

Time dilation or just emission frequency dilation? A new conclusion from re-examining historical experiments

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Abstract

The time dilation effect seems proven by numerous experiments and is widely accepted by the physics community. However, by re-examining the relevant historical experiments, this paper finds that the claimed time dilation effect is merely an emission frequency dilation of a moving light source or a source in a gravity field.

Key words: time dilation; relativity theory; transverse Doppler effect; gravitational redshift; emission frequency

1. Introduction

Time dilation is a key concept in Einstein's relativity theory. It seems that the effect of time dilation has been proven time and again by relativistic phenomena and experiments, such as the Ives and Stilwell (1938, 1941), Hay et al (1960), Kundig (1963), Kaivola et al (1985), Klein et al (1992). The application of GPS seems a living testimony of the time dilation effect in a gravity field. However, a detailed re-examination of historical experiments shows that the claimed time dilation effect is actually a decrease in emission frequency of a moving light source or a source in a gravity field.

There are numerous experiments testing the time dilation effect. This paper examines only representative ones. In section 2, we re-examine the time dilation effect of a moving atom clock measured by Ives and Stilwell (1938, 1941). Section 3 reviews and discusses the mixed experimental results on transverse Doppler effect. Section 4 focuses the time dilation effect in a

gravity field and assesses the implication of the experiment by Chou et al (2010). Section 5 concludes the paper.

2. Time dilation measured by Ives and Stilwell

Based on the time dilation effect, the special relativity suggested that there would be a relativistic or higher-order Doppler effect on the top of the following ordinary Doppler effect:

$$\frac{\lambda}{\lambda_0} = \frac{c-v*\cos\theta}{c} = 1 - \frac{v}{c} \cos\theta \quad (1)$$

where λ and λ_0 are the perceived and original wavelength of light, respectively; c is the speed of light, v the speed of the moving light source, and θ the angle between v and the light ray towards the observer.

The Lorentz transformation necessitates that for the moving object, time slows down by the amount of the Lorentz factor:

$$\gamma = 1/\sqrt{1 - \frac{v^2}{c^2}}$$

Incorporating this time-dilatoin effect into the ordinary Doppler effect, we have the formula of the full Doppler effect:

$$\frac{\lambda'}{\lambda_0} = \gamma(1 - \frac{v}{c} \cos\theta) = \frac{1 - \frac{v}{c} \cos\theta}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (2)$$

If the light source and the observer moves in a direction perpendicular to the light ray (i.e. $\theta=\pi/2$), the ordinary Doppler effect is zero (according to eq. 1), but there will still be a redshift of light frequency to be observed (according to eq.2), namely the transverse Doppler effect.

Einstein suggested experimental observations of this effect to confirm his theory. However, it is hard to observe a Doppler effect from the 90° angle. Ives and Stilwell (1938ⁱ, 1941ⁱⁱ) came up an ingenious idea of measuring the higher-order Doppler effect.

Ives and Stilwell used an electrical field to accelerate ions (H_2^+ and H_3^+) to very high speed which should cause a time dilation and thus a redshift of light the ions emitted. They used a mirror to reflect the light travelling opposite to the ions travelling direction, so they could measure the wavelength of both the directly-emitted light ($\theta=0$) and the reflected light ($\theta=\pi$). The full Doppler effect can be obtained quantitatively by applying binominal approximation to equation (2). For the direct light ray ($\theta=0$), the equation becomes:

$$\frac{\lambda_2}{\lambda_0} = \frac{1 - \frac{v}{c}}{\sqrt{1 - \frac{v^2}{c^2}}} \approx \left(1 - \frac{v}{c}\right) \left(1 + \frac{1}{2} \frac{v^2}{c^2}\right) = 1 - \frac{v}{c} + \frac{1}{2} \frac{v^2}{c^2} - \frac{1}{2} \frac{v^3}{c^3} \quad (3)$$

For the reflected light ($\theta=\pi$), we have:

$$\frac{\lambda_1}{\lambda_0} = \frac{1 + \frac{v}{c}}{\sqrt{1 - \frac{v^2}{c^2}}} \approx \left(1 + \frac{v}{c}\right) \left(1 + \frac{1}{2} \frac{v^2}{c^2}\right) = 1 + \frac{v}{c} + \frac{1}{2} \frac{v^2}{c^2} + \frac{1}{2} \frac{v^3}{c^3} \quad (4)$$

