Finding regions of correlated polarization directions of polarized starlight from 5830 sources in the Milky Way Galaxy

Richard Shurtleff *

Abstract

This article surveys polarized starlight in the Milky Way Galaxy. The data for the 5830 stars is taken from the agglomeration by Heiles 2000 and the Berdyugin 2014 catalog. The survey applies a particular test for transverse vector correlation, the Hub Test of spherical geodesic alignment with points on the sky. One finds a large collection of some 902 stars on the Galactic Disk from galactic longitude 90° to 150°, These stars are distinguished by having, by far, the most significantly aligned stellar polarizations in the survey. Thus the most obviously aligned regions are identified. On the subtle side, we identify two smaller collections, 67 and 75 stars, that are each exceedingly well aligned and adjacent on the sky. These two collections show that the method can be used to locate samples with distinct, but slight, differences in their properties. The selections show how samples can be identified by the Hub Test for further study. The appendix consists of a self-contained computer software program that performs the needed calculations.

Keywords: Alignment; Hub Test; Transverse Vectors; Polarized starlight; Survey

*Department of Sciences, Wentworth Institute of Technology, 550 Huntington Avenue, Boston, MA, USA, 02115, orcid.org/0000-0001-5920-759X, e-mail addresses: shurtleffr@wit.edu, momentummatrix@yahoo.com

0. Preface

The pdf version of this notebook is available online from the viXra archive. Search by title and author.
To find the ready-to-run notebook follow one of the links in Ref. 1.
The notebooks in this series were created using Wolfram Mathematica, Version Number: 12.1, Ref. 2.

Note(s):

(1) Some numerical quantities in the pdf version may differ from the ready-to-run version in Ref. 1 because the ready-to-run version may have been run after the pdf was produced. The ready-to-run version and the pdf version may be updated independently of one another.

(2) The notation is undergoing a change from “S” indicating significance to “*p*” standing for significance. Some of the “S” labels have most likely survived.

CONTENTS

Part I the Article

1. Introduction

Observations of an astronomical object may include quantities such as polarization and jets that can be represented as transverse vectors, vectors that are perpendicular to the direction from the detector to the object itself. Polarized starlight is one indicator of the direction of the Galactic magnetic field, see for example, Refs. 3 - 6.

In this article, a survey of polarized starlight evaluates correlations for many regions on the sky. Correlations of polarization directions are found by extending the polarization directions to make spherical geodesics, a.k.a. great circles. The calculated degree of correlation for a set of such transverse vectors depends on how well the great circles converge. A separate measure of correlation is associated with the divergence of the great circles.

The questions are: How well do the circles focus down on the best focus point on the Celestial Sphere? and How well do these circles avoid the point of most divergence? Those points are called hubs, a reference to the great circle paths of airliners inbound to centralized travel hubs. The test is the Hub Test.

This article surveys the local Galaxy, the Milky Way, for regions of significant alignment and avoidance, *i.e.* correlated behavior, of the polarization directions of partially linearly polarized stars. The regions surveyed have 5° radii. The experimental data for the stars that populate these 5° regions are from two catalogs, Heiles 2000 and Berdyugin 2014, Refs. 7 - 10. While the total number of catalogued stars is 11,647, we filter the data by percent polarization and experimental uncertainty leaving 5830 stars to analyze. Statistics of essentially the same list of stars that we survey is discussed by others in Ref. 11. The data to be analyzed is displayed in Fig. 1.

Section 2 provides a brief overview of the Hub Test. Section 3 describes the preparation of 5° radius regions. The significance of the regions is discussed in Sec. 4. We call one such collection a 'clump'. The selection process is presented in Sec. 5 for three collections of the 5° radii regions. There are many interesting areas of the survey that could have been chosen. No attempt is made to analyze the three clumps here. However, a few observations are made. The distances to the stars is noted and the change in polarization direction between the two adjacent clumps is noted.

One of the clumps contains 184 of the most significantly aligned 5° regions. Intriguingly, the 5°-radius regions combined in 'Clump 1' are orders of magnitude better aligned than the other 5° regions. See Figs. 7 and 8. The placement of the stars on the sky and their distances put them in the Orion Spur and Perseus Arms of the Galaxy. Their polarization directions overwhelmingly point along the Galactic Equator.

As a display of nuance, two adjacent clumps are selected, each composed of 5° radius regions whose alignment is exceptional. Fewer than one in 10^{21} regions with randomly directed polarizations would be as well aligned as any of the selected regions. The two clumps are separated by almost-as-significantly aligned regions, so their alignments may indicate some slight change in properties that is important to their alignment. The 75 and 67 stars in the two clumps, Clump 6 NE and SW, are roughly 100 to 300 parsecs away, with the NE clump about 50 pc closer on average than the SW clump. One expects that the differences in distance and polarization direction for these adjacent clumps of stars with exceptionally well-aligned polarization directions may be clues to help with deducing the magnetic structure of the Galaxy.

One motivation for including the Appendix is the hope that the program can serve as a template. If one possesses data from other sources, then, by putting that data in the same form as the data00 table entered in Sec. A2 below, one can run the program and get maps of the significance of alignment and avoidance for regions populated with the new data.

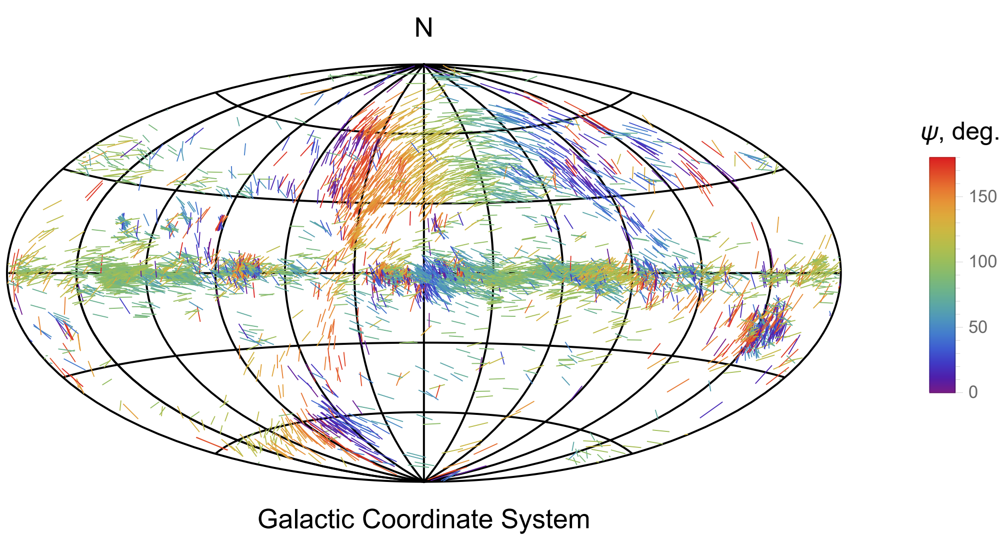


Figure 1. A whole-sphere pine-needle Aitoff plot of the polarized starlight surveyed in this article. The sources are part of the Heiles agglomeration of stellar polarization catalogs supplemented by the Berdyugin catalog. The plot is centered on (gLON,gLAT) = (0,0). East is to the left. Polarization directions are indicated by short line segments centered on each star. Large-scale flows of polarization directions are evident. Note that $\psi = 180°$ is the same as $\psi = 0°$ for polarization position angles, so the deepest red and the darkest violet indicate the same direction, *i.e.* ≈ .

2. The Hub Test

The Hub Test, Ref. 12, judges the alignment of transverse vectors. The test quantifies the alignment of the polarization directions with every point on the Celestial Sphere. By involving the direction to another point, the Hub test is indirect. For one source, the basic quantities are illustrated in Fig. 2. The "alignment angle" η is the acute angle η between two great circles at S, $0° \leq \eta \leq 90°$. The alignment angle η measures how well the polarization direction $\hat{v}_\psi$ matches the direction $\hat{v}_H$ toward the point H. Perfect alignment occurs when $\eta = 0°$ and the two great circles overlap. When these two great circles are perpendicular, $\eta = 90°$, that indicates maximum "avoidance" of the polarization direction $\hat{v}_\psi$ with the point H on the sphere. The halfway value, $\eta = 45°$, favors neither alignment nor avoidance.

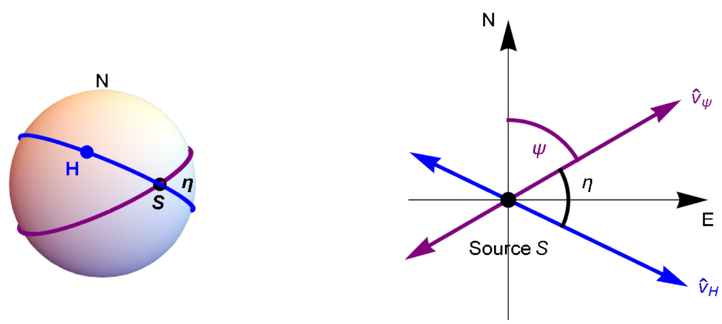


Figure 2: The Celestial sphere is pictured on the left and on the right is the plane tangent to the sphere at the source S. The linear polarization direction $\hat{v}_\psi$ lies in the tangent plane and determines the purple great circle on the sphere. A point H on the sphere together with the point S determine a second great circle, the blue circle drawn on the sphere. Clearly, H and S must be distinct in order to determine a unique great circle. The acute angle η measures the alignment of the polarization direction ψ with the point H.

