Five-dimensional cosmology
(*Fünfdimensionale Kosmologie*[1])

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Abstract. The thesis contains a comprehensive overview of various works of the author and his studies on 5D (projective) relativity theory and its application for the clarification of cosmology and star formation processes developed by Vefasser's earlier theory. The first part provides an explanation of the projective theory of relativity (which can be read independently of other work) in the expanded form developed by the author in 1944; the invariant $J = X^{\mu} X_{\mu}$, viewed as a constant in the older version of the theory, is treated as a scalar field. Even though in my first investigations this expansion of the theory seemed to lead to a considerable complication of the formula apparatus, a very simple implementation of the mathematical theory is now possible. The second part deals with the physical interpretation of the mathematical relationships thus obtained, with the establishment of the field equations – the variability of the size of J makes it possible to give corresponding mathematical precision for Dirac's idea of a gravitational *constant* that changes with cosmological time – and with the application to cosmology and star formation, according to the results of Ludwig and Müller, who provided the systematic execution of my model of the cosmos and star formation developed from *Dirac's principle*. Finally, relationships between the theory presented and the facts developed are discussed; my assessment of some of these points has changed somewhat.

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Translators Forward. This 2022 Vigier centenary issue of IOP JPCS provides the 1st English translations of the three 1946, 1947, 1948 Jordan papers with important insights in the development of Field equations for the Kaluza hypothesis - is a classical unified field theory of gravitation and electromagnetism. Pascual Jordan was a well-known German theoretical physicist who is one of the founders of quantum mechanics and quantum field theory. Together with Max Born and Werner Heisenberg, Jordan coauthored an important series of papers on quantum mechanics [1,2]. He went on

[1] Jordan P 1948 *Fünfdimensionale Kosmologie Astron. Nachr*. **276** 5–6: 193–208; doi:10.1002/asna.19482760502
[2] Pascual Jordan 18 October 1902 – 31 July 1980
[3] University of Göttingen (*Georg-August-Universität Göttingen*) Jordan earned his doctorate studying under Max Born.
[4] This work of translation generally adheres to literal meaning as stated by the author.

to pioneer early quantum field theory. However, in 1933 Jordan joined the Nazi party, if he had not done so; it is likely he would have won a Nobel Prize in Physics for his work with Max Born, who won in 1954 with Walther Bothe. The reason for translating is twofold: 1) These authors were key figures in the development of quantum theory, and (2 There is at present still no complete theory of Gravity; perhaps some interesting tidbit can still be gleaned to foster gravitation research as Kaluza's idea of a 5^{th} dimensions is considered an important precursor to String Theory.

Introduction

As is well known, Dirac introduced the new idea in the discussion of cosmological issues that the "gravitational constant" f is in truth changeable and may be a function of the age of the universe. This thought has been adopted in the author's cosmological considerations.

This leads to the task of generalization or expanding Einstein's general theory of relativity and gravity, for which the idea of f as a constant was essential. In the relativistic field theory, in addition to the previous field quantities, the *gravitational invariant* f or $x = 8\pi f / c^2$ must also be introduced as a scalar field quantity. In investigations that occupied me during the second half of the war, I recognized the 5D or projective form of the general Theory of Relativity as a particularly suitable starting point for such a generalization. The same idea had been pursued before in unpublished research by Einstein and Bergman [3]. Since then, my investigations have continued and were also significantly challenged by the work of G. Ludwig and C. Muller as well as through considerations of O. Heckmann. Because of time-related difficulties, these works are only partially published (and not in the order in which they were created), so that it is difficult for the reader to gain a clear overview [4]. Furthermore, the applications to cosmology and the problem of the details in star formation have undergone changes compared to my first publications [5], mainly due to information I owe to friendly communications from Baade. It should therefore be useful to give an overall presentation of the theory in the following, in such a form that it can be useful for anyone familiar with the general theory of relativity without consulting any other literature.

Part I. Mathematical Basics

1. The invariance group of the projective theory of relativity

The 5D form of the theory of relativity founded by Kaluza and Klein – whose aim is to incorporate the Maxwell field - was given a particularly elegant form by Veblen. This projective version of the theory introduced by Veblen was initially viewed as a certain modification of the conception of Kaluza and Klein; however, further development has shown that in reality the two versions in question are mathematically equivalent [6]. In the following, the projective version will be preferred, without us seeing it as more than a modification of the style of representation compared to Kaluza and Klein.

The projective version of the theory represents the four global coordinates as functions of the ratios of five coordinates. These latter are then not subjected to arbitrary, but only homogeneous transformations (first degree). The fact that this approach reveals deep-seated mathematical symmetry properties of the physical laws and allows a very simple summary of relationships, which in ordinary language appear quite complicated and confusing, is based on the fact that the invariance group of the general theory of relativity is essential in the five coordinates introduced in this way in a symmetrical and more beautiful form.

The usual 4D theory shows invariance not only with respect to coordinate transformations, but also with respect to gauge transformations. Let ϕ_k denotes the 4-potential. The transformation group $\mathfrak{G}$ for which the field equations are invariant is generated on the one hand by the gauge transformations

$$\left.\begin{aligned}\phi_k &= \phi'_k + \frac{\partial\phi\left(x^1,x^2,x^3,x^4\right)}{\partial x^k},\\ x^k &= x'^k,\end{aligned}\right\} \tag{1}$$

on the other hand, through the coordinate transformations

$$\left.\begin{aligned}\phi_k &= \phi'_l \frac{\partial x'^l}{\partial x^k},\\ x^k &= f^k\left(x'^1,x'^2,x'^3,x'^4\right).\end{aligned}\right\} \tag{2}$$

In the overall group $\mathfrak{G}$ the coordinate transformations eq. (2) form a subgroup, and the gauge transformations eq. (1) even form a normal divisor: Let T be a transformation $x'^k \to x''^k$ of the form (1), S a transformation $x''^k \to x'''^k$ of form (2), while the transformation $x^k \to x'^k$ signifies its inverse S^{-1}. Then the transformation $S^{-1}TS$ or $x^k \to x'''^k$ is evidently described by

$$\left.\begin{aligned}\phi^k &= \phi'''_k + \frac{\partial\phi\left(f^1(x), f^2(x), f^3(x), f^4(x)\right)}{\partial x^k},\\ x^x &= x'''^k.\end{aligned}\right\} \tag{3}$$

with $f^k(x) = f^k(x^1,x^2,x^3,x^4).$

So, you get the most general element of the group $\mathfrak{G}$ as a transformation of the form TS with T (1) according to eq. (2).

We now claim:

The group $\mathfrak{G}$ is isomorphic to group $\wp_5$ for all homogeneous transformations with five variables X^1, X^2, X^3, X^4, X^5.

The most general element of group $\wp_5$ is a transformation of form

$$X^\nu = X'^\nu F^\nu\left(\frac{X'^1}{X'^0},\frac{X'^2}{X'^0},\frac{X'^3}{X'^0},\frac{X'^4}{X'^0}\right);\quad \nu{=}0,1,\ldots,4. \tag{4}$$

This group has a normal divisor $\mathfrak{N}$, formed by such transformations (4), in which

$$F^\nu = F \quad \left(\text{for } \nu = 0,1,\ldots,4\right) \tag{5}$$

occurs. In the center of the group $\wp_5$ are the transformations (4), (5) for which

$$F = \text{const}; \tag{6}$$

Because of (6), $\mathfrak{B}$ is called the one-parameter group of these transformations.

We strengthen our assertion: $\mathfrak{G}$ is one-stage isomorphic with the factor group $\wp_5/\mathfrak{B}$.

First of all, $\mathfrak{N}/\mathfrak{B}$ is apparently one-step isomorphic with the gauge group (1): We can assign

$$F = \mathrm{const}\cdot \mathrm{e}^{\phi}; \tag{7}$$

With the note, that an additive constant ϕ has no influence on the transformation eq. (1).

