Properties of a possible unification algebra

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Abstract. An algebra providing a possible basis for the standard model is presented. The algebra is generated by combining the trigintaduonion Cayley-Dickson algebra with the complexified space-time Clifford algebra. Subalgebras are assigned to represent multivectors for transverse coordinates. When a requirement for isotropy with respect to spatial coordinates is applied to those subalgebras, the structure generated forms a pattern matching that of the fermions and bosons of the standard model.

Contents

1. Introduction

For the Clifford algebra for the dimensionality used in string theories, $Cl_{1,9}(R)$, patterns matching the symmetries of the standard model can found, but it is harder to find a pattern matching the asymmetries of the standard model. This leads to the conjecture that the observed universe is the result of a random choice by nature of one of many possible compactification topologies. A model which generates the features of the observed universe as an inevitable outcome from a relatively simple mathematical structure would be more useful. An algebra with the potential to provide a basis for such a model can be assembled by extending the Clifford algebra of space-time using a Cayley-Dickson algebra.

Cawagas et al[1] analysed the trigintaduonion loop T_L, finding four isomorphy classes of sedenion-type subloops having asymmetric octonion-type subloop composition. Combining a graded Clifford algebra of the same size, such as $Cl_{1,4}(R) \cong Cl_4(C) \cong M_4(C)$ with the trigintaduonion algebra $\mathbb{T}$ generates an algebra with a pattern of subalgebras having a complex combination of symmetry and asymmetry, suggesting that it could be provide a basis for a useful unification model.

In previous papers [2][3] the structure generated by subloops of the loop generated as the product of T_L with unit elements of $M_4(C)$ was analysed. In [2] $M_4(C) \otimes \mathbb{T}$ was labelled $\mathbb{U}$. The Cayley tables of unit elements of $M_4(C)$ and of T_L can be aligned so that, if the signs of products are ignored, they match each other. A subalgebra of $\mathbb{U}$, labelled $\mathbb{W}$, having unit elements, each being the product of a unit element of $M_4(C)$ with the element of T_L aligned with it, was investigated. In [3] the sedenion-type subloops of T_L, when required to be "spatially equivalent" were found display a possible correspondence with fundamental particles of the standard model.

In this paper a combination of these approaches is considered. The loop of unit elements of $M_4(C)$ is labelled M_L. The loop of unit elements of $\mathbb{U} \cong M_4(C) \otimes \mathbb{T}$ is designated $U_L \cong M_L \otimes T_L$. Elements of M_L are assigned to represent unit elements for the complexification of the space-time Clifford algebra for positive spatial signature, $Cl_{3,1}(R) \otimes \mathbb{C} \cong Cl_4(C)$ (using negative spatial signature would generate similar results). For that assignment, subloops of U_L of the same order as M_L having elements which are products of "spatially equivalent" alignments of T_L with M_L are considered. It is postulated that these subloops correspond to the equivalent of a multivector for transverse complexified space-time coordinates. These subloops can be arranged in sets having the same scalar, transverse spatial bivectors and tranverse pseudovector.

The Loops package[4] for GAP4[5] has been used to investigate isomorphisms and isotopisms for T_L and its subloops.

2. Notation

2.1. Notation used for T_L

The notation for T_L for the Cayley table shown in Appendix A is setout in table 1. Cawagas et al[1] labelled the isomorphism types of sedenionic-type subloops of T_L as S_γ, S_α, S_β, S_L. In this paper these have been labelled using uppercase greek letters with numbered subscripts: Γ_1, $A_{1..7}$, $B_{1..7}$, $\Sigma_{1..15}$, as shown in table 2.

Table 1. Notation used to label elements for T_L, the loop of unit elements of $\mathbb{T}$

e_0	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	e_{11}	e_{12}	e_{13}	e_{14}	e_{15}
σ_o	σ_ι	$\sigma_\jmath$	σ_κ	λ_o	λ_ι	$\lambda_\jmath$	λ_κ	μ_o	μ_ι	$\mu_\jmath$	μ_κ	ν_o	ν_ι	$\nu_\jmath$	ν_κ
e_{16}	e_{17}	e_{18}	e_{19}	e_{20}	e_{21}	e_{22}	e_{23}	e_{24}	e_{25}	e_{26}	e_{27}	e_{28}	e_{29}	e_{30}	e_{31}
α_o	α_ι	$\alpha_\jmath$	α_κ	β_o	β_ι	$\beta_\jmath$	β_κ	γ_o	γ_ι	$\gamma_\jmath$	γ_κ	δ_o	δ_ι	$\delta_\jmath$	δ_κ

Table 2. Unit elements for sedenion-type subloops of T_L

	σ_o	σ_ι	$\sigma_\jmath$	σ_κ	λ_o	λ_ι	$\lambda_\jmath$	λ_κ	μ_o	μ_ι	$\mu_\jmath$	μ_κ	ν_o	ν_ι	$\nu_\jmath$	ν_κ	α_o	α_ι	$\alpha_\jmath$	α_κ	β_o	β_ι	$\beta_\jmath$	β_κ	γ_o	γ_ι	$\gamma_\jmath$	γ_κ	δ_o	δ_ι	$\delta_\jmath$	δ_κ
Γ_0	■	■	■	■	■	■	■	■																	■	■	■	■	■	■	■	■
A_0	■	■	■	■					■	■	■	■									■	■	■	■					■	■	■	■
B_0	■	■	■	■									■	■	■	■					■	■	■	■	■	■	■	■				
A_1	■	■			■	■			■	■			■	■					■	■			■	■			■	■			■	■
A_2	■		■		■		■		■		■		■		■			■		■		■		■		■		■		■		■
A_3	■			■	■			■	■			■	■			■		■	■			■	■			■	■			■	■	
B_1	■	■			■	■					■	■			■	■			■	■			■	■	■	■			■	■		
B_2	■		■		■		■			■		■		■		■		■		■		■		■	■		■		■		■	
B_3	■			■	■			■		■	■			■	■			■	■			■	■		■			■	■			■
A_4	■	■					■	■	■	■					■	■			■	■	■	■					■	■	■	■		
A_5	■		■			■		■	■		■			■		■		■		■	■		■			■		■	■		■	
A_6	■			■		■	■		■			■		■	■			■	■		■			■		■	■		■			■
B_4	■	■					■	■			■	■	■	■					■	■	■	■			■	■					■	■
B_5	■		■			■		■		■		■	■		■			■		■	■		■		■		■			■		■
B_6	■			■		■	■			■	■		■			■		■	■		■			■	■			■		■	■	
Σ_0	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■																
Σ_1	■	■	■	■	■	■	■	■									■	■	■	■	■	■	■	■								
Σ_2	■	■	■	■					■	■	■	■					■	■	■	■					■	■	■	■				
Σ_3	■	■	■	■									■	■	■	■	■	■	■	■									■	■	■	■
Σ_4	■	■			■	■			■	■			■	■			■	■			■	■			■	■			■	■		
Σ_5	■		■		■		■		■		■		■		■		■		■		■		■		■		■		■		■	
Σ_6	■			■	■			■	■			■	■			■	■			■	■			■	■			■	■			■
Σ_7	■	■			■	■					■	■			■	■	■	■			■	■					■	■			■	■
Σ_8	■		■		■		■			■		■		■		■	■		■		■		■			■		■		■		■
Σ_9	■			■	■			■		■	■			■	■		■			■	■			■		■	■			■	■	
Σ_{10}	■	■					■	■	■	■					■	■	■	■					■	■	■	■					■	■
Σ_{11}	■		■			■		■	■		■			■		■	■		■			■		■	■		■			■		■
Σ_{12}	■			■		■	■		■			■		■	■		■			■		■	■		■			■		■	■	
Σ_{13}	■	■					■	■			■	■	■	■			■	■					■	■			■	■	■	■		
Σ_{14}	■		■			■		■		■		■	■		■		■		■			■		■		■		■	■		■	
Σ_{15}	■			■		■	■			■	■		■			■	■			■		■	■			■	■		■			■

2.2. Notation used for M_L, the group of unit elements of $M_4(C) \cong Cl_{3,1}(C) \cong Cl1,3(C)$

The notation for unit elements of M_L is setout in table 3, for the unit matrix assignments and Cayley table shown in Appendix A.

TABLE 3. Notation used to label elements for M_L the group of unit elements of $M_4(C)$

e_0	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	e_{11}	e_{12}	e_{13}	e_{14}	e_{15}
S	L	M	N	U	X	Y	Z	V	D	E	F	T	P	Q	R
e_{16}	e_{17}	e_{18}	e_{19}	e_{20}	e_{21}	e_{22}	e_{23}	e_{24}	e_{25}	e_{26}	e_{27}	e_{28}	e_{29}	e_{30}	e_{31}
iS	iL	iM	iN	iU	iX	iY	iZ	iV	iD	iE	iF	iT	iP	iQ	iR

2.3. Notation for elements of $U_L \cong T_L \otimes M_L$, unit elements of $\mathbb{U}$

All unit elements of $\mathbb{U}$ are the product of an element of T_L with an element of M_L, as listed in table 4.

TABLE 4. Notation used to label elements of U_L, the loop of unit elements of $\mathbb{U}$