With N sources S_i, $i = 1, ..., N$, there are N alignment angles η_{iH} at each point H. One can calculate an average alignment angle $\overline{\eta}$ at H,

$$\overline{\eta}(\text{H}) = \frac{1}{N} \sum_{i=1}^{N} \eta_{\text{iH}} , \qquad (1)$$

where

$$\cos(\eta_{\text{iH}}) = | \hat{v}_\psi . \hat{v}_H | . \qquad (2)$$

Given the positive value of the right in (2), the solution for the angle η_{iH} is taken to be the positive acute angle with $0° \leq \eta_{\text{iH}} \leq 90°$. Clearly, the average alignment angle $\overline{\eta}(\text{H})$ at the point H must also be acute. An example of the function $\overline{\eta}(\text{H})$ is presented in Figs. 3 and 4.

The alignment angle $\overline{\eta}(\text{H})$ is a function of position H on the sphere. The function $\overline{\eta}(\text{H})$ is symmetric across diameters, $\overline{\eta}(\text{H}) =$

$\overline{\eta}$(-H), because great circles are symmetric across diameters.

For random polarization directions, the average $\overline{\eta}$(H) should be near 45°, since each alignment angle η_{iH} is acute, 0° ≤ η_{iH} ≤ 90°, and random polarization directions should not favor large values or small values of η_{iH}, and, therefore, average to about 45°.

Points *H* where the average alignment angle $\overline{\eta}$(H) is smaller than 45°, the great circles tend to converge and where the angle $\overline{\eta}$(H) is larger than 45°, the great circles can be said to diverge. The extremes of the function $\overline{\eta}$(H) measure extreme convergence and extreme divergence of the great circles determined by the polarization directions. We use the term "alignment" for convergence and "avoidance" for divergence.

In this article and notebook, we often use "min" to label the smallest alignment angle $\overline{\eta}_{\text{min}}$, the minimum value of the function $\overline{\eta}$(H), Eq. (1). The points on the Celestial Sphere where the minimum occurs are the "hubs" H_{min} and $-H_{\text{min}}$. Thus "min" is associated with convergence of the polarization directions. For divergence, the hubs H_{max} and $-H_{\text{max}}$ locate places where the polarization directions most avoid, as indicated by the largest alignment angle $\overline{\eta}_{\text{max}}$, the maximum value of the function $\overline{\eta}$(H). Thus, we very often label an avoidance-related quantity with "max".

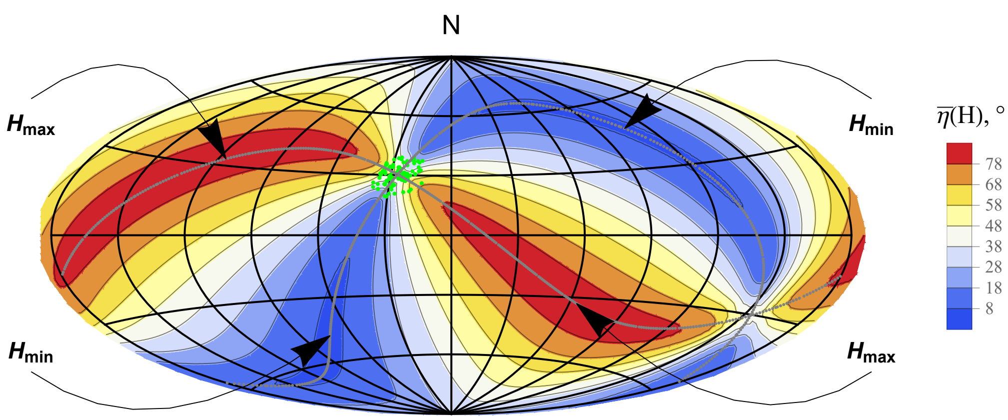


Figure 3: An example of the alignment angle function $\overline{\eta}$(H), here for 99 stars near the Clumps 6 NE and SW discussed below. (An Aitoff plot in Galactic Coordinates centered on (gLON,gLAT) = (0,0) , East to the left, copied with permission from Ref. 13.) The 99 sources are the dots, shaded green ■. The minimum of the function $\overline{\eta}$(H), the smallest alignment angle $\overline{\eta}_{\text{min}}$ = 7.01°, is located at the hubs H_{min} and $-H_{\text{min}}$, where the polarization directions converge best. By symmetry across diameters, the hubs H_{max} and $-H_{\text{max}}$ pinpoint the largest avoidance angle $\overline{\eta}_{\text{max}}$ = 83.12° where the great circles generated by the polarization directions most diverge.

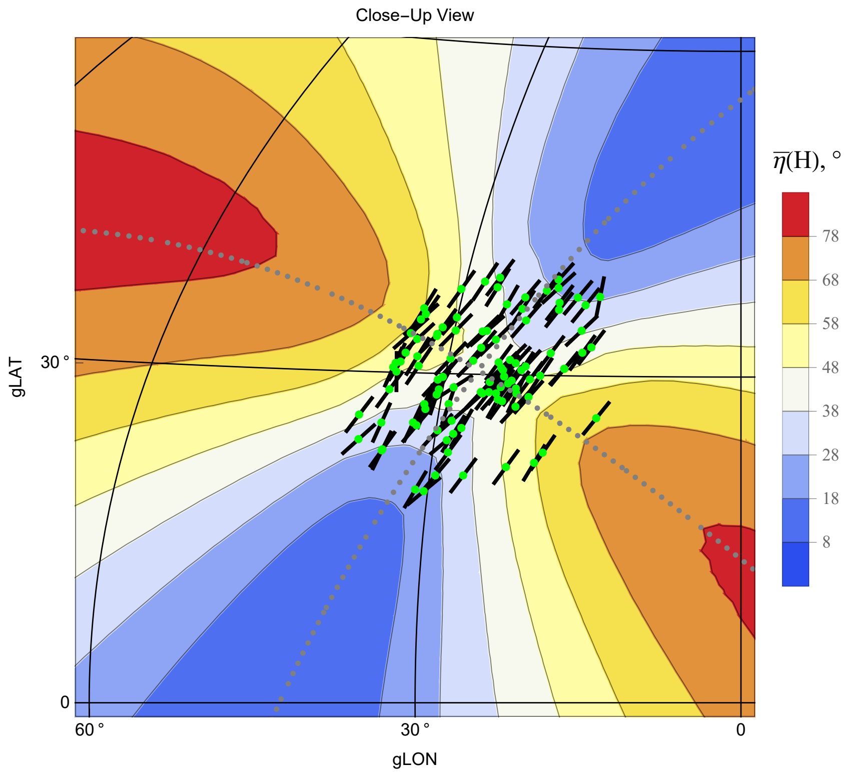


Figure 4: The region near the 99 sources, in green. The short black lines indicate the polarization directions. The gray dotted circles connect the center of the sources with the hubs $\pm H_{\mathrm{min}}$ and $\pm H_{\mathrm{max}}$. Measuring polarization directions ψ counterclockwise from North, with East to the Left, one sees that the angles ψ point in the general direction of $\psi = 150°$. Whether direct or indirect, any alignment test should find the 99 stars' polarization directions well-aligned.

By the plot of the sources and the alignment function $\overline{\eta}$(H) in Fig. 4, one sees that the polarization directions for this sample are parallel with one another. That observation is a direct comparison of the polarization directions one with the others. Such 'direct comparisons' are the basis for a different type of tests than the Hub Test, the 'S' and 'Z' tests, Refs. 14,15,16.

While the alignment of these polarization directions with one another is apparent, by applying the Hub Test, we were able to quantify the significance of the alignment of these polarization directions with the points H_{min} and $-H_{\mathrm{min}}$ in Fig. 3. The alignment angle $\overline{\eta}_{\mathrm{min}}$ is 18.9σs below the most likely random run value.

With the Hub Test, one determines alignment of the polarization directions with points on the Celestial Sphere, thereby making the Hub Test an indirect measure of alignment. The Hub Test supplements these other tests by finding alignments in cases where the polarization directions focus on a nearby point. Those cases have polarization directions with considerable parallax. Cases with nearby hubs and parallax would be uncovered by the indirect Hub Test, but not by the direct tests.

