Properties of a Special Partition of the Positive Integers

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Abstract We examine the properties of a partition of the positive integers and exhibit some formulas that describe the partition and place all positive integers in specific sequences that comprise the partition. The Collatz conjecture also partitions the positive integers.

1, 5, 9, … contains **1/2** all positive odd integers.

2, 6, 10, … contains **1/2** all positive even integers.

3, 11, 19, … contains **1/4** all positive odd integers.

4, 12, 20, … contains **1/4** all positive even integers.

7, 23, 39, … contains **1/8** all positive odd integers.

8, 24, 40, … contains **1/8** all positive even integers.

The first term of the ***nth*** positive (odd | even) number sequence is $(2^n – 1 \mid 2^n)$

The difference between terms in the ***nth*** sequence of positive (odd | even) numbers is 2^{n+1}

The ***nth*** sequence of (odd | even) numbers contains $1/2^n$ proportion of all positive (odd | even) numbers.

The proportion of positive (odd | even) numbers in the first ***m*** (odd | even) sequences is $(2^m – 1) / 2^m$.

The odd number ***j*** in the ***mth*** position of the ***nth*** odd number sequence is $j = (2^n – 1) + (m – 1)(2^{n+1})$.

The even number ***j*** in the ***mth*** position of the ***nth*** even number sequence is $j = (2^n) + (m – 1)(2^{n+1})$.

To find the sequence containing the (odd | even) number ***j***, sub ***1*** for ***n*** into the following (odd | even) number formula below. If the formula is not satisfied, keep substituting the next larger value ***s*** into the formula. The formula will be satisfied before $j > 2^{s+1}$.

$s \pmod{2^{s+1}} = (2^{s+1}– 1 \mid 0)$ for every (odd | even) $s < n$, where ***n*** satisfies the formula**.**

When the odd number $j \pmod{2^{n+1}} = 2^n – 1$, ***j*** will be in the ***nth*** odd number sequence.

When $m – 1= (j – (2^n – 1)) / (2^{n+1})$, ***j*** will be in the ***mth*** position of the ***nth*** odd number sequence.

When the even number $j \pmod{2^{n+1}} = 2^n$, ***j*** will be in the ***nth*** even number sequence.

When $m – 1= (j – (2^n)) / (2^{n+1})$, ***j*** will be in the ***mth*** position of the ***nth*** even number sequence.

The proportion of all positive (odd | even) integers in the partition:

$1/2 + 1/4 + 1/8 + \ldots$ is $1 = 1/2 / (1 – 1/2)$.

A proportion of one indicates that all positive (odd | even) integers are in the partition.

This is also confirmed by the formulas that exist to locate the position of every positive (odd | even) integer within one of the sequences that form the partition.

Collatz sequences are formed by starting with any even or odd positive integer. If it is even, divide by two until it is odd. Then multiply by three and add one to get an even number. The Collatz conjecture states that if this process is repeated you always get back to one.

The Collatz conjecture is true **[1] [2]** and it also partitions the positive integers into the Collatz structure. This is shown through a number of geometric series whose terms are proportions that all sum to one.

The Collatz Structure (displayed in the diagram below) consists of horizontal branches and vertical towers. Vertical arrows ↓ represent descending Collatz towers, where each term is half the previous term. Horizontal arrows ← indicate the Collatz algorithm is applied to move from term to term in the branch.

Collatz Structure Branches and Towers ↓ indicates a descending Collatz tower

		↓		↓				↓				↓				↓		↓							↓
↓		84		160	←	53	←	106	←	35	←	70	←	23	←	46	←	15							36
128		42		80					↓				↓				↓						↓		18
64	←	21		40←	13	←	26	←	52	←	17	←	34	←	11	←	22	←	7	←	14	←	28	←	9
32				20	↓																				
16	←	5	←	10←	3																				
8																									
4																									
2																									
1																									

[1] ***Proving the Collatz Conjecture.*** **https://vixra.org/pdf/2005.0084v1.pdf**

For more details see **[2]** ***Collatz Conjecture Proof.*** **https://vixra.org/pdf/1901.0227vE.pdf**

Thanks for your interest in this paper. If you wish to make comments send them to Jim Rock at collatz3106@gmail.com.