σ_oS	σ_oL	σ_oM	σ_oN	σ_oT	σ_oP	σ_oQ	σ_oR	σ_oU	σ_oX	σ_oY	σ_oZ	σ_oV	σ_oD	σ_oE	σ_oF	σ_oiS	σ_oiL	σ_oiM	σ_oiN	σ_oiT	σ_oiP	σ_oiQ	σ_oiR	σ_oiU	σ_oiX	σ_oiY	σ_oiZ	σ_oiV	σ_oiD	σ_oiE	σ_oiF
$\sigma_\iota S$	$\sigma_\iota L$	$\sigma_\iota M$	$\sigma_\iota N$	$\sigma_\iota T$	$\sigma_\iota P$	$\sigma_\iota Q$	$\sigma_\iota R$	$\sigma_\iota U$	$\sigma_\iota X$	$\sigma_\iota Y$	$\sigma_\iota Z$	$\sigma_\iota V$	$\sigma_\iota D$	$\sigma_\iota E$	$\sigma_\iota F$	$\sigma_\iota iS$	$\sigma_\iota iL$	$\sigma_\iota iM$	$\sigma_\iota iN$	$\sigma_\iota iT$	$\sigma_\iota iP$	$\sigma_\iota iQ$	$\sigma_\iota iR$	$\sigma_\iota iU$	$\sigma_\iota iX$	$\sigma_\iota iY$	$\sigma_\iota iZ$	$\sigma_\iota iV$	$\sigma_\iota iD$	$\sigma_\iota iE$	$\sigma_\iota iF$
σ_jS	σ_jL	σ_jM	σ_jN	σ_jT	σ_jP	σ_jQ	σ_jR	σ_jU	σ_jX	σ_jY	σ_jZ	σ_jV	σ_jD	σ_jE	σ_jF	σ_jiS	σ_jiL	σ_jiM	σ_jiN	σ_jiT	σ_jiP	σ_jiQ	σ_jiR	σ_jiU	σ_jiX	σ_jiY	σ_jiZ	σ_jiV	σ_jiD	σ_jiE	σ_jiF
$\sigma_\kappa S$	$\sigma_\kappa L$	$\sigma_\kappa M$	$\sigma_\kappa N$	$\sigma_\kappa T$	$\sigma_\kappa P$	$\sigma_\kappa Q$	$\sigma_\kappa R$	$\sigma_\kappa U$	$\sigma_\kappa X$	$\sigma_\kappa Y$	$\sigma_\kappa Z$	$\sigma_\kappa V$	$\sigma_\kappa D$	$\sigma_\kappa E$	$\sigma_\kappa F$	$\sigma_\kappa iS$	$\sigma_\kappa iL$	$\sigma_\kappa iM$	$\sigma_\kappa iN$	$\sigma_\kappa iT$	$\sigma_\kappa iP$	$\sigma_\kappa iQ$	$\sigma_\kappa iR$	$\sigma_\kappa iU$	$\sigma_\kappa iX$	$\sigma_\kappa iY$	$\sigma_\kappa iZ$	$\sigma_\kappa iV$	$\sigma_\kappa iD$	$\sigma_\kappa iE$	$\sigma_\kappa iF$
λ_oS	λ_oL	λ_oM	λ_oN	λ_oT	λ_oP	λ_oQ	λ_oR	λ_oU	λ_oX	λ_oY	λ_oZ	λ_oV	λ_oD	λ_oE	λ_oF	λ_oiS	λ_oiL	λ_oiM	λ_oiN	λ_oiT	λ_oiP	λ_oiQ	λ_oiR	λ_oiU	λ_oiX	λ_oiY	λ_oiZ	λ_oiV	λ_oiD	λ_oiE	λ_oiF
$\lambda_\iota S$	$\lambda_\iota L$	$\lambda_\iota M$	$\lambda_\iota N$	$\lambda_\iota T$	$\lambda_\iota P$	$\lambda_\iota Q$	$\lambda_\iota R$	$\lambda_\iota U$	$\lambda_\iota X$	$\lambda_\iota Y$	$\lambda_\iota Z$	$\lambda_\iota V$	$\lambda_\iota D$	$\lambda_\iota E$	$\lambda_\iota F$	$\lambda_\iota iS$	$\lambda_\iota iL$	$\lambda_\iota iM$	$\lambda_\iota iN$	$\lambda_\iota iT$	$\lambda_\iota iP$	$\lambda_\iota iQ$	$\lambda_\iota iR$	$\lambda_\iota iU$	$\lambda_\iota iX$	$\lambda_\iota iY$	$\lambda_\iota iZ$	$\lambda_\iota iV$	$\lambda_\iota iD$	$\lambda_\iota iE$	$\lambda_\iota iF$
λ_jS	λ_jL	λ_jM	λ_jN	λ_jT	λ_jP	λ_jQ	λ_jR	λ_jU	λ_jX	λ_jY	λ_jZ	λ_jV	λ_jD	λ_jE	λ_jF	λ_jiS	λ_jiL	λ_jiM	λ_jiN	λ_jiT	λ_jiP	λ_jiQ	λ_jiR	λ_jiU	λ_jiX	λ_jiY	λ_jiZ	λ_jiV	λ_jiD	λ_jiE	λ_jiF
$\lambda_\kappa S$	$\lambda_\kappa L$	$\lambda_\kappa M$	$\lambda_\kappa N$	$\lambda_\kappa T$	$\lambda_\kappa P$	$\lambda_\kappa Q$	$\lambda_\kappa R$	$\lambda_\kappa U$	$\lambda_\kappa X$	$\lambda_\kappa Y$	$\lambda_\kappa Z$	$\lambda_\kappa V$	$\lambda_\kappa D$	$\lambda_\kappa E$	$\lambda_\kappa F$	$\lambda_\kappa iS$	$\lambda_\kappa iL$	$\lambda_\kappa iM$	$\lambda_\kappa iN$	$\lambda_\kappa iT$	$\lambda_\kappa iP$	$\lambda_\kappa iQ$	$\lambda_\kappa iR$	$\lambda_\kappa iU$	$\lambda_\kappa iX$	$\lambda_\kappa iY$	$\lambda_\kappa iZ$	$\lambda_\kappa iV$	$\lambda_\kappa iD$	$\lambda_\kappa iE$	$\lambda_\kappa iF$
μ_oS	μ_oL	μ_oM	μ_oN	μ_oT	μ_oP	μ_oQ	μ_oR	μ_oU	μ_oX	μ_oY	μ_oZ	μ_oV	μ_oD	μ_oE	μ_oF	μ_oiS	μ_oiL	μ_oiM	μ_oiN	μ_oiT	μ_oiP	μ_oiQ	μ_oiR	μ_oiU	μ_oiX	μ_oiY	μ_oiZ	μ_oiV	μ_oiD	μ_oiE	μ_oiF
$\mu_\iota S$	$\mu_\iota L$	$\mu_\iota M$	$\mu_\iota N$	$\mu_\iota T$	$\mu_\iota P$	$\mu_\iota Q$	$\mu_\iota R$	$\mu_\iota U$	$\mu_\iota X$	$\mu_\iota Y$	$\mu_\iota Z$	$\mu_\iota V$	$\mu_\iota D$	$\mu_\iota E$	$\mu_\iota F$	$\mu_\iota iS$	$\mu_\iota iL$	$\mu_\iota iM$	$\mu_\iota iN$	$\mu_\iota iT$	$\mu_\iota iP$	$\mu_\iota iQ$	$\mu_\iota iR$	$\mu_\iota iU$	$\mu_\iota iX$	$\mu_\iota iY$	$\mu_\iota iZ$	$\mu_\iota iV$	$\mu_\iota iD$	$\mu_\iota iE$	$\mu_\iota iF$
μ_jS	μ_jL	μ_jM	μ_jN	μ_jT	μ_jP	μ_jQ	μ_jR	μ_jU	μ_jX	μ_jY	μ_jZ	μ_jV	μ_jD	μ_jE	μ_jF	μ_jiS	μ_jiL	μ_jiM	μ_jiN	μ_jiT	μ_jiP	μ_jiQ	μ_jiR	μ_jiU	μ_jiX	μ_jiY	μ_jiZ	μ_jiV	μ_jiD	μ_jiE	μ_jiF
$\mu_\kappa S$	$\mu_\kappa L$	$\mu_\kappa M$	$\mu_\kappa N$	$\mu_\kappa T$	$\mu_\kappa P$	$\mu_\kappa Q$	$\mu_\kappa R$	$\mu_\kappa U$	$\mu_\kappa X$	$\mu_\kappa Y$	$\mu_\kappa Z$	$\mu_\kappa V$	$\mu_\kappa D$	$\mu_\kappa E$	$\mu_\kappa F$	$\mu_\kappa iS$	$\mu_\kappa iL$	$\mu_\kappa iM$	$\mu_\kappa iN$	$\mu_\kappa iT$	$\mu_\kappa iP$	$\mu_\kappa iQ$	$\mu_\kappa iR$	$\mu_\kappa iU$	$\mu_\kappa iX$	$\mu_\kappa iY$	$\mu_\kappa iZ$	$\mu_\kappa iV$	$\mu_\kappa iD$	$\mu_\kappa iE$	$\mu_\kappa iF$
ν_oS	ν_oL	ν_oM	ν_oN	ν_oT	ν_oP	ν_oQ	ν_oR	ν_oU	ν_oX	ν_oY	ν_oZ	ν_oV	ν_oD	ν_oE	ν_oF	ν_oiS	ν_oiL	ν_oiM	ν_oiN	ν_oiT	ν_oiP	ν_oiQ	ν_oiR	ν_oiU	ν_oiX	ν_oiY	ν_oiZ	ν_oiV	ν_oiD	ν_oiE	ν_oiF
$\nu_\iota S$	$\nu_\iota L$	$\nu_\iota M$	$\nu_\iota N$	$\nu_\iota T$	$\nu_\iota P$	$\nu_\iota Q$	$\nu_\iota R$	$\nu_\iota U$	$\nu_\iota X$	$\nu_\iota Y$	$\nu_\iota Z$	$\nu_\iota V$	$\nu_\iota D$	$\nu_\iota E$	$\nu_\iota F$	$\nu_\iota iS$	$\nu_\iota iL$	$\nu_\iota iM$	$\nu_\iota iN$	$\nu_\iota iT$	$\nu_\iota iP$	$\nu_\iota iQ$	$\nu_\iota iR$	$\nu_\iota iU$	$\nu_\iota iX$	$\nu_\iota iY$	$\nu_\iota iZ$	$\nu_\iota iV$	$\nu_\iota iD$	$\nu_\iota iE$	$\nu_\iota iF$
ν_jS	ν_jL	ν_jM	ν_jN	ν_jT	ν_jP	ν_jQ	ν_jR	ν_jU	ν_jX	ν_jY	ν_jZ	ν_jV	ν_jD	ν_jE	ν_jF	ν_jiS	ν_jiL	ν_jiM	ν_jiN	ν_jiT	ν_jiP	ν_jiQ	ν_jiR	ν_jiU	ν_jiX	ν_jiY	ν_jiZ	ν_jiV	ν_jiD	ν_jiE	ν_jiF
$\nu_\kappa S$	$\nu_\kappa L$	$\nu_\kappa M$	$\nu_\kappa N$	$\nu_\kappa T$	$\nu_\kappa P$	$\nu_\kappa Q$	$\nu_\kappa R$	$\nu_\kappa U$	$\nu_\kappa X$	$\nu_\kappa Y$	$\nu_\kappa Z$	$\nu_\kappa V$	$\nu_\kappa D$	$\nu_\kappa E$	$\nu_\kappa F$	$\nu_\kappa iS$	$\nu_\kappa iL$	$\nu_\kappa iM$	$\nu_\kappa iN$	$\nu_\kappa iT$	$\nu_\kappa iP$	$\nu_\kappa iQ$	$\nu_\kappa iR$	$\nu_\kappa iU$	$\nu_\kappa iX$	$\nu_\kappa iY$	$\nu_\kappa iZ$	$\nu_\kappa iV$	$\nu_\kappa iD$	$\nu_\kappa iE$	$\nu_\kappa iF$
α_oS	α_oL	α_oM	α_oN	α_oT	α_oP	α_oQ	α_oR	α_oU	α_oX	α_oY	α_oZ	α_oV	α_oD	α_oE	α_oF	α_oiS	α_oiL	α_oiM	α_oiN	α_oiT	α_oiP	α_oiQ	α_oiR	α_oiU	α_oiX	α_oiY	α_oiZ	α_oiV	α_oiD	α_oiE	α_oiF
$\alpha_\iota S$	$\alpha_\iota L$	$\alpha_\iota M$	$\alpha_\iota N$	$\alpha_\iota T$	$\alpha_\iota P$	$\alpha_\iota Q$	$\alpha_\iota R$	$\alpha_\iota U$	$\alpha_\iota X$	$\alpha_\iota Y$	$\alpha_\iota Z$	$\alpha_\iota V$	$\alpha_\iota D$	$\alpha_\iota E$	$\alpha_\iota F$	$\alpha_\iota iS$	$\alpha_\iota iL$	$\alpha_\iota iM$	$\alpha_\iota iN$	$\alpha_\iota iT$	$\alpha_\iota iP$	$\alpha_\iota iQ$	$\alpha_\iota iR$	$\alpha_\iota iU$	$\alpha_\iota iX$	$\alpha_\iota iY$	$\alpha_\iota iZ$	$\alpha_\iota iV$	$\alpha_\iota iD$	$\alpha_\iota iE$	$\alpha_\iota iF$
α_jS	α_jL	α_jM	α_jN	α_jT	α_jP	α_jQ	α_jR	α_jU	α_jX	α_jY	α_jZ	α_jV	α_jD	α_jE	α_jF	α_jiS	α_jiL	α_jiM	α_jiN	α_jiT	α_jiP	α_jiQ	α_jiR	α_jiU	α_jiX	α_jiY	α_jiZ	α_jiV	α_jiD	α_jiE	α_jiF
$\alpha_\kappa S$	$\alpha_\kappa L$	$\alpha_\kappa M$	$\alpha_\kappa N$	$\alpha_\kappa T$	$\alpha_\kappa P$	$\alpha_\kappa Q$	$\alpha_\kappa R$	$\alpha_\kappa U$	$\alpha_\kappa X$	$\alpha_\kappa Y$	$\alpha_\kappa Z$	$\alpha_\kappa V$	$\alpha_\kappa D$	$\alpha_\kappa E$	$\alpha_\kappa F$	$\alpha_\kappa iS$	$\alpha_\kappa iL$	$\alpha_\kappa iM$	$\alpha_\kappa iN$	$\alpha_\kappa iT$	$\alpha_\kappa iP$	$\alpha_\kappa iQ$	$\alpha_\kappa iR$	$\alpha_\kappa iU$	$\alpha_\kappa iX$	$\alpha_\kappa iY$	$\alpha_\kappa iZ$	$\alpha_\kappa iV$	$\alpha_\kappa iD$	$\alpha_\kappa iE$	$\alpha_\kappa iF$
β_oS	β_oL	β_oM	β_oN	β_oT	β_oP	β_oQ	β_oR	β_oU	β_oX	β_oY	β_oZ	β_oV	β_oD	β_oE	β_oF	β_oiS	β_oiL	β_oiM	β_oiN	β_oiT	β_oiP	β_oiQ	β_oiR	β_oiU	β_oiX	β_oiY	β_oiZ	β_oiV	β_oiD	β_oiE	β_oiF
$\beta_\iota S$	$\beta_\iota L$	$\beta_\iota M$	$\beta_\iota N$	$\beta_\iota T$	$\beta_\iota P$	$\beta_\iota Q$	$\beta_\iota R$	$\beta_\iota U$	$\beta_\iota X$	$\beta_\iota Y$	$\beta_\iota Z$	$\beta_\iota V$	$\beta_\iota D$	$\beta_\iota E$	$\beta_\iota F$	$\beta_\iota iS$	$\beta_\iota iL$	$\beta_\iota iM$	$\beta_\iota iN$	$\beta_\iota iT$	$\beta_\iota iP$	$\beta_\iota iQ$	$\beta_\iota iR$	$\beta_\iota iU$	$\beta_\iota iX$	$\beta_\iota iY$	$\beta_\iota iZ$	$\beta_\iota iV$	$\beta_\iota iD$	$\beta_\iota iE$	$\beta_\iota iF$
β_jS	β_jL	β_jM	β_jN	β_jT	β_jP	β_jQ	β_jR	β_jU	β_jX	β_jY	β_jZ	β_jV	β_jD	β_jE	β_jF	β_jiS	β_jiL	β_jiM	β_jiN	β_jiT	β_jiP	β_jiQ	β_jiR	β_jiU	β_jiX	β_jiY	β_jiZ	β_jiV	β_jiD	β_jiE	β_jiF
$\beta_\kappa S$	$\beta_\kappa L$	$\beta_\kappa M$	$\beta_\kappa N$	$\beta_\kappa T$	$\beta_\kappa P$	$\beta_\kappa Q$	$\beta_\kappa R$	$\beta_\kappa U$	$\beta_\kappa X$	$\beta_\kappa Y$	$\beta_\kappa Z$	$\beta_\kappa V$	$\beta_\kappa D$	$\beta_\kappa E$	$\beta_\kappa F$	$\beta_\kappa iS$	$\beta_\kappa iL$	$\beta_\kappa iM$	$\beta_\kappa iN$	$\beta_\kappa iT$	$\beta_\kappa iP$	$\beta_\kappa iQ$	$\beta_\kappa iR$	$\beta_\kappa iU$	$\beta_\kappa iX$	$\beta_\kappa iY$	$\beta_\kappa iZ$	$\beta_\kappa iV$	$\beta_\kappa iD$	$\beta_\kappa iE$	$\beta_\kappa iF$
γ_oS	γ_oL	γ_oM	γ_oN	γ_oT	γ_oP	γ_oQ	γ_oR	γ_oU	γ_oX	γ_oY	γ_oZ	γ_oV	γ_oD	γ_oE	γ_oF	γ_oiS	γ_oiL	γ_oiM	γ_oiN	γ_oiT	γ_oiP	γ_oiQ	γ_oiR	γ_oiU	γ_oiX	γ_oiY	γ_oiZ	γ_oiV	γ_oiD	γ_oiE	γ_oiF
$\gamma_\iota S$	$\gamma_\iota L$	$\gamma_\iota M$	$\gamma_\iota N$	$\gamma_\iota T$	$\gamma_\iota P$	$\gamma_\iota Q$	$\gamma_\iota R$	$\gamma_\iota U$	$\gamma_\iota X$	$\gamma_\iota Y$	$\gamma_\iota Z$	$\gamma_\iota V$	$\gamma_\iota D$	$\gamma_\iota E$	$\gamma_\iota F$	$\gamma_\iota iS$	$\gamma_\iota iL$	$\gamma_\iota iM$	$\gamma_\iota iN$	$\gamma_\iota iT$	$\gamma_\iota iP$	$\gamma_\iota iQ$	$\gamma_\iota iR$	$\gamma_\iota iU$	$\gamma_\iota iX$	$\gamma_\iota iY$	$\gamma_\iota iZ$	$\gamma_\iota iV$	$\gamma_\iota iD$	$\gamma_\iota iE$	$\gamma_\iota iF$
γ_jS	γ_jL	γ_jM	γ_jN	γ_jT	γ_jP	γ_jQ	γ_jR	γ_jU	γ_jX	γ_jY	γ_jZ	γ_jV	γ_jD	γ_jE	γ_jF	γ_jiS	γ_jiL	γ_jiM	γ_jiN	γ_jiT	γ_jiP	γ_jiQ	γ_jiR	γ_jiU	γ_jiX	γ_jiY	γ_jiZ	γ_jiV	γ_jiD	γ_jiE	γ_jiF
$\gamma_\kappa S$	$\gamma_\kappa L$	$\gamma_\kappa M$	$\gamma_\kappa N$	$\gamma_\kappa T$	$\gamma_\kappa P$	$\gamma_\kappa Q$	$\gamma_\kappa R$	$\gamma_\kappa U$	$\gamma_\kappa X$	$\gamma_\kappa Y$	$\gamma_\kappa Z$	$\gamma_\kappa V$	$\gamma_\kappa D$	$\gamma_\kappa E$	$\gamma_\kappa F$	$\gamma_\kappa iS$	$\gamma_\kappa iL$	$\gamma_\kappa iM$	$\gamma_\kappa iN$	$\gamma_\kappa iT$	$\gamma_\kappa iP$	$\gamma_\kappa iQ$	$\gamma_\kappa iR$	$\gamma_\kappa iU$	$\gamma_\kappa iX$	$\gamma_\kappa iY$	$\gamma_\kappa iZ$	$\gamma_\kappa iV$	$\gamma_\kappa iD$	$\gamma_\kappa iE$	$\gamma_\kappa iF$
δ_oS	δ_oL	δ_oM	δ_oN	δ_oT	δ_oP	δ_oQ	δ_oR	δ_oU	δ_oX	δ_oY	δ_oZ	δ_oV	δ_oD	δ_oE	δ_oF	δ_oiS	δ_oiL	δ_oiM	δ_oiN	δ_oiT	δ_oiP	δ_oiQ	δ_oiR	δ_oiU	δ_oiX	δ_oiY	δ_oiZ	δ_oiV	δ_oiD	δ_oiE	δ_oiF
$\delta_\iota S$	$\delta_\iota L$	$\delta_\iota M$	$\delta_\iota N$	$\delta_\iota T$	$\delta_\iota P$	$\delta_\iota Q$	$\delta_\iota R$	$\delta_\iota U$	$\delta_\iota X$	$\delta_\iota Y$	$\delta_\iota Z$	$\delta_\iota V$	$\delta_\iota D$	$\delta_\iota E$	$\delta_\iota F$	$\delta_\iota iS$	$\delta_\iota iL$	$\delta_\iota iM$	$\delta_\iota iN$	$\delta_\iota iT$	$\delta_\iota iP$	$\delta_\iota iQ$	$\delta_\iota iR$	$\delta_\iota iU$	$\delta_\iota iX$	$\delta_\iota iY$	$\delta_\iota iZ$	$\delta_\iota iV$	$\delta_\iota iD$	$\delta_\iota iE$	$\delta_\iota iF$
δ_jS	δ_jL	δ_jM	δ_jN	δ_jT	δ_jP	δ_jQ	δ_jR	δ_jU	δ_jX	δ_jY	δ_jZ	δ_jV	δ_jD	δ_jE	δ_jF	δ_jiS	δ_jiL	δ_jiM	δ_jiN	δ_jiT	δ_jiP	δ_jiQ	δ_jiR	δ_jiU	δ_jiX	δ_jiY	δ_jiZ	δ_jiV	δ_jiD	δ_jiE	δ_jiF
$\delta_\kappa S$	$\delta_\kappa L$	$\delta_\kappa M$	$\delta_\kappa N$	$\delta_\kappa T$	$\delta_\kappa P$	$\delta_\kappa Q$	$\delta_\kappa R$	$\delta_\kappa U$	$\delta_\kappa X$	$\delta_\kappa Y$	$\delta_\kappa Z$	$\delta_\kappa V$	$\delta_\kappa D$	$\delta_\kappa E$	$\delta_\kappa F$	$\delta_\kappa iS$	$\delta_\kappa iL$	$\delta_\kappa iM$	$\delta_\kappa iN$	$\delta_\kappa iT$	$\delta_\kappa iP$	$\delta_\kappa iQ$	$\delta_\kappa iR$	$\delta_\kappa iU$	$\delta_\kappa iX$	$\delta_\kappa iY$	$\delta_\kappa iZ$	$\delta_\kappa iV$	$\delta_\kappa iD$	$\delta_\kappa iE$	$\delta_\kappa iF$