3. Working with the data and setting up the regions

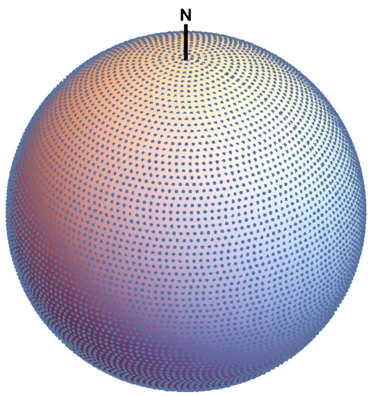


Figure 5. The grid. At a constant declination (latitude), the right ascension (longitude) of the grid points are spaced by 2°. The circles of constant declination are separated by 2°. Each of the regions analyzed is centered on one of the grid points.

The Hub Test needs the location of the sources as well as the polarization direction at each source or the directions of whatever transverse vectors are being surveyed. Also, the uncertainty in the measured polarization direction is needed to estimate error bars on the calculated quantities. That is enough information to draw great circles outward from the sources in the directions of the polarization vectors and propagate measurement errors through the calculations.

The Heiles agglomeration and the Berdyugin catalog gives us the locations of the sources and the polarization position angle ψ and its uncertainty. In the selection process, we use percent polarization and its uncertainty. The data also includes other interesting quantities such as distance.

Not all the stars listed in the original catalogs are included here. The Heile agglomeration has data for more than 9200 stars and the Berdyugin catalog has some 2400 stars, with a handful of stars listed in both. The total number of unique sources is 11647. We cut stars by applying three constraints: (1) the percent polarization must exceed 1%, (2) the fractional uncertainty in polarization percentage must not exceed one quarter, (3) the experimental uncertainty in polarization position angle must be less than seven degrees, $\sigma\psi \geq 7°$. Just 5830 stars qualify for the survey.

The needed data is collected from the original catalogs and placed in a table, called “data00”. The data table is contained in a ‘hidden cell’ in the computer program in Part II the Appendix. To open the cell and view the data00 table, one can download and run the program from the links provided in Ref. 1.

We partition the 5830 sources into 5° radius overlapping regions with centers on the 2° by 2° grid. The Hub Test statistics requires 7 or more sources in a sample, so underpopulated regions are ignored. There are 3558 sufficiently populated 5° regions.

Next, we apply the hub test to each region. The angle η in Fig. 2 is determined for every source in the region and every grid point. The average $\overline{\eta}(\mathrm{H})$ is calculated at every grid point H . The smallest alignment angle $\overline{\eta}_{\mathrm{min}}$ and the largest avoidance angle $\overline{\eta}_{\mathrm{max}}$ are noted, tabulated and saved. Following the method described in the next section, the significance of the region’s smallest alignment angle $\overline{\eta}_{\mathrm{min}}$ and its largest avoidance angle $\overline{\eta}_{\mathrm{max}}$ are calculated, tabulated, and saved.

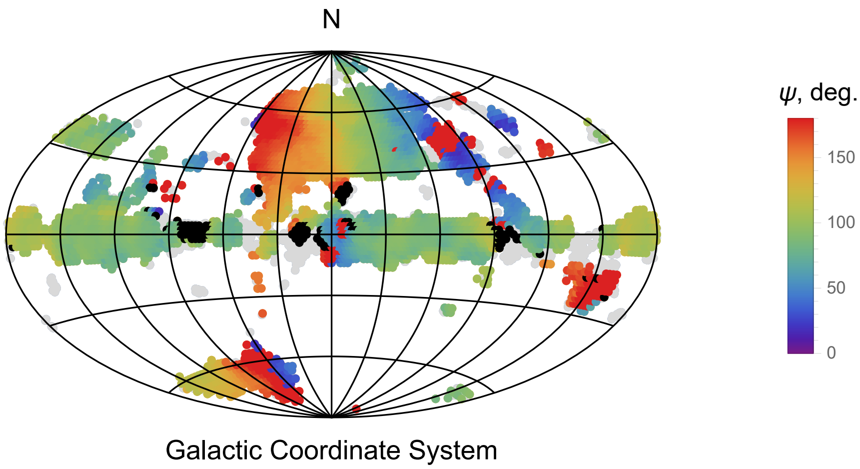


Figure 6. Color-coded average polarization directions $\overline{\psi}$ of the 5° radius regions surveyed. Regions in gray are not sufficiently populated to analyze, while the standard deviation of the polarization directions of the regions in black are too wide, $\Delta\psi > 40°$, to have a representative average. Note that, on the Galactic Disk, where the regions favor green, ■ or $\psi = 90°$, it means that ψ points along the Disk. This plot can be compared with a similar plot, Fig. 11 in Ref. 11.

Alignment, Significance exponent $a = -\log_{10} p$

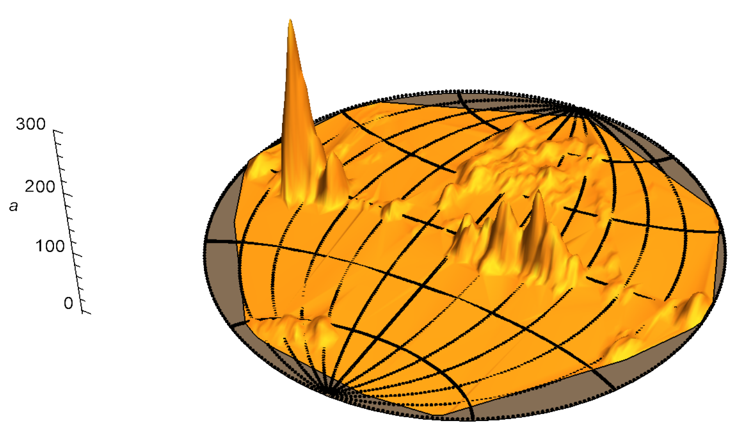


Figure 7. A map of the negative exponent a of the significance p, *i.e.* the logarithm $a = -\log_{10} p$ with $p = 10^{-a}$. Note the scale. The highest peak stands at $a = 294$, making the significance essentially nil, $p = 10^{-294}$. One should treat the value $a = 294$ with considerable skepticism. Yet, even with a large plus/minus, the value implies exceptional significance. However that may be, the importance of the map lies mainly in displaying the relative significance from one location to another, acting as a guide for choosing samples to study in more depth.

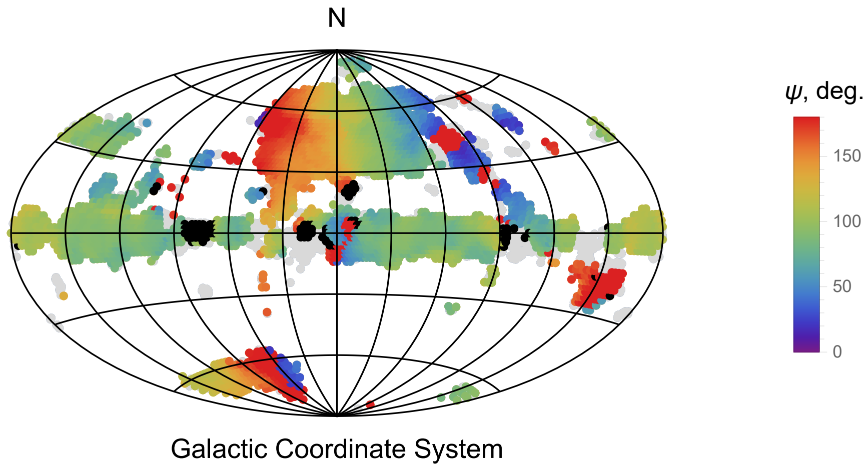


Figure 8. Very significantly aligned regions are are shaded in color. The centers of the other 5° radius regions that have at least 7 sources are plotted as gray points The points shaded in color and the grey points form a total of 3558 regions. There are 2806 regions shaded in color whose polarization directions align with a significance less than 1%, meaning that $p \leq 0.01$, are considered "very significantly aligned". The range of significance runs from $p = 10^{-294}$ to $p = 0.01$ (the very significant limit), giving the negative exponent a range of $2.0 \leq a = -\text{Log}_{10} p \leq 294$. As shown in Sec. A7e, Place Mark A, the uncertainty in $-\text{Log}_{10} p$ is about ±20 for large a, and about ±0.25 for the low end at $a \approx 2$ (the very significant limit).

4. The significance of the regions' alignments

The significance of the smallest alignment angle $\overline{\eta}_{\min}$ is defined as the likelihood that randomly directed polarization vectors would produce a smaller value of $\overline{\eta}_{\min}$. We denote the significance by 'p' or the 'p-value' and sometimes 'S.' Therefore, by definition, one way to determine significance is to start with randomly directed vectors and repeat the process of making the Great Circles from the random polarization directions. Then continue the process by calculating the alignment function $\overline{\eta}$(H) in Eq. (1), and finding its minimum value $\overline{\eta}_{\min}$. Completing one such procedure with random ψs makes a "random run".

That is the most reliable method of determining significance. Here it is dismissed for being both time-consuming and unnecessary for the purpose of the survey. We simply seek sufficiently accurate information to identify samples for more comprehensive study.

