Analyse the relation between Graph Matching and Edge Cover

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ABSTRACT

In Graph Theory, there is a concept of the matching the graph and the covering of edge I haveto find the relation between both of them however there is no restriction which specifications we have to take either maximum matching or minimum edge cover and maximum edge coveror minimum matching and any other like perfect matching. I had taken a simple and a adjustable set functions of (maximum and minimum) which will be helpful in the counter partof the calculations. The assumption which we have taken is maximum graph matching and minimal edge cover in the arbitrary graph. After doing all the calculations for this situation which we have taken we got the sum of the maximum matching and minimum edge cover is less than equal to the vertices.

I. INTRODUCTION

A matching graph is a subgraph of a graph where there are no edges adjacent to each other.Simply, there should not be any common vertex between any two edges.

An edge cover of a graph is a set of edges such that every vertex of a graph is incident to atleast one edge of the set.

We have consider two situation the maximal matching and minimal edge cover which means:The ***maximal matching*** is: A matching M of a graph 'G' is said to be maximal if no other edges of 'G' can be added to M.

The ***minimal edge*** is: It is not a proper subset of any other edge cover.

So, by relating them we should get a mathematical relation which we got the sum of themaximum matching and minimal edge cover is less than the vertex of the graph.

II. EXPLANATION

Let G be a graph and M a match with maximum size and F an edge cover with minimal size. There is no restrictions for getting any maximum and minimal size of any part of a graph, weassume to be a maximum and minimum for the match and edge cover respectively.

Let $\rho\ (G)$ denote the maximum size of a matching in G = (V, E) and $\rho(G)$ the minimum number of edges needed to cover all the vertices of G. Note that we need to assume that G has no isolated vertices for otherwise G does not contain a set of edges covering all vertices.Let, $M \subseteq E$ be a maximum matching of G. The edges in M cover $2M$ vertices. Each of the remaining $|V| - 2|M|$ vertices has degree at least 1 and so can be covered by some edge.

Hence, G has an edge covering of cardinality $|M| + |V| - 2|M| = |V| - \ |M|$. This implies that the size of a minimum edge cover is $\rho(G) \leq |V| - |M|$. Since $|M| = \rho\ (G)$.

Let's see the reverse inequality, let $F \subseteq E$ be a minimum edge cover of G. Observe that thesubgraph (V, F) contains no isolated vertices because F is an edge cover. Each connected component of the subgraph (V, F) is a star i,e. is isomorphic to $K_{1,r}$ for some $r \geq 1$. This is because, suppose some vertex x in one of the connected components has degree > 1. If each neighbor of x has degree at least 2. So (V, F) contains a walk of length 3 of the form a, x, y,a, or a, x, y, b and the middle edge of this walk can be removed to give a smaller edge cover,contradicting the minimality of $|F|$.

We showed that (V, F) is a vertex-disjoint union of stars. The number of edges in a star $K_{1,r}$ is r-1. If (V, F) consists of l stars, then the number of edges in (V, F) is

$|F| = |V| - l$.

Hence $l = |V| - |F|$. Since (V, F) is the vertex-disjoint union of l stars and each star can contribute one edge to a matching, the maximum size of a matching is $\rho\ (G) \geq l$.

Hence, $\mathrm{P}(G) \geq |V| - |F| = |V| - \rho(G)$.

This is the equation we have given out for our assumption of maximum size of a matchingand minimum number of edges is $\mathrm{P}(G) + \rho(G) \geq |V|$.

III. CONCLUSION

In this paper, we have concluded that the sum of maximum matching and the minimum number of the edges is greater than equal to vertices of the graph.

IV. REFERENCES

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