Furthermore, $\wp_5$ has a subgroup $\mathfrak{A}$ such that $\mathfrak{A}/\mathfrak{B}$ is one-step isomorphic to group (2): We assign the following single-parameter family of transformations (4) to the transformation eq. (2):

$$\begin{aligned} &F^0 = C = \mathrm{const}, \\ &F^\lambda = C\frac{X'^0}{X'^\lambda} f^\lambda\left(\frac{X'^1}{X'^0}, \frac{X'^2}{X'^0}, \frac{X'^3}{X'^0}, \frac{X'^4}{X'^0}\right) \quad \text{for } \lambda > 0, \end{aligned} \tag{8}$$

or

$$\begin{aligned} &X^0 = CX'^0, \\ &\frac{X^\lambda}{X^0} = f^\lambda\left(\frac{X'^1}{X'^0}, \frac{X'^2}{X'^0}, \frac{X'^3}{X'^0}, \frac{X'^4}{X'^0}\right) \quad \text{for } \lambda > 0. \end{aligned} \tag{9}$$

This completely defines the assignment of $\mathfrak{G}$ and $\wp_5/\mathfrak{B}$; the correctness of our assertion results from the following remark: If the transformation (7) is designated by $\overline{T}$ and the transformation (8) by $\overline{S}$, then $\overline{S}^{-1}\overline{TS}$ is the assigned transformation to (3), also to $S^{-1}TS$.

Of course, the assignment made is arbitrary, insofar as any automorphism could still be carried out in $\wp_5$. If one chooses in particular an inner automorphism, mediated by a coordinate transformation (2), then according to (3) this obviously amounts for the fact that the coordinates x^k are not directly determined by the ratios X^λ / X^0 but by any four others representing homogeneous functions of the zeroth degree of X^ν.

The isomorphism of $\mathfrak{G}$ with $\wp_5/\mathfrak{B}$ determined above would have to be replaced by the simpler determination of the immediate isomorphism of $\mathfrak{G}$ and $\wp_5$ itself, if in the gauge transformation (1) an additive constant of the function ϕ would not drop out. It therefore seems very remarkable that quantum mechanical wave functions φ are (1) according to the transformation,

$$\varphi = \varphi' \exp\left(\frac{2\pi i e}{hc}\phi\right) \tag{10}$$

where e is the charge on the electron; so that the additive constant also really matters here. However, with the restriction that integer multiples of hc/e in (10) remain irrelevant: the restriction of the electrical charges to integer multiples of the elementary charge means that the transformation group $\wp_5$ is not exactly one-step isomorphic with the transformation group of the theory of relativity. This probably supports the assumption (emphasized by 0. Klein in a letter) that an understanding of the elementary electrical charge might require a deepening of the five-dimensional theory that goes beyond the projective formulation.

2. Five-dimensional and four-dimensional metrics

In the following, Greek tensor indices should always run through five values, Latin only four. If F is a function of the five coordinates X^{μ} respectively or the four x^{k}, then $F_{|\mu}$ respectively or $F_{|k}$ should mean the derivative according to X^{μ} respectively or x^{k}. The fact that x^{k} are homogeneous functions of the X^{μ} of degree zero is expressed in

$$x^{k}_{|\mu} X^{\mu} = 0. \tag{1}$$

Furthermore, more restriction to the transformation group $\wp_5$ can be expressed by

$$X'^{\nu}_{|\mu} X^{\mu} = X'^{\nu}; \tag{2}$$

evidently this means that the X^{μ} behaves like a vector in these transformations.

We call the large $T^{\nu_1 \dots \nu_n}_{\mu_1 \dots \mu_m}$ a projector if they a) transform themselves with respect to the transformations $\wp_5$ like a tensor, b) are homogeneous functions to the $(n-m)^{th}$ degree of X^{μ}:

$$T^{\nu_1 \dots \nu_n}_{\mu_1 \dots \mu_m | \lambda} X^{\lambda} = (n-m) T^{\nu_1 \dots \nu_n}_{\mu_1 \dots \mu_m}. \tag{3}$$

This restriction obviously means that knowing a projector field - as a function of the five coordinates X^{μ} - does not mean more than knowing a field in a 4D manifold.

If spinor fields are taken into account, the homogeneity requirement must be abandoned; these circumstances, which cannot be discussed here, are dealt with in detail from the standpoint of the theory represented in a monograph by G. Ludwig to be published shortly.

A metric $g_{\mu\nu}$ is now introduced in the 5D area; we use the $g_{\mu\nu}$, which are supposed to form a projector in the sense of (3), in the usual way to move the tensor indices up and down. We want to denote the tensor δ^{ν}_{μ}, which arises from $g_{\mu\nu}$ by pulling up the ν: $\delta^{\nu}_{\mu} = g_{\mu\lambda} g^{\lambda\nu}_{\cdot}$, logically with g^{ν}_{μ}.

The invariant

$$J = g_{\mu\nu} X^{\mu} X^{\nu} = X_{\nu} X^{\nu} \tag{4}$$

always $\neq 0$. We can then uniquely decompose a vector a^{ν} into a part parallel to X^{ν} and a part orthogonal to X^{ν}:

$$\alpha^{\nu} = \alpha^{(\nu)} + X^{\nu} \frac{\alpha^{\sigma} X_{\sigma}}{J}, \tag{5}$$

or

$$\alpha_{\nu} = \alpha_{(\nu)} + X_{\nu} \frac{\alpha_{\sigma} X_{\sigma}}{J}; \tag{5'}$$

$$\alpha^{(\nu)} X_{\nu} = \alpha_{(\nu)} X^{\nu} = 0. \tag{6}$$

In particular, for every scalar S according to (3):

$$S_{|\nu} = S_{|(\nu).} \tag{7}$$

This notation can easily be applied to any tensors. It becomes for example

$$\left.\begin{aligned} g_{\mu}^{(\nu)} &= g_{\mu}^{\nu} - X^{\nu}\frac{g_{\mu}^{e}X_{e}}{J} \\ &= g_{\mu}^{\nu} - X^{\nu}\frac{g^{\nu}X_{\mu}}{J}; \end{aligned}\right\} \tag{8}$$

$$g_{(\mu)(\nu)} g_{\mu\nu} - \frac{X_{\mu}X^{\nu}}{J} = g_{(\mu)\nu}. \tag{8'}$$

Definition: The abbreviation of the 5D vector a^{μ} was referred to as 4D vector

$$\alpha^{k} = X_{|\mu}^{k}\alpha^{\mu}. \tag{9}$$

We can also designate eq. (1) like this:

$$X^{k} = 0. \tag{10}$$

According to (10), the shortening of a vector a^{ν} depends solely on its part X^{ν} which is orthogonal to $a^{(k)}$:

$$\alpha^{k} = \alpha^{(k)}. \tag{11}$$

Accordingly, we define abbreviations for projectors with any number of initially upper indices; for example

$$g_{\nu}^{k} = x_{|\mu}^{k} g_{\nu}^{\mu} = x_{|\mu}^{k}; \tag{12}$$

$$g^{k\nu} = x_{|\mu}^{k} g^{\mu\nu} = g_{\mu}^{k} g^{\mu\nu}; \tag{13}$$

$$g^{kl} = x_{|\mu}^{k} x_{|\nu}^{l} g^{\mu\nu} = g_{\mu}^{k} g_{\nu}^{l} g^{\mu\nu}. \tag{14}$$

The components of the complete shortening of a projector - with only Latin indices - are homogeneous functions of the X^{μ} of degree zero, i.e. functions of the $x_{\cdot}^{k}$; namely, they form a tensor in the 4D sense.

From the fact that x^{k} are four independent functions of the X^{μ}, one can easily deduce: a) From $x^{k} = 0$ it follows that the vector a^{μ} is parallel to $a^{\mu} =$ const X^{μ}. b) From the fact Det $\left|g^{\mu\nu}\right| \neq 0$ follows also Det $\left|g^{kl}\right| \neq 0$. We can and want to use the g^{kl} respectively

$$\left(g_{kl}\right) = \left(g^{kl}\right)^{-1} \tag{15}$$

defined by g_{kl} as a metric of the 4D world.

Thus, the indices of the 4D tensors created by shortening can also be lowered:

$$\alpha_k = g_{kl}\alpha^l = g_{kl} g^l_\mu \alpha^\mu . \tag{16}$$

In particular, the meaning is the character g^μ_k can be attributed to;

$$g^\mu_k = g_{kl} g^{\mu l} \tag{17}$$

and then eq. (16) can also be in the form

$$\alpha_k = g^\mu_k \alpha_\mu \tag{18}$$

to be written; analogous to (9) or

$$\alpha^k = g^k_\mu \alpha^\mu \tag{19}$$

The same applies to any projectors.

From these considerations it follows without further explanation, that

$$g^e_k g^l_e = g^l_k = \delta^l_k \tag{20}$$

occurs because the left side is created by shortening the projector $g^e_\mu g^\nu_e = g^\nu_\mu$. On the other hand

$$g^\mu_i g^i_\nu = g^\mu_\nu - \frac{X^\mu X_\nu}{J}, \tag{21}$$

follows.

