Ordinary scalars will undergo topological phase transitions under $f(R,\phi)$

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In this article, we make a simulation, when the boundary conditions are preset, the ratio of the temperature of the two systems is a complex number, which is consistent with $f(R,\phi)$ theory, then a new solution (prediction) appears, ordinary scalars will undergo topological phase transitions under $f(R,\phi)$.

Keywords: topological phase transitions, $f(R,\phi)$,temperature

1. INTRODUCTION

Since the discovery of the quantum Hall effect, many topological phases have been predicted theoretically and verified experimentally, however, direct measurement of topological numbers experimentally remains a challenge.

Topology: Topology is a branch of mathematics. Its main research content is the properties of geometric shapes that do not change in continuous deformation. For example, a teapot with a handle continuously changes into a tire instead of a ball.

Phase transition: A phase transition is a process in which a substance suddenly changes from one phase to another phase when the external conditions change continuously, such as melting ice into water.

The most common phases in daily life are gaseous, liquid and solid. Under some extreme conditions, such as extremely high or extremely low temperatures, many more exotic states will appear. The phase transitions that people see are the result of molecules changing together at the microscopic level. For example, on the macroscopic level, ice melts into water and then evaporates into water vapor. On the microscopic level, the molecules and molecules are firstly arranged neatly like a phalanx, and on the macroscopic level, the state of ice is shown. When the temperature rises, the soldiers move freely in the vicinity, no longer neatly lined up, but still next to each other, showing the form of water on a macro level; when the temperature rises again, the soldiers move completely freely, showing the appearance of water. The state of water vapor.

And David Solis and Michael Kosterlitz also proposed the BKT phase transition (Berezinskii–Kosterlitz–Thouless transition), which is microscopically like this: a group of soldiers circles several officers. In order to keep going, there must be a group of soldiers going clockwise, and there must be a group of soldiers going counterclockwise. In the beginning, each anti-clockwise officer was paired with a clockwise officer, and the soldiers led by each pair of clockwise/counter-clockwise officers would only complement each other; later each pair of officers was separated and moved at will , and the soldiers they lead are no longer giving only to each other, but all others, so the topology changes, resulting in a phase transition. Unlike water, however, the BKT phase transition describes matter in two dimensions.

David Solis, Duncan Haldane and Michael Kosterlitz used advanced mathematics to study some special phases or states of matter, such as superconductors, superfluids, and magnetic thin films. Their bold application of topological concepts to physics played a decisive role in their later discoveries. Topology is a branch of mathematics that is often used to describe some gradually changing properties. In the early 1970s, Michael Kosterlitz and David Solis overturned the theory at the time that superconductivity and superfluidity could not arise in thin layers. They demonstrated that superconductivity can occur at low temperatures, and explained the mechanism by which superconductivity can also occur at higher temperatures—phase transitions. Then, in the 1980s, Solis successfully explained an earlier experiment in which conductance in ultrathin conducting layers could be measured precisely to whole numbers. He proved that these integers are in a topological state in natural properties. Meanwhile, Duncan Haldane discovered that topology could be used to understand the properties of chains of small magnets in certain materials.

We can immediately integrate the Ricci scalar $f_R(r)$ to obtain the general form of $f(R,\phi)$ theory[4]

$$f_R(r) = 1 + \alpha r \to f(R) = R + \alpha \int^R r(R)dR + C, \tag{1}$$

Where C is an integral constant, the unit is $[L]^{-2}$, which is related to the cosmological constant. This expression shows that, in addition to the Einstein-Hilbert term, a geometric correction term appears, while there is no immediate scalar field in the $f(R)$ model.

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2. TEMPERATURE SOLUTION UNDER $F(R,\phi)$ THEORY

We make a hypothesis:Thermodynamic phase transition. - The state equation of a charged AdS black hole displays a vdW-like thermodynamic behavior. The SBH-LBH coexistence curve has a parametric form [1]

$$\frac{P}{P_c} = \sum_i a_i \left(\frac{T}{T_c}\right)^i . \tag{2}$$

The concept of entropy was proposed by the German physicist Clausius in 1865. Kjeldahl defines the increase and decrease of entropy in a thermodynamic system: the total amount of heat used at a constant temperature in a reversible process, and can be expressed as:

$$\Delta S = \frac{\Delta Q}{T} \tag{3}$$

if$\frac{T}{T_c} = z$,when z is plural. The Laurent series of the function f(z) with respect to point c is given by:

