Conservation of Mass-Energy and Reinterpretation of the Einstein Field Equations

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Energy is everywhere. Energy propagates by wave and light, and has no mass. So light is a wavelength. Einstein introduced a particle called photon to explain the photoelectric effect, and it is said that light causes the photoelectric effect. Therefore, the conclusion so far is that "Light is both a particle and a wavelength". Energy is an independent entity with a physical quantity and is quantized according to Planck's law. Energy can be measured in terms of temperature, and can also be expressed in terms of mass-energy according to the mass-energy equivalence principle. Energy can raise the temperature of a matter, or it can provide energy for a matter to move. They are related to each other but act independently. All matters have potential energy and thermal energy separately inside. The internal potential energy (E_p) of a matter does not interact with thermal energy or external kinetic energy (E_k). Particles in the microscopic world can either emit or absorb energy, or they can release energy through mass deficits that release parts of matter. It can be seen that Einstein's field equation is an equation to which the law of conservation of mass-energy applies. Therefore,from Einstein's field equations, we can derive out the matter-dominated universe and energy-dominated universe, respectively.

I. ENERGY

A vacuum is the absence of anything. In general, we refer to a state in which there is no matter as a vacuum. But we also know that our universe is full of energy[1], and that energy works many things. Light, including all wavelengths, is the means by which energy is transmitted. Energy does a lot of work. Energy itself has magnitude of energy, but it is not to have mass because it has no mass. However, although energy has no comparable magnitude to mass or momentum, it has a physical quantity called energy. Energy (E) has no difference from the well-known physical quantity called mass (M). Whether light has a lot of energy or not, but since it is quantized, physical quantity (E) can be measured separately in a given space like mass.

Let's consider there are two rooms here. The rooms can fit anything we can imagine. That is, one room contains our universe and the other contains our sun. Now, when we take the universe out of the room that contains it, the room becomes an empty room with nothing. In this room, we cannot measure all the physical quantities we know, such as energy, matter and even temperature. This room is not a black-body that absorbs all electromagnetic radiation, nor a white-body that perfectly reflects all incident light. The room is a completely empty vacuum. Likewise, from the room containing the sun, remove all matter, including dark matter, on which the sun and gravity act. In this case, all matters, that gravity is involved in, removed, but unlike the removal of the universe from the other room, it is still full of energy called radiant heat. To remove this energy, the temperature is to be reduced to absolute zero. When the temperature lowered, energy escapes to the outside through radiation or heat conduction through the walls of the room, and the room reaches absolute zero. These two empty spaces are exactly the same vacuum space where neither energy nor matter exist.

Now, when heat is applied from the outside of an empty room, the inside of the space is heated, generating radiant heat and heat conduction, so that radiant heat with energy fills the inside of the room. Observers inside the room are unaware that the energy is coming from outside, so they know that energy is being created. If we continue to heat it from the outside, the energy inside the room will continue to increase. The observer in the room discovers that the state of increasing energy can be understood as an equation.
For energy can be expressed in terms of temperature, one may find the following equation

$$\frac{P}{A} = \sigma T^4, \tag{1.1}$$

where, P is the power, A is the surface area, $\sigma = 5.670374 \times 10^{-8} W m^{-2} K^{-4}$ and T is the temperature in kelvin. This is known as the Stefan-Boltzmann's Law.

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[1] Energy, here, refers to all energies, including dark energy discovered by cosmologists. Dark energy is energy itself, not any other form of energy.

However, this equation has slight difference from Planck's law of black-body radiation. [2]
Planck's law of black-body radiation U:

$$U(\nu, T) = \frac{8\pi h\nu^3}{c^3}\frac{1}{e^{\frac{h\nu}{kT}} - 1}, \tag{1.2}$$

can be used to obtain the total energy density per unit volume by integration with respect to ν,

$$\begin{aligned} \frac{U}{V} &= \int_0^\infty \frac{8\pi h\nu^3}{c^3}\frac{1}{e^{\frac{h\nu}{kT}} - 1} d\nu \\ &= \omega T^4, \end{aligned} \tag{1.3}$$

where $\frac{U}{V}$ is the total energy density per unit volume and T is the effective temperature in Kelvin. And omega(ω)[2] represents the radiation density constant [6]

