

Presented at the Conference on Theoretical and Foundational Problems in Information Studies, IS4SI Summit 2021, online, 12–19 September 2021.

Obtaining information about nature with finite mathematics

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Abstract

The main goal of this note is to explain that classical mathematics is a special degenerate case of finite mathematics in the formal limit $p \rightarrow \infty$, where p is the characteristic of the ring or field in finite mathematics. This statement is not philosophical but has been rigorously proved mathematically in our publications. We also describe phenomena which finite mathematics can explain but classical mathematics cannot. Classical mathematics involves limits, infinitesimals, continuity etc., while finite mathematics involves only finite numbers.

1 Problems with describing nature by classical mathematics

More than 300 years ago Newton and Leibniz proposed the calculus of infinitesimals, and this mathematics turned out to be very successful in explaining many physical phenomena. The idea of infinitesimals was in the spirit of existed experience that any macroscopic object can be divided into arbitrarily large number of arbitrarily small parts, and, even in the 19th century, people did not know about atoms and elementary particles. But now we know that when we reach the level of atoms and elementary particles then standard division loses its usual meaning and in nature there are no arbitrarily small parts and no continuity.

For example, typical energies of electrons in modern accelerators are millions of times greater than the electron rest energy, and such electrons experience many collisions with different particles. If it were possible to break the electron into parts, then it would have been noticed long ago. Another example is that if we draw a line on a sheet of paper and look at this line by a microscope then we will see that the line is strongly discontinuous because it consists of atoms. That is why standard geometry (the concepts of continuous lines and surfaces) can work well only in the approximation when sizes of atoms are neglected, standard macroscopic theory can work well only in this approximation etc.

The development of quantum theory has shown that there are phenomena which classical mathematics cannot explain because for them standard quantum theory gives divergent expressions. As the famous physicist and the Nobel Prize laureate Steven Weinberg writes in his book [1]:

Disappointingly this problem appeared with even greater severity in the early days of quantum theory, and although greatly ameliorated by subsequent improvements in the theory, it remains with us to the present day.

The title of Weinberg's paper [2] is "Living with infinities".

2 How to compare two theories

The main goal of this note is to explain the following

Statement: Classical mathematics is a special degenerate case of finite mathematics in the formal limit $p \rightarrow \infty$, where p is the characteristic of the ring or field in finite mathematics.

For proving this statement, we should define a concept when theory A is more general (fundamental) than theory B. In Ref. [4] we propose the following

Definition: Let theory A contain a finite non-zero parameter and theory B be obtained from theory A in the formal limit when the parameter goes to zero or infinity. Suppose that, with any desired accuracy, A can reproduce any result of B by choosing a value of the parameter. On the contrary, when, the limit is already taken, one cannot return back to A and B cannot reproduce all results of A. Then A is more general than B and B is a special degenerate case of A.

Known examples are that:

- 1) nonrelativistic theory (NT) is a special degenerate case of relativistic one (RT) in the formal limit $c \rightarrow \infty$ (where c is the speed of light).

This implies that RT can explain any result of NT if c is chosen sufficiently large but there are phenomena which only RT can explain because for the explanation it is important that c is finite and not infinite.

- 2) classical theory is a special degenerate case of quantum one (QT) in the formal limit $\hbar \rightarrow 0$ (where \hbar is the Planck constant);
- 3) RT is a special degenerate case of de Sitter invariant theories in the formal limit $R \rightarrow \infty$ where R is the parameter of contraction from the de Sitter groups or Lie algebras to the Poincare group or Lie algebra, respectively.

In the literature those facts are explained from physical considerations but, as shown in the famous Dyson's paper "Missed Opportunities" [3] and in our book [4], those statements can be proved mathematically because they follow from *Definition*. So, for proving the main **Statement** we have to show that it also follows from *Definition*.

3 Why finite mathematics is more general than classical one

In classical mathematics the ring of integers Z is involved from the very beginning and, even in standard textbooks, it is not even posed a problem whether Z should be treated as a limit of finite rings. Moreover, Z is the starting point for constructing the sets of rational, real and complex numbers and sets with greater and greater cardinalities.