Nevertheless, we should outline the definition-driven method in more detail. In Ref. 17, that method is called "Direct Method A". Following the definition of significance, one generates many random runs with randomly directed transverse vectors ψ assigned to the sources. Each run produces a minimum $\overline{\eta}_{\min}$ and a maximum $\overline{\eta}_{\max}$ of the alignment function $\overline{\eta}$(H). A histogram of the many random-based results for $\overline{\eta}_{\min}$ is then approximated by a suitable fitting function. Aside from a scale factor to normalize the distribution, the fitting function of the histogram is the probability distribution of the random results $\overline{\eta}_{\min}$. Having found a function that approximates the probability distribution, one estimates the likelihood that random runs return better results than the observed $\overline{\eta}_{\min}{}^{\text{obs}}$ and that is the significance of $\overline{\eta}_{\min}{}^{\text{obs}}$. Similar comments apply for avoidance. That concludes our description of Direct Method A.

It would be terribly inconvenient to apply Direct Method A to each of the regions in the survey. In this survey, there are 3558 regions. That is just too many to treat individually with the best statistics.

Instead, we introduce a “Library” of preprocessed data that can be used to reconstitute the probability distributions for a range of samples with various number of sources and with various sizes. The Library data is used in two ways, as “Interpolation Method B” and as “Function Method C”. The Methods are discussed briefly where they are introduced in Sec. A7a,b,c,d. For a more complete discussion, see Ref. 17.

The Library has a collection of parameters to generate probability distributions that can be utilized to obtain the significance of the alignment of the sources in a region. The Library works on the assumption that the significance of the smallest alignment angle $\overline{\eta}_{\min}$ depends mainly on the number of sources N and the root-mean-square radius ρRMS of the region. Then, given the values of N and ρRMS for one of the 3558 regions, we can estimate the probability distribution of $\overline{\eta}_{\min}$ for randomly directed sources by finding the distribution for N and ρRMS in a Library of distributions. Having estimated the probability distribution, the significance can be determined by integration.

The Library is hidden in a Mathematica cell in Sec. A7a of the Appendix and can be viewed by running the program in Ref. 1 and opening the cell.

The Library can be used in two ways. One can find significance by Interpolation Method B or Function Method C, as explained in detail in Ref. 17. Basically, we use interpolation to average the Library data if N and ρRMS are within the boundaries of the Library. That is Method B. But, when N and ρRMS are outside the range of Library data, there are fitting functions that extend reach of the Library, Method C.

By applying Methods B and C to the survey of polarization starlight, we find that 2806 regions have very significant alignment, *i.e.* $p < 10^{-2}$ or $a > 2$. The significances of these regions are displayed in raised relief in Fig. 7 and plotted in color in Fig. 8. Since there are 3558 regions, 2806 regions is close to 80% of the total number of regions analyzed. Thus the Galaxy shows a high degree of significant local alignment. From all these well-aligned regions, we next show a couple of ways to choose samples suitable for more detailed in depth study. The studies themselves are beyond the scope of this article and may appear elsewhere.

5. Combining regions to make samples for further study

In this section we combine the stars in some of the 5° radius regions to make samples for further investigation. The distinct peak in Figs. 7 and 8 is a prominent feature. But also the hills to the North of the Galactic Center suggest there may be subtle differences between the polarization direction alignments of adjacent areas. The method of selecting samples sections off an area and selects regions based on some requirement involving the significance of the regions.

The prominent peak in Fig. 7 is a stand-out feature making it the first sample we collect. Call it ‘Clump 1.’ In Fig. 9, the relevant stars are sectioned off by location and by the significance of the 5° regions. We take the cutoff at power $a = 9$, meaning significances p of the 5° regions obey the inequality $p \leq 10^{-9}$. The side walls of the protrusion in Fig. 7 are so steep that essentially the same batch of stars would be collected with a cutoff at $a = 15$. Clump 1 contains 902 stars.

The distances to the stars are binned in Fig. 10, a histogram that shows the distribution of the distances. By Ref. 18, it looks like the nearer stars are in the ‘Orion Spur’ and the more distant stars are in the ‘Perseus Arm’.

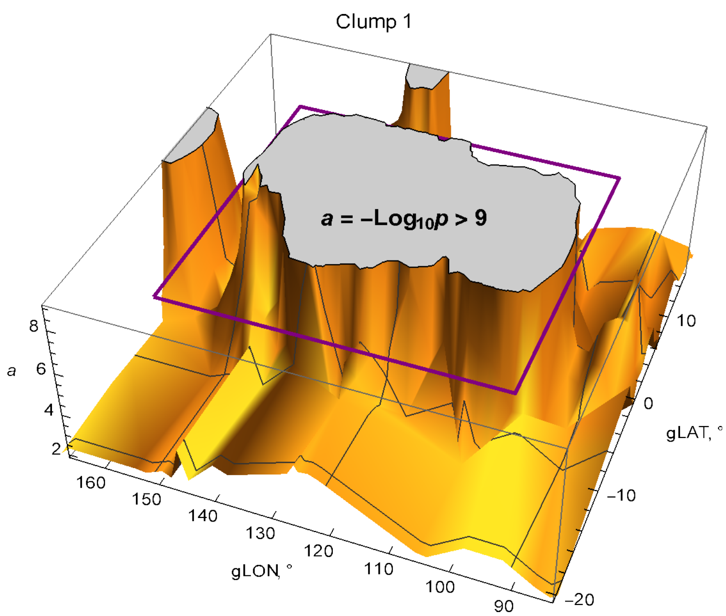


Figure 9. Selecting Clump 1, the tallest peak toward the East on the Disk in Fig. 7. Here, Clump 1 consists of the stars in all of the 5° regions in the purple rectangle with significance better than one in a billion, $p = 10^{-a}$ with $a > 9$. The plot is cut off at heights $a = 9$, but the selected regions run to $a \approx 290$ with significance $p \approx 10^{-290}$.

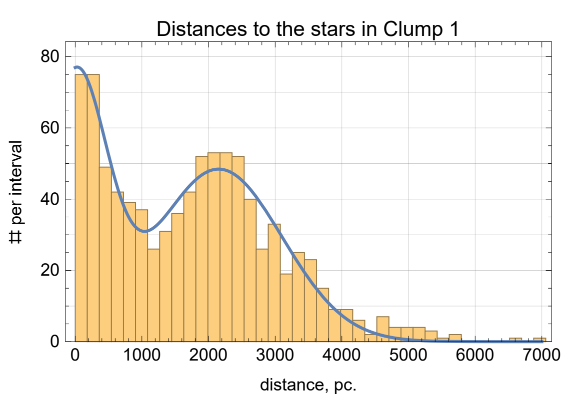


Figure 10. Distances to the stars in Clump 1. While the survey is 2-dimensional, finding alignments on the Celestial Sphere, the locations of the sources in 3-D may be important to understanding the mechanisms that produce stellar polarization.

Turn now to an example of nuance. We select and compare adjacent hills in the rolling significance topology near (gLAT, gLON) = (30°, 30°), which is Northeast of the Galactic Center. See Figs. 7 and 8. Most of the 5° regions in the neighborhood of these hills are very significantly aligned. To distinguish the adjacent hilltops, we climb to a cutoff at $a > 21$, or significance $p <$

10^{-21} , so fewer than one in 10^{21} randomly directed regions are better aligned. See Fig. 11. The two clumps are called Clump 6NE and Clump 6SW based on their relative locations in the sky. Clumps 2 - 4 and those numbered beyond 6 may be treated elsewhere.

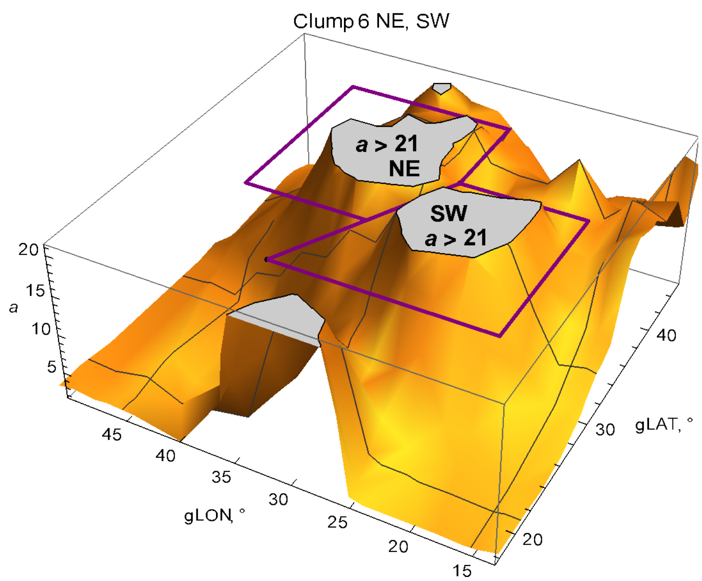


Figure 11. Selecting Clump 6NE and Clump 6SW. The survey can distinguish samples that might be missed by examination of the plotted polarization directions as in Fig. 1.