2.4. Grading of elements of M_L when used to represent unit multivector elements of $Cl_{3,1}(R)$

Elements of M_L can be assigned to represent unit multivector elements for $Cl_{3,1}(R)$ as follows:

Unit Scalar: S
Unit spatial vectors: X, Y, Z
Unit temporal vector: T
Spatial bivectors: L,M,N
Space/time bivectors: D,E,F
Spatial trivector: U
Space/time trivectors: P,Q,R
Pseudoscalar: V

3. Electroweak sector

T_L subloops with have identical participation for elements subscripted ι, $\jmath$ and κ are shown in table 5.

Table 5. Unit elements for spatially equivalent sedenion-type subloops

	σ_o	σ_ι	$\sigma_\jmath$	σ_κ	λ_o	λ_ι	$\lambda_\jmath$	λ_κ	μ_o	μ_ι	$\mu_\jmath$	μ_κ	ν_o	ν_ι	$\nu_\jmath$	ν_κ	α_o	α_ι	$\alpha_\jmath$	α_κ	β_o	β_ι	$\beta_\jmath$	β_κ	γ_o	γ_ι	$\gamma_\jmath$	γ_κ	δ_o	δ_ι	$\delta_\jmath$	δ_κ
Γ_0	■	■	■	■	■	■	■	■																	■	■	■	■	■	■	■	■
A_0	■	■	■	■					■	■	■	■									■	■	■	■					■	■	■	■
B_0	■	■	■	■									■	■	■	■					■	■	■	■	■	■	■	■				
Σ_0	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■																
Σ_1	■	■	■	■	■	■	■	■									■	■	■	■	■	■	■	■								
Σ_2	■	■	■	■					■	■	■	■					■	■	■	■					■	■	■	■				
Σ_3	■	■	■	■									■	■	■	■	■	■	■	■									■	■	■	■

If elements subscripted ι, $\jmath$ and κ are excluded, the automorphism group for the unit imaginary octonions that remain, $[\lambda_o, \mu_o, \nu_o, \alpha_o, \beta_o, \gamma_o, \delta_o]$, is G2. Including the ι, $\jmath$ and κ elements breaks that symmetry, but some symmetry remains. Σ_1, Σ_2 and Σ_3 are related by a quaterionic symmetry, so the automorphism group for their complexification can be $SU(2) \otimes U(1)$.

Most unification models using the octonions[6] [7] [8] [9] [10] [11] [12] [13] [14] [15] [16] [17] [18] [19] [20] [21] [22] [23] [24] [25] [26] include quarks and gluons. However, G2 has been used in modelling the electroweak/lepton sector alone by Carone and Rastogi[27] [28], extending the SU(3) model for electroweak unification proposed by Dimopoulos and Kaplan[29].

As noted in [3], the α_o element of T_L and the unit imaginary of M_L have unique status. This suggests identifying them with unit imaginary elements of a complex doublet for the Brout-Englert-Higgs mechanism[30][31][32]. Σ_1, Σ_2 and Σ_3 include $\alpha_o, \alpha_\iota, \alpha_\jmath, \alpha\kappa$, whereas Σ_0 does not. This suggests assignment of Σ_1, Σ_2 and Σ_3 to electroweak vector bosons that gain mass by the Brout-Englert-Higgs mechanism, and Σ_0 to a vector boson that remains massless.

Having $\Sigma_0, \Sigma_1, \Sigma_2$ and Σ_3 assigned to electroweak vector bosons leaves A_0, B_0 and Γ_0 available to be assigned to electroweak fermions. This suggests assignation of one of these subloops to generate three generations of one chirality of the neutrino family, and the other two subloops to generate three generations of two chiralities of electron/muon/tau family.

4. Fermions

4.1. Basis for the Dirac equation

In [3] spatially equivalent assignment of elements of T_L were identified as ones for which the subloop $[\sigma_o\sigma_\iota\sigma_\jmath\sigma_\kappa]$ of T_L is aligned with the subloop $[SLMN]$ of M_L. In this paper this concept is extended by postulating that spatially equivalent application of elements of T_L to unit multivector elements of the space-time Clifford algebra generates an algebra, $\mathbb{U}$ with sublagebras which can be used to represent graded multivector-type subalgebras for "unphysical" transverse coordinates for excitations of quantum fields.

For the complexified sedenionic-type subloop labelled Γ_0, a possible spatially equivalent alignment generates the loop of unit transverse multivector elements shown in table 6.

TABLE 6. Transverse multivector elements for a spatially equivalent Γ_0 alignment

e_0	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	e_{11}	e_{12}	e_{13}	e_{14}	e_{15}	e_{16} to e_{31}
$\sigma_o S$	$\sigma_\iota L$	$\sigma_\jmath M$	$\sigma_\kappa N$	$\lambda_o U$	$\lambda_\iota X$	$\lambda_\jmath Y$	$\lambda_\kappa Z$	$\gamma_o V$	$\gamma_\iota D$	$\gamma_\jmath E$	$\gamma_\kappa F$	$\delta_o T$	$\delta_\iota P$	$\delta_\jmath Q$	$\delta_\kappa R$	$i \times [e_1$ to $e_{15}]$

If elements $[e_1..e_{15}]$ are graded according to the grading of their M_L components when $M_4(C)$ is assigned to represent the complexified space-time Clifford algebra, the result shown in Table 7.

TABLE 7. Grading for transverse multivector unit elements for a Γ_0 alignment

scalar	vector				bivector						trivector				pseudo
	spatial			t	spatial			spatio-t			spatio-t			xyz	scalar
$\sigma_o S$	$\lambda_\iota X$	$\lambda_\jmath Y$	$\lambda_\kappa Z$	$\delta_o T$	$\sigma_\iota L$	$\sigma_\jmath M$	$\sigma_\kappa N$	$\gamma_\iota D$	$\gamma_\jmath E$	$\gamma_\kappa F$	$\delta_\iota P$	$\delta_\jmath Q$	$\delta_\kappa R$	$\lambda_o U$	$\gamma_o V$

The even multivector components in table 7 could also be generated by imaginary counterparts of the vector elements as shown in Table 8.

TABLE 8. Grading for alternative spatially equivalent transverse multivector unit elements

scalar	vector				bivector						trivector				pseudo
	spatial			t	spatial			spatio-t			spatio-t			xyz	scalar
$\sigma_o S$	$\lambda_\iota iX$	$\lambda_\jmath iY$	$\lambda_\kappa iZ$	$\delta_o iT$	$\sigma_\iota L$	$\sigma_\jmath M$	$\sigma_\kappa N$	$\gamma_\iota D$	$\gamma_\jmath E$	$\gamma_\kappa F$	$\delta_\iota iP$	$\delta_\jmath iQ$	$\delta_\kappa iR$	$\lambda_o iU$	$\gamma_o V$

This suggests assembly of a version of the Dirac equation for this alignment using $\lambda_\iota, \lambda_\jmath, \lambda_\kappa, \delta_o$ instead of the gamma matrices. This assembly can be labelled $\Gamma_{0\gamma}^{\lambda\delta}$, with the subscript indicating the T_L element applied to the pseudoscalar, the first superscript indicating the T_L elements applied to the spatial vectors and the second superscript indicating the T_L element applied to the time-like vector.

For the space-time Clifford algebra multivector, Hestenes [?] uses the unit pseudoscalar as a substitute for the unit imaginary, generating a version of the Dirac equation for one handing of a fermion. Comparing this with $[\sigma_o S, \sigma_\iota L, \sigma_\jmath M, \sigma_\kappa N, \gamma_\iota D, \gamma_\jmath E, \gamma_\kappa F, \gamma_o V]$, for all elements except the scalar there is a reversal of signature and commutation/anticommutation properties, and Lie brackets are interchanged with Jordan brackets. This means that the equivalent of the pseudoscalar, $[\gamma_o V]$, anticommutes with the bivector elements and squares to the positive scalar. an alternative way to assemble spinors could be to use odd multivector components, as $[\lambda_\iota X, \lambda_\jmath Y, \lambda_\kappa Z, \delta_o T] \times \gamma_o V = [\delta_\iota P, \delta_\jmath Q, \delta_\kappa R, \lambda_o U]$. This suggests that similar equations to those obtained by Hestenes can be generated from each of $\Gamma_{0\gamma}$, $\Gamma_{0\delta}$, $\Gamma_{0\lambda}$, corresponding different flavors for one chirality for type of fermion.

4.2. Three generations for families of fermions

For combined $T_L \otimes M_L$ subloops, rotations and reflections with respect to unit vectors of $Cl_{3,1}(R)$ correspond to permutations of $\iota \to \jmath \to \kappa$ subscripts. Permuting $\lambda_{o\iota\jmath\kappa} \to \gamma_{o\iota\jmath\kappa} \to \delta_{o\iota\jmath\kappa}$ reorientates subloops with respect to grading, but not with respect to spatial orientation.

For the spatially equivalent transverse multivector subloops based on each of the Γ_0, A_0 and B_0 subloops of T_L, ignoring rotations and reflections, there are 6 possible spatially equivalent orientations. They can be grouped in sets which share the same even multivector components, as shown in tables 9 and 10.

The families of fermions are:
1 chirality for the neutrino family
2 chiralities for the electron/muon/tau family
2 chiralities x 3 colors for the up/charm/top quark family
2 chiralities x 3 colors for the down/strange/bottom quark family
As noted in [3], this, together with the observation that sets of three subloops can, as a combination, display spatial equivalence, suggests assignment of subloops to families of fermions as follows:
Γ_0 to 1 chirality for the neutrino family
A_0 to one chirality and B_0 to a second chirality for the electron/muon/tau family
A_{1-6} to one chirality with 3 colors two families of quarks
B_{1-6} to a second chirality with 3 colors for two families of quarks

Distinct representations for different generations within these families can be found when these T_L subloops are combined with M_L subloops in different spatially equivalent alignments and are to represent a type of multivector for transverse coordinates for an unphysical space. The notation for elements of U_L, unit elements of $\mathbb{U}$ is set out in table 5.

