962, 1.81144, 0.16092, 1, 0, 0, 0)$

$v_6 = (0, 0, -1, 0, 0, 0, 0, 1)$

$v_7 = (0, -1, 0, 0, 0, 0, 1, 0)$

$v_8 = (-1, 0, 0, 0, 0, 1, 0, 0)$