However, finite mathematics rejects infinities from the beginning. This mathematics starts from the ring $R_p = (0, 1, 2, \dots, p-1)$ where addition, subtraction and multiplication are performed as usual but modulo p , and p is called the characteristic of the ring. The founder of finite mathematics is Evarist Galois who was killed in a duel in 1832 when he was 20 years old. In his 20s, he has already done a lot and probably would have done much more if he had not been killed. Probably, this is one of the reasons why finite mathematics is not so popular as classical mathematics. In computers finite mathematics is natural because any computer can operate with the maximum number of bits it has. However, physicists typically think that finite mathematics is exotic which has nothing to do with physics.

We have proved [4] that, as follows from *Definition*, theory with R_p is more general than the theory with Z . This implies that any result in Z can be obtained in R_p if p is chosen to be sufficiently large. On the other hand, in Z there are no operations modulo a number, and therefore some results in R_p cannot be recovered in Z .

This implies that, even from purely mathematical point of view, the concept of infinity is not fundamental since, when we introduce infinity, we get a degenerate theory where all operations modulo a number disappear.

Now we describe quantum states not by elements of Hilbert spaces but by elements of spaces over a finite ring $R_p + iR_p$. We call this theory FQT - finite quantum theory. As follows from the abovementioned, FQT is more general (fundamental) than standard quantum theory (SQT).

4 Examples when finite mathematics can solve problems which classical mathematics cannot

In our book [4] we consider phenomena where it is important that p is finite. They cannot be described by SQT, and this is analogous to the fact that nonrelativistic theory cannot describe phenomena in which it is important that c is finite.

- **Example 1:** Derivation of the empirical Newton gravitational law.

This law is only phenomenological. Standard quantum theory cannot derive this law because the theory is nonrenormalizable. But in our approach, this law can be derived as a consequence of FQT in semiclassical approximation

[4]. In the Newton law, the gravitational constant G is taken from the outside, but in our approach it is not a constant but a function which depends on p as $1/\ln(p)$. By comparing the result with the experimental value, one gets that p is a huge number of the order of $\exp(10^{80})$ or more. In the formal limit $p \rightarrow \infty$, G becomes zero and gravity disappears. Therefore, in our approach, gravity is a consequence of finiteness of nature.

- **Example 2:** Standard theory cannot explain the experimental fact that a particle and its antiparticle have the same masses. As shown in Ref. [4], in quantum theory over finite mathematics, this fact is obvious.

- **Example 3:** Explanation of the Dirac vacuum energy problem.

This problem is as follows. The vacuum energy should be zero, but in standard theory the sum for this energy diverges. In Ref. [4], we take the standard expression for this sum; then we explicitly calculate this sum in finite mathematics without any assumptions, and, since all the calculations are modulo p , we get zero as it should be.

- **Example 4:** Explanation of baryon asymmetry problem.

This problem is formulated as follows. At early stages of the universe the amounts of matter and antimatter were the same, and therefore from the law of conservation of the baryon number, those amounts should be the same now. But now the amount of matter is much greater than the amount of antimatter. However, we have shown [4] that in FQT the laws of conservation of electric charge and baryon number are only approximate. At the present stage of the universe, symmetry is such that those laws are valid with a very high accuracy. But at early stages, symmetry is essentially different and those laws cannot be valid. So the baryon asymmetry problem does not arise.

- **Example 5:** Arguments are given [4] that in FQT only Dirac's singletons are the only true elementary particles.

- **Example 6:** Arguments are given [4] that the ultimate quantum theory should be based on a ring but not a field. In other words, only addition, subtraction and multiplication are fundamental mathematical operations while division is not.

These examples demonstrate that there are phenomena which can be explained only in finite mathematics because for them it is important that p is finite and not infinitely large. So, we have an analogy with the case that relativistic theory can explain phenomena where it is important that c is finite while nonrelativistic theory cannot explain such phenomena.

5 Conclusion

Let us note that in FQT there are no infinities in principle and that is why divergences are absent in principle. The conclusion from the above consideration can be formulated as:

Mathematics describing nature at the most fundamental level involves only a finite number of numbers, while the concepts of limit, infinitesimals and continuity are needed only in calculations describing nature approximately.

This situation is a good illustration of the famous Kronecker's expression: "God made the natural numbers, all else is the work of man". In view of the above discussion, I propose to reformulate this expression as: "God made only finite sets of natural numbers, all else is the work of man".

Acknowledgements: I am grateful to Mark Burgin who invited me to this conference and told me about his book with Czachor [5] where classical mathematics is a special case of a more general mathematics.

References

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