TABLE 9. Graded spatially equivalent Γ_0 orientations

Ref	scalar	vector				bivector						trivector				pseudo	
		spatial			t	spatial			spatio-t			spatio-t			xyz	scalar	$[e_{16}$ to $e_{31}]$
$\Gamma_{0\gamma}^{\lambda\delta}$	$\sigma_o S$	$\lambda_\iota X$	$\lambda_\jmath Y$	$\lambda_\kappa Z$	$\delta_o T$	$\sigma_\iota L$	$\sigma_\jmath M$	$\sigma_\kappa N$	$\gamma_\iota D$	$\gamma_\jmath E$	$\gamma_\kappa F$	$\delta_\iota P$	$\delta_\jmath Q$	$\delta_\kappa R$	$\lambda_o U$	$\gamma_o V$	$i \times [e_1$ to $e_{15}]$
	$\sigma_o S$	$\lambda_\iota iX$	$\lambda_\jmath iY$	$\lambda_\kappa iZ$	$\delta_o iT$	$\sigma_\iota L$	$\sigma_\jmath M$	$\sigma_\kappa N$	$\gamma_\iota D$	$\gamma_\jmath E$	$\gamma_\kappa F$	$\delta_\iota iP$	$\delta_\jmath iQ$	$\delta_\kappa iR$	$\lambda_o iU$	$\gamma_o V$	$i \times [e_1$ to $e_{15}]$
$\Gamma_{0\gamma}^{\delta\lambda}$	$\sigma_o S$	$\delta_\iota X$	$\delta_\jmath Y$	$\delta_\kappa Z$	$\lambda_o T$	$\sigma_\iota L$	$\sigma_\jmath M$	$\sigma_\kappa N$	$\gamma_\iota D$	$\gamma_\jmath E$	$\gamma_\kappa F$	$\lambda_\iota P$	$\lambda_\jmath Q$	$\lambda_\kappa R$	$\delta_o U$	$\gamma_o V$	$i \times [e_1$ to $e_{15}]$
	$\sigma_o S$	$\delta_\iota iX$	$\delta_\jmath iY$	$\delta_\kappa iZ$	$\lambda_o iT$	$\sigma_\iota L$	$\sigma_\jmath M$	$\sigma_\kappa N$	$\gamma_\iota D$	$\gamma_\jmath E$	$\gamma_\kappa F$	$\lambda_\iota iP$	$\lambda_\jmath iQ$	$\lambda_\kappa iR$	$\delta_o iU$	$\gamma_o V$	$i \times [e_1$ to $e_{15}]$
$\Gamma_{0\delta}^{\lambda\gamma}$	$\sigma_o S$	$\lambda_\iota X$	$\lambda_\jmath Y$	$\lambda_\kappa Z$	$\gamma_o T$	$\sigma_\iota L$	$\sigma_\jmath M$	$\sigma_\kappa N$	$\delta_\iota D$	$\delta_\jmath E$	$\delta_\kappa F$	$\gamma_\iota P$	$\gamma_\jmath Q$	$\gamma_\kappa R$	$\lambda_o U$	$\delta_o V$	$i \times [e_1$ to $e_{15}]$
	$\sigma_o S$	$\lambda_\iota iX$	$\lambda_\jmath iY$	$\lambda_\kappa iZ$	$\gamma_o iT$	$\sigma_\iota L$	$\sigma_\jmath M$	$\sigma_\kappa N$	$\delta_\iota D$	$\delta_\jmath E$	$\delta_\kappa F$	$\gamma_\iota iP$	$\gamma_\jmath iQ$	$\gamma_\kappa iR$	$\lambda_o iU$	$\delta_o V$	$i \times [e_1$ to $e_{15}]$
$\Gamma_{0\delta}^{\gamma\lambda}$	$\sigma_o S$	$\gamma_\iota X$	$\gamma_\jmath Y$	$\gamma_\kappa Z$	$\lambda_o T$	$\sigma_\iota L$	$\sigma_\jmath M$	$\sigma_\kappa N$	$\delta_\iota D$	$\delta_\jmath E$	$\delta_\kappa F$	$\lambda_\iota P$	$\lambda_\jmath Q$	$\lambda_\kappa R$	$\gamma_o U$	$\delta_o V$	$i \times [e_1$ to $e_{15}]$
	$\sigma_o S$	$\gamma_\iota iX$	$\gamma_\jmath iY$	$\gamma_\kappa iZ$	$\lambda_o iT$	$\sigma_\iota L$	$\sigma_\jmath M$	$\sigma_\kappa N$	$\delta_\iota D$	$\delta_\jmath E$	$\delta_\kappa F$	$\lambda_\iota iP$	$\lambda_\jmath iQ$	$\lambda_\kappa iR$	$\gamma_o iU$	$\delta_o V$	$i \times [e_1$ to $e_{15}]$
$\Gamma_{0\lambda}^{\gamma\delta}$	$\sigma_o S$	$\gamma_\iota X$	$\gamma_\jmath Y$	$\gamma_\kappa Z$	$\delta_o T$	$\sigma_\iota L$	$\sigma_\jmath M$	$\sigma_\kappa N$	$\lambda_\iota D$	$\lambda_\jmath E$	$\lambda_\kappa F$	$\delta_\iota P$	$\delta_\jmath Q$	$\delta_\kappa R$	$\gamma_o U$	$\lambda_o V$	$i \times [e_1$ to $e_{15}]$
	$\sigma_o S$	$\gamma_\iota iX$	$\gamma_\jmath iY$	$\gamma_\kappa iZ$	$\delta_o iT$	$\sigma_\iota L$	$\sigma_\jmath M$	$\sigma_\kappa N$	$\lambda_\iota D$	$\lambda_\jmath E$	$\lambda_\kappa F$	$\delta_\iota iP$	$\delta_\jmath iQ$	$\delta_\kappa iR$	$\gamma_o iU$	$\lambda_o V$	$i \times [e_1$ to $e_{15}]$
$\Gamma_{0\lambda}^{\delta\gamma}$	$\sigma_o S$	$\delta_\iota X$	$\delta_\jmath Y$	$\delta_\kappa Z$	$\gamma_o T$	$\sigma_\iota L$	$\sigma_\jmath M$	$\sigma_\kappa N$	$\lambda_\iota D$	$\lambda_\jmath E$	$\lambda_\kappa F$	$\gamma_\iota P$	$\gamma_\jmath Q$	$\gamma_\kappa R$	$\delta_o U$	$\lambda_o V$	$i \times [e_1$ to $e_{15}]$
	$\sigma_o S$	$\delta_\iota iX$	$\delta_\jmath iY$	$\delta_\kappa iZ$	$\gamma_o iT$	$\sigma_\iota L$	$\sigma_\jmath M$	$\sigma_\kappa N$	$\lambda_\iota D$	$\lambda_\jmath E$	$\lambda_\kappa F$	$\gamma_\iota iP$	$\gamma_\jmath iQ$	$\gamma_\kappa iR$	$\delta_o iU$	$\lambda_o V$	$i \times [e_1$ to $e_{15}]$

Table 10. Graded spatially equivalent A_0 and B_0 orientations

Ref	scalar	vector				bivector						trivector				pseudo	
		spatial			t	spatial			spatio-t			spatio-t			xyz	scalar	$[e_{16}$ to $e_{31}]$
$A_{0\beta}^{\mu\delta}$	$\sigma_o S$	$\mu_\iota X$	$\mu_\jmath Y$	$\mu_\kappa Z$	$\delta_o T$	$\sigma_\iota L$	$\sigma_\jmath M$	$\sigma_\kappa N$	$\beta_\iota D$	$\beta_\jmath E$	$\beta_\kappa F$	$\delta_\iota P$	$\delta_\jmath Q$	$\delta_\kappa R$	$\mu_o U$	$\beta_o V$	$i \times [e_1$ to $e_{15}]$
	$\sigma_o S$	$\mathrm{i}\mu_\iota X$	$\mathrm{i}\mu_\jmath Y$	$\mathrm{i}\mu_\kappa Z$	$\mathrm{i}\delta_o T$	$\sigma_\iota L$	$\sigma_\jmath M$	$\sigma_\kappa N$	$\beta_\iota D$	$\beta_\jmath E$	$\beta_\kappa F$	$\mathrm{i}\delta_\iota P$	$\mathrm{i}\delta_\jmath Q$	$\mathrm{i}\delta_\kappa R$	$\mathrm{i}\mu_o U$	$\beta_o V$	$i \times [e_1$ to $e_{15}]$
$A_{0\beta}^{\delta\mu}$	$\sigma_o S$	$\delta_\iota X$	$\delta_\jmath Y$	$\delta_\kappa Z$	$\mu_o T$	$\sigma_\iota L$	$\sigma_\jmath M$	$\sigma_\kappa N$	$\beta_\iota D$	$\beta_\jmath E$	$\beta_\kappa F$	$\mu_\iota P$	$\mu_\jmath Q$	$\mu_\kappa R$	$\delta_o U$	$\beta_o V$	$i \times [e_1$ to $e_{15}]$
	$\sigma_o S$	$\mathrm{i}\delta_\iota X$	$\mathrm{i}\delta_\jmath Y$	$\mathrm{i}\delta_\kappa Z$	$\mathrm{i}\mu_o T$	$\sigma_\iota L$	$\sigma_\jmath M$	$\sigma_\kappa N$	$\beta_\iota D$	$\beta_\jmath E$	$\beta_\kappa F$	$\mathrm{i}\mu_\iota P$	$\mathrm{i}\mu_\jmath Q$	$\mathrm{i}\mu_\kappa R$	$\mathrm{i}\delta_o U$	$\beta_o V$	$i \times [e_1$ to $e_{15}]$
$A_{0\delta}^{\beta\mu}$	$\sigma_o S$	$\beta_\iota X$	$\beta_\jmath Y$	$\beta_\kappa Z$	$\mu_o T$	$\sigma_\iota L$	$\sigma_\jmath M$	$\sigma_\kappa N$	$\delta_\iota D$	$\delta_\jmath E$	$\delta_\kappa F$	$\mu_\iota P$	$\mu_\jmath Q$	$\mu_\kappa R$	$\beta_o U$	$\delta_o V$	$i \times [e_1$ to $e_{15}]$
	$\sigma_o S$	$\mathrm{i}\beta_\iota X$	$\mathrm{i}\beta_\jmath Y$	$\mathrm{i}\beta_\kappa Z$	$\mathrm{i}\mu_o T$	$\sigma_\iota L$	$\sigma_\jmath M$	$\sigma_\kappa N$	$\delta_\iota D$	$\delta_\jmath E$	$\delta_\kappa F$	$\mathrm{i}\mu_\iota P$	$\mathrm{i}\mu_\jmath Q$	$\mathrm{i}\mu_\kappa R$	$\mathrm{i}\beta_o U$	$\delta_o V$	$i \times [e_1$ to $e_{15}]$
$A_{0\delta}^{\mu\beta}$	$\sigma_o S$	$\mu_\iota X$	$\mu_\jmath Y$	$\mu_\kappa Z$	$\beta_o T$	$\sigma_\iota L$	$\sigma_\jmath M$	$\sigma_\kappa N$	$\delta_\iota D$	$\delta_\jmath E$	$\delta_\kappa F$	$\beta_\iota P$	$\beta_\jmath Q$	$\beta_\kappa R$	$\mu_o U$	$\delta_o V$	$i \times [e_1$ to $e_{15}]$
	$\sigma_o S$	$\mathrm{i}\mu_\iota X$	$\mathrm{i}\mu_\jmath Y$	$\mathrm{i}\mu_\kappa Z$	$\mathrm{i}\beta_o T$	$\sigma_\iota L$	$\sigma_\jmath M$	$\sigma_\kappa N$	$\delta_\iota D$	$\delta_\jmath E$	$\delta_\kappa F$	$\mathrm{i}\beta_\iota P$	$\mathrm{i}\beta_\jmath Q$	$\mathrm{i}\beta_\kappa R$	$\mathrm{i}\mu_o U$	$\delta_o V$	$i \times [e_1$ to $e_{15}]$
$A_{0\mu}^{\delta\beta}$	$\sigma_o S$	$\delta_\iota X$	$\delta_\jmath Y$	$\delta_\kappa Z$	$\beta_o T$	$\sigma_\iota L$	$\sigma_\jmath M$	$\sigma_\kappa N$	$\mu_\iota D$	$\mu_\jmath E$	$\mu_\kappa F$	$\beta_\iota P$	$\beta_\jmath Q$	$\beta_\kappa R$	$\delta_o U$	$\mu_o V$	$i \times [e_1$ to $e_{15}]$
	$\sigma_o S$	$\mathrm{i}\delta_\iota X$	$\mathrm{i}\delta_\jmath Y$	$\mathrm{i}\delta_\kappa Z$	$\mathrm{i}\beta_o T$	$\sigma_\iota L$	$\sigma_\jmath M$	$\sigma_\kappa N$	$\mu_\iota D$	$\mu_\jmath E$	$\mu_\kappa F$	$\mathrm{i}\beta_\iota P$	$\mathrm{i}\beta_\jmath Q$	$\mathrm{i}\beta_\kappa R$	$\mathrm{i}\delta_o U$	$\mu_o V$	$i \times [e_1$ to $e_{15}]$
$A_{0\mu}^{\beta\delta}$	$\sigma_o S$	$\beta_\iota X$	$\beta_\jmath Y$	$\beta_\kappa Z$	$\delta_o T$	$\sigma_\iota L$	$\sigma_\jmath M$	$\sigma_\kappa N$	$\mu_\iota D$	$\mu_\jmath E$	$\mu_\kappa F$	$\delta_\iota P$	$\delta_\jmath Q$	$\delta_\kappa R$	$\beta_o U$	$\mu_o V$	$i \times [e_1$ to $e_{15}]$
	$\sigma_o S$	$\mathrm{i}\beta_\iota X$	$\mathrm{i}\beta_\jmath Y$	$\mathrm{i}\beta_\kappa Z$	$\mathrm{i}\delta_o T$	$\sigma_\iota L$	$\sigma_\jmath M$	$\sigma_\kappa N$	$\mu_\iota D$	$\mu_\jmath E$	$\mu_\kappa F$	$\mathrm{i}\delta_\iota P$	$\mathrm{i}\delta_\jmath Q$	$\mathrm{i}\delta_\kappa R$	$\mathrm{i}\beta_o U$	$\mu_o V$	$i \times [e_1$ to $e_{15}]$
$B_{0\beta}^{\nu\gamma}$	$\sigma_o S$	$\nu_\iota X$	$\nu_\jmath Y$	$\nu_\kappa Z$	$\gamma_o T$	$\sigma_\iota L$	$\sigma_\jmath M$	$\sigma_\kappa N$	$\beta_\iota D$	$\beta_\jmath E$	$\beta_\kappa F$	$\gamma_\iota P$	$\gamma_\jmath Q$	$\gamma_\kappa R$	$\nu_o U$	$\beta_o V$	$i \times [e_1$ to $e_{15}]$
	$\sigma_o S$	$\mathrm{i}\nu_\iota X$	$\mathrm{i}\nu_\jmath Y$	$\mathrm{i}\nu_\kappa Z$	$\mathrm{i}\gamma_o T$	$\sigma_\iota L$	$\sigma_\jmath M$	$\sigma_\kappa N$	$\beta_\iota D$	$\beta_\jmath E$	$\beta_\kappa F$	$\mathrm{i}\gamma_\iota P$	$\mathrm{i}\gamma_\jmath Q$	$\mathrm{i}\gamma_\kappa R$	$\mathrm{i}\nu_o U$	$\beta_o V$	$i \times [e_1$ to $e_{15}]$
$B_{0\beta}^{\gamma\nu}$	$\sigma_o S$	$\gamma_\iota X$	$\gamma_\jmath Y$	$\gamma_\kappa Z$	$\nu_o T$	$\sigma_\iota L$	$\sigma_\jmath M$	$\sigma_\kappa N$	$\beta_\iota D$	$\beta_\jmath E$	$\beta_\kappa F$	$\nu_\iota P$	$\nu_\jmath Q$	$\nu_\kappa R$	$\gamma_o U$	$\beta_o V$	$i \times [e_1$ to $e_{15}]$
	$\sigma_o S$	$\mathrm{i}\gamma_\iota X$	$\mathrm{i}\gamma_\jmath Y$	$\mathrm{i}\gamma_\kappa Z$	$\mathrm{i}\nu_o T$	$\sigma_\iota L$	$\sigma_\jmath M$	$\sigma_\kappa N$	$\beta_\iota D$	$\beta_\jmath E$	$\beta_\kappa F$	$\mathrm{i}\nu_\iota P$	$\mathrm{i}\nu_\jmath Q$	$\mathrm{i}\nu_\kappa R$	$\mathrm{i}\gamma_o U$	$\beta_o V$	$i \times [e_1$ to $e_{15}]$
$B_{0\gamma}^{\beta\nu}$	$\sigma_o S$	$\beta_\iota X$	$\beta_\jmath Y$	$\beta_\kappa Z$	$\nu_o T$	$\sigma_\iota L$	$\sigma_\jmath M$	$\sigma_\kappa N$	$\gamma_\iota D$	$\gamma_\jmath E$	$\gamma_\kappa F$	$\nu_\iota P$	$\nu_\jmath Q$	$\nu_\kappa R$	$\beta_o U$	$\gamma_o V$	$i \times [e_1$ to $e_{15}]$
	$\sigma_o S$	$\mathrm{i}\beta_\iota X$	$\mathrm{i}\beta_\jmath Y$	$\mathrm{i}\beta_\kappa Z$	$\mathrm{i}\nu_o T$	$\sigma_\iota L$	$\sigma_\jmath M$	$\sigma_\kappa N$	$\gamma_\iota D$	$\gamma_\jmath E$	$\gamma_\kappa F$	$\mathrm{i}\nu_\iota P$	$\mathrm{i}\nu_\jmath Q$	$\mathrm{i}\nu_\kappa R$	$\mathrm{i}\beta_o U$	$\gamma_o V$	$i \times [e_1$ to $e_{15}]$
$B_{0\gamma}^{\nu\beta}$	$\sigma_o S$	$\nu_\iota X$	$\nu_\jmath Y$	$\nu_\kappa Z$	$\beta_o T$	$\sigma_\iota L$	$\sigma_\jmath M$	$\sigma_\kappa N$	$\gamma_\iota D$	$\gamma_\jmath E$	$\gamma_\kappa F$	$\beta_\iota P$	$\beta_\jmath Q$	$\beta_\kappa R$	$\nu_o U$	$\gamma_o V$	$i \times [e_1$ to $e_{15}]$
	$\sigma_o S$	$\mathrm{i}\nu_\iota X$	$\mathrm{i}\nu_\jmath Y$	$\mathrm{i}\nu_\kappa Z$	$\mathrm{i}\beta_o T$	$\sigma_\iota L$	$\sigma_\jmath M$	$\sigma_\kappa N$	$\gamma_\iota D$	$\gamma_\jmath E$	$\gamma_\kappa F$	$\mathrm{i}\beta_\iota P$	$\mathrm{i}\beta_\jmath Q$	$\mathrm{i}\beta_\kappa R$	$\mathrm{i}\nu_o U$	$\gamma_o V$	$i \times [e_1$ to $e_{15}]$
$B_{0\nu}^{\gamma\beta}$	$\sigma_o S$	$\gamma_\iota X$	$\gamma_\jmath Y$	$\gamma_\kappa Z$	$\beta_o T$	$\sigma_\iota L$	$\sigma_\jmath M$	$\sigma_\kappa N$	$\nu_\iota D$	$\nu_\jmath E$	$\nu_\kappa F$	$\beta_\iota P$	$\beta_\jmath Q$	$\beta_\kappa R$	$\gamma_o U$	$\nu_o V$	$i \times [e_1$ to $e_{15}]$
	$\sigma_o S$	$\mathrm{i}\gamma_\iota X$	$\mathrm{i}\gamma_\jmath Y$	$\mathrm{i}\gamma_\kappa Z$	$\mathrm{i}\beta_o T$	$\sigma_\iota L$	$\sigma_\jmath M$	$\sigma_\kappa N$	$\nu_\iota D$	$\nu_\jmath E$	$\nu_\kappa F$	$\mathrm{i}\beta_\iota P$	$\mathrm{i}\beta_\jmath Q$	$\mathrm{i}\beta_\kappa R$	$\mathrm{i}\gamma_o U$	$\nu_o V$	$i \times [e_1$ to $e_{15}]$
$B_{0\nu}^{\beta\gamma}$	$\sigma_o S$	$\beta_\iota X$	$\beta_\jmath Y$	$\beta_\kappa Z$	$\gamma_o T$	$\sigma_\iota L$	$\sigma_\jmath M$	$\sigma_\kappa N$	$\nu_\iota D$	$\nu_\jmath E$	$\nu_\kappa F$	$\gamma_\iota P$	$\gamma_\jmath Q$	$\gamma_\kappa R$	$\beta_o U$	$\nu_o V$	$i \times [e_1$ to $e_{15}]$
	$\sigma_o S$	$\mathrm{i}\beta_\iota X$	$\mathrm{i}\beta_\jmath Y$	$\mathrm{i}\beta_\kappa Z$	$\mathrm{i}\gamma_o T$	$\sigma_\iota L$	$\sigma_\jmath M$	$\sigma_\kappa N$	$\nu_\iota D$	$\nu_\jmath E$	$\nu_\kappa F$	$\mathrm{i}\gamma_\iota P$	$\mathrm{i}\gamma_\jmath Q$	$\mathrm{i}\gamma_\kappa R$	$\mathrm{i}\beta_o U$	$\nu_o V$	$i \times [e_1$ to $e_{15}]$