Since measuring the wavelength of a spectrum line is more difficult and less accurate than measuring the amount of shift of the spectrum line, Ives and Stilwell measured the average of the two shifted spectrum lines and compared it with the original spectrum line caused by atoms at rest. The average wavelength of the Doppler shifts of both rays can be calculated as:

$$\bar{\lambda} = \frac{\lambda_1 + \lambda_2}{2} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \lambda_0 \approx \left(1 + \frac{1}{2} \frac{v^2}{c^2}\right) \lambda_0 > \lambda_0$$

For $0 < v < c$, the average wavelength is greater than the original wavelength λ_0 , indicating that the spectrum line of the reflected ray redshifts more than the blue-shift of the direct ray. This asymmetric shift can be expressed explicitly by the difference between this average and the original wavelength, which consisted of the relativistic Doppler effect in Ives and Stilwell (1938):

$$\Delta\lambda = (\bar{\lambda} - \lambda_0) \approx \frac{1}{2} \frac{v^2}{c^2} \lambda_0$$

The positive relativistic Doppler effect $\Delta\lambda$ is regarded as time dilation effect. The Ives and Stilwell experiment confirmed the asymmetrical Doppler shift and also showed that the size of relativistic shift from the original wavelength is consistent with the prediction of the time dilation

effect shown in equation (2). As a result, this experiment is viewed as a confirmation of time dilation.

However, as Christov (2010)ⁱⁱⁱ pointed out, the only problem with the analysis of Ives and Stilwell (1938) is that they assumed that the light frequency emitted by the atoms was independent of the speed of the atoms. Their assumption simply ruled out the possibility of light emission frequency/wavelength change when the emitter is moving. As an alternative hypothesis, we can assume that the wavelength of light emitted by objects moving at speed of v increases according to the following equation:

$$\lambda_M = \lambda_0 / \sqrt{1 - \frac{v^2}{c^2}} \quad (5)$$

where λ_0 is the wavelength from a stationary light source and λ_M is the wavelength from a moving light source.

The change in wavelength described by equation (5) will also be perceived by the observer, so the ordinary Doppler effect in equation (1) should be upgraded to:

$$\lambda' = \frac{\lambda}{\lambda_0} * \lambda_M = \frac{c - v * \cos\theta}{c} \frac{\lambda_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1 - \frac{v}{c} * \cos\theta}{\sqrt{1 - \frac{v^2}{c^2}}} \lambda_0 \quad (6)$$

This equation is exactly the same as equation (2), which is derived based on time dilation. Using equation (6), we can produce the same relativistic effect as that from the special relativity theory. As such, the results from Ives-Stilwell experiment can be caused by time dilation or simply by an emission frequency decrease.

Theoretically, there is a way to find out what really is the cause behind the experimental results. If the observer/detector can be in the same reference frame as the moving ions (i.e. moving at the same velocity as the ions), based on time dilation hypothesis, the time for detector should be dilated by the same degree as that for the ions/emitter. As such, the detector cannot detect any frequency change of emitted light because the time elapses at the same pace for both the detector and the emitter. However, based on the emission frequency dilation hypothesis, the detector that moves at the same velocity as the emitter will register a longer wavelength. In practice, a rapid

moving detector may affect the detection stability and accuracy, so one needs suitable technology to overcome this problem.

Fortunately, there are other experiments that can tell if the experimental results was caused by time dilation or simply by an emission frequency decrease.

3. Explaining mixed experimental results on transverse Doppler effect

The transverse Doppler effect proposed by Einstein was eventually directly confirmed by emission experiments (e.g. Hay et al, 1960^{iv}; Kundig, 1963^v; Kaivola et al, 1985^{vi} ; Klein et al, 1992^{vii}). However, when Jennison and Davies (1974^{viii}, 1975^{ix}) attempted to measure the transverse Doppler effect from a rotating mirror, they got a null result. With the advancement of technology, Thim (2003^x) tried to measure accurately the Doppler shift of microwave reflected from a transversely moving/rotating antennas but also reported absence of transverse Doppler shift. This section is to explain these mixed experimental results.

Both the time-dilation hypothesis and the emission frequency dilation hypothesis can explain the positive experimental results convincingly. The emission frequency dilation can also explain the null results easily: in the reflection experiments by Jennison and Davies (1974, 1975) and Thim (2002), the emitter is stationary so there is no change in frequency of emission. This leads to the experimental null results.

On the other hand, the special relativity cannot explain logically the null results of reflection experiments. Since the reflector (i.e. the transversely moving mirror/surface) and the light source are moving relatively, so the time would be dilated when the photons arrive the reflector and thus the light frequency should decrease. In the same reasoning, when the photons move from the reflector to the observer, there is relative movement between the reflector and the observer, so there must be another time dilation effect, causing another redshift. Adding two redshifts, the observed transverse Doppler effect would be twice as much as that observed from emission experiments. As a result, the expectation from the special relativity is at odds with reflection experiments.

