# Toward a proof of the Riemann Hypothesis

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**Abstract:** The Möbius function $\mu(j) = -1,0,1$ depending on whether $j$ has an odd number of factors, a square factor, or an even number of factors. The Mertens function $m(n)$ is $j = 1$ *to* $n$, $\sum \mu(j)$. For all $n$, $|m(n)| < 2n^{1/2}$. $|m(n)| = O(n^{1/2})$, and therefore the Riemann Hypothesis is true.

Set $f_n = (1/2)(2/3)(3/4)\ldots([n^{1/2}]/([n^{1/2}]+1))(n) = n/([n^{1/2}]+1) < n^{1/2}$.

Set $S_n = (4/3)(9/8)(16/15)(25/24)\ \ldots(s^2/(s^2-1)),\ \ s^2 \leq [n^{1/2}]+1.\ \ S_n = 2n/(n+1)$ Proof by induction.

$n=2\ \ S_n = (4/3)$ assume $S_n = 2n/n+1\ \ S_{n+1} = (2n/n+1)(n+1)^2 / ((n+1)^2 - 1) = 2(n+1)/((n+1)+1)$

$\boldsymbol{|m(n)| < (S_n)(f_n) < (2)(n^{1/2})}$**.**

A negative cycle is an interval in which $m(s) \leq 0$ for all values of $s$ and a positive cycle is an interval in which $m(s) \geq 0$ for all values of $s$.

For every $s \geq 1$, $m(s)$ is in a positive or negative cycle or possibly both if $m(s)=0$.

The Mertens function $m(n)$ is applied to the first $n$ positive integers as a set. The reciprocal of each of the $\boldsymbol{s}$ non-square integers up to $[n^{1/2}]+1$ is a Mertens proportionality factor. The MPF are applied repeatedly to the fractional part of $n$. $(1 - 1/f_1)(n) = n_1,\ (1 - 1/f_2)(n_1) = n_2,\ \ldots\ (1 - 1/f_s)(n_{s-1}) = n_s < 2n^{1/2}$ $2 = f_1$ thru $f_s \leq [n^{1/2}]+1$. Collectively, the MPF are a measure of the proportion of elements in the Mertens function set whose Möbius function always has a combined value of zero. $m(n)$ has a maximum/minimum possible value depending on $m(n)$ being in a positive/negative cycle.