For each of Γ_0, A_0 and B_0 three sets of four of alignments share common even multivector components, suggesting that the even transverse multivectors for each set correspond to a spinor for a fundamental fermion. The Jordan bracket for their subalgebra matches the Lie bracket for the even $Cl^+_{3,1}$ subalgebra, so this may be consistent with Hestenes version of the Dirac equation[33].

For the assignment of A_0 to one chirality and B_0 to the opposite chirality, inspection of table 10 reveals that, for the change to the opposite chiralities, $[x, y, z, t]$ transverse vectors are factored by unit octonions from a quaternionic subloop, e.g. for:
$A_{0\beta}^{\mu\delta} \leftrightarrow B_{0\beta}^{\nu\gamma}$ and $A_{0\beta}^{\delta\mu} \leftrightarrow B_{0\beta}^{\gamma\nu}$: $[xyzt]$ are factored by λ_o
$A_{0\beta}^{\mu\delta} \leftrightarrow B_{0\beta}^{\gamma\nu}$ and $A_{0\beta}^{\delta\mu} \leftrightarrow B_{0\beta}^{\nu\gamma}$: $[xyzt]$ are factored by α_o
For these subloops vectors are factored by either α_o or λ_o from the $[\sigma_o, \lambda_o, \alpha_o, \beta_o]$ subloop, and for:
$A_{0\delta}^{\beta\mu} \leftrightarrow B_{0\gamma}^{\beta\nu}$: $[xyz]$ are factored by σ_o, $[t] \times \lambda_o$
$A_{0\delta}^{\beta\mu} \leftrightarrow B_{0\gamma}^{\nu\beta}$: $[xyz]$ are factored by γ_o, $[t] \times \delta_o$
$A_{0\delta}^{\mu\beta} \leftrightarrow B_{0\gamma}^{\nu\beta}$: $[xyz]$ are factored by λ_o, $[t] \times \sigma_o$
$A_{0\delta}^{\mu\beta} \leftrightarrow B_{0\gamma}^{\beta\nu}$: $[xyz]$ For these subloops vectors are factored by δ_o, $[t] \times \gamma_o$
vectors are factored by elements from the $[\sigma_o, \lambda_o, \gamma_o, \delta_o]$ subloop.
Subloops for all such chirality changes include λ_o, so it is associated with a chirality operator

5. Strong nuclear sector

Elements subscripted $\iota, \jmath$ and κ, when complexified, can be associated with rotations in in three complex dimensions, for which $SU(3)$ is applicable symmetry group. When $[\sigma_o, \iota_o, \jmath_o, \kappa_o]$ are multiplied by α_o, the result is a loop of unit octonions. So, if α_o is fixed the associated automorphism group becomes the $SU(3)$ subgroup of $G2$.

5.1. Colored fermions - quarks

In section 4.2 subloops $A_{1..6}$ and $B_{1..6}$ were assigned to quarks. Inspection of table 2 suggests that they can be combined in sets of three so that spatial equivalence is achieved for the combination, as elements subscripted $\iota, \jmath$ and κ would be included in a balanced way. Also, whilst a single subloop representing a single color quark would be unbalanced with respect to these subscripted elements, in a combination with a subloop representing its antiquark the imbalance can be eliminated.

5.2. Colored Bosons - Gluons

Subloops Σ_4, Σ_5 and Σ_6 do not feature internal spatial equivalence, so are associated with a single color, and are subject to the strong nuclear force. For a given subloop/color, their $\sigma_{o\iota\jmath\kappa}$, $\lambda_{o\iota\jmath\kappa}$, $\mu_{o\iota\jmath\kappa}$, $\nu_{o\iota\jmath\kappa}$, $\alpha_{o\iota\jmath\kappa}$, $\beta_{o\iota\jmath\kappa}$, $\gamma_{o\iota\jmath\kappa}$,and $\delta_{o\iota\jmath\kappa}$ contents all match each other, so gluons are not subject to the electroweak force.

5.3. Flavor/Colored bosons - U bosons

With respect to subloops Σ_4, Σ_5 and Σ_6, subloops Σ_7 to Σ_{15} have a similar relationship to that for subloops Σ_1, Σ_2 and Σ_3 with respect to Σ_o. This suggests that they will acquire mass by a Brout-Englert-Higgs type mechanism generating bosons similar to the lepto-quark bosons featuring in Pati-Salam models[34].

5.4. Higgs Boson

An algebra assembled as $[S, T, U, V] \otimes [\sigma_o, \lambda_o, \mu_o, \nu_o] \otimes [\sigma_o, i\sigma_o, \alpha_o, i\alpha_o]$ has only scalar components with respect to the spatial dimensions for $CL_{3,1}(R)$ so could accommodate a subalgebra repesenting the Higgs boson.

6. Charge

In [3], charge was postulated as being associated with the proportion of $[\beta_\iota, \beta_\jmath, \beta_\kappa]$ present. This was dictated by the assignment of A_0 and B_0 to the electron/muon/tau family and Γ_0 to the neutrino family. However, this rule only confers charge for one electroweak boson subloop. A rule that generates charge assignments that are consistent with the fermion families' charges, and which confers charges for two electroweak boson subloops, is: to associate neutrality with the proportion of $[\lambda_\iota, \lambda_\jmath, \lambda_\kappa]$ present.

For this rule, the Γ_0, Σ_0 and Σ_2 families are neutral, the A_0 and B_0 families have unit charge, the $A_{1..3}$ families and $B_{1..3}$ families have 1/3 charge and the $A_{4..6}$ families and $B_{1..3}$ families have 2/3 charge. The $\Sigma_{1..3}$ subloops would also have 1/3 charge, but this need not prevent these subloops from being used for gluons, as gluons are combinations of colors with anticolors, so combinations can have zero electric charge.

7. Supersymmetry

For each fermionic subalgebra, there is a bosonic subalgebra with the same $[\sigma_{o\iota\jmath\kappa}, \lambda_{o\iota\jmath\kappa}, \mu_{o\iota\jmath\kappa}, \nu_{o\iota\jmath\kappa}]$ content, but inverted $[\alpha_{o\iota\jmath\kappa}, \beta_{o\iota\jmath\kappa}, \gamma_{o\iota\jmath\kappa}, \delta_{o\iota\jmath\kappa}]$ content. This suggests a form of supersymmetry. The bosonic subalgebra Σ_0 is left without a supersymmetric partner.

8. Dark matter

$\mathbb{U}$ does not offer subalgebras that could represent fermionic dark matter. This suggests that the bosonic subalgebras postulated as representing U bosons may be responsible for the existence of dark matter, possibly combining to create bosonic dark matter particles in a similar way to that in which gluons can combine to create glueballs.

9. Measurement/collapse

For a model based on a non-associative algebra, it may be possible to ascribe the phenomenon of quantum measurement/collapse to its non-associativity. For a model based on $\mathbb{T} \otimes Cl_{3,1}(C)$, this suggests a universe propagating as a Huygens wave for a conformal space embedded in five dimensions with a $Cl_{0,5}(R) \cong Cl_{3,1}(C)$ multivector combined with absolute time. Between measurements particle trajectories would be deterministic, but, because handings and order of algebraic operations could differ for different points of observation, calculations of trajectories between events would be ambiguous. As a result, any attempt to predict events could only be probabilistic. Events would be loci for which calculations for all points of observation generate a consistent but not necessarily identical result, some latitude being possible subject to limits imposed by the Heisenberg uncertainty principle. Loci with inconsistent results would be occupied by "quantum foam". A theory based on this approach may require application of the principles of chaos theory.

10. Conclusion

It seems reasonable to suppose that, in the tradition of the periodic table and Bohr's model of the atom, there might be a simplistic mathematical pattern with a structure similar to that of the standard model. Just such a pattern can be found for $\mathbb{U}$. This may be a coincidence, but the similarities are striking, so the speculative analysis presented in this paper may assist in finding a path to a deeper understanding of the basis of reality.