$$\begin{aligned} \omega &= \frac{8\pi^5 k^4}{15c^3h^3} \\ &= 7.56573 \times 10^{-16} Jm^{-3}K^{-4}, \end{aligned} \tag{1.4}$$

where k is the Boltzmann constant, h is the Planck's constant and c is the speed of light in vacuum. The most important difference between the Stefan-Boltzmann constant and omega(ω) is that the former is a relationship between power and surface area, whereas the latter is a relationship between total energy and volume.
From the equation (1.3), we get

$$U = \omega T^4 V, \tag{1.5}$$

where, U is the total energy of the universe if V is the volume of the universe and T is the average temperature of the universe.

If we continue to heat the room from the outside, the temperature inside the room will continue to rise, but no matter will be created. Now, when the external heating is stopped, the radiant heat is in equilibrium and the energy inside the room has a constant value. It is seemed that the interior of the black-hole also maintains this state. If the radiant heat in the room does not escape to the outside through radiation or heat conduction, then the energy in the room is conserved. If the room starts to expand, the temperature will drop as the volume of the room increases, and if it contracts, the temperature will rise. This is, in fact, possible only when there is a force that tries to expand or contract from the inside of the room. Now if we're in the room, like scientists, we can find energy is in equilibrium and ubiquitous. However we don't know if it was created inside or if it came from outside. We define that the law of conservation of energy applies. When an external pressure is applied to the above energy-filled room, it can be compressed into an infinitely small point. If a state in which a lot of energy is compressed in a very small particle is called to be a cosmic seed. If the cosmic seed with full of energy causes a big-bang by internal radiation pressure or an unknown trigger, that is the starting point of our universe. So far, we have assumed that a room can reach an infinitely small state by heating and applying pressure to increase the temperature of the room by external heating. It is thought to have started from a very small cosmic seed full of energy.

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[2] In Gesetz der Energieverteilung im Normalspectrum[2] p. 562, Planck showed the spatial density of the total radiant energy by termwise integration from (1.2) as

$$\begin{aligned} u &= \frac{8\pi h}{c^3} \times 6\left(\frac{k}{h}\right)^4 \left(1 + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \dots\right) \\ &= \frac{48\pi k^4}{c^3h^3} \times 1.0823, \end{aligned}$$

where u represents the radiation density constant. But he didn't materialize the coupling constant.

II. LAW OF CONSERVATION OF MASS-ENERGY

A moving matter does not move by itself, but it obtains energy necessary to move from outside. In other words, kinetic energy follows the law of equivalence of mass-energy. Energy has three forms: potential energy (or atomic energy, which may not mean the same), thermal energy and kinetic energy. Potential energy is absorbed by the matter and does not interact with thermal energy or kinetic energy. The more thermal energy a matter has, the less energy it takes to move. When a rest mass is converted into energy, the sum of all these energies is equal to the converted energy of the rest mass. Since kinetic energy has the most part of a moving particle, kinetic energy is mainly treated. The kinetic energy $E_k = \frac{1}{2}m_0v^2$ is transmitted from the outside and moves the matter at the speed v. So the total energy (E) of a given space is

$$E = E_k + \frac{1}{2}m_0v^2, \tag{2.1}$$

where E_k is the kinetic energy used by the matter m_0 to move at a speed of v. So the law of conservation of mass-energy applies. To see the law of conservation of mass-energy more simply, we can rewrite as follows

$$E_k - \frac{1}{2}m_0v^2 = 0, \tag{2.2}$$

which is the law of conservation of mass-energy. Taking our sun as an example, the sun has mass $m_\odot$, and the escape velocity from the sun is $v_\odot$, so the total energy the sun has is $E_k = \frac{1}{2}m_\odot v_\odot^2$. Here we can look at the following situation.
In case,

$$E_k - \frac{1}{2}m_0v^2 > 0. \tag{2.3}$$

This is the case when extra energy is being supplied or being created from the outside. If our universe is in this state, it is evidence that energy is being created. However, as the observation of cosmologists confirms that our universe has much more energy than all matter has converted into energy, the early universe may have had all the energy, or energy may be being created as the universe expands.
In another case,