First of all, we prove

$$\left.\begin{aligned} \alpha^{(\nu)} &= g^\nu_k \alpha^k ; \\ \alpha_{(\nu)} &= g^k_\nu \alpha_k . \end{aligned}\right\} \tag{22}$$

The difference $a^{(\nu)} - g^\nu_k a^k$ has on the one hand, because of eqs. (20) and (11) the reduction to zero, so according to the above remark a) it is parallel to X^ν; on the other hand, because of (6) and (19) it is also orthogonal to X^ν.

Theorem: It is

$$a^k b_k = a^{(e)} b_{(e)} . \tag{23}$$

If the right side is equal to $a^{(e)} g^k_e b_k$, according to eq. (22) then according to eqs. (11) and (19) it is equal to the left side.

The proof of (21) follows from $g^\mu_i g^i_\nu = g^\mu_{(e)} g^{(e)}_\nu$, using (8). Because $\mathrm{d}x^k = g^k_\nu \mathrm{d}X^\mu$ one can express the relation $g^{kl} g^k_\mu g^l_\nu = g^k_\mu g_{k\nu} = g_{\mu\nu} - \left(X_\mu X_\nu\right)/J$ which is equivalent to eq. (21), as follows:

$$g_{kl}\mathrm{d}x^k \mathrm{d}x^l = g_{\mu\nu}\mathrm{d}X^\mu \mathrm{d}X^\nu - \frac{1}{J}\left(X_\mu \mathrm{d}X^\mu\right)^2 . \tag{24}$$

According to the above, a 5D projector equation $a^{\nu}=0$ is equivalent to a 4D equation system, consisting of the vector equation $a^{k}=0$ and the scalar equation $a^{\nu}X_{\nu}=0$. The same applies to any 5D projector equations: For example, an equation $R_{\mu\nu}=0$ with a symmetrical projector $R_{\mu\nu}=R_{\nu\mu}$ would be equivalent to the 4D equations

$$\left.\begin{aligned} R_{kl}=0 \quad &\left(\text{d. h. } g_{k}^{\mu}g_{l}^{\nu}R_{\mu\nu}=0\right); \\ R_{k\nu}X^{\nu}=0 \quad &\left(\text{d. h. } g_{k}^{\mu}R_{\mu\nu}X^{\nu}=0\right); \\ R_{\mu\nu}X^{\mu}X^{\nu}=0. & \end{aligned}\right\} \tag{25}$$

3. Four-dimensional covariant differentiation

In order to see in which way 4D differential equations are expressed by 5D (or vice versa), we have to examine the relationship between 4D and 5D covariant differentiation. It is advisable, however, to allow this investigation to be preceded by a closer examination of the 4D differentiation, since a more precise clarification of its properties than is usually sought in the literature makes the connection in question easier to determine.

The usual definition, according to which the covariant derivative $T_{l...\|i}^{k...}$ is the tensor field which at point P agrees with the usual derivative $T_{l...|i}^{k...}$, provided that the coordinate system used at this point P is a plane one (with $g_{kl|i}=0$), let (after one has been convinced that it In fact, for every point P there is a plane coordinate system in P, which - incidentally - is possible without the usual cumbersome use of geodetic lines), recognizing the following properties of the covariant derivative:

(I) The usual rules for differentiating the sums and products of tensor components apply.

(II) If the vector field a^{k} (or a_{k}) vanishes at the point P, then there is $a_{\|i}^{k}=a_{|i}^{k}$ (or $a_{k\|i}=a_{k|i}$).

(III) It is $a_{k\|i}-a_{i\|k}=a_{k|i}-a_{i|k}$.

(IV) It is $g_{kl|i}=0$, so that the operation of covariant differentiation is interchangeable with that of moving the tensor indices up and down.

It is important for the latter that these properties can also serve as a sufficient definition of the covariant derivative. The proof of this remark is as follows: First we get from (I), (II), is that

$$\left.\begin{aligned} a_{\|i}^{k}&=a_{|i}^{k}+\Gamma_{li}^{k}a^{l}, \\ a_{k\|i}&=a_{k|i}-\Lambda_{ki}^{l}a_{l}. \end{aligned}\right\} \tag{1}$$

Furthermore, for each scalar

$$S_{\|k}=S_{|k}; \tag{2}$$

because we apply (I) to the vector Sa^{k}. Consideration of the scalar then yields $S=a^{k}b_{k}$

$$\Lambda_{ki}^{l}=\Gamma_{ki}^{l}. \tag{3}$$

From axiom (III) it follows

$$\Gamma_{ki}^{l}=\Gamma_{ik}^{l}; \tag{4}$$

and then from eq. (5) the formula

$$\Gamma^l_{ki} = \frac{1}{2} g^{lm} \left(g_{km|i} + g_{im|k} - g_{ki|m} \right). \tag{5}$$

Auxiliary clause: Every tensor $T^{k\ldots}_{l\ldots}$ can be represented as the sum of products of vectors. (Proof trivial).

Therefore, according to axiom (I), the covariant derivative of any tensor is then also clearly defined, in the sense of the well-known explicit formula, with variables we can dispense with.

These considerations, the usefulness of which will appear later, may be followed by some remarks on the Riemann curvature tensor. The difference $a_{k||l||i} - a_{k||i||l}$ is in any case a linear form in the a_m and the $a_{m||n}$; but because

$$2\left(a_{k||l||i} - a_{k||i||l}\right) a^k = \left(a_k a^k\right)_{||l||i} - \left(a_k a^k\right)_{||i||l} = 0 \tag{6}$$

- it holds for every scalar

$$S_{||l||i} - S_{||i||l} = 0 \tag{7}$$

because of axiom (III) and eq. (2) – the linear form in question actually only contains the a_m, and in

$$a_{k||l||i} - a_{k||i||l} = -G^{m}_{\cdot\, kli} a_m \tag{8}$$

is also

$$G_{mkli} = -G_{kmli} \text{ (and) } G_{kmli} = -G_{kmil}. \tag{9}$$

In this way we get the property of symmetry

$$\left\{ G_{mkli} \right\}_{[li]} = 0, \tag{10}$$

- being the left side, the sum of the expressions resulting from the cyclical exchange of k, l, i should mean – also without calculation from the following consideration: From (III) it follows that for skew-symmetric tensors $H_{kl} = -H_{lk}$ the agreement

$$\left\{ H_{kl||i} \right\} = \left\{ H_{kl|i} \right\}_{[kli]} \tag{11}$$

holds – because of the above proposition, it is sufficient to confirm this for the case $H_{kl} = a_k b_l - a_l b_k$.

After (II) and (III) then becomes

$$\left\{ a_{k||l||i} - a_{k||i||l} \right\}_{[kil]} = \left\{ (a_{k||l} - a_{l||k})_{[kli]} \right\} = \left\{ (a_{k|l} - a_{l|k})_{|i} \right\}_{[kli]} = 0, \tag{12}$$

with which (10) is proven. From eqs. (10) and (9) one gets as a consequence

$$G_{mkli} = G_{limk}. \tag{13}$$

Similarly, one also gets the right relationship

$$\left.\begin{aligned}\left(G^{kl}-\frac{1}{2}g^{kl}G\right)_{\|k}=0,\\ G_{kl}=G^{m}_{\bullet kml};\quad G=G_{kl}g^{kl}\end{aligned}\right\} \tag{14}$$

effortless if one completely avoids calculating with the complicated explicit expression for G_{mkli}.

4. Five-dimensional covariant differentiation

In 5D, among the permitted coordinate systems, there is none that has a plane at any point P, because

$$g_{\mu\nu|\lambda}X^{\lambda}=-2g_{\mu\nu} \tag{1}$$

of $g_{\mu\nu|\lambda}=0$ incompatibility. Axioms (I) to (IV) can, however, also be used here as the definition of the covariant derivative; and the covariant derivative of a projector is again a projector. The symmetry properties of the 4D curvature tensor G_{mkli} therefore also apply to the 5D curvature projector $R_{\mu e\sigma\tau}$.

For every projector b_ν applies

$$X^{\nu}_{\|\mu}b_{\nu}+X^{\nu}b_{\mu\|\nu}=0. \tag{2}$$

Because it is according to eq. (3), Sec. 2:

$$X^{\nu}_{|\mu}b_{\nu}+X^{\nu}b_{\mu|\nu}=0; \tag{2'}$$

and because of (III) and eq. (2), Sec. 3 the left side of eq. (2') is the same

$$\left(X^{\nu}b_{\nu}\right)_{\|\mu}+X^{\nu}\left(b_{\mu\|\nu}-b_{\nu\|\mu}\right),$$

Thus, equal to the left side of eq. (2).