$$f(z) = \sum_{n=-\infty}^{\infty} a_n (z-c)^n \tag{4}$$

It is defined by the following curve integral, which is a generalization of the Cauchy integral formula:

$$a_n = \frac{1}{2\pi i} \oint_\gamma \frac{f(z)dz}{(z-c)^{n+1}} \tag{5}$$

Since the algebra of real quaternions is the only fourdimensional division algebra, we introduce the fourdimensional quaternion manifold,[2]

$$\tau^4 = (\hat{\tau}_0, \vec{\tau}_1, \vec{\tau}_2, \vec{\tau}_3) = (\hat{\imath}_0\tau_0, \vec{\imath}_1\tau_1, \vec{\imath}_2\tau_2, \vec{\imath}_3\tau_3) \tag{6}$$

$$\begin{cases} \hat{\imath}_0\hat{\imath}_0 = \hat{\imath}_0 = 1 \\ \vec{\imath}_1\vec{\imath}_1 = \vec{\imath}_2\vec{\imath}_2 = \vec{\imath}_3\vec{\imath}_3 = \vec{\imath}_1\vec{\imath}_2\vec{v}_3 = -\hat{\imath}_0 = -1 \\ \overrightarrow{\imath_1}\overrightarrow{\imath_2} = \overrightarrow{\imath_3}, \quad \overrightarrow{\imath_2}\overrightarrow{\imath_3} = \overrightarrow{\imath_1}, \quad \overrightarrow{\imath_3}\overrightarrow{\imath_1} = \overrightarrow{\imath_2}, \\ \overrightarrow{\imath_2}\overrightarrow{\imath_1} = -\overrightarrow{\imath_3}, \quad \overrightarrow{\imath_3}\overrightarrow{\imath_2} = -\vec{\imath}_1, \quad \vec{\imath}_1\overrightarrow{\imath_3} = -\overrightarrow{\imath_2} \end{cases} \tag{7}$$

$$\boldsymbol{t} = \left(\hat{\imath}_0 t_0, \vec{\imath}_1\frac{x_1}{c}, \vec{\imath}_2\frac{x_2}{c}, \vec{\imath}_3\frac{x_3}{c}\right) \tag{8}$$

$$\begin{cases} t = t\left(\frac{t_0}{t}, \frac{\vec{v}}{c}\right) = t(\cos\theta, \vec{\imath}\sin\theta) = t\exp(\vec{\imath}\theta) \\ \bar{t} = t\left(\frac{t_0}{t}, -\frac{\vec{v}}{c}\right) = t(\cos\theta, -\vec{\imath}\sin\theta) = t\exp(-\vec{\imath}\theta) \end{cases} \tag{9}$$

$$\begin{cases} t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}\exp(\vec{\imath}\theta) \\ \bar{t} = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}\exp(-\vec{\imath}\theta) \end{cases} \tag{10}$$

We can pre-set the boundary conditions $\mu = z\omega$(z is a complex number)[5].

Spherical quantum solution in vacuum state.

In this theory, the general relativity theory's field equation is written completely.

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\frac{8\pi G}{c^4}T_{\mu\nu} \tag{11}$$

The Ricci tensor is by $T_{\mu v}=0$ in vacuum state.

$$R_{\mu v}=0 \tag{12}$$

The proper time of spherical coordinates is

$$d\tau^2=A(t,r)dt^2-\frac{1}{c^2}\left[B(t,r)dr^2+r^2d\theta^2+r^2\ \sin\theta d\phi^2\right] \tag{13}$$

If we use Eq, we obtain the Ricci-tensor equations.

$$R_{tt}=-\frac{A''}{2B}+\frac{A'B'}{4B^2}-\frac{A'}{Br}+\frac{A'^2}{4AB}+\frac{\ddot{B}}{2B}-\frac{\dot{B}^2}{4B^2}-\frac{\dot{A}\dot{B}}{4AB}=0 \tag{14}$$

$$R_{rr}=\frac{A''}{2A}-\frac{A'^2}{4A^2}-\frac{A'B'}{4AB}-\frac{B'}{Br}-\frac{\ddot{B}}{2A}+\frac{\dot{A}\dot{B}}{4A^2}+\frac{\dot{B}^2}{4AB}=0, \tag{15}$$

$$R_{\theta\theta}=-1+\frac{1}{B}-\frac{rB'}{2B^2}+\frac{rA'}{2AB}=0, R_{\phi\phi}=R_{\theta\theta}\sin^2\theta=0, R_{tr}=-\frac{\dot{B}}{Br}=0, R_{t\theta}=R_{t\phi}=R_{r\theta}=R_{r\phi}=R_{\theta\phi}=0 \tag{16}$$