$$E_k - \frac{1}{2}m_0v^2 < 0. \tag{2.4}$$

Since energy is not destroyed, it is evidence that energy has escaped to the outside. This is because the energy held by matter is also released. If all the mass ($M_\odot$) of the sun is converted into energy, it becomes $E_k = \frac{1}{2}M_\odot c^2$. Adding as much energy as E_k is applied to $M_\odot$, all matters are converted into energy. This is because matter does change itself into energy without the help of outer energy as much as it can move. Therefore, the total energy (E) of a matter moving at the speed of light is

$$E_{tot} = E_k + \frac{1}{2}M_0c^2 = M_0c^2. \tag{2.5}$$

This proves that total energy is consisted of kinetic energy including thermal and potential energy as well as the rest mass energy converted into energy.
This also applies to the case where mass deficits in microscopic nuclear fusion or fission reactions release a lot of energy. However, in this case, potential energy stored inside the particle itself is released so that the mass loss can be converted into energy, rather than being supplied with energy from the outside of the particle to cause a mass loss to occur and be converted into energy. From this it can be seen that some potential energy is adsorbed onto a particle. This is a different form of energy from externally supplied kinetic energy. In addition, in the microscopic world, an exothermic reaction ($+E_k$) without mass deficit or an endothermic reaction ($-E_k$) without an increase of mass is because an atom emits some of the basic potential energy or absorbs energy from the outside to increase potential energy.

From the equation (1.5) and above, we get

$$U = \omega T^4 V = \Omega M c^2, \tag{2.6}$$

where, Ω represents the rate of the total energy of the universe per the total energy of the mass of the universe. According to the current measurements, the rest energy consists of 68% of the total energy of the universe. If we measure the coupling constant Ω, we get the average temperature of the universe by using the equation (2.6),

$$T = \left(\frac{\Omega M c^2}{\omega V}\right)^{\frac{1}{4}}. \tag{2.7}$$

Ω has the following values:
$\Omega > 1$, our universe belongs to this case, and the value can be measured by observation.
$\Omega = 1$, a black-hole belongs to this case. Extra energy is released to the outside during the process of compression by gravity, and energy of $E = mc^2$ remains inside the black-hole. It also includes the process of atomic fusion or fission reaction, by which matter is converted into energy.
$\Omega = \frac{1}{2}$, all celestial bodies and matters, including our sun, belong to this case, and have energy $E = \frac{1}{2}mv^2$.

III. REINTERPRETATION OF EINSTEIN FIELD EQUATION

Since Einstein's field equation follows the principle of mass-energy equivalence, it is interpreted using tensor calculus as an equation to describe the universe using the law of conservation of mass-energy. Einstein's tensor equation can be solved by integrating the formula provided by the differential equation of the four-metric tensors, even though the process of solving it is very complex. However, various interpretations are possible using the equation obtained by the first-order integral.
Einstein's field equation is described as

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa T_{\mu\nu}, \tag{3.1}$$

where μ and ν represent the four-metric tensors (r, θ, ϕ, t). Hereinafter, I refer κ and the speed of light c to be unity for convenience, unless otherwise mentioned. If we arrange this using a line-element:

$$ds^2 = -e^{\sigma}dt^2 + e^{\omega}dr^2 + e^{\mu}(d\theta^2 + \sin^2\theta d\phi^2), \tag{3.2}$$

where σ, ω, and μ are equations of r and t, respectively.
The equations in the above can be simplified by using the Ricci tensor as follows