The proposition of Sec. 3 applies – as can be easily seen – for projectors; and consequently, a general formula for any projectors can be derived from eq. (2) because of (I). For projectors $B_{e\sigma}$ with two indices will be

$$X^{\nu}_{\|e}B_{\nu\sigma}+X^{\nu}_{\|\sigma}B_{e\nu}+X^{\nu}B_{e\sigma\|\nu}=0; \tag{3}$$

and for $B_{e\sigma}=g_{e\sigma}$ this gives the *Killing equation*

$$X_{\sigma\|e}+X_{e\|\sigma}=0. \tag{4}$$

We use the term

$$X_{e\sigma}=2X_{\sigma\|e}=X_{\sigma|e}-X_{e|\sigma}, \tag{5}$$

and write eq. (2) now as

$$X^{\nu} b_{\mu\|\sigma} = \frac{1}{2} X_{\nu\mu} b^{\nu}. \tag{6}$$

By the way is

$$\left\{ X_{\sigma\|e\|\nu} \right\}_{[\sigma e\nu]} = 0 \tag{7}$$

and

$$X^{\nu} X_{e\sigma\|\nu} = 0. \tag{7'}$$

With the definition

$$a_{\mu\|\sigma\|\tau} - a_{\mu\|\tau\|\sigma} = -R^{\nu}_{\bullet\mu\sigma\tau} a_{\nu} \tag{8}$$

we get for $a_{\mu} = X_{\mu}$ because of eqs. (4) and (7):

$$R^{\nu}_{\bullet\mu\sigma\tau} X_{\nu} = \frac{1}{2} X_{\tau\sigma\|\mu}; \tag{9}$$

$$R_{\nu\mu\sigma\tau} X^{\nu} X^{\sigma} = \frac{1}{2} J_{\|\tau\|\mu} + \frac{1}{4} X_{\tau\sigma} X^{\sigma}_{\bullet\mu}; \tag{10}$$

for eq. (10) is the fact

$$J_{|\lambda} = J_{\|\lambda} = 2 X_{\mu\|\lambda} X^{\mu} = X_{\lambda\mu} X^{\mu} \tag{11}$$

is used, from which one also obtains

$$J_{\|\mu\|\nu} X^{\mu} X^{\nu} = \frac{1}{2} J_{|\mu} J_{|\nu} g^{\mu\nu}. \tag{11'}$$

The tapered curvature projector then gives $R_{\mu\nu} = R^{\lambda}_{\cdot\mu\lambda\nu}$

$$R_{\mu\nu} X^{\mu} X^{\nu} = \frac{1}{2} J_{\|\mu\|\nu} g^{\mu\nu} - \frac{1}{4} X^{\mu\nu} X_{\mu\nu}; \tag{12}$$

$$R_{\mu\nu} X^{\nu} = \frac{1}{2} X^{\nu}_{\bullet\mu\|\nu}. \tag{12'}$$

We also put together the following formulas:

$$\left.\begin{aligned} X_{(\nu)\mu} &= X_{\nu\mu} + \frac{1}{J} X_{\nu} J_{|\mu}, \\ X_{(\nu)(\mu)} &= X_{\nu\mu} + \frac{1}{J} [X_{\nu} J_{|\mu} - X_{\mu} J_{|\nu}]. \end{aligned}\right\} \tag{13}$$

Because

$$a^{(\lambda)} b_{(\lambda)} = a^{\lambda} b_{(\lambda)} = a^{\lambda} b_{\lambda} - \frac{1}{J} \left(X_{\sigma} a^{\sigma} \right) \left(X^{e} b_{e} \right)$$

will eventually

$$\left. \begin{aligned} X_{(\nu)\lambda} X^{(\nu)}_{\bullet\mu} &= X_{\nu\lambda} X^{\nu}_{\bullet\mu} - \frac{1}{J} J_{|\lambda} J_{|\mu}; \\ X_{(\nu)(\mu)} X^{(\nu)(\mu)} &= X_{\nu\mu} X^{\nu\mu} - \frac{2}{J} J_{|\nu} J_{|\mu} - X_{\mu} J_{|\nu} g^{\mu\nu}. \end{aligned} \right\} \tag{13'}$$

We want to call two tensors $A^{\nu\ldots}_{\mu\ldots}, B^{\nu\ldots}_{\mu\ldots}$ congruent if their simplifications are equal:

$$A^{\nu\ldots}_{\mu\ldots} \equiv B^{\nu\ldots}_{\mu\ldots} \ \text{ means } \ A^{k\ldots}_{l\ldots} = B^{k\ldots}_{l\ldots}. \tag{14}$$

Congruences can be added and multiplied like equations; but covariant differentiation cannot be carried out in them.

Therefore, we consider a new differentiation process, which fulfills the axioms (I), (II) and (IV) and is then given uniquely by

$$a_{\mu|||\lambda} = a_{\mu||\lambda} + \frac{1}{2J} \left(X_{\mu\lambda} X^{\sigma} - X^{\sigma}_{\bullet\lambda} X_{\mu} \right) a_{\sigma}. \tag{15}$$

Since, according to (Iv), this differentiation should be interchangeable with the operation of pulling the tensor indices up and down:

$$g_{\mu\nu||\lambda} = 0, \tag{16}$$

so μ in eq. (15) above can also be written:

$$a^{\mu}_{|||\lambda} = a^{\mu}_{||\lambda} + \frac{1}{2J} \left(X^{\mu}_{\bullet\lambda} X_{\sigma} - X_{\sigma\lambda} X^{\mu} \right) a^{\sigma}. \tag{17}$$

We get from eqs. (15), (17):

$$\left. \begin{aligned} a^{\mu}_{|||\lambda} &= a^{\mu}_{||\lambda} + X^{\mu}_{\bullet\lambda} \frac{X_{\sigma} a^{\sigma}}{2J} \equiv a^{(\mu)}_{||\lambda}, \\ a_{\mu|||\lambda} &\equiv a_{\mu||\lambda} + X_{\mu\lambda} \frac{X^{\sigma} a_{\sigma}}{2J} \equiv a_{(\mu)||\lambda}, \end{aligned} \right\} \tag{18}$$

which, by the way, generally results in

$$T^{\nu_1 \ldots \nu_\mu}_{\mu_1 \ldots \mu_m|||\lambda} \qquad T^{(\nu_1)\ldots(\nu_\mu)}_{(\mu_1)\ldots(\mu_m)||\lambda}. \tag{18'}$$

Because $X^{(\mu)} = 0$ becomes

$$X^{\mu}_{\bullet|||\lambda} \equiv 0. \tag{19}$$

This differentiation may therefore be carried out in congruences: It also follows from eq. (14)

$$A^{\nu....}_{\mu...|||\lambda} \equiv B^{\nu....}_{\mu...|||\lambda}. \quad (20$$

Furthermore, because of eq. (7), Sec. 2:

$$S_{|\mu|||\nu} = S_{|\mu||\nu}; \tag{21}$$

is finite

$$\left\{ \left(\frac{X_{\mu\nu}}{J} \right)_{|||\lambda} \right\}_{|\mu\nu\lambda} = 0; \tag{22}$$

in addition, see eq. (7).

5. Relationship of the curvature tensor

Analogous to eq. (8), Sec. 3 we deduce (using eq. (21), Sec. 4):

$$a_{\mu|||\sigma|||\tau} - a_{\mu|||\tau|||\sigma} \equiv -G^{\nu}_{\bullet\mu\sigma\tau} a_{\nu}; \tag{1}$$

and we claim that the shortening of the $G_{\cdot\mu\sigma\tau}$ occurring here is equal to the 4D curvature tensor $G^{m}_{\cdot kli}$. The proof is as follows:

The shortening of $T^{\nu...}_{\mu...|||\lambda}$ obviously only depends on the shortening of $T^{\nu...}_{\mu...|}$: More precisely, it should be noted that it is equal to the 4D covariant derivative of this latter shortening.

We prove this main theorem by showing that the abbreviation of $T^{\nu...}_{\mu...|||\lambda}$ fulfills those four axioms from axioms (I) to (IV) which uniquely characterize the covariant (4D) derivative.