Due to the tremendous success of relativity in predicting other phenomena, the null experimental results of the transverse Doppler effect from a rotating mirror are largely neglected. The only

discussion about them is an explanation by Sfarti (2010^{xi}) that the null result in Thim's experiment is expected because both the light source and detector are stationary. Mathematically, Sfarti's explanation can be expressed by a Lorentz transformation and inverse-Lorentz transformation.

Let the reflector moves on the x axis while the photons travel on the y axis. For photons move from the light source A to the reflector A', the following 2D Lorentz transformation formula to transform the stationary frame of A (txy) to the moving frame of A' (t'x'y')

$$x' = \gamma(x - vt), \quad y' = y, \quad t' = \gamma\left(t - \frac{vx}{c^2}\right), \quad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (7)$$

From the above formulas we can calculate the time t' in the new reference frame A' as:

$$t' = \gamma\left(t - \frac{vx}{c^2}\right) = \frac{t}{\gamma} - \frac{vx'}{c^2} \quad (8)$$

Using A' itself as the new reference frame, point A' is stationary, so x' is constant for point A and we have $\Delta t' = \gamma \Delta t$. Since v is less than c, $\gamma > 1$, so $\Delta t' = \gamma \Delta t > \Delta t$. This means smaller amount of change in t' is equivalent to more change in t, i.e. the time in reference frame (t'x'y') elapses slower than in the frame (txy). This is the time dilation effect claimed in special relativity.

When the photons move from the reflector A' to the observer A'', an inverse Lorentz transformation seems applicable because the observer A'' and the light source both are in the stationary frame. Applying the following inverse Lorentz transformation

$$x = \gamma(yx' + vt'), \quad y = y', \quad t = \gamma\left(t' + \frac{vx'}{c^2}\right), \quad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (9)$$

we can obtain:

$$t = \beta\left(t' + \frac{vy'}{c^2}\right) = \frac{t'}{\beta} + \frac{vy}{c^2} \quad (10)$$

Plugging the t' in eq. (7) into (10), we have a null transverse Doppler effect of reflected photons:

$$t = \beta\left(t' + \frac{vy'}{c^2}\right) = \frac{t'}{\beta} + \frac{vy}{c^2} = t - \frac{vy}{c^2} + \frac{vy}{c^2} = t$$

Since t has not changed, it seems that the special relativity provides a perfect explanation for the null results of the reflection experiments.

A similar but more advanced Lorentz transformation is to use a four-dimensional Lorentz boost to the energy momentum of the photon $(E, p_x, p_y, p_z) = (1, 0, 1, 0)$:

$$\begin{pmatrix} \gamma & -\gamma v/c & 0 & 0 \\ -\gamma v/c & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \gamma \\ -\gamma v/c \\ 1 \\ 0 \end{pmatrix}$$

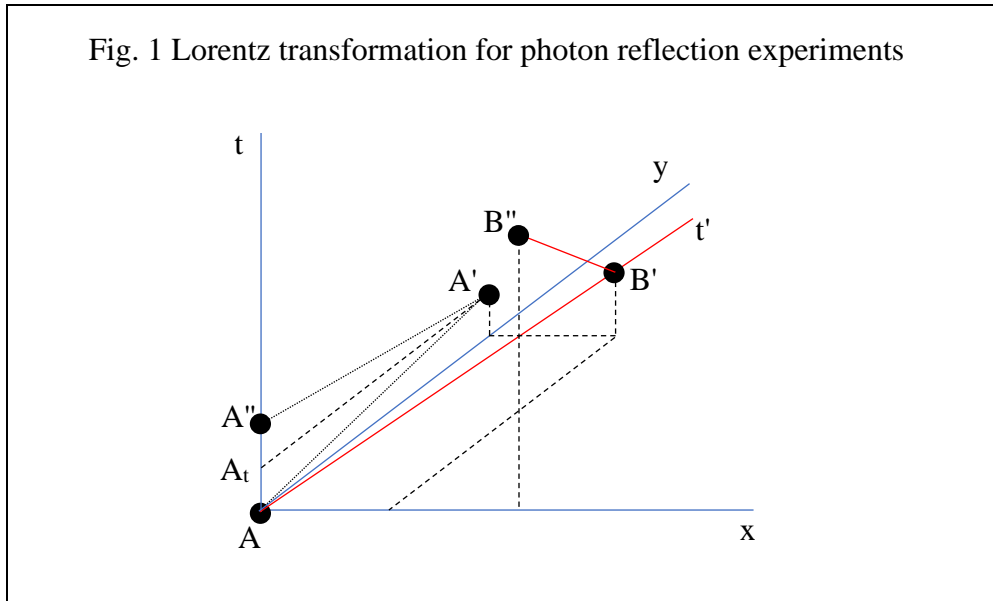
The results show that the photon's energy and x-direction momentum have changed but the momentum/speed in the y direction does not change. When the photon is reflected back from the mirror, the y-direction momentum changes sign. Using an inverse Lorentz boost, we obtain:

