Alice meets Bob! or: The association of infinity and finiteness within the Schwarzschild metric

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The Schwarzschild metric is the basic description of a gravitational field, but it is more than that: It provides us with some hints about the way how the universe is working. One main feature of the Schwarzschild metric is the association of finite and infinite time structures, and it includes even proposals for the solution of the so-called "information paradox" of black holes and the supposed "breakdown of general relativity" near singularities.

1. The association of infinity and finiteness

The Schwarzschild metric is the basic description of a gravity field, but it is more than that: It provides us with some precious information and with some hints about the structure of the universe.

At the event horizon of a black hole, infinity is meeting finiteness, due to gravitational time dilation. An external observer will come to the conclusion that nothing will ever reach the event horizon, whereas an infalling observer is reaching it within finite time. Currently, the infalling observer is considered to refute the observations of external observers **[1]**, and for this reason, the Schwarzschild coordinates are supposed to be uncomplete, and alternative coordinates have been developed which are going beyond the event horizon, such as the Kruskal-Szekeres coordinates which we will use here.

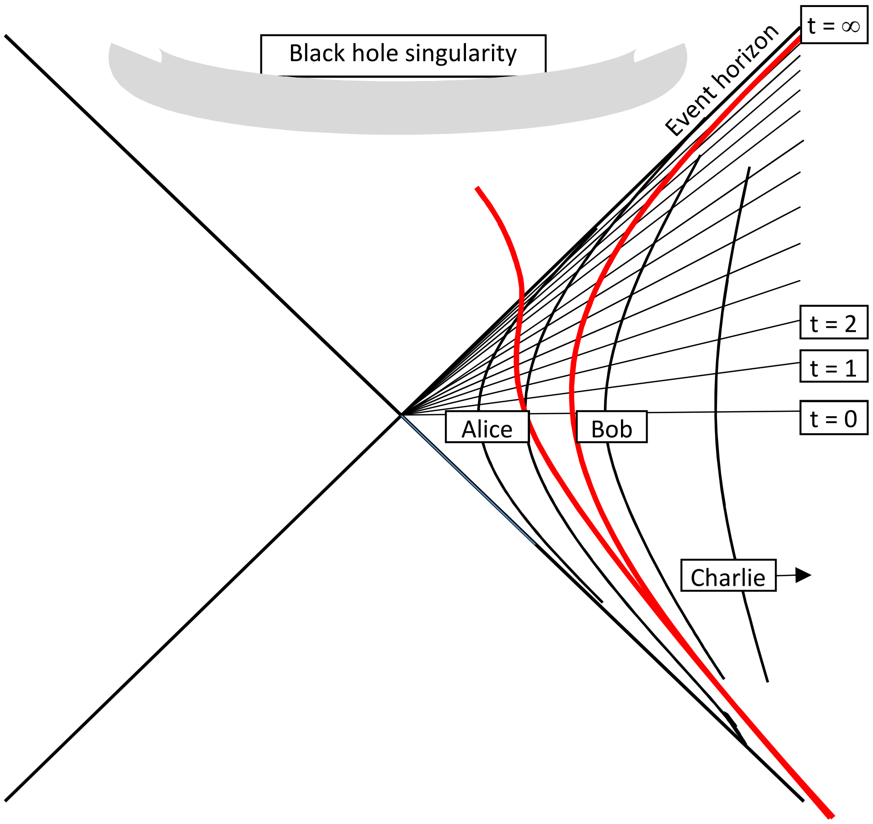

Fig. 1: The Kruskal-Szekeres coordinates

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In the Kruskal diagram, the particle A (Alice) is an infalling particle, the particle B (Bob) is hovering at a constant distance from the event horizon, and we may add Charlie as a far-away observer at a place without any gravitational potential. The lines of simultaneity are counting time from zero to infinity, and they are directed towards the center of the diagram, whereas the lines of equal distance from the event horizon are hyperbolic lines.

If we follow the logic of the Kruskal coordinates with their simultaneity lines, the universe is getting older, approaching infinity step by step, but it is never ending because whatever is the coordinate time we chose, we may always find a finite coordinate time which is happening later. The event horizon is the limit, the ceiling which is never reached.

The Kruskal coordinates correspond to the view of a far-away observer outside of the gravitational potential, but the main principle of the infinity at the event horizon applies also to any external observer and to the whole external universe: External observers are subject to time dilation, but nevertheless they will agree that the event horizon represents the simultaneity line of infinity and the end of our time.