Axiom (I) needs no discussion; Axiom (IV) is done by referring to eq. (16), Sec.4; We confirm axiom (II) as follows: Firstly

$$a_{\mu|||\lambda} \equiv a_{(\mu)|||\lambda} \equiv a_{(\mu)||\lambda};$$

in a point P, where $a_k = 0$, so will $a_{\mu|||\lambda} \equiv a_{(\mu)|\lambda}$; and $a_{(\mu)|\lambda} = \left(g^{k}_{\mu} a_k \right)_{|\lambda} = g^{k}_{\mu} a_{k|\lambda} = g^{k}_{\mu} g^{l}_{\lambda} a_{k|l}$. But the shortening of this is $a_{k|l}$.

Axiom (III) is confirmed as follows: The shortening of $a_{\mu|||\nu} - a_{\nu|||\mu}$ becomes $a_{k|l} - a_{l|k}$. Because of eq. (12), Sec. 2, we have $g^{k}_{\mu|\nu} = g^{k}_{\nu|\mu}$, so

$$a_{(\mu)|||\nu} - a_{(\nu)|||\mu} \equiv a_{(\mu)|\nu} - a_{(\nu)|\mu} \equiv g^{k}_{\mu} a_{k|\nu} - g^{l}_{\nu} a_{l|\mu} \equiv g^{k}_{\mu} g^{l}_{\nu} \left(a_{k|l} - a_{l|k} \right)$$

This proves the main theorem mentioned.

As an application: by eq. (22), Sec. 4, is

$$\left\{\left(\frac{X_{kl}}{J}\right)_{\|l}\right\}_{[kli]} = 0; \tag{2}$$

so that the 6-vector $X_{kl} = -X_{lk}$ is the rotation of a 4-vector.

The relation of the 4D curvature tensor to the 5D congruence differentiation asserted in axiom (I) requires only the following consideration after the proof of the main theorem: The coefficient system $G^{\nu}_{\cdot\mu\sigma\tau}$ in axiom (I) is already determined in the sense of congruence by the fact that axiom (I) holds in particular for all vectors $a_\nu = a_{(\nu)}$ orthogonal to X_ν. We can therefore write the right-hand side axiom (I) after the theorem of Sec. 2 as follows:

$$-G^{(\nu)}_{\bullet\mu\sigma\tau}a_{(\nu)} = --G^{k}_{\bullet\mu\nu\tau}a_k. \tag{3}$$

The rest follows from the above. –

We calculate

$$a_{\mu\|\|\sigma\|\|\tau} \equiv a_{(\mu)\|\sigma\|\|\tau} \equiv a_{(\mu)\|\sigma\|\tau} + \frac{1}{2J}\left[X_{\mu\tau}X^e a_{(e)\|\sigma} + X_{\sigma\tau}X^e a_{(\mu)\|e}\right];$$

With eq. (6) Sec. 4 and $X^e a_{(e)} = 0$ gives the further

$$a_{\mu\|\|\sigma\|\|\tau} \equiv a_{(\mu)\|\sigma\|\tau} + \frac{1}{4J}\left[X_{\mu\tau}X^{e}_{\bullet\sigma} + X_{\sigma\tau}X^{e}_{\bullet\mu}\right]a_{(e)}. \tag{4}$$

After that will finally

$$G^{\nu}_{\bullet\mu\sigma\tau} \equiv R^{\nu}_{\bullet\mu\sigma\tau} - \frac{1}{4J}\left[X_{\mu\tau}X^{\nu}_{\bullet\sigma} - X_{\mu\sigma}X^{\nu}_{\bullet\tau} + 2X_{\sigma\tau}X^{\nu}{}_{\mu}\right]. \tag{5}$$

We also list the following applications of the above:

$$J_{|\mu}J_{|\nu}g^{\mu\nu} = J_{|(\mu)}J_{|(\nu)}g^{\mu\nu} = J_{|k}J_{|l}g^{kl}; \tag{6}$$

$$\left.\begin{aligned} J_{\|\mu\|\nu}g^{\mu\nu} &= J_{\|\mu\|\nu}g^{(\mu)(\nu)} + \frac{1}{J}J_{\|\mu\|\nu}X^{\nu}X^{\nu} \\ &= J_{\|\|\mu\|\|\nu}g^{(\mu)(\nu)} + \frac{1}{2J}J_{|\mu}J_{|\nu}g^{\mu\nu} \\ &= J_{\|k\|l}g^{kl} + \frac{1}{2J}J_{|k}J_{|l}g^{kl}; \end{aligned}\right\} \tag{7}$$

For (7) see eq. (8') Sec. 2 and eq. (11') Sec. 3.

Part II. Physical Application

6. Physical interpretation

After the introduction of suitably dimensioned units, we interpret the $g_{kl|}$ as Einstein's components of the gravitational field and the 6-vector

$$F_{kl} = \frac{X_{kl}}{J} \tag{1}$$

the electromagnetic field strengths.

Finally, we set

$$\frac{Jc^2}{2} = x = \frac{8\pi f}{c^2}. \tag{2}$$

Field equations are then to be set up, and in analogy to Einstein's field equations $G_{kl} = 0$ of the pure gravitational field one will first take

$$R_{\mu\nu} = 0 \tag{3}$$

to consider what is equivalent to $15 = 10 + 4 + 1$ as 4D equations in the manner described in eq. (25), Sec. 2, which we now want to develop.

From eq. (5), Sec. 5 we take:

$$G^{(\nu)}_{\bullet\mu(\nu)\tau} \equiv R^{(\nu)}_{\bullet\mu(\nu)\tau} - \frac{3}{4J} X_{(\nu)\tau} X^{\nu}_{\bullet\mu}. \tag{4}$$

Now according to the formulas of eqs. (10), (13'), (21) from Sec. 4:

$$\left.\begin{aligned} R^{(\nu)}_{\bullet\mu(\nu)\tau} &= R^{\nu}_{\bullet\mu\nu\tau} - \frac{1}{J} X^{\nu} X^{\sigma} R_{\nu\mu\sigma\tau} \\ &= R_{\mu\tau} - \frac{1}{2J} J_{\|\mu\|\tau} - \frac{1}{4J} X_{\tau\sigma} X^{\sigma}_{\bullet\mu} \\ &\equiv R_{\mu\tau} - \frac{1}{2J} J_{\|\|\mu\|\|\tau} - \frac{1}{4J} X_{\tau(\sigma)} X^{(\sigma)}_{\bullet\mu} + \frac{1}{4J^2} J_{|\tau} J_{|\mu}; \end{aligned}\right\} \tag{5}$$

also

$$G_{kl} = R_{kl} - \frac{J}{2} F_{ki} F^{l}_{i} - \frac{1}{2} \frac{J_{\|k\|l}}{J} + \frac{1}{4} \frac{J_{|k} J_{|l}}{J^2} \tag{6}$$

On the other hand, we calculate according to the formulas (12'), (7'), (18) of Sec. 3:

$$\left.\begin{aligned} R_{\mu\nu} X^{\nu} &= \frac{1}{2} X_{\lambda\mu\|\nu} g^{\lambda\nu} = \frac{1}{2} X_{\lambda\mu\|\nu} g^{(\lambda)(\nu)} \\ &\equiv \frac{1}{2}\left[X_{\lambda\mu\|\|\nu} - X_{\mu\nu} \frac{X^{\sigma} X_{\lambda\sigma}}{2J} \right] g^{(\lambda)(\nu)} \\ &\equiv \frac{1}{2}\left[X_{\lambda\mu\|\|\nu} - \frac{1}{2J} X_{\mu\nu} J_{|\lambda} \right] g^{(\lambda)(\nu)} \end{aligned}\right\} \tag{7}$$

or

$$2R_{k\nu}X^{\nu} = \left(JF^{i}_{\bullet k}\right)_{\|i} + \frac{1}{2}F^{i}_{\bullet k}J_{|i}. \tag{8}$$

Finally, from eqs. (12), (13’), Sec. 4 and eqs. (6), (7), Sec. 5 we get:

$$\left.\begin{aligned} R_{\mu\nu}X^{\mu}X^{\nu} &= \frac{1}{2}J_{\|\mu\|\nu}g^{(\mu)(\nu)} - \frac{1}{4J}J_{|\mu}J_{|\nu}g^{\mu\nu} - \frac{1}{4}X^{(\mu)(\nu)}X_{(\mu)(\nu)} \\ &= \frac{1}{2}J_{\|k\|l}g^{kl} - \frac{1}{4J}J_{|k}J_{|l}g^{kl} - \frac{J^2}{4}F^{kl}F_{kl}. \end{aligned}\right\} \tag{9}$$