As shown in Fig. 12, the two clumps have polarization directions that differ by about 8°. We see that NW tend to have larger polarization position angles ψ than the SW ψs. In Fig. 13, we see that the SW clump is on the order of 50 pc further away than the NE clump, with some overlap. Both NE and SW stars are about the same distance from us as the closest stars in Clump 1, by Fig. 10.

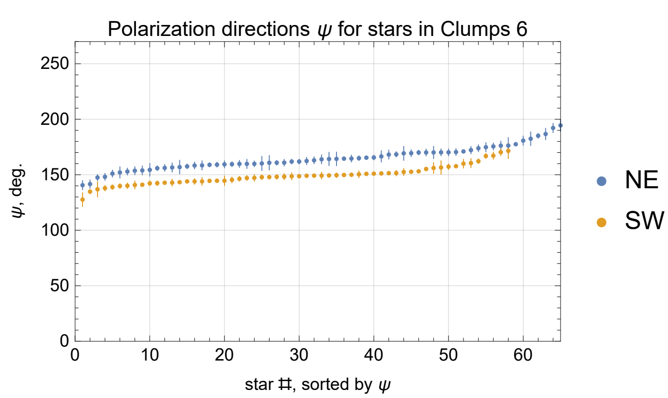


Figure 12. Comparing the polarization direction for the NE and SW Clumps. The difference is about 8° with the ψs for NE greater than the ψs for SW. The measurement uncertainties are shown as vertical bars.

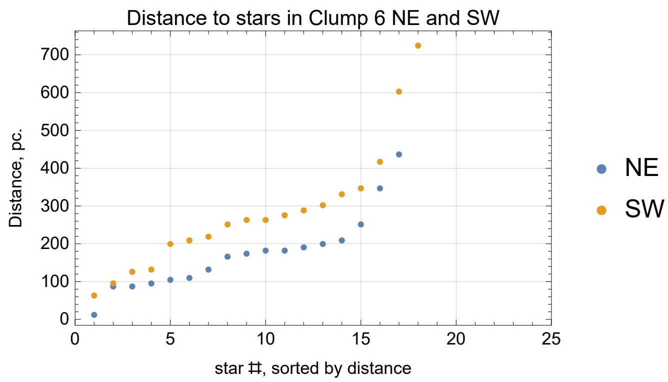


Figure 13. Comparing the distances to the NE and SW Clumps. The stars that NE and SW have in common are removed. Clearly, one set is closer than the other. The survey can assist in locating well-aligned populations that have slightly different properties.

6. Concluding Remarks

(1) When confronted with thousands of transverse vectors as in Fig. 1, a survey that maps the significance of the alignment in a grid of overlapping regions may help organize the data and identify areas to investigate further. Thus, a survey can help locate the sources with well-aligned polarization directions in a large catalog.

There are no guarantees, of course. Some overlooked small area may contain sources with interesting alignment properties, overlooked because the alignment is diluted in a region that is too big. A 5° radius region survey might be blind to a well-aligned collections of sources confined to 1° samples. Conversely, one suspects that a 1° region survey might miss some of the alignments that a 5° survey uncovers. Maybe the answer is to conduct more surveys.

(2) Since finding the smallest alignment angle $\overline{\eta}_{\mathrm{min}}$ and the largest avoidance angle $\overline{\eta}_{\mathrm{max}}$ are such similar processes, this article treats only alignment in any detail. Yet avoidance may be the more important property of polarization directions for some sets of data. What if the polarization direction is perpendicular to some local feature, perhaps perpendicular to some stream of stars? Then correlations of perpendiculars, *i.e.* avoidance and $\overline{\eta}_{\mathrm{max}}$, take center stage.

Part II the Appendix treats alignment and avoidance equally. Remarkably, the significance of avoidance for the regions tracks almost exactly the significance of alignment. Such behavior can be expected for 5° radius regions when the polarization directions are parallel, comparing the directions one to another directly. When a set of polarization directions focus down on a nearby alignment hub H_{min}, there is necessarily considerable parallax. Parallax decouples alignment and avoidance. It may be that the few regions in the survey with different alignment and avoidance significances hold special interest.

7. References

1. R. Shurtleff, the ready-to-run Mathematica version of this notebook is available at the following URL:
   https://www.dropbox.com/s/gx96p5105hz76dw/20220113SurveyStarsGALa.nb?dl=0
   https://www.wolframcloud.com/obj/shurtleffr/Published/20220113SurveyStarsGALa.nb
2. Wolfram Research, Inc., Mathematica, Version 12.1, Champaign, IL (2020).
3. Boulanger, F., Ensslin, T., Fletcher, A., et al., JCAP, 08, 049 (2018).
4. Ian W. Stephens et al, The Galactic Magnetic Field’s Effect in Star-forming Regions, ApJ 728 99 (2011), Fig. 4.
5. Han, J. (2012). Magnetic fields in our Milky Way Galaxy and nearby galaxies. Proceedings of the International Astronomical Union, 8(S294), 213-224. doi:10.1017/S1743921313002561
6. A. Lazarian and Thiem Hoang, Alignment and Rotational Disruption of Dust, ApJ 908 12 (2021).
7. Heiles, C., An agglomeration of stellar polarization catalogs, Astron. J. 119, 923 (2000).
8. Heiles, C., The catalog, ReadMe, and other files related to Ref. 7 are also available online at VizieR; https://cdsarc.unistra.fr/viz-bin/cat/II/226 (2000).
9. Berdyugin A., Piirola V., Teerikorpi P., Astron. Astrophys. 561, A24 DOI: 10.1051/0004-6361/201322604 (2014).
10. Berdyugin A., *et al.*, The catalog, ReadMe, and other files related to Ref. 9 are available online at VizieR; http://cdsarc.u-strasbg.fr/viz-bin/ReadMe/J/A+A/561/A24?format=html&tex=true
11. Fosalba, P., *et al.*, Statistical Properties of Galactic Starlight Polarization, ApJ 564 762 (2002).
12. R. Shurtleff, “Indirect polarization alignment with points on the sky, the Hub Test” , https://vixra.org/abs/2011.0026 (2020).
13. R. Shurtleff, Evaluating the Alignment of the Polarized Starlight from 99 Stars in a Region off the Disk of the Milky Way, viXra:2104.0074v2 (2021).
14. Hutsemekers, D., “Evidence for very large-scale coherent orientations of quasar polarization vectors”, Astronomy and Astrophysics 332, pp. 410-428 (1998).
15. Jain, P., Narain, G. and Sarala, S., “Large-scale alignment of optical polarizations from distant QSOs using coordinate-invariant statistics”, MNRAS 347, pp. 394-402, doi: 10.1111/j.1365-2966.2004.07169.x, eprint: astro-ph/0301530 (2004).
16. Pelgrims, V. and Cudell, J. R., “A new analysis of quasar polarization alignments”, MNRAS 442, pp. 1239-1248, doi : 10.1093/mnras/stu966, eprint: arXiv:1402.4313 (2014).
17. R. Shurtleff, “Alternate methods for judging the significance of Hub Test results” , eprint: viXra 2112.0109v1 (2021).
18. File:Galactic longitude.JPG. (2015, June 5). Wikimedia Commons, the free media repository. Retrieved 13:50, February 17, 2022 from https://commons.wikimedia.org/w/index.php?title=File:Galactic_longitude.JPG& oldid=162642561.

Part II the Appendix

CONTENTS of Part II

A1. Introduction

The Appendix is the computer program, a “notebook” written in the Wolfram Mathematica language. The inputs to the program in Sec. A1 can be changed so the survey can deal with new data.

The Appendix treats alignment and avoidance equally, what it finds for one, it finds for the other. Avoidance may be important if the polarization direction turns out to be perpendicular to some feature such as a jet or some other structure. Then it would be important to find correlations of directions that are perpendicular to the polarization vectors and that would be revealed by gauging avoidance.

A2. User Input

This notebook may be used as a template to evaluate new data.

1. The new data should conform to the format of the table “data00” displayed below.
2. You may want to furnish a home directory so the program can find and save data files.
3. The grid spacing can be chosen by the user below in this section.
4. The regions to be analyzed are circular with a radius that can be chosen by the user in this section.

Definitions:

homeDirectory a place on the computer to store and retrieve files.
gridSpacing in degrees, the angular separation of grid points along a circle of constant latitude and the angular separa-

tion of the circles of constant latitude. See Sec. A5.
rgnRadius radius of regions in degrees

data00 This data table contains the information about the sources that produces the rest of the notebook.