2. The infalling observer Alice

So, if the whole universe agrees on infinity at the event horizon, what does that mean for the infalling observer Alice?

The problem of the infalling observer was described by Penrose in 1969 with respect to a collapsing star:

> *"(…) it would seem that the surface of the star can never cross to within the r = 2m region. However, this is misleading. For suppose an observer were to follow the surface of the star in a rocket ship, down to r = 2m. He would find (assuming that the collapse does not differ significantly from free fall) that the total proper time that he would experience as elapsing, as he finds his way down to r = 2m, is in fact finite."* **[1]**

First of all, we must be aware of the fact that an infalling observer is also an external observer as long as she does not reach the event horizon, and as such Alice will agree - based on her own spacetime diagram - that the event horizon represents infinity, until the very moment when she is reaching it.

How exactly is it possible that Alice may reach the event horizon? The answer is located in the zone at infinitesimal small distance to the event horizon. Before reaching this final zone, nothing distinguishes Alice from other external observers, and for her the event horizon lies in infinity. Only while she is crossing this final zone right before the event horizon, she is exposed to infinite gravitational time dilation, and she is crossing the gap between the position of an external observer and the event horizon.

That means that Alice is reaching the event horizon abruptly. On her whole way towards the event horizon there is a quantitative increase of time dilation, she is crossing every single simultaneity line of the Kruskal coordinates one by one, but she does not lose her quality of an external observer. In particular, Alice could change her mind at any time on her way to the event horizon, or she could be pushed back by the collision with some other particle. It is only during the final step when she is exposed to infinite time dilation that she is abruptly reaching the event horizon.

On her travel approaching the event horizon, the universe is running faster and faster from her point of view, and before her final step to the event horizon the universe has nearly come to its end. She is reaching the event horizon simultaneously with the end of our universe, the end of the time coordinate of our universe (all outside observers agree). When crossing the event horizon, she is going beyond our infinite time coordinate, and therefore her time coordinates do no longer correspond to any of our time coordinates, that means: We don't know where she arrives, but for sure she has left our universe of spacetime.

By consequence, in the example mentioned by Penrose, the point of view of the external observer is not incompatible with the point of view of the infalling observer: Before reaching the event horizon, the infalling observer is an external observer. But also the very last step when reaching the event horizon is compatible if we understand the word "never" in the sense of "never within our spacetime" because it happens at the end of the time of our spacetime.

That means that the Schwarzschild coordinates are <u>not</u> uncomplete: They are representing our spacetime which is ending at the event horizon, and it is completely useless to look for a continuation of spacetime inside the event horizon, and the infalling observer is not a "proof" of the contrary because she is leaving our spacetime. This is documented very precisely by the Kruskal coordinates where our spacetime is limited to the right quarter of the diagram, and the event horizon is the limit to which spacetime is converging. In the upper quarter of the diagram, beyond the event horizon, there might be something after the end of our time, but it is not part of our spacetime.
The Schwarzschild metric introduces the possibility of a "universe after our universe", but we must accept that our spacetime is referring only to our universe, and it would be a mistake to think that we can describe what happens after spacetime by the means of spacetime.

We may distinguish two different reference frames. Bob represents the point of view of an arbitrary observer within our spacetime. In contrast, when Alice reaches the event horizon, she is adopting a sort of "supra-universal" reference frame which is creating a sort of link between the outside and the inside of the event horizon. On the one hand we have the intrinsic evolution of a particle within our universe until infinity, on the other hand we have the "supra-universal" evolution of the particle within a cyclic cosmology, from one universe to the next.

And this is the reason of the current confusion on event horizons: Currently it is supposed that the hovering observer Bob and the observer reaching the event horizon Alice are both subject to the spacetime of general relativity, with unresolvable paradoxes. But the Kruskal diagram shows clearly that our spacetime is limited to the quarter on the right, and at the event horizon, Alice is reaching a point where she is leaving spacetime. So, the black hole complementarity is not intrinsic to spacetime, it is the complementarity of the reference frame of spacetime on the one hand and the "supra-universal" reference frame on the other. Our spacetime reveals to be well-closed and perfectly consistent. Difficulties arise only if we try to explore the zones beyond spacetime with the intrinsic tools of spacetime.

3. Alice meets Bob

We saw that the event horizon is a simultaneity line - that implies that all objects and all particles that will ever fall into the black hole will reach this line simultaneously. If one later day, Bob should fall into the black hole, he becomes an infalling observer, after Alice. But the surprising result of the Schwarzschild metric is, that, even if he starts to follow Alice 1000 years later, he will join Alice at the simultaneity line of the event horizon.