If the field equations in 3D have the simple form $R_{\mu\nu} = 0$, this means according to 4D eqs. (6), (8), (9), if we now introduce eq. (2):

$$G_{kl} + \frac{x}{c^2}F_{ki}F^{i}_{l} = -\frac{1}{2x}\left(x_{\|k\|l} - \frac{x_{|k}x_{|l}}{2x}\right); \tag{6*}$$

$$\frac{x}{c^2}F_{ki}F^{kl} = \frac{g^{kl}}{x}\left(x_{\|k\|l} - \frac{x_{|k}x_{|l}}{2x}\right); \tag{9*}$$

$$F^{i}_{\bullet k\|i} = -\frac{3}{2}\frac{x_{|i}}{x}F^{i}_{\bullet k}. \tag{8*}$$

From (8*) we get Maxwell’s equations in the case for $X =$ const. On the other hand, we can combine eqs. (6*), (9*) into

$$G_{kl} - \frac{1}{2}g_{kl}G + \frac{x}{c^2}\left(F_{ki}F^{i}_{l} - \frac{1}{4}g_{kl}F_{im}F^{im}\right) = -\frac{1}{2x}\left(x_{\|k\|l} - \frac{x_{|k}x_{|l}}{2x}\right) + \frac{g^{kl}}{2x}g^{im}\left(x_{\|i\|m} - \frac{x_{|i}x_{|m}}{2x}\right) \tag{10}$$

which if the right side is neglected - again corresponding to $x =$ const - leads to the field equations of the combined Einstein-Maxwell theory.

However, the field equations $R_{\mu\nu} = 0$ only have Einstein's gravitational equations $G_{kl} = 0$ together with $x =$ const for the solution when the electromagnetic field $F_{kl} = 0$ exact; otherwise, i.e. at $F_{kl} \neq 0$, there will always be small changes in x, which, however, are beyond practical measurability.

As is well known, the equations $G_{kl} = 0$ are the Lagrangian equations of the invariant variation problem with the integrant $G\sqrt{-g}$; and correspondingly the equations $R_{\mu\nu} = 0$ belong to

$$\delta\int R\sqrt{-\overset{5}{g}}\mathrm{d}X = 0; \qquad \overset{5}{g} = \text{ Det } |\, g_{\mu\nu} \,|. \tag{11}$$

The older version of the projective theory of relativity [7] added the secondary condition $J =$ const, to its field equations in order to take into account the assumption $x =$ const. The field equations were then determined in such a way that (11) should apply with this secondary condition. It resulted in

$$P_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

only for eq. (14) instead of eq. (15)

$$P_{\mu\nu} = X_{\mu} X_{\nu} \frac{P_{e\sigma} X^{e} X^{\sigma}}{J^{2}},$$

and that is equivalent to the classical equations of the combined Einstein-Maxwell theory that result from eqs. (8) and (10) by $x =$ const.

In the context of the current theory, it must be emphasized that there is no compelling reason to mark the variation problem (11), since in addition to the invariant R there are three other simple invariants J and $J_{|\mu} J_{|\nu} g^{\mu\nu}$ and $X_{\mu\nu} X^{\mu\nu}$ available. For example, instead of eq. (11) we can take into account the variation problem;

$$\delta \int \sqrt{J} R \sqrt{-\overset{5}{g}} \mathrm{d}X = 0 \tag{12}$$

which results in the field equations in place of $R_{\mu\nu} = 0$.

$$R_{\mu\nu} + \frac{J_{||\mu||\nu}}{2J} - \frac{J_{|\mu|\nu}}{4J^{2}} = \frac{X_{\mu} X_{\nu}}{J} \cdot \frac{R}{2} \tag{13}$$

Although the authors concerned with the problem of the “unified field theory” generally saw their goal in determining a clearly distinguished form of the field equations or the corresponding problem of variation, we should radically take the view that this goal is unattainable. Theoretical approaches, which ignore not only quantum phenomena but also the existence of spinor fields, should not set so far ambitious goals. We should therefore regard the empirical point of view as the only appropriate one, and determine the choice of field equations from experience albeit by utilizing the fundamental conviction that the field equations are simpler in the 5D formulation than in their equivalent 4D version.

According to G. Ludwig [4], a simple recipe can be given according for which a 5D variation problem

$$\delta \int L \sqrt{-\overset{5}{g}} \mathrm{d}X = 0, \tag{14}$$

where L is an invariant formed from the $g_{\mu\nu}$ (as independent functions) and must be converted directly into an equivalent 4D variation principle, one can replace eq. (14) with

$$\delta \int \sqrt{x} L \sqrt{-\overset{4}{g}} \mathrm{dx} = 0, \tag{15}$$

where L is now expressed as a function of the following independent variables: g_{kl}, x and ϕ_k = 4-potential (whose rotation is F_{kl}).

For example, in the case of $L = R$ in eq. (13), the formula is

$$R = G + \frac{x}{2c^2} F_{kl} F^{kl} + \frac{x_{\|k\|l} g^{kl}}{x} - \frac{x_{|k} x_{|l} g^{kl}}{2x^2} \tag{16}$$

to use, which correspond to the ones already used above can be obtained.

7. Inductive cosmology

In order to be able to test the statements of our field equations about the gravitational invariant x against experience, we have to base a certain idea of the cosmological model realized in nature. Although the facts available for this have so far found many different assessments, a largely clear picture emerges, provided that a principle is used which we want to call the Dirac principle [8]. After that, the measures characterizing the cosmos find their analogous expression as multiples of the microphysical elementary values: quantum of action h, elemental length 10^{-13} cm, elementary time 10^{-23} sec. (The associated mass value is then somewhat smaller than the proton mass m_p). Age of the cosmos, that is, the period of about 4 to 5 billion years, then receives the approximate numerical value 10^{40}; the world radius as well, the cosmic mass about 10^{80}. The average or maximum mass of the stars is about 10^{60}. These gigantic dimensionless numbers appearing here are now basically supposed to be functions - namely as powers - of the age of the world 10^{40}: This is the only way to avoid the extremely unsatisfactory idea that even dimensionless coefficients of an enormous order of magnitude occur in the elementary laws of nature.

Dirac has explained the gravitational invariant x, in this sense, since it, made dimensionless, has the order of magnitude 10^{-40}, as inversely proportional to the age of the world. With the same justification, however, we have to consider the mean mass density in the cosmos: $\mu \approx 10^{-28}$ g cm^{-3}, also inversely proportional to the age of the world, since it is also about 10^{-40} in elementary measure. On the other hand, the order of magnitude of the world radius requires proportionality with time - with a factor of order c, in agreement with the Hubble effect. According to this (or also directly by reference to its specified order of magnitude), growth with the square of the age of the world can be inferred for the world mass. The average or maximum star mass should increase accordingly with $t^{3/2}$ (t = age of the world), and the number 10^{20} of the existing stars with $t^{1/2}$.

The overall picture that emerges is that of a closed Riemannian space whose radius of curvature increases proportionally with the age of the world, while the total mass within it increases with t^2. The number of stars populating it increases with $t^{1/2}$, and the newly emerging stars each have a mass proportional to $t^{3/2}$. This picture, however, requires more detailed elaboration with regard to the spiral nebulae, the number of which should logically be set as proportional to $t^{1/4}$ while the subsequent emergence of spiral nebulae to be expected would result in star numbers in the largest spiral nebulae, which were also proportional to $t^{1/4}$. Incidentally (after friendly verbal advice from Mr. Gora) the angular momentum of spiral nebulae in the case of the Milky Way and the Andromeda Nebula reaches about the order of magnitude $10^{100} \sim t^{5/2}$. In the case of single stars or double stars (or the planetary system), on the other hand, we find angular momentum which is variable around the order of magnitude $10^{80} \sim t^2$.

The doubts that initially arise against the idea of a growing world mass on the basis of the energy law can be eliminated for the time being by the reference, that in the illustrated picture the positive rest energy is just of the same order of magnitude as the amount of the negative potential gravitational energy. However, only a precise calculation of cosmological models on the basis of a corresponding field theory can clarify whether this reference is actually made denoting a way to solve the problem.

An application of Dirac's principle that is independent of the cosmological context is offered in nuclear physics. The meson has a lifetime that is almost the order of magnitude 10^{20} greater than the elementary time, and therefore (according to Blackett) than should be assumed proportionally with $t^{1/2}$.