1. cat. ID # (HD,BD,CD, or CPD) 2. Gal. Long. (rad) 3. Gal. Lat. (rad) 4. ψ (Galactic, rad.) 5. $\sigma\psi$ (rad) [6. Distance 7. RA (rad) 8. Dec. (rad) 9. ψ (Equatorial, rad.) 10. p (%) 11. σp (%)

In[1]:=
```
homeDirectory =
  "C:\\Users\\shurt\\Dropbox\\HOME_DESKTOP-0MRE5OJ\\SendXXX_CJP_CEJPetc\\SendViXra\\
    20200715AlignmentMethod\\20211221MapsOfSignificance\\20220113MapTheStars"
```
Out[1]=
```
C:\Users\shurt\Dropbox\HOME_DESKTOP-0MRE5OJ\SendXXX_CJP_CEJPetc\
  SendViXra\20200715AlignmentMethod\20211221MapsOfSignificance\20220113MapTheStars
```

In[2]:=
```
 (*Set the grid spacing in degrees*)
gridSpacing = 2;
(*Set the radius of the regions in degrees*)
rgnRadius = 5.;
```

The following cell has the data00 table with the information about the sources. It is very large and, therefore, it is hidden from view. To see it go to “Cell Properties” and click “Open”.

In[5]:=
```
(*If you got it, you can get data00 from a file:*)
(*SetDirectory[homeDirectory]
   data00=Get["20220116data00HBCatalogConvertedCutandReordered.dat"];
*)
Print["Table data00 contains information on ", Length[data00], " sources."]
Print["For example, record #16 in the table data00: ", data00[[16]], "."]
```
```
Table data00 contains information on 5830 sources.

For example, record #16 in the table data00: {137387. HD, -0.805415,
  -0.244547, 2.74715, 0.0191986, 418., -2.21869, -1.28089, 2.1293, 0.93, 0.035}.
```

A3. Preliminary

Definitions:

cos, sin, tan, arccos, arcsin, arctan trig functions for angles in degrees have lower case names
mean the arithmetic average of a set of numbers, $\frac{1}{N}\sum_{i=1}^{N} n_i$ The mean and standard deviation are convenient functions.
stanDev the standard deviation. Given a set of *N* numbers n_i with mean value *m*, the standard deviation is $\left(\frac{1}{N}\sum_{i=1}^{N}(n_i-m)^2\right)^{1/2}$, the square root of the average of the squares of the differences of the numbers with the mean. Note that we divide by *N* to get the average of the deviations squared.
lp00QUANTITY a list plot of the values in the table data00 for the quantity QUANTITY
er, eN, eE are unit vectors in a 3D Cartesian coordinate system from Origin to Source,
(gLON,gLAT) = galactic longitude and latitude of the source. We use radians for the angles.
er(gLON,gLAT) = unit vectors from Origin to Source
eN(gLON,gLAT) = local North at Source

eE(gLON,gLAT) = local East at Source
gLONFROMr(er) = gLON determined by radial unit vector er
gLATFROMr(er) = gLAT determined by radial unit vector er

Aitoff Plot Functions
gLONH(gLON,gLAT) , xH(gLON,gLAT) , yH(gLON,gLAT), where xH, yH is centered on gLON = 0.
xH180(gLON,gLAT) , yH180(gLON,gLAT), where xH is centered on gLON = 180°.

In[7]:=
```
(*We work with degrees, so define convenient functions.*)
cos[θ_] := cos[θ] = Cos[θ (2. π/360.)];
sin[θ_] := sin[θ] = Sin[θ (2. π/360.)];
tan[θ_] := tan[θ] = Tan[θ (2. π/360.)];
arccos[x_] := arccos[x] = ArcCos[x] (360./2. π);
arcsin[x_] := arcsin[x] = ArcSin[x] (360./2. π);
arctan[x_] := arctan[x] = ArcTan[x] (360./2. π)
```

In[10]:=
```
mean[data_] := mean[data] = (1/Length[data]) Sum[data[[i4]], {i4, Length[data]}];
(* arithmetic average *)
stanDev[data_] :=
 stanDev[data] = ((1/Length[data]) Sum[(data[[i5]] - mean[data])^2, {i5, Length[data]}])^(1/2)
  (*standard deviation*)
```

In[12]:=
```
(*List plots of the data:*)
lp00gLON = ListPlot[Sort[Table[data00[[i, 2]], {i, Length[data00]}]],
   PlotLabel → "Galactic Longitude", PlotRange → All];
```

In[13]:=
```
lp00gLAT = ListPlot[Sort[Table[data00[[i, 3]], {i, Length[data00]}]],
   PlotLabel → "Galactic Latitude", PlotRange → All];
lp00ψGAL = ListPlot[Sort[Table[data00[[i, 4]], {i, Length[data00]}]],
   PlotLabel → "ψ (galactic, rad)", PlotRange → All];
```

In[15]:=
```
lp00σψ = ListPlot[Sort[Table[data00[[i, 5]], {i, Length[data00]}]],
   PlotLabel → "σψ (rad)", PlotRange → All];
lp00dist = ListPlot[Sort[Table[data00[[i, 6]], {i, Length[data00]}]],
   PlotLabel → " Distance ", PlotRange → All];
```

In[17]:=
```
lp00RA = ListPlot[Sort[Table[data00[[i, 7]], {i, Length[data00]}]],
   PlotLabel → "RA (rad)", PlotRange → All];
lp00dec = ListPlot[Sort[Table[data00[[i, 8]], {i, Length[data00]}]],
   PlotLabel → "dec (rad)", PlotRange → All];
```

In[19]:=
```
lp00ψEqui = ListPlot[Sort[Table[data00[[i, 9]], {i, Length[data00]}]],
   PlotLabel → "ψ (equatorial, rad)", PlotRange → All];
```

```
In[20]:= lp00p = ListPlot[Sort[Table[data00[[i, 10]], {i, Length[data00]}]],
           PlotLabel → "p (%)", PlotRange → {0, 0.5}];
         lp00Fracσp = ListPlot[Sort[Table[data00[[i, 11]] / data00[[i, 10]], {i, Length[data00]}]],
           PlotLabel → "σp / p ", PlotRange → All];

In[22]:= (* For a Source at (gLON,gLAT) = (gLON,gLAT): er, eN,
         eE are unit vectors from Origin to Source, local North, local East, resp.  *)
         er[gLON_, gLAT_] := er[gLON, gLAT] = {cos[gLON] × cos[gLAT], sin[gLON] × cos[gLAT], sin[gLAT]}
         eN[gLON_, gLAT_] := eN[gLON, gLAT] = {-cos[gLON] × sin[gLAT], -sin[gLON] × sin[gLAT], cos[gLAT]}
         eE[gLON_, gLAT_] := eE[gLON, gLAT] = {-sin[gLON], cos[gLON], 0}
         {"Check er.er = 1, er.eN = 0, er.eE = 0, eN.eN
           = 1, eN.eE = 0,eE.eE = 1, erXeE = eN, eEXeN = er, eNXer = eE: ",
          {0} == Union[Flatten[Simplify[{er[gLON, gLAT].er[gLON, gLAT] - 1, er[gLON, gLAT].eN[gLON, gLAT],
              er[gLON, gLAT].eE[gLON, gLAT], eN[gLON, gLAT].eN[gLON, gLAT] - 1, eN[gLON, gLAT].
               eE[gLON, gLAT], eE[gLON, gLAT].eE[gLON, gLAT] - 1, Cross[er[gLON, gLAT], eE[gLON, gLAT]] -
               eN[gLON, gLAT], Cross[eE[gLON, gLAT], eN[gLON, gLAT]] - er[gLON, gLAT],
              Cross[eN[gLON, gLAT], er[gLON, gLAT]] - eE[gLON, gLAT]}]]]}

Out[25]= {Check er.er = 1, er.eN = 0, er.eE = 0, eN.eN = 1,
           eN.eE = 0,eE.eE = 1, erXeE = eN, eEXeN = er, eNXer = eE: , True}
```

Get (gLON,gLAT) in degrees from radial vector r:

```
In[26]:= gLONFROMr[r_] := N[arctan[Abs[r[[2]] / r[[1]]]]] /; (r[[2]] ≥ 0 && r[[1]] > 0)
         gLONFROMr[r_] := N[180 - arctan[Abs[r[[2]] / r[[1]]]]] /; (r[[2]] ≥ 0 && r[[1]] < 0)
         gLONFROMr[r_] := N[-180 + arctan[Abs[r[[2]] / r[[1]]]]] /; (r[[2]] < 0 && r[[1]] < 0)
         gLONFROMr[r_] := N[-arctan[Abs[r[[2]] / r[[1]]]]] /; (r[[2]] < 0 && r[[1]] > 0)
         gLONFROMr[r_] := 90. /; (r[[2]] ≥ 0 && r[[1]] == 0)
         gLONFROMr[r_] := -90. /; (r[[2]] < 0 && r[[1]] == 0)

In[32]:= gLATFROMr[r_] := N[arctan[r[[3]] / (√(r[[1]]^2 + r[[2]]^2))]] /; (√(r[[1]]^2 + r[[2]]^2) > 0)
         gLATFROMr[r_] := Sign[r[[3]]] (90.) /; (√(r[[1]]^2 + r[[2]]^2) == 0)
```