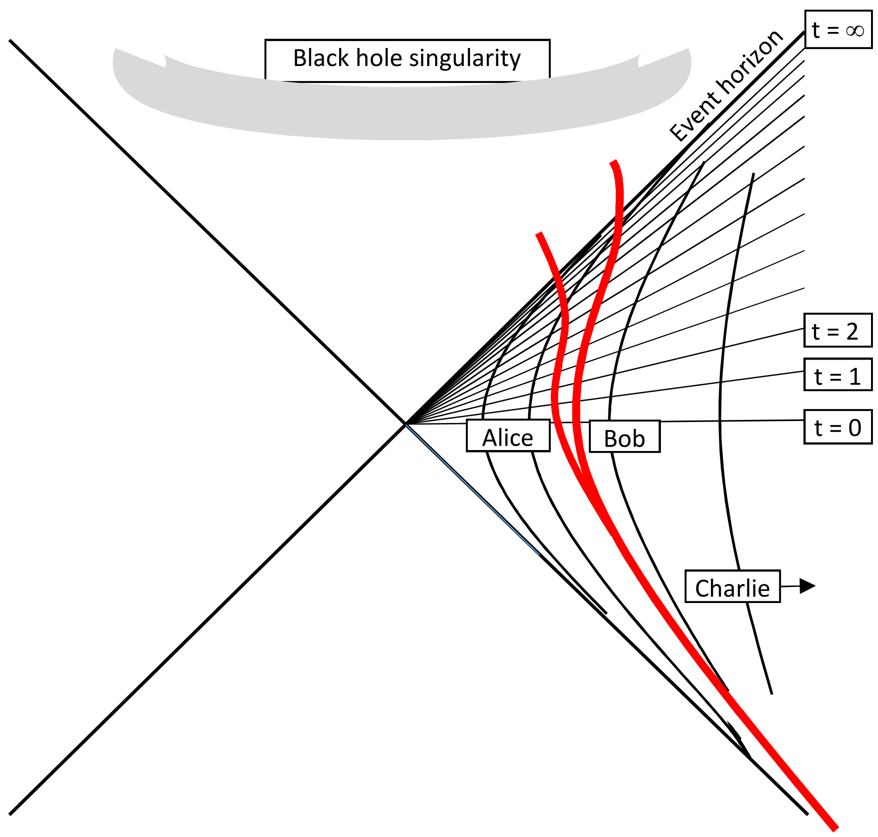

Fig. 2: Alice meets Bob

This is an effect of time dilation, and we will check this with an example, we presume that one simultaneity line corresponds to 250 years. So Alice is always ahead of Bob by 4 simultaneity lines, that is 4 time units. However, on her way to the event horizon, the distance between these simultaneity lines in the diagram is shrinking to zero, and even 1000 years are melting to nothing. As there is an infinite amount of time units, their spatial representation in the diagram is getting infinitely small near the event horizon, converging towards zero.

And this result is perfectly confirmed by the diagram: The Kruskal diagram is showing that both of them are crossing every single simultaneity line, one after the other, until infinity. She is always 4 time units ahead, but at infinity this is compensated by the fact that $\infty + 4 = \infty$.

Moreover, we must remember that Alice when she is reaching the event horizon, she does this abruptly. One infinitesimal time unit before, she was outside the event horizon, and by consequence, from her point of view, the event horizon represented still infinity, until the very last moment. That means that the last step of Alice before reaching the event horizon is equivalent to an infinite time interval, but dilated by infinite time dilation! Obviously, the time interval between Alice and Bob of 1000 years is nothing against this infinite time interval.

How can it be that Bob who started later than Alice is reaching the event horizon within the same time? The answer is the time dilation: As Alice is always ahead by 4 time units, she is always nearer to the source of gravity, and she is exposed to a stronger time dilation than Bob. It is this time dilation difference which sums up to 1000 years.

But how is it possible that there is one single event "Alice meets Bob" if in the diagram, Alice and Bob are not reaching the event horizon at the same point?
Answer: The Kruskal diagram is in no way isometric, it does not preserve lengths. The event horizon is an extreme limit case, in two different aspects:
- On the one hand, it is the limit case of the simultaneity lines where $t = \infty$, so time is the same on the whole event horizon.
- On the other hand, it is the limit of the hyperbolic lines of spatial equidistance where the radial distance equals the Schwarzschild radius, so also the radial distance is the same on the whole event horizon, and if time is the same and space is the same, that means that the event horizon in the diagram represents only one point. - We may not forget that in the Kruskal coordinates, the two other space coordinates are not represented, and they are not needed because we are considering radially infalling movements here.