$$\begin{pmatrix} \gamma & \gamma v/c & 0 & 0 \\ \gamma v/c & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \gamma \\ -\gamma v/c \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$$

It is apparent that the energy and momentum is unchanged, except a sign change for the momentum in the photon travel axis. This also seems to be consistent with the null results of reflection experiments.

However, the above explanation involves an inappropriate application of the inverse Lorentz transformation. The inverse transformation formula is used to transform the same event back to the original reference frame, but they apply this formula to a new event. To use the (inverse) Lorentz transformation, one must satisfy a condition that the two reference frames have a common initial point. This can be seen from equations 7 and 9. If we let $t=x=0$, we have $t'=x'=0$, and vice versa. A new event like the reflected photon does not satisfy this condition. This can be demonstrated by a Minkowsky spacetime diagram:

Fig. 1 Lorentz transformation for photon reflection experiments



In Fig. 1, the stationary lab reference frame is indicated by spacetime (t,x,y) . The reflection experiment in this reference frame can be described as an incoming light from A to A' and then a reflected light from A' to A'' . All events occur in the yt plane because photon reflection occurs only on the y axis. In the frame of the moving mirror (t', x', y') , the reflection experiment can be described as an incoming light from A to B' and then a reflected light from B' to B'' . The Lorentz transformation can transform event AA' from the lab reference frame to the AB' in the moving mirror frame and the inverse Lorentz transformation can transform AB' back to AA' . However, one cannot transform the new event $B'B''$ back to $A'A''$ by inverse Lorentz transformation: the starting points of the new events (A' and B') are different and are not at the origin of two reference frames, thus the (inverse) Lorentz transformation is not applicable. An appropriate Lorentz transformation can be used only when one set the initial point (i.e. the reflection point) as the common origin of the two reference frames involved.

The other possible argument in defending the time dilation explanation of this case may be that the spinning mirror/plate is not an initial frame so one should apply the general relativity rather than the special relativity. This argument is not valid for the reflection experiments. Because the reflected photons (or electromagnetic waves) contact only the surface of the mirror/plate, they have not entered the spinning system and have experienced no attractive force from the system. The part of touching surface moves at an almost constant speed in the transverse direction, so it should be viewed as an inertial frame.

4. Clear evidence from comparing signals from two identical atom clocks

Based on Einstein's equivalence principle, the light frequency f' in a gravity field (at a distance R from the mass centre M) will be less than the light frequency f outside the gravity field (infinitely away from the mass centre). The two frequencies are related by the following ratio:

$$\frac{f'}{f} = 1/(1 + \frac{GM}{c^2R}) \approx 1 - \frac{GM}{c^2R} \quad (11)$$

The equation shows that the locally emitted photons appear to be redshifted by $\frac{GM}{c^2R}$. If the light sources are at different distance R_1 and R_2 from the mass centre, their light frequencies f_1 and f_2 satisfy:

$$\frac{f_1}{f_2} = \frac{1 + \frac{GM}{c^2R_2}}{1 + \frac{GM}{c^2R_1}} \approx 1 + \frac{GM}{c^2R_2} - \frac{GM}{c^2R_1}$$

In terms of frequency shift, the above equation can be rewritten as:

$$\frac{\Delta f}{f_2} = \frac{GM}{c^2R_2} - \frac{GM}{c^2R_1} \quad (12)$$

This redshift is quantitatively proven by the application of GPS. Since the cause of this redshift is attributed to time dilation in a gravity field, the time dilation effect seems proven by the application of GPS. However, it is possible that the redshift may be caused by an emission frequency change at different strengths of gravity field. Namely, if the emission frequency at the radius of R_1 and R_2 in a gravity field is dilated by $\frac{GM}{c^2R_1}$ and $\frac{GM}{c^2R_2}$, respectively, we should be able to obtain the same amount of redshift as in the general relativity. We need an experiment that can categorically differentiate the two hypotheses. The work of Chou et al (2010^{xii}) is such an experiment.

