

## **Further Thoughts on the Thermodynamics of Antimatter via Santilli's Isodualities.**

Jeremy Dunning-Davies,  
Departments of Mathematics and Physics (retd),  
University of Hull, England;  
Institute for Basic Research, Palm Harbor, Florida, USA.  
[masjd@masjd.karoo.co.uk](mailto:masjd@masjd.karoo.co.uk)

### **Abstract.**

The property of isoselfduality has been shown previously to pervade much of thermodynamics. Here it is shown that this property does not extend to partial derivatives of the well-known basic thermodynamic functions such as internal energy. This raises several issues which will need to be borne in mind in future for those considering the thermodynamic properties of antimatter via Santilli's isodualities.

### **Introduction.**

In an earlier publication<sup>1</sup>, it was shown that many of the well-known basic results of thermodynamics are isoselfdual. Included in this were all the fundamental equations representing the First and Second Laws, that representing a combination of those two laws (even when extended to apply to open as well as closed systems), the Gibbs-Duhem relation, and others. However, some relations may be written in several different forms and some alternative forms depend on partial derivatives. These forms have not, as yet, been examined separately. It is the purpose of this note to remedy that failing.

### **Discussion and Conclusion.**

Consider, as a typical example, the internal energy  $U$ .

$$dU \rightarrow d^d U^d = -d(-U) = dU$$

so that the total derivative  $dU$  is isoselfdual.

However, the internal energy  $U$  is a dependent variable and if  $U$  is taken to be a function of temperature  $T$  and volume  $V$ , then that total derivative may be written

$$dU = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV \quad (1)$$

and, for consistency, it follows that the right-hand side of this equation must surely be isoselfdual also but is that so?

It is worth noting at this point that it is customary in thermodynamics to use the notation adopted here for partial derivatives and this entails noting clearly which variables are being held constant during the indicated operation. However, the actual operation being discussed is differentiation and it is only the manner of carrying that out which is being examined here. Incidentally, if the internal energy was taken to be a function of the number of particles as well as on temperature and volume, the equation for  $dU$  would become

$$dU = \left(\frac{\partial U}{\partial T}\right)_{V,N} dT + \left(\frac{\partial U}{\partial V}\right)_{T,N} dV + \left(\frac{\partial U}{\partial N}\right)_{T,V} dN \quad (2)$$

and, in each partial derivative, two variables would be being held constant but the operation of so doing would be just one operation – that of keeping all variables constant except the one with respect to which the derivative was being *taken*.

Consider, as an example, the first term on the right-hand side of equation (1). Under Santilli's isodualities

$$dT \rightarrow d^d T^d = dT,$$

while the actual partial derivative will change as follows

$$\partial U / \partial T \rightarrow (\partial^d U^d / \partial^d T^d) = -\partial U / \partial T,$$

but this still leaves open the question of the action of keeping a variable, in this case  $V$ , constant during the differentiation. However, if it is noted that, when considering antimatter via Santilli's isodualities, the dependent variables, in this case  $V$  and  $T$ , become  $-V$  and  $-T$  and so, for the case under consideration here, the action of holding  $V$  constant simply becomes holding  $-V$  constant. This holds true if more than one variable is to be held constant, as in equation (2), as well. Of course, holding a variable constant and holding its negative constant are the same and so it should come as no surprise that this action has no immediately observable effect on expressions when considering antimatter via Santilli's isodualities. In

truth, the action of holding one or more variables constant while differentiating partially is simply part of the single process of partial differentiation. This observation, together with the knowledge that the partial derivative is *multiplied* by  $dT$ , leads finally to the result that, under Santilli's isodualities, terms such as those on the right-hand sides of equations (1) and (2) remain unchanged and so, consistency is upheld. However, the actual partial derivatives themselves do not remain unchanged but suffer a sign change and this has consequences for thermodynamics where so many functions are defined as partial derivatives.

An important point to note from this discussion concerns the well-known heat capacities. For example, the constant volume heat capacity is defined by

$$\left(\frac{\partial Q}{\partial T}\right)_V = \left(\frac{\partial U}{\partial T}\right)_V,$$

where  $Q$  represents heat and it might be seen that the second partial derivative here appears in equation (1) above and has been clearly shown not to be invariant under Santilli's isodualities. Hence, the heat capacities for antimatter as discussed under Santilli's theory are negative, although it must be remembered that these values are referred to negative units of heat capacity.

This discussion concerning partial derivatives also gives a much firmer footing to an assertion in the earlier article<sup>1</sup> that the important Maxwell thermodynamic relations are isoselfdual. This follows because each of the said relations is the equality of two partial derivatives and the discussion here merely introduces a negative sign to each side of that equality, thereby not altering the actual equality itself.

It is worth noting that, because of the notation used for partial derivatives, thermodynamics has a slight advantage over other areas in that it always states explicitly which variables are being held constant and hence its notation draws immediate attention to the fact that, in all partial differentiation, the action of keeping one or more variables constant is present. This seemingly smallish point is important because it is known the basic thermodynamic equations, representing the important First and Second Laws, are isoselfdual and so, if the above breakdown of  $dU$  is necessary in an example, that isoselfduality might be expected to remain. The above is reassuring on this point.

Here attention has been drawn to pure partial derivatives and functions, such as heat capacity, which depend only on such derivatives but other thermodynamic functions such as, for example, the coefficient of volume expansion at constant volume as given by

$$\frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_p,$$

depend on variables as well as partial derivatives, in this case the reciprocal of the volume. However, it is simple to see that, provided one notes that the function is  $\frac{1}{V}$  multiplied by the partial derivative, this function too changes under Santilli's isodualities. This argument applies equally well to such functions as the isothermal bulk modulus of elasticity and, therefore, the isothermal compressibility..

Although it seemed likely at one point<sup>1</sup> that all thermodynamic functions and, indeed, all of thermodynamics are isoselfdual, that has been shown here not to be the case. In fact, it has been shown here via illustrations that it seems sensible to check behaviour under Santilli isodualities in each case when any new thermodynamic function is encountered.

#### **Reference.**

1. Dunning-Davies, J., 1999, *Hadronic Journal*, **22**, 607-615
2. Santilli, R. M., 2006, *Isodual Theory of Antimatter*, Springer, Dordrecht.