As Alice and Bob are meeting in one reference frame, they are meeting in any frame, also in the respective reference frames of Alice and Bob.

4. Alice and Bob are not alone…

We saw, that Bob meets Alice at the event horizon, and we may say that all infalling particles that will ever fall into this black hole will reach the event horizon simultaneously - the event horizon is a line of simultaneity. But that is not all - we may also suppose that all infalling particles which will ever fall into <u>any</u> black hole of the universe will do this simultaneously, at the end of our time, and it seems that this is a moment where something special is happening.

5. The emptiness of black holes and the membrane paradigm

We saw that the whole universe agrees that nothing may ever cross the event horizon of black holes within the time of our spacetime. The inside of event horizons is not part of our spacetime of general relativity, black holes are empty, they are hollow spheres, their inside may be considered as the "future after our universe".

That implies that black holes have no mass! Instead, all the mass of a black hole is consisting of the infalling matter which is approaching the event horizon.

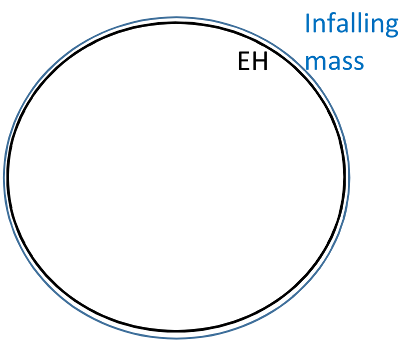

Fig. 3: Black holes are empty

The closer a zone is to the event horizon, the longer the time a particle will be found in this zone. That means that an important amount of the infalling matter is found at an infinitesimal distance to the event horizon, and still steadily approaching.

This concept corresponds to another important concept on black holes which is the membrane paradigm of the stretched horizon, considered as a formalism for the description of several properties of a black hole **[2]**. However, as we saw, the membrane concept is not only a formalism but it corresponds to an approximation of reality as the mass is effectively approaching the form of a stretched horizon, at infinitesimal distance to the event horizon.

The concept of empty black holes resolves the information paradox, because no information is ever crossing the event horizon before the end of time (see also **section 8**).

6. The merging of black holes

How are black holes forming and merging? At the encounter of two black holes, the two shells of matter around both event horizons (in blue) are merging, whatever is the size of the black hole:

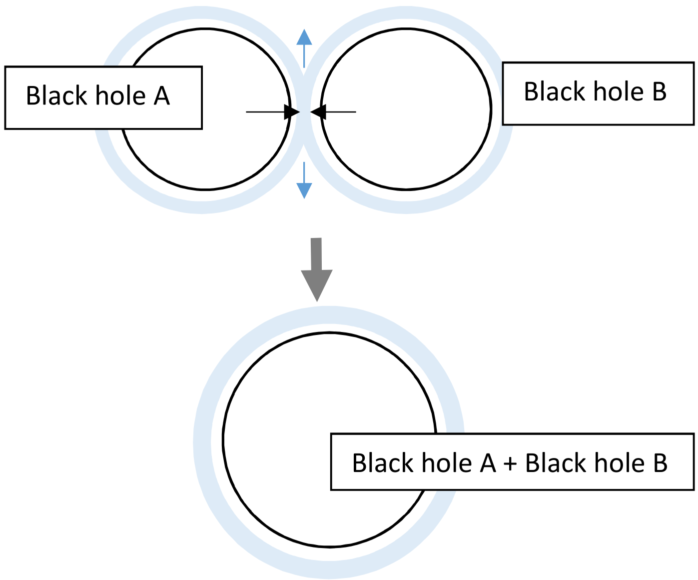

Fig. 4: The merging of two black holes

The merging process is driven by the attraction of gravity (the horizontal black arrows) and by the repulsive effect of gravitational time dilation (the vertical blue arrows) near the event horizon.

Both event horizons are merging and forming one big event horizon, and mass is remaining outside.

7. What is happening at "the end of the universe"?

Now we come to cosmology:

Physical theories predict the "Big Rip" or the "Big Crunch" at the end of the time of our universe. The Schwarzschild metric is a good basis for the Big Crunch theory: All black holes would have merged to one cosmic black hole, all particles and radiation would reach simultaneously the event horizon, crossing it and entering possibly into a new universe.

