# Combinatorial Relation of Optimized Combination with Permutation

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**Abstract**: This paper presents the relations of optimized combination with traditional combination and permutation. Combinatorics is a collection of several computing techniques used in science and management. In this research paper, new results on both combination and permutation are introduced for the researchers who are involving to find solutions for the scientific problems.

**Keywords:** combinatorics, counting technique, optimized combination

## I. Introduction

The combinatorics consisting of combinations and permutations provides various numerical techniques of computation for finding solutions to the real world problems. The optimized combination [1, 2] focuses on new developments for the improvement of traditional combination.

## II. Optimized Combination

The optimized combination of combinatorics is

$$V_r^n = \frac{(r+1)(r+2)(r+3)\cdots\cdots(r+n-1)(r+n)}{n!} \; (n, r \in N, n \geq 1, r \geq 0, \& \; r \leq n),$$

where $N = \{0, 1, 2, 3, 4, 5, \ldots\ldots\}$ is the set of natural numbers including the element 0.

The conversion of traditional combination into optimized combination is given below:

$$nC_r = \frac{n!}{r!\,(n-r)!} = (V_0^r)(V_r^{n-r}) = V_r^{n-r} \text{ where } V_0^r = 1 \; (r \geq 1).$$

Also,

$$nC_{n-r} = \frac{n!}{(n-r)!\,r!} = nC_r = V_r^{n-r}, \qquad \mathrm{n}C_n = \mathrm{n}C_0 = \frac{n!}{n!} = V_0^n = 1, \quad and \;\; (n+r)C_r = V_r^n.$$

For examples,

$nC_1 = nC_{n-1} = V_1^{n-1}, \quad nC_2 = nC_{n-2} = V_2^{n-2}, \quad nC_3 = nC_{n-3} = V_3^{n-3}, \quad \cdots\cdots$

Some results [1, 2] of the optimized combination are provided below.

Result 1: $V_0^n = 1 \; (n \geq 1 \;\; \& \;\; n \in N)$

Result 2: $V_r^{n+1} - V_r^n = V_{r-1}^n$

Result 3: $1 + V_1^1 + V_1^2 + V_1^3 \cdots\cdots V_1^n = V_2^n$

Result 4: $V_r^n = V_n^r \;\; (n, r \geq 1 \;\&\; n, r \in N)$

Result 5: $V_n^n = 2V_{n-1}^n$

Result 6: $V_0^n + V_1^n + V_2^n + V_3^n \cdots + V_{r-1}^n + V_r^n = V_r^{n+1}$

## III. Relation of Optimized Combination with Permutation

The relation of optimized combination with permutation is $r!\, V_r^{n-r} = nPr$,

where $nPr = \dfrac{n!}{(n-r)!}$.

Proof. $r!\, V_r^{n-r} = \dfrac{r!\,(r+1)(r+2)) \cdots\cdots (r+n-r)}{(n-r)!} = \dfrac{n!}{(n-r)!} = nPr,$

where $nC_r = \dfrac{n!}{r!\,(n-r)!} = \dfrac{1}{r!}(nPr) = V_r^{n-r} \; (n, r \in N, n \geq 1, r \geq 0 \& \; r \leq n\,).$

## IV. Conclusion

In this paper, the optimized combination and its relation to traditional combination and permutation have been introduced with some results that are useful for the researchers who are involving to solve the scientific problems and meet today's challenges.

## V. References


[1] Annamalai C (2020) "Novel Computing Technique in combinatorics, Archive ouverte HAL. https://hal.archives-ouvertes.fr/hal-02862222

[2] Annamalai C (2020) "Optimized Computing Technique for Combination in Combinatorics, Archive ouverte HAL. https://hal.archives-ouvertes.fr/hal-02865835