According to this, the determination of the age of very old geological layers had to result in different values if one used an alpha emitter on the one hand and a beta emitter on the other hand as a clock. This point has been discussed in more detail by Houtermans and the author [9]. The measurements that Houtermans and Haxel have carried out on the rubidium since then have shown that both types of clocks are in agreement within the accuracy that has been achieved up to now. If one nevertheless considers Dirac's principle to be correct in this particular application, one must conclude that there is still a noticeable difference between the age of the world and that of the earth (about 3 billion years) (about 30-50 %). But in this case too, it should not be impossible to detect the difference in question if the measurements are further sharpened. It would greatly enrich our evidence for a quantitative theory of the early stages of our cosmos.

The order of magnitude 10^{20} also occurs when the moment of rotation of the earth or the sun is divided by the corresponding magnetic dipole moment; and Blackett's [10] conjecture of a fundamental law that is present here - the occurrence of a magnetic moment which is approximately equal to a $\sqrt{x}$ torque for every rotating ball - poses particularly attractive problems for physics.

The general character of the field equations considered above - both the simple $R_{\mu\nu} = 0$ and the equations (13), Sec. 6 - agrees well with the requirements of the model explained, insofar as they suggest such approaches to their solution, in which in a homogeneous space x is assumed to be the power t^{-a} of the world age t. Because then in eqs. (6*), (9*) Sec. 6, the right-hand sides are proportional to t^{-2}; and for G_{kl} proportionality with t^{-2} is precisely the expression of an increase in the world radius proportional to t. For the components of the tensor $F_{ki}F_{l}^{\cdot i}$ or their mean values, the result is proportionality with t^{a-2}, and that is precisely what appears plausible on the basis of a primitive energy balance: an energy density $\varepsilon \sim t^{a-2}$, i.e. total energy $\sim \varepsilon R_0^3 \sim t^{a+1}$ ($R_0 =$ world radius), became a negative potential gravitational energy $\sim x\left(\varepsilon R_0^3\right)^2 R_0^{-1} \sim t^{a+1}$ standing opposite in a compensating manner.

These considerations seem to show that we are on the right track. Only the complete calculation of cosmological models can decide whether the field equations can already be regarded as final or whether they still require modifications.

8. The cosmological link

As is well known, Einstein's field equations have, $G_{kl} = 0$ the unwelcome property that they - even in their generalization corresponding to the presence of matter - do not lead to a closed Riemannian space as a solution. The modified vacuum field equation

$$G_{kl} - \frac{1}{2} g_{kl} G = \lambda g_{kl}, \tag{1}$$

a problem of variation

$$\delta \int (G - \lambda) \sqrt{-g} \, \overset{4}{\mathrm{d}x} = 0 \tag{2}$$

accordingly, is well suited to explain the meaning of Dirac's principle: In order to obtain Einstein's cylinder world as the solution to the field equations, one must (in addition to the matter tensor) insert a coefficient into the field equations, which determines the order of magnitude of the world radius $R_0 : \lambda$

in eq. (1) is known to be equal to R_0^{-2}. So, in these field equations (1), if they are written in natural microphysical units, a dimensionless number of the order of 10^{80} occurs.

The projective field equations $R_{\mu\nu} = 0$ however, as was first recognized by Heckmann, likewise do not allow any positively curved space as a solution and must therefore still be changed according to our program. The form of the field equations discussed by Ludwig and Muller [4] corresponds to a variation problem

$$\delta \int \sqrt{J} \left(R - \lambda \frac{J_{|\mu} J_{|\nu} g^{\mu\nu}}{J^2} \right) \sqrt{-\overset{5}{g}} \mathrm{d}X = 0, \tag{3}$$

with a coefficient λ, which has an effect similar to that of Einstein's in eq. (1), but with the fundamental difference from our point of view that this λ is easily a dimensionless number to which the order of magnitude (1) can be assigned. There is no reason to fix a certain value λ but it turns out that λ must be $> 3/2$ in order to get a Riemannian space as a possible solution. Furthermore, in contrast to eq. (2), (3) is not tailored from the outset to a cylinder world, but to an expanding one (corresponding to the fact that Dirac's principle basically excludes a cylinder world). Because the approach $J \sim t^{-a}$ already results from eq. (3) that $R \sim t^{-2}$ must be.

Since the projective relativity theory, in contrast to 4D, includes the electromagnetic field from the start, we can use the field equations arising from eq. (3) to construct a cosmological model in which only electromagnetic radiation is present instead of matter (light quantum world). Since the calculation of this model is easily accessible in the work of Ludwig and Muller [4]. Details of the calculation will not be discussed here. We will briefly discuss the main results in the next paragraph.

The setting up of a model, which corresponds to the cosmos in its real form, requires a consideration of the matter, and naturally one must first be content with a method of treatment which has the character of the preliminary to a much higher degree than what has been discussed so far.

Since the problems of matter and its atomistics have long been regarded as essential tasks of the "unified field theory", our position on this should be briefly specified. Compared to the older view that it is the task of continuum field theory to explain the material elementary particles as (real or approximate) singularities of the field, two objections must be raised on the basis of the results of quantum theory. For example, the existence of corpuscular electrons can be understood as fundamentally analogous to the existence of corpuscular light quanta; a theory that disregards the quantification of the electromagnetic field is guilty of inconsequentiality if it nevertheless tries to describe matter as corpuscular. Second, we know that spinor fields are required to describe electrons and other particles with half-integer spin: A theory that only works with tensors is therefore unable to give an account of such particles from the outset. The problem of the structure of matter can only be attacked as a quantum theoretical problem; however, the investigation of the singularities of the solutions to the field equations remains of considerable importance in this context.

A consistent inclusion of matter in quantum free field theory can only take place in such a way that the wave functions of matter are also taken into account. Whether or to what extent this program can be carried out in the sense of an expansion of the geometry (for which Schrödinger's ideas with regard to the meson field seem to give meaningful approaches) is a question that goes so far that it seems appropriate to completely address the questions of cosmology pursued here to separate.

It is therefore entirely logical that Ludwig introduced matter into the theory presented above in a provisional way in such a way that he presented it as the simplest possibility using a scalar wave field.

After this it is possible to fully calculate a model that corresponds to our actual cosmos.

9. Cosmology and star formation

The complete calculation presented by Ludwig and Müller has led to the following results: If we assume that the matter is practically immobile (in a corresponding coordinate system), then the radius e of the 4D sphere, whose 3D surface is the Riemannian space, increases according to measure

$$e = ct\frac{2}{\sqrt{2\lambda - 3}}; \tag{1}$$

is furthermore, with a normalization constant a_0 that remains open:

$$x = 8 \cdot \frac{\lambda - 1}{2\lambda - 3} \cdot \frac{1}{ea_0}, \tag{2}$$

and the mass density becomes

$$\mu = \frac{a_0}{e}, \tag{3}$$

which gives a total world mass $M = 2\pi^2 e^2 a_0$.

In fact, the result is that picture of the cosmos as a whole which we explained above as the probable one from the empirical point of view: Both x and μ are inversely proportional to the world factor t; and the world mass grows with t^2.

It was emphasized above that these proportionalities are reasonable with regard to the energy principle, because an elementary consideration shows that here the negative potential energy of gravity is constantly proportional to the world mass (= sum of all stellar masses), so that a compensation to a total energy of zero can be considered conceivable. But of course, only the real calculation according to the presupposed, exact relativistic field equations could clarify that the possibility suggested by these elementary considerations can really be carried out without any problems.

But if we now assume that matter is at such a high temperature that it practically moves with the speed of light - or if we consider a matter-free "light quantum world" - we get instead:

$$e = ct\sqrt{\frac{3}{2(\lambda - 1)}}; \tag{4}$$

$$x = \frac{1}{e^2} \cdot \frac{6}{b_0}; \tag{5}$$

$$\mu = b_0 = \text{ const}; \tag{6}$$

Thus, total world mass $M = 2\pi^2 e^3 b_o$.

So here the gravitational invariant decreases faster, namely with t^{-2}; and in a manner corresponding to the elementary energy consideration, the law adapted to this results in a constancy of the mass density, i.e. an increase in the positive total mass with t^3. If, taking into account Dirac's principle, as explained above, we also want to ascribe a certain real significance to this second cosmological model, we must obviously assume the approximate order of magnitude of the nuclear density for μ. In doing so, we are using the second model in question as the foundation of a theory of star formation.