$$\begin{aligned}
R^r{}_r - \frac{1}{2}R &= e^{-\sigma}(\ddot{\mu} - \frac{1}{2}\dot{\mu}\dot{\sigma} + \frac{3}{4}\dot{\mu}^2) + e^{-\mu} - e^{-\omega}(\frac{1}{2}\acute{\mu}\acute{\sigma} + \frac{1}{4}\acute{\mu}^2), && (3.3)\\
R^\theta{}_\theta - \frac{1}{2}R &= R^\phi{}_\phi - \frac{1}{2}R && (3.4)\\
&= -\frac{1}{4}e^{-\omega}(2\acute{\acute{\sigma}} + \acute{\sigma}^2 + 2\acute{\acute{\mu}} + \acute{\mu}^2 + \acute{\mu}\acute{\sigma} - \acute{\mu}\acute{\omega} - \acute{\sigma}\acute{\omega})\\
&\quad + \frac{1}{4}e^{-\sigma}(2\ddot{\omega} + \dot{\omega}^2 + 2\ddot{\mu} + \dot{\mu}^2 + \dot{\mu}\dot{\omega} - \dot{\mu}\dot{\sigma} - \dot{\sigma}\dot{\omega}),\\
R^t{}_t - \frac{1}{2}R &= e^{-\sigma}(\frac{1}{2}\dot{\mu}\dot{\omega} + \frac{1}{4}\dot{\mu}^2) + e^{-\mu} - e^{-\omega}(\acute{\acute{\mu}} - \frac{1}{2}\acute{\mu}\acute{\omega} + \frac{3}{4}\acute{\mu}^2), && (3.5)\\
R^r{}_t &= \frac{1}{2}e^{-\omega}(2\dot{\acute{\mu}} + \acute{\mu}\dot{\mu} - \acute{\mu}\dot{\omega} - \dot{\mu}\acute{\sigma}), && (3.6)\\
R^t{}_r &= -\frac{1}{2}e^{-\sigma}(2\dot{\acute{\mu}} + \acute{\mu}\dot{\mu} - \acute{\mu}\dot{\omega} - \dot{\mu}\acute{\sigma}), && (3.7)
\end{aligned}$$

where an acute represents differentiation with respect to r and a dot with respect to t.
We can integrate the equation (3.3) with the help of the equation (3.6) with respect to t,

$$\begin{aligned}
&\dot{\mu}^2 e^{-\sigma} - \acute{\mu}^2 e^{-\omega} + 4e^{-u} && (3.8)\\
&= 2e^{\frac{-3\mu}{2}}\int \dot{\mu}e^{\frac{3\mu}{2}}T^r{}_r dt - 2e^{\frac{-3\mu}{2}}\int \acute{\mu}e^{\frac{3\mu}{2}}T^r{}_t dt + Ce^{\frac{-3\mu}{2}},
\end{aligned}$$

where C is the constant of integration.
We can get similar solution from (3.5) and (3.7) by integration with respect to r,

$$\begin{aligned} &\dot{\mu}^2 e^{-\sigma} - \acute{\mu}^2 e^{-\omega} + 4e^{-u} \\ &= 2e^{\frac{-3\mu}{2}} \int \acute{\mu} e^{\frac{3\mu}{2}} T^t{}_t dr - 2e^{\frac{-3\mu}{2}} \int \dot{\mu} e^{\frac{3\mu}{2}} T^t{}_r dr + Ce^{\frac{-3\mu}{2}}. \end{aligned} \tag{3.9}$$

From the above equations, which are integral of Einstein's field equations, several practical structures of the universe can be described. In other words, it can be known by examining the terms of $T^r{}_r$ and $T^t{}_t$ or the constant of integration C, and the solution may vary depending on whether the geometric tensor is a coupling coefficient of r or t only, or a coefficient of both r and t. It is assumed that shear stress tensors $T^r{}_t$ and $T^t{}_r$ to be equal to zero.

A. Schwarzschild Solution

First, in case $T^r{}_r = 0$ of the equation (3.8) or $T^t{}_t = 0$ of the equation (3.9) we get the following result

$$\dot{\mu}^2 e^{-\sigma} - \acute{\mu}^2 e^{-\omega} + 4e^{-\mu} = Ce^{\frac{-3\mu}{2}}. \tag{3.10}$$

Plugging in $e^{\mu} = r^2$ and $C = \frac{8GM}{c^2}$, we get

$$e^{-\omega} = 1 - \frac{2GM}{rc^2} \quad \text{and} \quad e^{\sigma} = 1 - \frac{2GM}{rc^2}. \tag{3.11}$$

From this, we immediately identify the Schwarzschild line-element becomes

$$ds^2 = -\left(1 - \frac{2GM}{rc^2}\right) dt^2 + \left(1 - \frac{2GM}{rc^2}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{3.12}$$

This model represents an empty space-time, because we put the energy-momentum tensor $T^r{}_r = 0$ or $T^t{}_t = 0$, and the momentum tensor is $\frac{2GM}{R}$, so the law of conservation of mass-energy applies.