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Appendix A. Cayley tables for M_L and T_L

A.1. Cayley tables

The Cayley tables of both $M_4(C)$ and $\mathbb{T}$ can be assembled as normalised latin squares with elements ordered so that bit-wise 'exclusive or' (XOR) of binary representations of two elements' numbering generates the numbering of their product. As a result, if the sign of products is ignored, their Cayley tables are the same and, for subalgebras that include the negative of the identity, their subalgebra inventory is the same. To detail a scheme of ultra-complexification of $M_4(C)$, notation such as $e_0, e_1....e_{31}$ could be used for its elements. However, as one requirement for a unification algebra is consistency with respect to the principle of equivalence of spatial dimensions, an alternative approach to labelling unit elements has been adopted.

A.2. Notation for $M_4(C)$ unit elements

Table 11. Notation used to label 4×4 unit matrices

$$S = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \quad P = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad M = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$Y = \begin{bmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \quad E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad T = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad X = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \quad N = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$F = \begin{bmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad Z = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad L = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

$$U = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad V = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad Q = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Note: the forms of these matrices differ from those used in previous papers by this author[35][2]. Positive forms have been chosen to allow $[S, L, M, N]$ to represent unit elements for a right isoclinic quaternion algebra $\mathbb{H}_{\mathbb{R}}$, and $[S, T, U, V]$ to represent unit elements for a left isoclinic quaternion algebra $\mathbb{H}_{\mathbb{L}}$, as used by Van Elfrinkhof[36].

The multiplication table for $M_4(C)$ using these labels is shown in Table 12.

TABLE 12. Labels and Cayley table for $\mathbb{M} \cong M_4(C)$

	e_0	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	e_{11}	e_{12}	e_{13}	e_{14}	e_{15}	e_{16}	e_{17}	e_{18}	e_{19}	e_{20}	e_{21}	e_{22}	e_{23}	e_{24}	e_{25}	e_{26}	e_{27}	e_{28}	e_{29}	e_{30}	e_{31}
	S	L	M	N	T	P	Q	R	U	X	Y	Z	V	D	E	F	iS	iL	iM	iN	iT	iP	iQ	iR	iU	iX	iY	iZ	iV	iD	iE	iF
S	+S	+L	+M	+N	+T	+P	+Q	+R	+U	+X	+Y	+Z	+V	+D	+E	+F	+iS	+iL	+iM	+iN	+iT	+iP	+iQ	+iR	+iU	+iX	+iY	+iZ	+iV	+iD	+iE	+iF
L	+L	−S	−N	+M	+P	−T	−R	+Q	+X	−U	−Z	+Y	+D	−V	−F	+E	+iL	−iS	−iN	+iM	+iP	−iT	−iR	+iQ	+iX	−iU	−iZ	+iY	+iD	−iV	−iF	+iE
M	+M	+N	−S	−L	+Q	+R	−T	−P	+Y	+Z	−U	−X	+E	+F	−V	−D	+iM	+iN	−iS	−iL	+iQ	+iR	−iT	−iP	+iY	+iZ	−iU	−iX	+iE	+iF	−iV	−iD
N	+N	−M	+L	−S	+R	−Q	+P	−T	+Z	−Y	+X	−U	+F	−E	+D	−V	+iN	−iM	+iL	−iS	+iR	−iQ	+iP	−iT	+iZ	−iY	+iX	−iU	+iF	−iE	+iD	−iV
T	+T	+P	+Q	+R	−S	−L	−M	−N	+V	+D	+E	+F	−U	−X	−Y	−Z	+iT	+iP	+iQ	+iR	−iS	−iL	−iM	−iN	+iV	+iD	+iE	+iF	−iU	−iX	−iY	−iZ
P	+P	−T	−R	+Q	−L	+S	+N	−M	+D	−V	−F	+E	−X	+U	+Z	−Y	+iP	−iT	−iR	+iQ	−iL	+iS	+iN	−iM	+iD	−iV	−iF	+iE	−iX	+iU	+iZ	−iY
Q	+Q	+R	−T	−P	−M	−N	+S	+L	+E	+F	−V	−D	−Y	−Z	+U	+X	+iQ	+iR	−iT	−iP	−iM	−iN	+iS	+iL	+iE	+iF	−iV	−iD	−iY	−iZ	+iU	+iX
R	+R	−Q	+P	−T	−N	+M	−L	+S	+F	−E	+D	−V	−Z	+Y	−X	+U	+iR	−iQ	+iP	−iT	−iN	+iM	−iL	+iS	+iF	−iE	+iD	−iV	−iZ	+iY	−iX	+iU
U	+U	+X	+Y	+Z	−V	−D	−E	−F	−S	−L	−M	−N	+T	+P	+Q	+R	+iU	+iX	+iY	+iZ	−iV	−iD	−iE	−iF	−iS	−iL	−iM	−iN	+iT	+iP	+iQ	+iR
X	+X	−U	−Z	+Y	−D	+V	+F	−E	−L	+S	+N	−M	+P	−T	−R	+Q	+iX	−iU	−iZ	+iY	−iD	+iV	+iF	−iE	−iL	+iS	+iN	−iM	+iP	−iT	−iR	+iQ
Y	+Y	+Z	−U	−X	−E	−F	+V	+D	−M	−N	+S	+L	+Q	+R	−T	−P	+iY	+iZ	−iU	−iX	−iE	−iF	+iV	+iD	−iM	−iN	+iS	+iL	+iQ	+iR	−iT	−iP
Z	+Z	−Y	+X	−U	−F	+E	−D	+V	−N	+M	−L	+S	+R	−Q	+P	−T	+iZ	−iY	+iX	−iU	−iF	+iE	−iD	+iV	−iN	+iM	−iL	+iS	+iR	−iQ	+iP	−iT
V	+V	+D	+E	+F	+U	+X	+Y	+Z	−T	−P	−Q	−R	−S	−L	−M	−N	+iV	+iD	+iE	+iF	+iU	+iX	+iY	+iZ	−iT	−iP	−iQ	−iR	−iS	−iL	−iM	−iN
D	+D	−V	−F	+E	+X	−U	−Z	+Y	−P	+T	+R	−Q	−L	+S	+N	−M	+iD	−iV	−iF	+iE	+iX	−iU	−iZ	+iY	−iP	+iT	+iR	−iQ	−iL	+iS	+iN	−iM
E	+E	+F	−V	−D	+Y	+Z	−U	−X	−Q	−R	+T	+P	−M	−N	+S	+L	+iE	+iF	−iV	−iD	+iY	+iZ	−iU	−iX	−iQ	−iR	+iT	+iP	−iM	−iN	+iS	+iL
F	+F	−E	+D	−V	+Z	−Y	+X	−U	−R	+Q	−P	+T	−N	+M	−L	+S	+iF	−iE	+iD	−iV	+iZ	−iY	+iX	−iU	−iR	+iQ	−iP	+iT	−iN	+iM	−iL	+iS
S	+iS	+iL	+iM	+iN	+iT	+iP	+iQ	+iR	+iU	+iX	+iY	+iZ	+iV	+iD	+iE	+iF	−S	−L	−M	−N	−T	−P	−Q	−R	−U	−X	−Y	−Z	−V	−D	−E	−F
L	+iL	−iS	−iN	+iM	+iP	−iT	−iR	+iQ	+iX	−iU	−iZ	+iY	+iD	−iV	−iF	+iE	−L	+S	+N	−M	−P	+T	+R	−Q	−X	+U	+Z	−Y	−D	+V	+F	−E
M	+iM	+iN	−iS	−iL	+iQ	+iR	−iT	−iP	+iY	+iZ	−iU	−iX	+iE	+iF	−iV	−iD	−M	−N	+S	+L	−Q	−R	+T	+P	−Y	−Z	+U	+X	−E	−F	+V	+D
N	+iN	−iM	+iL	−iS	+iR	−iQ	+iP	−iT	+iZ	−iY	+iX	−iU	+iF	−iE	+iD	−iV	−N	+M	−L	+S	−R	+Q	−P	+T	−Z	+Y	−X	+U	−F	+E	−D	+V
T	+iT	+iP	+iQ	+iR	−iS	−iL	−iM	−iN	+iV	+iD	+iE	+iF	−iU	−iX	−iY	−iZ	−T	−P	−Q	−R	+S	+L	+M	+N	−V	−D	−E	−F	+U	+X	+Y	+Z
P	+iP	−iT	−iR	+iQ	−iL	+iS	+iN	−iM	+iD	−iV	−iF	+iE	−iX	+iU	+iZ	−iY	−P	+T	+R	−Q	+L	−S	−N	+M	−D	+V	+F	−E	+X	−U	−Z	+Y
Q	+iQ	+iR	−iT	−iP	−iM	−iN	+iS	+iL	+iE	+iF	−iV	−iD	−iY	−iZ	+iU	+iX	−Q	−R	+T	+P	+M	+N	−S	−L	−E	−F	+V	+D	+Y	+Z	−U	−X
R	+iR	−iQ	+iP	−iT	−iN	+iM	−iL	+iS	+iF	−iE	+iD	−iV	−iZ	+iY	−iX	+iU	−R	+Q	−P	+T	+N	−M	+L	−S	−F	+E	−D	+V	+Z	−Y	+X	−U
U	+iU	+iX	+iY	+iZ	−iV	−iD	−iE	−iF	−iS	−iL	−iM	−iN	+iT	+iP	+iQ	+iR	−U	−X	−Y	−Z	+V	+D	+E	+F	+S	+L	+M	+N	−T	−P	−Q	−R
X	+iX	−iU	−iZ	+iY	−iD	+iV	+iF	−iE	−iL	+iS	+iN	−iM	+iP	−iT	−iR	+iQ	−X	+U	+Z	−Y	+D	−V	−F	+E	+L	−S	−N	+M	−P	+T	+R	−Q
Y	+iY	+iZ	−iU	−iX	−iE	−iF	+iV	+iD	−iM	−iN	+iS	+iL	+iQ	+iR	−iT	−iP	−Y	−Z	+U	+X	+E	+F	−V	−D	+M	+N	−S	−L	−Q	−R	+T	+P
Z	+iZ	−iY	+iX	−iU	−iF	+iE	−iD	+iV	−iN	+iM	−iL	+iS	+iR	−iQ	+iP	−iT	−Z	+Y	−X	+U	+F	−E	+D	−V	+N	−M	+L	−S	−R	+Q	−P	+T
V	+iV	+iD	+iE	+iF	+iU	+iX	+iY	+iZ	−iT	−iP	−iQ	−iR	−iS	−iL	−iM	−iN	−V	−D	−E	−F	−U	−X	−Y	−Z	+T	+P	+Q	+R	+S	+L	+M	+N
D	+iD	−iV	−iF	+iE	+iX	−iU	−iZ	+iY	−iP	+iT	+iR	−iQ	−iL	+iS	+iN	−iM	−D	+V	+F	−E	−X	+U	+Z	−Y	+P	−T	−R	+Q	+L	−S	−N	+M
E	+iE	+iF	−iV	−iD	+iY	+iZ	−iU	−iX	−iQ	−iR	+iT	+iP	−iM	−iN	+iS	+iL	−E	−F	+V	+D	−Y	−Z	+U	+X	+Q	+R	−T	−P	+M	+N	−S	−L
F	+iF	−iE	+iD	−iV	+iZ	−iY	+iX	−iU	−iR	+iQ	−iP	+iT	−iN	+iM	−iL	+iS	−F	+E	−D	+V	−Z	+Y	−X	+U	+R	−Q	+P	−T	+N	−M	+L	−S

A.3. Notation for $\mathbb{T}$ unit elements

As for $M_4(C)$, for $\mathbb{T}$ an alternative approach to labelling unit elements has been adopted. Greek letters with greek subscripts have been chosen. The subscripts relate unit elements in sets in a scheme similar to that relating sets of unit elements for $M_4(C)$ to the set chosen to represent a right isoclinc quaternion algebra as used by Van Elfrinkhof. The usual labelling for $\mathbb{T}$, as used by Cawagas et al, features the labels $e_0, e_1 ... e_{31}$. Those unit elements have been relabelled as shown in table 13.

TABLE 13. Labels for $\mathbb{T}$ basis elements

e_0	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	e_{11}	e_{12}	e_{13}	e_{14}	e_{15}
σ_o	σ_ι	$\sigma_\jmath$	σ_κ	λ_o	λ_ι	$\lambda_\jmath$	λ_κ	μ_o	μ_ι	$\mu_\jmath$	μ_κ	ν_o	ν_ι	$\nu_\jmath$	ν_κ

e_{16}	e_{17}	e_{18}	e_{19}	e_{20}	e_{21}	e_{22}	e_{23}	e_{24}	e_{25}	e_{26}	e_{27}	e_{28}	e_{29}	e_{30}	e_{31}
α_o	α_ι	$\alpha_\jmath$	α_κ	β_o	β_ι	$\beta_\jmath$	β_κ	γ_o	γ_ι	$\gamma_\jmath$	γ_κ	δ_o	δ_ι	$\delta_\jmath$	δ_κ

The multiplication table for $\mathbb{T}$ using these labels is shown in Table 14.