The following Aitoff Plot formulas can be found in, for example, Wikipedia contributors. “Aitoff projection.” Wikipedia, The Free Encyclopedia. Wikipedia, The Free Encyclopedia, 25 May. 2017. Web. 3 Jan. 2018.
I didn’t bother to define the function Sinc for arguments in degrees, so the degrees are converted to radians in the Sinc function.

```
In[34]:= αH[gLON_, gLAT_] := αH[gLON, gLAT] = arccos[cos[gLAT] × cos[gLON / 2.]]
         xH[gLON_, gLAT_] :=
          xH[gLON, gLAT] = (2. cos[gLAT] × sin[gLON / 2.]) / Sinc[((2. π) / 360.) αH[gLON, gLAT]]
         yH[gLON_, gLAT_] := yH[gLON, gLAT] = sin[((2. π) / 360.) gLAT] / Sinc[((2. π) / 360.) αH[gLON, gLAT]]
```

Using the following functions produces an Aitoff Plot that is centered on gLON = 180°.

```
In[37]:= (*EQUATORIAL COORDINATES are not used! The following are kept for porting to other notebooks.*)
         xH180[gLON_, gLAT_] := xH180[gLON, gLAT] =
           (2. cos[gLAT] × sin[(gLON - 180.) / 2.]) / Sinc[((2. π) / 360.) αH[(gLON - 180.), gLAT]]
         yH180[gLON_, gLAT_] := yH180[gLON, gLAT] = sin[gLAT] / Sinc[((2. π) / 360.) αH[(gLON - 180.), gLAT]]
```

For Galactic Coordinates, the following functions produces an Aitoff Plot that is centered on gLON = 0° and the gLON axis runs backwards from +180° on the left to 0° at the center to -180° on the right. The viewpoint is inside the Celestial Sphere, looking out.

In[39]:=
```
(*The plots of the sky in Galactic coordinates have the gLON axis running from +
 180° on the left to -180° on the right. Angles gLON and gLAT are in degrees*)
xHGal[gLON_, gLAT_] := xHGal[gLON, gLAT] =
  (2. cos[gLAT] × sin[-gLON / 2.]) / Sinc[((2. π) / 360.) αH[-gLON, gLAT]]
yHGal[gLON_, gLAT_] := yHGal[gLON, gLAT] = sin[gLAT] / Sinc[((2. π) / 360.) αH[-gLON, gLAT]]
```

A4. Managing the Source Data

The source data table “data00” can be found in a hidden cell in Sec. A2. In this section, source related quantities are placed in tables for use in calculations.

Definitions:

data00

1. cat. ID # (HD,BD,CD, or CPD) 2. Gal. Long. (rad) 3. Gal. Lat. (rad) 4. ψ (Galactic, rad.) 5. $\sigma\psi$ (rad) 6. Distance 7. RA (rad) 8. Dec. (rad) 9. ψ (Equatorial, rad.) 10. p (%) 11. σp (%)

```
(*From data00. CONVERT ANGLES TO DEGREES*)
nCAT[i]     ID of star (HD, BD, CD, or CPD)
gLONi[i]    Galactic longitude
gLATi[i]    Galactic latitude
ψGALi[i]    PPA, polarization position angle ψ in the galactic coordinate system
σψi[i]      uncertainty of PPA ψi
disti[i]    distance, parsecs
rai[i]      Right Ascension of ith source, equatorial coordinates
deci[i]     declination
ψi[i]       PPA, polarization position angle : equatorial,
clockwise from North with East to the right.
  pi[i]     percent polarization p %, units : %
σpi[i]      uncertainty in percent polarization p %, units : %
```

Calculated from data00 quantities:

ri[i] unit vector from Origin to ith Source on Celestial Sphere, Galactic Coordinates

vN[i] local North

vE[i] local East

vψi[i] unit vector in direction of PPA ψ

nSxψi[i] r Cross vψ, a unit vector

xyAitoffSources 2-D coordinates of the sources for an Aitoff plot

rPlusψ[i,d] radial vector to line segment endpoints representing polarization direction

crossesOverPlus, MinusiD #s of sources so close to the Aitoff plot edges that the polarization line segments go over the edge

polarLinesNoCrossing[d] polarization line segments when there is no problem with the Aitoff edge

polarLinesCrossingPlus, Minus[d] line segments for sources close to the Aitoff edge
mapOfSources pine-needle plot of the stars and the polarization line segments, a full-sphere 2-D Aitoff plot

In[41]:=
```
(*From data00. CONVERT DATA TO DEGREES*)
nCAT[i_] := nCAT[i] = data00[[i, 1]] (*catalog # for the ith source*)
gLONi[i_] := gLONi[i] = data00[[i, 2]] (360. / (2. π)) (*Galactic longitude*)
gLATi[i_] := gLATi[i] = data00[[i, 3]] (360. / (2. π)) (*Galactic latitude*)
ψGALi[i_] := ψGALi[i] = data00[[i, 4]] (360. / (2. π)) (*PPA,
polarization position angle ψ in the galactic coordinate system *)
σψi[i_] := σψi[i] = data00[[i, 5]] (360. / (2. π)) (* uncertainty in PPA ψi *)
disti[i_] := disti[i] = data00[[i, 6]] (*distance, pc *)
rai[i_] := rai[i] = data00[[i, 7]] (360. / (2. π))
(*RA of ith source. EQUATORIAL COORDINATES are not used*)
deci[i_] := deci[i] = data00[[i, 8]] (360. / (2. π)) (*dec. EQUATORIAL COORDINATES are not used*)
ψi[i_] := ψi[i] = data00[[i, 9]] (360. / (2. π)) (*EQUATORIAL COORDINATES are not used! PPA,
polarization position angle: clockwise from North with East to the right. *)
pi[i_] := pi[i] = data00[[i, 10]] (* percent polarization p%, units: % *)
σpi[i_] := σpi[i] = data00[[i, 11]] (* uncertainty inpercent polarization p%,  units: % *)
```

In[52]:=
```
ri[i_] := ri[i] = er[gLONi[i], gLATi[i]]
(*unit vector from Origin to ith Source on Celestial Sphere*)
vNi[i_] := vNi[i] = eN[gLONi[i], gLATi[i]] (*North*)
vEi[i_] := vEi[i] = eE[gLONi[i], gLATi[i]] (*East*)
vψi[i_] := vψi[i] = cos[ψGALi[i]] × vNi[i] + sin[ψGALi[i]] × vEi[i]
(*unit vector in direction of PPA*)
nSxψi[i_] := nSxψi[i] = sin[ψGALi[i]] × vNi[i] - cos[ψGALi[i]] × vEi[i] (* r Cross vψ *)
```

In[57]:=
```
(*Plot sources*)
xyAitoffSources =
  Table[{xHGal[ gLONi[i], gLATi[i] ], yHGal[ gLONi[i], gLATi[i] ]}, {i, Length[data00]}];
```

In[58]:=
```
(*Plot polarization directions*)
rPlusψ[i_, d_] :=
 rPlusψ[i, d] = (ri[i] + d vψi[i]) / ((ri[i] + d vψi[i]).(ri[i] + d vψi[i]))^(1/2)
crossesOverPlus = {}; crossesOverMinus = {};
For[i = 1, i ≤ Length[data00], i++,
 If[ gLONFROMr[ rPlusψ[i, 0.05]] - gLONi[i] < (*-200*) - 350, AppendTo[crossesOverPlus, i]];
 If[ gLONFROMr[ rPlusψ[i, -0.05]] - gLONi[i] > (*200*) 350.,
  AppendTo[crossesOverMinus, i]]]
noCrossing = Complement[Range[Length[data00]], Union[crossesOverPlus, crossesOverMinus]];
```

```
In[62]:= (*Plot polarization directions and color-code them.*)
polarLinesNoCrossing[d_] := (*polarLinesNoCrossing[d]=*)
 Table[{ColorData["Rainbow"][(ψGALi[i]) / 180.],
   Line[{{xHGal[gLONFROMr[ rPlusψ[i, d]], gLATFROMr[ rPlusψ[i, d]]],
      yHGal[gLONFROMr[ rPlusψ[i, d]], gLATFROMr[ rPlusψ[i, d]]]},
     {xHGal[gLONFROMr[ rPlusψ[i, -d]], gLATFROMr[ rPlusψ[i, -d]]],
      yHGal[gLONFROMr[ rPlusψ[i, -d]], gLATFROMr[ rPlusψ[i, -d]]]}}]}, {i, noCrossing}]
polarLinesCrossingPlus[d_] := (*polarLinesCrossingPlus[d]=*)
 Table[{ColorData["Rainbow"][(ψGALi[i]) / 180.],
   Line[{{xHGal[gLONi[i], gLATi[i]], yHGal[gLONi[i], gLATi[i]]},
     {xHGal[gLONFROMr[ rPlusψ[i, -d]], gLATFROMr[ rPlusψ[i, -d]]], yHGal[
       gLONFROMr[ rPlusψ[i, -d]], gLATFROMr[ rPlusψ[i, -d]]]}}]}, {i, crossesOverPlus}]
polarLinesCrossingMinus[d_] := (*polarLinesCrossingMinus[d]=*)
 Table[{ColorData["Rainbow"][(ψGALi[i]) / 180.],
   Line[{{xHGal[gLONFROMr[ rPlusψ[i, d]], gLATFROMr[ rPlusψ[i, d]]],
      yHGal[gLONFROMr[ rPlusψ[i, d]], gLATFROMr[ rPlusψ[i, d]]]},
     {xHGal[gLONi[i], gLATi[i]], yHGal[gLONi[i], gLATi[i]]}}]},
  {i, crossesOverMinus(*noCrossing*)}]

In[65]:= setUPplotForMap = Show[{ParametricPlot[{xHGal[150, δ], yHGal[150, δ]},
     {δ, -90, 90}, PlotStyle → {Black, Thickness[0.002]},
     PlotLegends → BarLegend[{"Rainbow", {0, 180.}}, LegendLabel → "ψ, deg."],
     PlotPoints → 60, PlotRange → {{-4.0, 3.5}, (7.5 / 11.0) {-3, 3}}, Axes -> False,
     Frame → False], Table[ParametricPlot[{xHGal[α, δ], yHGal[α, δ]},
      {δ, -90, 90}, PlotStyle → {Black, Thickness[0.002]}, PlotPoints → 60,
      PlotRange → {{-4.0, 3.5}, (7.5 / 11.0) {-3, 3}}, Axes -> False, Frame → False],
     {α, -180, 180, 30}], Table[ParametricPlot[{xHGal[α, δ], yHGal[α, δ]}, {α, -179, 179},
      PlotStyle → {Black, Thickness[0.002]}, PlotPoints → 60], {δ, -60, 60, 30}]}];
```