B. Matter dominated Universe

This is a solution to Einstein's field equation because the value of the energy tensor is a non-zero constant. From the equation (3.9), let $T^t{}_t = 3H^2$, which is reciprocal of time($H = 1/t$) and the integral constant C is zero, we get

$$\dot{\mu}^2 e^{-\sigma} - \acute{\mu}^2 e^{-\omega} + 4e^{-\mu} = 4H^2. \tag{3.13}$$

Plugging $e^{\mu} = R^2$ of the above, we have

$$e^{-\omega} = 1 - H^2R^2 \quad \text{and} \quad e^{\sigma} = 1 - H^2R^2. \tag{3.14}$$

The line-element becomes

$$ds^2 = -(1 - H^2R^2)dT^2 + (1 - H^2R^2)^{-1}dR^2 + R^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{3.15}$$

This model represents the spherically symmetric matter-dominated space-time. Here HR represents the escape velocity of a celestial body. For our universe, $HR = c$, where c is the speed of light, brings $H = H_0$ represents the Hubble constant and R represents the radius of our universe. For the sun, $H_\odot R_\odot = v_\odot$ represents the escape velocity from the sun.
Since $3H^2$ is equal to $\kappa T^t{}_t = 8\pi G\rho$ of the equation (3.9), multiplying both sides by $R^2/3$, we get

$$\begin{aligned} H^2R^2 &= 8\pi G\rho R^2/3 \\ &= \frac{8}{3}\pi R^2 GM/V \\ &= \frac{2GM}{R}. \end{aligned} \tag{3.16}$$

From the equation of mass energy of a moving particle, we get the same result,

$$\frac{1}{2}mv^2 = \frac{GmM}{R}. \tag{3.17}$$

Whence we have,

$$v^2 = \frac{2GM}{R}, \tag{3.18}$$

i.e.,

$$H^2R^2 = v^2 = \frac{2GM}{R}. \tag{3.19}$$

This phenomenon also occurs in a black-hole when $v = c$. This is a system in which the law of conservation of mass-energy operates.

C. Energy Dominated Universe

From the equation (3.9), let $T^t{}_t = 3Q$ and $e^\mu = R^2$, we get

$$e^{-\omega} = 1 - QR^2, \quad \text{and} \quad e^\sigma = 1 - QR^2. \tag{3.20}$$

where Q represents the total energy in a given space per total energy converted into mass per square time ($Q = 1/t^2$). From the above, we get the line-element as follows;

$$ds^2 = -(1 - QR^2)dt^2 + (1 - QR^2)^{-1}dR^2 + R^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{3.21}$$

This energy-dominated universe is filled of energy and mass.
From the equation (1.3), (1.5) and $T^t{}_t = 3Q$ above, we get Q:

$$QR^2 = v^2 = \frac{\omega T^4 V}{3M}. \tag{3.22}$$

V represents the volume of a sphere, and M represents the total rest mass of the given space. This is equal to the total energy of the universe if $v = c$ and V represents the volume of the universe.
Comparing the above with the equation (2.7), we get the average temperature of the universe by using $\Omega = 3$,

$$\begin{aligned} T &= \left(\frac{3Mc^2}{\omega V}\right)^{\frac{1}{4}} \\ &\simeq 42K, \end{aligned} \tag{3.23}$$

where V is the volume and M is the total mass of the universe.

[1] https://en.wikipedia.org/wiki/Dark_energy
[2] von Max Planck, *"Ueber das Gesetz der Energieverteilung im Normalspectrum"*, p. 553 - 563
[3] https://en.wikipedia.org/wiki/Stefan-Boltzmann_law
[4] https://ko.wikipedia.org/wiki/Thermodynamics
[5] https://en.wikipedia.org/wiki/Thermodynamics
[6] https://en.wikipedia.org/wiki/Stefan-Boltzmann_constant
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[21] https://en.wikipedia.org/wiki/Cosmological_constant