TABLE 14. Labels and Cayley table for $\mathbb{T}$ basis elements

	e_0	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	e_{11}	e_{12}	e_{13}	e_{14}	e_{15}
	σ_o	σ_ι	$\sigma_\jmath$	σ_κ	λ_o	λ_ι	$\lambda_\jmath$	λ_κ	μ_o	μ_ι	$\mu_\jmath$	μ_κ	ν_o	ν_ι	$\nu_\jmath$	ν_κ
σ_o	$+\sigma_o$	$+\sigma_\iota$	$+\sigma_\jmath$	$+\sigma_\kappa$	$+\lambda_o$	$+\lambda_\iota$	$+\lambda_\jmath$	$+\lambda_\kappa$	$+\mu_o$	$+\mu_\iota$	$+\mu_\jmath$	$+\mu_\kappa$	$+\nu_o$	$+\nu_\iota$	$+\nu_\jmath$	$+\nu_\kappa$
σ_ι	$+\sigma_\iota$	$-\sigma_o$	$+\sigma_\kappa$	$-\sigma_\jmath$	$+\lambda_\iota$	$-\lambda_o$	$-\lambda_\kappa$	$+\lambda_\jmath$	$+\mu_\iota$	$-\mu_o$	$-\mu_\kappa$	$+\mu_\jmath$	$-\nu_\iota$	$+\nu_o$	$+\nu_\kappa$	$-\nu_\jmath$
$\sigma_\jmath$	$+\sigma_\jmath$	$-\sigma_\kappa$	$-\sigma_o$	$+\sigma_\iota$	$+\lambda_\jmath$	$+\lambda_\kappa$	$-\lambda_o$	$-\lambda_\iota$	$+\mu_\jmath$	$+\mu_\kappa$	$-\mu_o$	$-\mu_\iota$	$-\nu_\jmath$	$-\nu_\kappa$	$+\nu_o$	$+\nu_\iota$
σ_κ	$+\sigma_\kappa$	$+\sigma_\jmath$	$-\sigma_\iota$	$-\sigma_o$	$+\lambda_\kappa$	$-\lambda_\jmath$	$+\lambda_\iota$	$-\lambda_o$	$+\mu_\kappa$	$-\mu_\jmath$	$+\mu_\iota$	$-\mu_o$	$-\nu_\kappa$	$+\nu_\jmath$	$-\nu_\iota$	$+\nu_o$
λ_o	$+\lambda_o$	$-\lambda_\iota$	$-\lambda_\jmath$	$-\lambda_\kappa$	$-\sigma_o$	$+\sigma_\iota$	$+\sigma_\jmath$	$+\sigma_\kappa$	$+\nu_o$	$+\nu_\iota$	$+\nu_\jmath$	$+\nu_\kappa$	$-\mu_o$	$-\mu_\iota$	$-\mu_\jmath$	$-\mu_\kappa$
λ_ι	$+\lambda_\iota$	$+\lambda_o$	$-\lambda_\kappa$	$+\lambda_\jmath$	$-\sigma_\iota$	$-\sigma_o$	$-\sigma_\kappa$	$+\sigma_\jmath$	$+\nu_\iota$	$-\nu_o$	$+\nu_\kappa$	$-\nu_\jmath$	$+\mu_\iota$	$-\mu_o$	$+\mu_\kappa$	$-\mu_\jmath$
$\lambda_\jmath$	$+\lambda_\jmath$	$+\lambda_\kappa$	$+\lambda_o$	$-\lambda_\iota$	$-\sigma_\jmath$	$+\sigma_\kappa$	$-\sigma_o$	$-\sigma_\iota$	$+\nu_\jmath$	$-\nu_\kappa$	$-\nu_o$	$+\nu_\iota$	$+\mu_\jmath$	$-\mu_\kappa$	$-\mu_o$	$+\mu_\iota$
λ_κ	$+\lambda_\kappa$	$-\lambda_\jmath$	$+\lambda_\iota$	$+\lambda_o$	$-\sigma_\kappa$	$-\sigma_\jmath$	$+\sigma_\iota$	$-\sigma_o$	$+\nu_\kappa$	$+\nu_\jmath$	$-\nu_\iota$	$-\nu_o$	$+\mu_\kappa$	$+\mu_\jmath$	$-\mu_\iota$	$-\mu_o$
μ_o	$+\mu_o$	$-\mu_\iota$	$-\mu_\jmath$	$-\mu_\kappa$	$-\nu_o$	$-\nu_\iota$	$-\nu_\jmath$	$-\nu_\kappa$	$-\sigma_o$	$+\sigma_\iota$	$+\sigma_\jmath$	$+\sigma_\kappa$	$+\lambda_o$	$+\lambda_\iota$	$+\lambda_\jmath$	$+\lambda_\kappa$
μ_ι	$+\mu_\iota$	$+\mu_o$	$-\mu_\kappa$	$+\mu_\jmath$	$-\nu_\iota$	$+\nu_o$	$+\nu_\kappa$	$-\nu_\jmath$	$-\sigma_\iota$	$-\sigma_o$	$-\sigma_\kappa$	$+\sigma_\jmath$	$-\lambda_\iota$	$+\lambda_o$	$+\lambda_\kappa$	$-\lambda_\jmath$
$\mu_\jmath$	$+\mu_\jmath$	$+\mu_\kappa$	$+\mu_o$	$-\mu_\iota$	$-\nu_\jmath$	$-\nu_\kappa$	$+\nu_o$	$+\nu_\iota$	$-\sigma_\jmath$	$+\sigma_\kappa$	$-\sigma_o$	$-\sigma_\iota$	$-\lambda_\jmath$	$-\lambda_\kappa$	$+\lambda_o$	$+\lambda_\iota$
μ_κ	$+\mu_\kappa$	$-\mu_\jmath$	$+\mu_\iota$	$+\mu_o$	$-\nu_\kappa$	$+\nu_\jmath$	$-\nu_\iota$	$+\nu_o$	$-\sigma_\kappa$	$-\sigma_\jmath$	$+\sigma_\iota$	$-\sigma_o$	$-\lambda_\kappa$	$+\lambda_\jmath$	$-\lambda_\iota$	$+\lambda_o$
ν_o	$+\nu_o$	$+\nu_\iota$	$+\nu_\jmath$	$+\nu_\kappa$	$+\mu_o$	$-\mu_\iota$	$-\mu_\jmath$	$-\mu_\kappa$	$-\lambda_o$	$+\lambda_\iota$	$+\lambda_\jmath$	$+\lambda_\kappa$	$-\sigma_o$	$-\sigma_\iota$	$-\sigma_\jmath$	$-\sigma_\kappa$
ν_ι	$+\nu_\iota$	$-\nu_o$	$+\nu_\kappa$	$-\nu_\jmath$	$+\mu_\iota$	$+\mu_o$	$+\mu_\kappa$	$-\mu_\jmath$	$-\lambda_\iota$	$-\lambda_o$	$+\lambda_\kappa$	$-\lambda_\jmath$	$+\sigma_\iota$	$-\sigma_o$	$+\sigma_\kappa$	$-\sigma_\jmath$
$\nu_\jmath$	$+\nu_\jmath$	$-\nu_\kappa$	$-\nu_o$	$+\nu_\iota$	$+\mu_\jmath$	$-\mu_\kappa$	$+\mu_o$	$+\mu_\iota$	$-\lambda_\jmath$	$-\lambda_\kappa$	$-\lambda_o$	$+\lambda_\iota$	$+\sigma_\jmath$	$-\sigma_\kappa$	$-\sigma_o$	$+\sigma_\iota$
ν_κ	$+\nu_\kappa$	$+\nu_\jmath$	$-\nu_\iota$	$-\nu_o$	$+\mu_\kappa$	$+\mu_\jmath$	$-\mu_\iota$	$+\mu_o$	$-\lambda_\kappa$	$+\lambda_\jmath$	$-\lambda_\iota$	$-\lambda_o$	$+\sigma_\kappa$	$+\sigma_\jmath$	$-\sigma_\iota$	$-\sigma_o$
α_o	$+\alpha_o$	$-\alpha_\iota$	$-\alpha_\jmath$	$-\alpha_\kappa$	$-\beta_o$	$-\beta_\iota$	$-\beta_\jmath$	$-\beta_\kappa$	$-\gamma_o$	$-\gamma_\iota$	$-\gamma_\jmath$	$-\gamma_\kappa$	$-\delta_o$	$-\delta_\iota$	$-\delta_\jmath$	$-\delta_\kappa$
α_ι	$+\alpha_\iota$	$+\alpha_o$	$-\alpha_\kappa$	$+\alpha_\jmath$	$-\beta_\iota$	$+\beta_o$	$+\beta_\kappa$	$-\beta_\jmath$	$-\gamma_\iota$	$+\gamma_o$	$+\gamma_\kappa$	$-\gamma_\jmath$	$+\delta_\iota$	$-\delta_o$	$-\delta_\kappa$	$+\delta_\jmath$
$\alpha_\jmath$	$+\alpha_\jmath$	$+\alpha_\kappa$	$+\alpha_o$	$-\alpha_\iota$	$-\beta_\jmath$	$-\beta_\kappa$	$+\beta_o$	$+\beta_\iota$	$-\gamma_\jmath$	$-\gamma_\kappa$	$+\gamma_o$	$+\gamma_\iota$	$+\delta_\jmath$	$+\delta_\kappa$	$-\delta_o$	$-\delta_\iota$
α_κ	$+\alpha_\kappa$	$-\alpha_\jmath$	$+\alpha_\iota$	$+\alpha_o$	$-\beta_\kappa$	$+\beta_\jmath$	$-\beta_\iota$	$+\beta_o$	$-\gamma_\kappa$	$+\gamma_\jmath$	$-\gamma_\iota$	$+\gamma_o$	$+\delta_\kappa$	$-\delta_\jmath$	$+\delta_\iota$	$-\delta_o$
β_o	$+\beta_o$	$+\beta_\iota$	$+\beta_\jmath$	$+\beta_\kappa$	$+\alpha_o$	$-\alpha_\iota$	$-\alpha_\jmath$	$-\alpha_\kappa$	$-\delta_o$	$-\delta_\iota$	$-\delta_\jmath$	$-\delta_\kappa$	$+\gamma_o$	$+\gamma_\iota$	$+\gamma_\jmath$	$+\gamma_\kappa$
β_ι	$+\beta_\iota$	$-\beta_o$	$+\beta_\kappa$	$-\beta_\jmath$	$+\alpha_\iota$	$+\alpha_o$	$+\alpha_\kappa$	$-\alpha_\jmath$	$-\delta_\iota$	$+\delta_o$	$-\delta_\kappa$	$+\delta_\jmath$	$-\gamma_\iota$	$+\gamma_o$	$-\gamma_\kappa$	$+\gamma_\jmath$
$\beta_\jmath$	$+\beta_\jmath$	$-\beta_\kappa$	$-\beta_o$	$+\beta_\iota$	$+\alpha_\jmath$	$-\alpha_\kappa$	$+\alpha_o$	$+\alpha_\iota$	$-\delta_\jmath$	$+\delta_\kappa$	$+\delta_o$	$-\delta_\iota$	$-\gamma_\jmath$	$+\gamma_\kappa$	$+\gamma_o$	$-\gamma_\iota$
β_κ	$+\beta_\kappa$	$+\beta_\jmath$	$-\beta_\iota$	$-\beta_o$	$+\alpha_\kappa$	$+\alpha_\jmath$	$-\alpha_\iota$	$+\alpha_o$	$-\delta_\kappa$	$-\delta_\jmath$	$+\delta_\iota$	$+\delta_o$	$-\gamma_\kappa$	$-\gamma_\jmath$	$+\gamma_\iota$	$+\gamma_o$
γ_o	$+\gamma_o$	$+\gamma_\iota$	$+\gamma_\jmath$	$+\gamma_\kappa$	$+\delta_o$	$+\delta_\iota$	$+\delta_\jmath$	$+\delta_\kappa$	$+\alpha_o$	$-\alpha_\iota$	$-\alpha_\jmath$	$-\alpha_\kappa$	$-\beta_o$	$-\beta_\iota$	$-\beta_\jmath$	$-\beta_\kappa$
γ_ι	$+\gamma_\iota$	$-\gamma_o$	$+\gamma_\kappa$	$-\gamma_\jmath$	$+\delta_\iota$	$-\delta_o$	$-\delta_\kappa$	$+\delta_\jmath$	$+\alpha_\iota$	$+\alpha_o$	$+\alpha_\kappa$	$-\alpha_\jmath$	$+\beta_\iota$	$-\beta_o$	$-\beta_\kappa$	$+\beta_\jmath$
$\gamma_\jmath$	$+\gamma_\jmath$	$-\gamma_\kappa$	$-\gamma_o$	$+\gamma_\iota$	$+\delta_\jmath$	$+\delta_\kappa$	$-\delta_o$	$-\delta_\iota$	$+\alpha_\jmath$	$-\alpha_\kappa$	$+\alpha_o$	$+\alpha_\iota$	$+\beta_\jmath$	$+\beta_\kappa$	$-\beta_o$	$-\beta_\iota$
γ_κ	$+\gamma_\kappa$	$+\gamma_\jmath$	$-\gamma_\iota$	$-\gamma_o$	$+\delta_\kappa$	$-\delta_\jmath$	$+\delta_\iota$	$-\delta_o$	$+\alpha_\kappa$	$+\alpha_\jmath$	$-\alpha_\iota$	$+\alpha_o$	$+\beta_\kappa$	$-\beta_\jmath$	$+\beta_\iota$	$-\beta_o$
δ_o	$+\delta_o$	$-\delta_\iota$	$-\delta_\jmath$	$-\delta_\kappa$	$-\gamma_o$	$+\gamma_\iota$	$+\gamma_\jmath$	$+\gamma_\kappa$	$+\beta_o$	$-\beta_\iota$	$-\beta_\jmath$	$-\beta_\kappa$	$+\alpha_o$	$+\alpha_\iota$	$+\alpha_\jmath$	$+\alpha_\kappa$
δ_ι	$+\delta_\iota$	$+\delta_o$	$-\delta_\kappa$	$+\delta_\jmath$	$-\gamma_\iota$	$-\gamma_o$	$-\gamma_\kappa$	$+\gamma_\jmath$	$+\beta_\iota$	$+\beta_o$	$-\beta_\kappa$	$+\beta_\jmath$	$-\alpha_\iota$	$+\alpha_o$	$-\alpha_\kappa$	$+\alpha_\jmath$
$\delta_\jmath$	$+\delta_\jmath$	$+\delta_\kappa$	$+\delta_o$	$-\delta_\iota$	$-\gamma_\jmath$	$+\gamma_\kappa$	$-\gamma_o$	$-\gamma_\iota$	$+\beta_\jmath$	$+\beta_\kappa$	$+\beta_o$	$-\beta_\iota$	$-\alpha_\jmath$	$+\alpha_\kappa$	$+\alpha_o$	$-\alpha_\iota$
δ_κ	$+\delta_\kappa$	$-\delta_\jmath$	$+\delta_\iota$	$+\delta_o$	$-\gamma_\kappa$	$-\gamma_\jmath$	$+\gamma_\iota$	$-\gamma_o$	$+\beta_\kappa$	$-\beta_\jmath$	$+\beta_\iota$	$+\beta_o$	$-\alpha_\kappa$	$-\alpha_\jmath$	$+\alpha_\iota$	$+\alpha_o$