We see the main task of any theory of star formation in answering the question why there is a very clearly defined upper limit of the star masses in the cosmos. This task of the theory of star formation had not yet been clearly recognized - rather, one has repeatedly tried to understand the upper limit of the stellar masses for reasons of stability, in such a way that one imagined that even heavier stars would no longer be able to exist stationary for a long time, but rather tend to explode. This is undoubtedly a complete misunderstanding of the situation. On the one hand, it can be determined purely empirically that stars which noticeably exceed the normal limit mass are also capable of permanent existence - since there are indeed some exceptional cases of such stars. (Not to mention the controversial Trumpler stars). Incidentally, one can perhaps assume that the stars in question were only formed through the subsequent union of several normal stars. - On the other hand, theoretical attempts to deduce an instability or inability to exist in "excessively heavy" stars turn out to be unsuccessful on closer examination. This applies, for example, to the thesis, derived from the admirable Chandrasekar theory of degenerate stars, that stars that exceed a certain limit mass are not capable of degenerating [11].

So, we can only expect an answer to the real main question of star theory - why the stellar masses do not exceed about $10^{60} m_p$ - only from a theoretical analysis of the conditions in which stars are formed. In doing so, we completely abandon the traditional idea of the formation of stars through the condensation of dust and gas matter; It therefore means an important clarification that this idea has recently been deprived of its most important support insofar as Baade [12] was able to show that the supposedly "unresolved" parts of the spiral nebulae adjacent to us actually consist of individual stars - so that it is spiral nebulae which "had not yet condensed into stars" apparently do not exist. The theory of star formation now to be discussed is the only one which makes an attempt to attack this central problem of the mass value $10^{60} m_p$ in a way which is in full harmony with Dirac's principle.

The spacetime manifold of a cosmos, which is a Riemannian space growing with constant speed (radius proportional to time), obviously represents a cone. Let us now imagine that this cone, considered more closely, besides its one tip many other smaller ones, secondary points that grow out of the cone shell everywhere (think parallel with the main point). A plane $t =$ const will then, under certain circumstances, intersect this "cone with secondary points" in a multi-part 3D space; i.e., the 3D section will not only consist of the normal, large cosmos space, but also of a small one that exists independently of it "Special cosmos", which only connects with the great cosmos at later time values and then finally merges completely with it.

Let us now assume in particular that this special cosmos corresponds to the second model above during the duration of its separate existence, in its "prehistory" or its embryonic state.

During this prehistory, a time t_1 is found, counting from the origin of the Riemannian space in question proportional increase in its radius e_1 takes place; the gravitational constant x is (except for number factors of the order of magnitude 1) equal to t_1^{-2}, if x and t_1 are measured in microphysical units. The mass density μ must be constant of the order of magnitude 1 (nuclear density), the temperature is so high that the elementary particles have practically the speed of light. The mass of the special cosmos is $\sim t_1^3$. during its prehistory. The merging with the large cosmos, in which $x \sim t^{-1}$ is, can take place when x is the same size in both spaces, that is,

$$t_1^{-2} = t^{-1} \tag{7}$$

so that with "today's" values t even $t_1 = 10^{20}$ elementary times $\left(\simeq 10^{-3} \text{ sec}\right)$; and the mass M is then equal to $10^{60} m_p$.

We thus have a model for a conceivable process of star formation.

10. About the supernova problem

The work carried out by Baade and Minkowski [13] in the last few years gives us today a much more solid basis for assessing the problems of star formation, as were available in the first presentation [14] of the cosmology developed above. We are now definitely certain that the spiral nebulae also in the so-called "unresolved" parts consist of single stars and not of diffuse gas masses - the traditional idea of star formation by agglomeration of gas masses loses its strongest support. On the other hand, our knowledge of the three historical galactic supernovae of type I. In addition to the work mentioned, I am grateful to friendly communications from Mr. Baade for the knowledge of unpublished facts which have significantly influenced my views.

Unsöld [15] has enriched "cosmological physics" with the two surprising thoughts that 1. new stars are still emerging today and 2. supernovae may indicate star generation.

In particular, because of their chemical composition and brightness, the B-stars should be regarded as youthful stars; Genetic connections with the supernovae phenomenon are indicated by the parallel occurrence in the spiral arms. Unsöld's opinion gave me the courage to regard the supernovae as production in the sense of the above theory, which should therefore initially lead to the formation of B-stars. However, I no longer think that is likely.

It is precisely the supernovae heaped in the spiral arms of type II, which perhaps only includes particularly powerful ordinary novae; and, according to Baade, there are strong reasons for the fact that the "population I" that occurs in the spiral arms, to which the B and O stars belong, only emerged from "population II" through a subsequent transformation process - whereby perhaps it is not a question of additional new formation of stars, but rather, for example, a strong increase in the mass of certain stars through the collection of dark matter; this could probably explain those facts which Unsöld's reason for the low age of the B-stars.

I would therefore now prefer the idea that population II, which occurs in the globular clusters (especially, but not only their centers) and in the parts of the spiral nebula that do not belong to the sparse arms, is to be regarded as original, so that any new formations to it are to be considered fit. Now supernovae 1 do indeed belong to population II; and in any case, this includes the planetary nebulae, which can probably be interpreted as traces of Supernovae I. The hypothesis that these Supernovae I are indeed new generation seems debatable [16].

However, the theory advocated here does not make it necessary that new generations still occur in the spiral nebulae that we can observe. For the spiral nebula one will have to assume a method of formation similar to that explained above for the single star: That is, the formation of an initially isolated special cosmos, in which then at $x \sim \tau^{-4/3}$ had to be. (A theoretical interpretation of this exponent 4/3 has not yet been given). In this special cosmos the stars - each from a secondary special cosmos - should then arise in the manner described above; and it must first remain open whether later, subsequent incorporation of the spiral nebula into the general cosmos, any subsequent star formation at all. (Once this integration - initially in the form of a turbulent, exploding heap of "star gas" - has taken place, then understanding the further development into the typical spiral nebula according to von Weizsäcker [17] no longer presents any difficulties).

Regardless of this question, two points seem to speak strongly in favor of the theory of star formation considered here.

1. Investigations into the cosmic prevalence of the elements and the theoretical possibilities for interpretating the assumption of an initial state which combines a high density of matter - not far from the density of nuclear matter - with high temperature, and which then underwent dispersion and cooling through rapid expansion. Compare the considerations of Unsöld [15], Gamow [18], Jensen and Suess [19], Klein, Beskow and Treffenberg [20]. This is in good agreement with the theory represented here. On the other hand, it is uncertain whether it could be reconciled with a theory which relocates the formation of the chemical elements to a hypothetical original union of all matter present today (and imagined as already existing then).

2. After it has been proven (Ehmert, Forbush) that eruptive processes on the sun make small contributions to high-frequency radiation, a reasonable interpretation of this phenomenon will hardly be able to avoid reverting to the old Baade-Zwicky theory. In relation to visible radiation, novae will provide considerably more high radiation than the sun; and supernovae even more. However, since the energy density of high radiation is known to be 100 times that of extra-galactic light, each of the stars primarily responsible for high radiation must then deliver about 10^6 times more high radiation than visible light. The required energy supply is now so great that a regeneration process other than the one discussed here could hardly be sufficient.

At the time, Baade and Zwicky estimated with impressive precision that even with a type-1 supernova the visible light emission should be offset by a short-wave energy supply that is 10^3 times or even 10^6 times larger. The only process on an already existing star that could supply this energy would be the total conversion into a neutron star, which was considered by these authors at the time. But Minkowski's investigation of the Crab Nebula and its central star has clearly shown that although this is a white dwarf (extreme temperature), it is by no means a neutron star: these circumstances seem to suggest that we must indeed interpret the type-1 supernovae as new formations.

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[5] See Jordan P 1947 *The origin of the stars* Stuttgart and the work mentioned there
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[15] Unsöld A 1948 *Z. Astrophys*. **24** 278. The decisive points of this work were kindly made to me by Mr. Unsöld 1942 **96** 199; 1943 **97** 128. Made known in 1944.
[16] Bok B J und Reilly F E (Astrophys. J. **107** 255 [1947]) have pointed to the not uncommon occurrence of small isolated pieces of dark matter, with diameters of 1 to 10 light years. The opinion of these authors that the structures in question are spherical or elliptical is, however, according to the friendly communication of Mr. Baade, incorrect. Perhaps these isolated heaps of matter could have been created by the same process as the planetary nebulae, only with the difference that it came to a complete burst without a remaining central star.
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