Chou et al examined the time dilation effect from both the speed of the light source and from the strength of gravity field. They built two nearly identical optical clocks and compared the clock tick frequency under two scenarios: (1) setting the ions in one clock in harmonic motion by a RF electrical field while keeping the other on stationary at the same elevation, (2) raising one clock to a higher elevation (about 33cm) than the other. The experiment shows that the frequency of

the clock at higher elevation is greater than that of the other clock while the clock in motion ticks slower than the stationary one. The amount of tick frequency change was consistent with Einstein's prediction.

Since the clock frequency indicates time, it seems to suggest that the time is dilated for the moving clock and for the clock at a position closer to ground. The experiment results seem to support Einstein's time dilation, however, if we go to the details of the experiment, we will find it actually supports the emission frequency change hypothesis.

Optical clock has three major components. First, a highly stable reference frequency or 'clock transition', which is provided by optical absorption of transition of different states of atoms or ions. Second, a laser or local oscillator that can stabilize its frequency to the clock transition, third, a femtosecond comb that can count the frequency of local oscillator, or clock ticks. Based on Rosenband et al (2008)^{xiii}, Chou et al (2010a, b^{xiv}), the transition frequency of their atom clocks is the $^1S_0 - ^3P_0$ transition frequency of Al^+ ions in a trap. The probing laser (local oscillator) utilizes a reference laser transported to the ion traps through optical fiber. The clock signals from the two atomic clocks then transmitted through optical fibers to femtosecond comb for comparison.

Since the measured effect of moving ions in Chou et al (2010) is similar to that of Ives and Stilwell (1938), we focus on the effect of different elevations. If the expectation from the general relativity is correct that the time in a weaker gravity field dilates less, the 'clock transition' frequency at the higher elevation should be higher. Meanwhile, the frequency of the fiber-transmitted probing laser should also increase when the laser reaches the higher elevation. The degree of change in two frequencies should be the same because time dilation is equal to all components at the same height. As such, the less dilated probing laser will match the less dilated clock transition and no effort is needed to change the probing laser frequency (i.e., no error signal is produced to the laser lock). Once the less dilated (and thus higher frequency) laser at higher elevation is transmitted to femtosecond comb at lower elevation for comparison, the higher frequency will be dilated back to the original frequency thanks to the increased gravity strength. As a result, there should be no frequency difference for two clocks. This expectation based on time dilation conflicts with experimental outcomes.

On the other hand, if the gravity causes only an emission frequency change, the experimental result is very easy to explain. Since the gravity field at higher elevation increases only the light frequency of emitter, the probing laser frequency has to be increased by the experiment system in order to match the clock transition frequency. When the probing laser signals are transmitted to the femtosecond comb at lower elevation, the increased frequency does not change and thus is recorded. Compared with the clock frequency at the lower elevation, we find the frequency at higher elevation is blue-shifted. This is what observed in Chou et al (2010)

In short, the results of Chou et al is at odd with the time dilation hypothesis, but can be easily explained by an emission frequency change induced by gravity strength. A more clear-cut experiment can be done by checking the frequency of same type of light source in a space station and on earth. If the frequencies measured at two locations are the same, the experiment supports time dilation hypothesis. Otherwise, it rejects the time dilation and supports the emission frequency change hypothesis.

4. Conclusions

After a careful examination of relevant historical experiments, we find that the popularly accepted concept of time dilation actually means only an emission frequency dilation. The implication of this finding on the relativity theory is profound. Although the predictions from Einstein's relativity theory are proven by many experiments and observations, its explanations based on time dilation still present a problem. One may doubt this claim based on the belief that a theory proven by numerous experiments must be correct. However, the scientific history shows otherwise. A typical example is Fresnel's partial aether dragging theory (1818)^{xv}. In this theory, Fresnel derived a formula for the velocity variation of light travelling in a moving medium. Fizeau (1851)^{xvi} and Michael (1886)^{xvii} conducted the water tube experiments and proved Fresnel's formula, but the aether dragging theory is discarded.

The finding in this paper shows a necessity of a new theory that can explain relativistic phenomena but without the time dilation concept. The author actually developed a photon theory of inertia and gravity, which can explain why the photon emission frequency changes according to eq.6 when the emitter is in motion, and according to eq. 12 when the emitter is in a gravity field. The theory can also quantitatively explain other relativistic phenomena such as the

precession of planetary orbit, gravitational lensing, decay of pulsar orbit, gravitational waves, and blackholes. This theory will be presented in a separate paper.

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