	e_{16}	e_{17}	e_{18}	e_{19}	e_{20}	e_{21}	e_{22}	e_{23}	e_{24}	e_{25}	e_{26}	e_{27}	e_{28}	e_{29}	e_{30}	e_{31}
	α_o	α_ι	$\alpha_\jmath$	α_κ	β_o	β_ι	$\beta_\jmath$	β_κ	γ_o	γ_ι	$\gamma_\jmath$	γ_κ	δ_o	δ_ι	$\delta_\jmath$	δ_κ
σ_o	$+\alpha_o$	$+\alpha_\iota$	$+\alpha_\jmath$	$+\alpha_\kappa$	$+\beta_o$	$+\beta_\iota$	$+\beta_\jmath$	$+\beta_\kappa$	$+\gamma_o$	$+\gamma_\iota$	$+\gamma_\jmath$	$+\gamma_\kappa$	$+\delta_o$	$+\delta_\iota$	$+\delta_\jmath$	$+\delta_\kappa$
σ_ι	$+\alpha_\iota$	$-\alpha_o$	$-\alpha_\kappa$	$+\alpha_\jmath$	$-\beta_\iota$	$+\beta_o$	$+\beta_\kappa$	$-\beta_\jmath$	$-\gamma_\iota$	$+\gamma_o$	$+\gamma_\kappa$	$-\gamma_\jmath$	$+\delta_\iota$	$-\delta_o$	$-\delta_\kappa$	$+\delta_\jmath$
$\sigma_\jmath$	$+\alpha_\jmath$	$+\alpha_\kappa$	$-\alpha_o$	$-\alpha_\iota$	$-\beta_\jmath$	$-\beta_\kappa$	$+\beta_o$	$+\beta_\iota$	$-\gamma_\jmath$	$-\gamma_\kappa$	$+\gamma_o$	$+\gamma_\iota$	$+\delta_\jmath$	$+\delta_\kappa$	$-\delta_o$	$-\delta_\iota$
σ_κ	$+\alpha_\kappa$	$-\alpha_\jmath$	$+\alpha_\iota$	$-\alpha_o$	$-\beta_\kappa$	$+\beta_\jmath$	$-\beta_\iota$	$+\beta_o$	$-\gamma_\kappa$	$+\gamma_\jmath$	$-\gamma_\iota$	$+\gamma_o$	$+\delta_\kappa$	$-\delta_\jmath$	$+\delta_\iota$	$-\delta_o$
λ_o	$+\beta_o$	$+\beta_\iota$	$+\beta_\jmath$	$+\beta_\kappa$	$-\alpha_o$	$-\alpha_\iota$	$-\alpha_\jmath$	$-\alpha_\kappa$	$-\delta_o$	$-\delta_\iota$	$-\delta_\jmath$	$-\delta_\kappa$	$+\gamma_o$	$+\gamma_\iota$	$+\gamma_\jmath$	$+\gamma_\kappa$
λ_ι	$+\beta_\iota$	$-\beta_o$	$+\beta_\kappa$	$-\beta_\jmath$	$+\alpha_\iota$	$-\alpha_o$	$+\alpha_\kappa$	$-\alpha_\jmath$	$-\delta_\iota$	$+\delta_o$	$-\delta_\kappa$	$+\delta_\jmath$	$-\gamma_\iota$	$+\gamma_o$	$-\gamma_\kappa$	$+\gamma_\jmath$
$\lambda_\jmath$	$+\beta_\jmath$	$-\beta_\kappa$	$-\beta_o$	$+\beta_\iota$	$+\alpha_\jmath$	$-\alpha_\kappa$	$-\alpha_o$	$+\alpha_\iota$	$-\delta_\jmath$	$+\delta_\kappa$	$+\delta_o$	$-\delta_\iota$	$-\gamma_\jmath$	$+\gamma_\kappa$	$+\gamma_o$	$-\gamma_\iota$
λ_κ	$+\beta_\kappa$	$+\beta_\jmath$	$-\beta_\iota$	$-\beta_o$	$+\alpha_\kappa$	$+\alpha_\jmath$	$-\alpha_\iota$	$-\alpha_o$	$-\delta_\kappa$	$-\delta_\jmath$	$+\delta_\iota$	$+\delta_o$	$-\gamma_\kappa$	$-\gamma_\jmath$	$+\gamma_\iota$	$+\gamma_o$
μ_o	$+\gamma_o$	$+\gamma_\iota$	$+\gamma_\jmath$	$+\gamma_\kappa$	$+\delta_o$	$+\delta_\iota$	$+\delta_\jmath$	$+\delta_\kappa$	$-\alpha_o$	$-\alpha_\iota$	$-\alpha_\jmath$	$-\alpha_\kappa$	$-\beta_o$	$-\beta_\iota$	$-\beta_\jmath$	$-\beta_\kappa$
μ_ι	$+\gamma_\iota$	$-\gamma_o$	$+\gamma_\kappa$	$-\gamma_\jmath$	$+\delta_\iota$	$-\delta_o$	$-\delta_\kappa$	$+\delta_\jmath$	$+\alpha_\iota$	$-\alpha_o$	$+\alpha_\kappa$	$-\alpha_\jmath$	$+\beta_\iota$	$-\beta_o$	$-\beta_\kappa$	$+\beta_\jmath$
$\mu_\jmath$	$+\gamma_\jmath$	$-\gamma_\kappa$	$-\gamma_o$	$+\gamma_\iota$	$+\delta_\jmath$	$+\delta_\kappa$	$-\delta_o$	$-\delta_\iota$	$+\alpha_\jmath$	$-\alpha_\kappa$	$-\alpha_o$	$+\alpha_\iota$	$+\beta_\jmath$	$+\beta_\kappa$	$-\beta_o$	$-\beta_\iota$
μ_κ	$+\gamma_\kappa$	$+\gamma_\jmath$	$-\gamma_\iota$	$-\gamma_o$	$+\delta_\kappa$	$-\delta_\jmath$	$+\delta_\iota$	$-\delta_o$	$+\alpha_\kappa$	$+\alpha_\jmath$	$-\alpha_\iota$	$-\alpha_o$	$+\beta_\kappa$	$-\beta_\jmath$	$+\beta_\iota$	$-\beta_o$
ν_o	$+\delta_o$	$-\delta_\iota$	$-\delta_\jmath$	$-\delta_\kappa$	$-\gamma_o$	$+\gamma_\iota$	$+\gamma_\jmath$	$+\gamma_\kappa$	$+\beta_o$	$-\beta_\iota$	$-\beta_\jmath$	$-\beta_\kappa$	$-\alpha_o$	$+\alpha_\iota$	$+\alpha_\jmath$	$+\alpha_\kappa$
ν_ι	$+\delta_\iota$	$+\delta_o$	$-\delta_\kappa$	$+\delta_\jmath$	$-\gamma_\iota$	$-\gamma_o$	$-\gamma_\kappa$	$+\gamma_\jmath$	$+\beta_\iota$	$+\beta_o$	$-\beta_\kappa$	$+\beta_\jmath$	$-\alpha_\iota$	$-\alpha_o$	$-\alpha_\kappa$	$+\alpha_\jmath$
$\nu_\jmath$	$+\delta_\jmath$	$+\delta_\kappa$	$+\delta_o$	$-\delta_\iota$	$-\gamma_\jmath$	$+\gamma_\kappa$	$-\gamma_o$	$-\gamma_\iota$	$+\beta_\jmath$	$+\beta_\kappa$	$+\beta_o$	$-\beta_\iota$	$-\alpha_\jmath$	$+\alpha_\kappa$	$-\alpha_o$	$-\alpha_\iota$
ν_κ	$+\delta_\kappa$	$-\delta_\jmath$	$+\delta_\iota$	$+\delta_o$	$-\gamma_\kappa$	$-\gamma_\jmath$	$+\gamma_\iota$	$-\gamma_o$	$+\beta_\kappa$	$-\beta_\jmath$	$+\beta_\iota$	$+\beta_o$	$-\alpha_\kappa$	$-\alpha_\jmath$	$+\alpha_\iota$	$-\alpha_o$
α_o	$-\sigma_o$	$+\sigma_\iota$	$+\sigma_\jmath$	$+\sigma_\kappa$	$+\lambda_o$	$+\lambda_\iota$	$+\lambda_\jmath$	$+\lambda_\kappa$	$+\mu_o$	$+\mu_\iota$	$+\mu_\jmath$	$+\mu_\kappa$	$+\nu_o$	$+\nu_\iota$	$+\nu_\jmath$	$+\nu_\kappa$
α_ι	$-\sigma_\iota$	$-\sigma_o$	$-\sigma_\kappa$	$+\sigma_\jmath$	$-\lambda_\iota$	$+\lambda_o$	$+\lambda_\kappa$	$-\lambda_\jmath$	$-\mu_\iota$	$+\mu_o$	$+\mu_\kappa$	$-\mu_\jmath$	$+\nu_\iota$	$-\nu_o$	$-\nu_\kappa$	$+\nu_\jmath$
$\alpha_\jmath$	$-\sigma_\jmath$	$+\sigma_\kappa$	$-\sigma_o$	$-\sigma_\iota$	$-\lambda_\jmath$	$-\lambda_\kappa$	$+\lambda_o$	$+\lambda_\iota$	$-\mu_\jmath$	$-\mu_\kappa$	$+\mu_o$	$+\mu_\iota$	$+\nu_\jmath$	$+\nu_\kappa$	$-\nu_o$	$-\nu_\iota$
α_κ	$-\sigma_\kappa$	$-\sigma_\jmath$	$+\sigma_\iota$	$-\sigma_o$	$-\lambda_\kappa$	$+\lambda_\jmath$	$-\lambda_\iota$	$+\lambda_o$	$-\mu_\kappa$	$+\mu_\jmath$	$-\mu_\iota$	$+\mu_o$	$+\nu_\kappa$	$-\nu_\jmath$	$+\nu_\iota$	$-\nu_o$
β_o	$-\lambda_o$	$-\lambda_\iota$	$-\lambda_\jmath$	$-\lambda_\kappa$	$-\sigma_o$	$-\sigma_\iota$	$-\sigma_\jmath$	$-\sigma_\kappa$	$-\nu_o$	$-\nu_\iota$	$-\nu_\jmath$	$-\nu_\kappa$	$+\mu_o$	$+\mu_\iota$	$+\mu_\jmath$	$+\mu_\kappa$
β_ι	$-\lambda_\iota$	$+\lambda_o$	$+\lambda_\kappa$	$-\lambda_\jmath$	$+\sigma_\iota$	$-\sigma_o$	$+\sigma_\kappa$	$-\sigma_\jmath$	$-\nu_\iota$	$+\nu_o$	$-\nu_\kappa$	$+\nu_\jmath$	$-\mu_\iota$	$+\mu_o$	$-\mu_\kappa$	$+\mu_\jmath$
$\beta_\jmath$	$-\lambda_\jmath$	$-\lambda_\kappa$	$-\lambda_o$	$+\lambda_\iota$	$+\sigma_\jmath$	$-\sigma_\kappa$	$-\sigma_o$	$+\sigma_\iota$	$-\nu_\jmath$	$+\nu_\kappa$	$+\nu_o$	$-\nu_\iota$	$-\mu_\jmath$	$+\mu_\kappa$	$+\mu_o$	$-\mu_\iota$
β_κ	$-\lambda_\kappa$	$+\lambda_\jmath$	$-\lambda_\iota$	$-\lambda_o$	$+\sigma_\kappa$	$+\sigma_\jmath$	$-\sigma_\iota$	$-\sigma_o$	$-\nu_\kappa$	$-\nu_\jmath$	$+\nu_\iota$	$+\nu_o$	$-\mu_\kappa$	$-\mu_\jmath$	$+\mu_\iota$	$+\mu_o$
γ_o	$-\mu_o$	$+\mu_\iota$	$+\mu_\jmath$	$+\mu_\kappa$	$+\nu_o$	$+\nu_\iota$	$+\nu_\jmath$	$+\nu_\kappa$	$-\sigma_o$	$-\sigma_\iota$	$-\sigma_\jmath$	$-\sigma_\kappa$	$-\lambda_o$	$-\lambda_\iota$	$-\lambda_\jmath$	$-\lambda_\kappa$
γ_ι	$-\mu_\iota$	$-\mu_o$	$+\mu_\kappa$	$-\mu_\jmath$	$+\nu_\iota$	$-\nu_o$	$-\nu_\kappa$	$+\nu_\jmath$	$+\sigma_\iota$	$-\sigma_o$	$+\sigma_\kappa$	$-\sigma_\jmath$	$+\lambda_\iota$	$-\lambda_o$	$-\lambda_\kappa$	$+\lambda_\jmath$
$\gamma_\jmath$	$-\mu_\jmath$	$-\mu_\kappa$	$-\mu_o$	$+\mu_\iota$	$+\nu_\jmath$	$+\nu_\kappa$	$-\nu_o$	$-\nu_\iota$	$+\sigma_\jmath$	$-\sigma_\kappa$	$-\sigma_o$	$+\sigma_\iota$	$+\lambda_\jmath$	$+\lambda_\kappa$	$-\lambda_o$	$-\lambda_\iota$
γ_κ	$-\mu_\kappa$	$+\mu_\jmath$	$-\mu_\iota$	$-\mu_o$	$+\nu_\kappa$	$-\nu_\jmath$	$+\nu_\iota$	$-\nu_o$	$+\sigma_\kappa$	$+\sigma_\jmath$	$-\sigma_\iota$	$-\sigma_o$	$+\lambda_\kappa$	$-\lambda_\jmath$	$+\lambda_\iota$	$-\lambda_o$
δ_o	$-\nu_o$	$-\nu_\iota$	$-\nu_\jmath$	$-\nu_\kappa$	$-\mu_o$	$+\mu_\iota$	$+\mu_\jmath$	$+\mu_\kappa$	$+\lambda_o$	$-\lambda_\iota$	$-\lambda_\jmath$	$-\lambda_\kappa$	$-\sigma_o$	$+\sigma_\iota$	$+\sigma_\jmath$	$+\sigma_\kappa$
δ_ι	$-\nu_\iota$	$+\nu_o$	$-\nu_\kappa$	$+\nu_\jmath$	$-\mu_\iota$	$-\mu_o$	$-\mu_\kappa$	$+\mu_\jmath$	$+\lambda_\iota$	$+\lambda_o$	$-\lambda_\kappa$	$+\lambda_\jmath$	$-\sigma_\iota$	$-\sigma_o$	$-\sigma_\kappa$	$+\sigma_\jmath$
$\delta_\jmath$	$-\nu_\jmath$	$+\nu_\kappa$	$+\nu_o$	$-\nu_\iota$	$-\mu_\jmath$	$+\mu_\kappa$	$-\mu_o$	$-\mu_\iota$	$+\lambda_\jmath$	$+\lambda_\kappa$	$+\lambda_o$	$-\lambda_\iota$	$-\sigma_\jmath$	$+\sigma_\kappa$	$-\sigma_o$	$-\sigma_\iota$
δ_κ	$-\nu_\kappa$	$-\nu_\jmath$	$+\nu_\iota$	$+\nu_o$	$-\mu_\kappa$	$-\mu_\jmath$	$+\mu_\iota$	$-\mu_o$	$+\lambda_\kappa$	$-\lambda_\jmath$	$+\lambda_\iota$	$+\lambda_o$	$-\sigma_\kappa$	$-\sigma_\jmath$	$+\sigma_\iota$	$-\sigma_o$

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