Such assumptions are speculation, but they would be a coherent interpretation of the asymptotical logic and geometry of the Schwarzschild metric. However, astronomical observations go rather in the direction of the Big Rip. For the Big Rip, the application of the Schwarzschild metric is clearly less self-explanatory, but we could imagine that during the Big Rip, the universe is approaching some sort of external event horizon, at the end of the time of our spacetime.

Of course, the Schwarzschild metric should also be taken into account with respect to the creation of our universe, the "Big Bang": Currently it is not clear which physical laws might have governed the beginning of a universe emerging from one point of singularity, and how this could harmonize with general relativity. The assumption of an event horizon of a white hole avoids the concept of a single point and would mitigate this problem.

We do not know what is happening with an infalling observer Alice at the other side of the event horizon of a black hole, but there is one candidate: The event horizon could be the separation between a black hole on the outside and a white hole on the inside. We could suppose that a new big bang is happening, introducing a new infinite universe after the end of our infinite universe.

At this point, there is one important question arising: If the final black hole after the Big Crunch is generating a white hole, in which direction does the white hole go, towards the inside or towards the outside of the event horizon?

The argument in favor of the inside direction of the Big Bang is the dynamics of the infalling observer Alice. From her point of view, she is continually accelerating towards the central singularity, in accordance with Newton's law, and it seems coherent to refer for this question to her "supra-universal" reference frame. However, her momentum towards the center could be stopped at the event horizon by the environment, because we must remember that the matter of the whole universe is reaching the event horizon exactly simultaneously with her! The horizontal pressure could inhibit the entry of matter through the event horizon and push it back outside. We will not try to predict the outcome of the high-energy phenomena which could happen at this final moment of our universe, but nevertheless we want to mention this open question.

8. The second law of thermodynamics

What about entropy at the end of our universe, at the Big Crunch, at the final black hole and in the new universe after our universe? According to the second law of thermodynamics, entropy is steadily increasing, but the question arises here if the second law of thermodynamics is a law of our spacetime which is limited to spacetime, or if it is a supra-universal law which would apply also to the next universe.

Indeed, it would not be very romantic if the new universe after our universe would begin with heat death, it would mean that after our universe, nothing would happen anymore in later universes.

But there are good reasons to think that the Big Crunch could be a sort of "reset" of entropy. According to the model of the Schwarzschild metric, at the end of the universe, the whole matter of the universe which has taken the form of a membrane near the event horizon is reaching simultaneously the final event horizon - the membrane with its approximate form becomes a perfect hollow sphere, a shell which is exactly positioned on the event horizon. By this final process which is taking infinite time from the point of view of outside observers, the energy of the universe would adopt by itself a perfectly ordered, homogeneous structure, without any information left, with entropy close to zero, comparable with the reinitialization of a vast game.

9. Outlook

In this paper, I tried to show by the means of the Kruskal-Szekeres coordinates the conclusions we may draw from the structure of the Schwarzschild metric, in particular:

Spacetime is not "breaking down" at black holes because their event horizons are natural limits of spacetime. There is "cosmic censorship" because the inside of black holes is not defined within our spacetime.

It would be a crucial mistake to think that the view of Alice is contradicting the view of external observers. Before reaching the event horizon, Alice is an external observer. The event horizons are the limit for all observers of the whole universe, even including Alice while she is outside the event horizon. Infalling observers are leaving our spacetime after the end of our universe and may be considered as "supra-universal" reference frames, going beyond our spacetime. The complementarity of black holes is not intrinsic to spacetime but it is a complementarity between spacetime and the "supra-universal" reference frames.

The current concept of black holes is clearly incompatible with the Schwarzschild metric: the inside of an event horizon must be empty because, by time dilation, all approaching matter is eternally slowed down.

All mass which will ever enter any black hole of the universe will do this simultaneously, at the end of the time of our universe.

All these conclusions are following directly from the Schwarzschild metric. Based on these conclusions, we may speculate about a cyclic cosmological model: One infinite universe is followed by the next infinite universe, and the final Big Crunch in one single black hole corresponds to the white hole of the Big Bang of the next universe.

[1] Roger Penrose: Gravitational Collapse: The Role of General relativity, 1969, Rivista di Nuovo Cimento, (1) p. 252-276

[2] Richard H. Price, Kip S. Thorne: Membrane viewpoint on black holes: Properties and evolution of the stretched horizon", 1986, Physical Review D, 33, 915 with further references