In this time, $\ '=\frac{\partial}{\partial r}\quad, \cdot=\frac{1}{c}\frac{\partial}{\partial t}$,

$$\dot{B}=0 \tag{17}$$

We see that,

$$\frac{R_{tt}}{A}+\frac{R_{rr}}{B}=-\frac{1}{Br}\left(\frac{A'}{A}+\frac{B'}{B}\right)=-\frac{(AB)'}{rAB^2}=0 \tag{18}$$

Hence, we obtain this result.

$$A=\frac{1}{B} \tag{19}$$

If,

$$R_{\theta\theta}=-1+\frac{1}{B}-\frac{rB'}{2B^2}+\frac{rA'}{2AB}=-1+\left(\frac{r}{B}\right)'=0 \tag{20}$$

If we solve the Eq,

$$\frac{r}{B}=r+C\rightarrow\frac{1}{B}=1+\frac{C}{r} \tag{21}$$

When r tends to infinity, and we set C=ze^{-z}, Therefore,

$$A=\frac{1}{B}=1-\frac{z}{r}\Sigma, \Sigma=e^{-z} \tag{22}$$

$$d\tau^2=\left(1-\frac{z}{r}\sum\right)dt^2 \tag{23}$$

In this time, if particles' mass are m_i, the fusion energy is e,

$$E=Mc^2=m_1c^2+m_2c^2+\ldots+m_nc^2+e. \tag{24}$$

3. SCALAR HEAT CAPACITY HAS THE POTENTIAL TO DIVERGE

Using the relations[5] $\Box F = \frac{1}{\sqrt{-g}} \partial_\mu \left[\sqrt{-g} g^{\mu\nu} \partial_\nu F\right]$ and assuming the geometry of a static spherically symmetric black hole. $f(R_0) = 3ze^{-z}/r^3$,

$$\mathcal{T}_1^1 = \frac{1}{F} \left[\frac{1}{2}(f - RF) - \frac{1}{2} g' F' - \frac{2}{r} g F'\right]. \tag{25}$$

We find a special solution, so that the special solution has a potential barrier.

Hawking temperature can be calculated as

$$T_H = \frac{A'(r_h)}{4\pi} \tag{26}$$

which uses the relation $A(r_h) = 0$.

We find the derivative of the energy and momentum with respect to z:

$$X = \frac{d\mathcal{T}_1^1}{dz} = \left\{\frac{1}{2}\left(-z - \frac{e^{z^2}}{-e^z + e^{z^2})}\right)\right\} \tag{27}$$

$$C_q = T(\frac{\partial S}{\partial T})_q \to X = \frac{d\mathcal{T}_1^1}{dz} = \left\{\frac{1}{2}\left(-z - \frac{e^{z^2}}{-e^z + e^{z^2})}\right)\right\} \tag{28}$$

Because z is a complex number, we find that the scalar heat capacity has the potential to diverge.

4. SUMMARY

In this paper, when the preset boundary condition-the ratio of the temperature of the two systems is a complex number, the entropy can be constructed into the ring structure of an algebraic system.The ratio of the temperature of the two systems is a complex number, which is consistent with $f(R, \phi)$ theory, then a new solution (prediction) appears, ordinary scalars will undergo topological phase transitions under $f(R, \phi)$.

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[1] Wei, Shao-Wen, and Yu-Xiao Liu. "Clapeyron equations and fitting formula of the coexistence curve in the extended phase space of charged AdS black holes." Physical Review D 91.4 (2015): 044018.
[2] Ariel, Viktor. "Quaternion Space-Time and Matter." arXiv preprint arXiv:2106.06394 (2021).
[3] Murata K , Soda J . Hawking Radiation from Rotating Black Holes and Gravitational Anomalies[J]. Physical Review D, 2006, 74(4):200-206.
[4] Karakasis, Thanasis, et al. "$(2 + 1)$-Dimensional Black Holes in $f(R, \phi)$ Gravity." arXiv preprint arXiv:2201.00035 (2021).
[5] Chen, Wen-Xiang. "The Schwarzschild black hole in f (R) can exist superradiation phenomenon." arXiv preprint arXiv:2202.01060 (2021).