In[66]:=
```
mapOfSources =
  Show[{setUPplotForMap, Graphics[
     {PointSize[0.004], Text[StyleForm["N", FontSize -> 14, FontWeight -> "Plain"], {0, 1.85}],
      Text[StyleForm["Galactic Coordinate System", FontSize -> 14, FontWeight -> "Plain"],
       {0, -1.85}], {polarLinesNoCrossing[0.05]},
      {polarLinesCrossingPlus[0.01]}, {polarLinesCrossingMinus[0.01]}
          }]}, ImageSize → 1.2 × 432]
```

Out[66]=

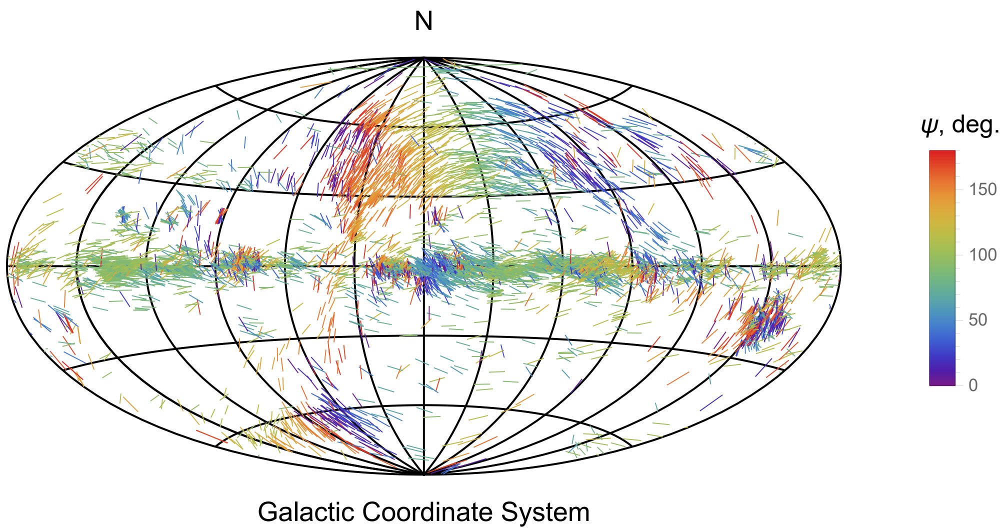


Figure A1. Pine needle plot of the transverse vectors of the sources. There are 5830 sources.

A5. Building the Grid

There are an infinite number of points on the surface of a sphere. We seek to represent the behavior of the alignment function $\overline{\eta}(H)$ in the surface with a finite number of grid points.

As the number of grid points grows larger, the better the approximation to the continuous sphere. But the more grid points, the longer the computer takes to run the software. It is a balance.

The spacing of points on the grid can be adjusted to whatever value one wishes in Sec. A2. For this article, the grid spacing is 2°. Another user-definable quantity that affects computer time is the region radius. Regions in this article have 5° radii.

The grid scatters 10356 grid points uniformly over the Celestial Sphere. The grid takes into account the shrinking radii of circles of constant latitude as they approach the poles. Circles of constant latitude, gLAT, decrease in size as they approach the poles and have fewer grid points along their circumferences. Adjacent circles of constant latitude are separated by 2° in the North/South direction. The grid is oriented with respect to Galactic North. See Fig. 5.

Definitions:

gridSpacing = separation in degrees between grid points on a constant latitude circle and separation of constant latitude circles.
gLONpointH,gLATpointH gLON and gLAT of the grid points
dθ1 = gridSpacing grid spacing in degrees
gridN, gridS, grid North, South hemisphere and whole sphere grid tables. See item-by-item description.
gLATj, ai, idN, j, dummy indices

gLON(or LAT)pointH   Longitude/latitude of grid point H, degrees
nGrid   number of grid points
rHj(j)   unit radial vector to jth grid point H_j
gLONGrid   longitude of jth grid point H_j
gLATGrid   latitude of jth grid point H_j
rGrid   3D radial unit vector to the jth grid point H_j
vHij(i,j)   unit vector tangent to the great circle connecting the ith source with Hj in tangent space of the ith source
nSxHij(i,j)   unit vector perpendicular to the plane of the great circle containing the ith source and the jth grid point Hj
ηiHj(i,j)   alignment angle in the tangent space at the ith source between the polarization direction and the great circle toward Hj.
[See Ref. 12 for details. The two unit vectors nSxψi and nSxHij are perpendicular to vψ and vHij, but the angle ηiHj between vψ and vHij is the same as the angle between nSxψi and nSxHij .]

Tables:

gridN and ***gridS*** and ***grid***
1. sequential point #   2. gLON index 3. gLAT index   4. gLON (rad) 5. gLAT (rad)   6. rH Cartesian coordinates of the point

We need to calculate the alignment angle η_{iH} = arccos[| $\hat{n}_{Sx\psi}.\hat{n}_{SxH}$|] . The angle η_{iH} is the acute angle between the polarization vector for the ith source S and the direction from S to the grid point H. Both vectors are in the tangent plane at the ith source.

We have $\hat{n}_{Sx\psi}$ = nSxψi for ith source. It only depends on data00 and was determined above in Sec. A4.

We also need $\hat{n}_{SxH}$ = nSxHij = ri×rH for ith source and jth grid point H_j. This depends on both data00 and the location of allowed points H_j on the grid. It is calculated below in this section.

```
In[67]:= Print["The space between grid points is gridSpacing = ",
          gridSpacing, "°. It was chosen in Sec. A2."];
```

```
The space between grid points is gridSpacing = 2°. It was chosen in Sec. A2.
```

```
In[68]:= dΘ1 = gridSpacing ;(* another name for grid Spacing in degrees*)
```

```
In[69]:= (*KEEP this cell - DO NOT DELETE*)
(*The Northern Grid "gridN". *)
gridN = {}; idN = 1;

For[gLATj = 0., gLATj < 90. / dΘ1, gLATj ++, gLATpointH = gLATj dΘ1;
 For[ ai = 0., ai < Ceiling[(360. / dΘ1) (cos[gLATpointH] (*+0.01*))],
  ai ++, gLONpointH = ai dΘ1/ (cos[gLATpointH] (*+0.01*));
  AppendTo[gridN, {idN, ai, gLATj, gLONpointH, gLATpointH, er[gLONpointH, gLATpointH]}];
  idN = idN + 1
 ]]
```

In[71]:=
```
(*KEEP this cell - DO NOT DELETE*)
(*The Southern Grid "gridS". *)
gridS = {}; idN = 1;

For[gLATj = 1., gLATj < 90. / dθ1, gLATj ++, gLATpointH = -gLATj dθ1;
 For[ ai = 0., ai < Ceiling[(360. / dθ1) (cos[gLATpointH] (*+0.01*))],
  ai ++, gLONpointH = ai dθ1/ (cos[gLATpointH] (*+0.01*));
  AppendTo[gridS, {idN, ai, gLATj, gLONpointH, gLATpointH, er[gLONpointH, gLATpointH]}];
  idN = idN + 1
 ]]
```

In[73]:=
```
(*KEEP this cell - DO NOT DELETE*)
grid = {}; j = 1;
For[jN = 1, jN ≤ Length[gridN], jN ++, AppendTo[grid, {j, gridN[[jN, 2]], gridN[[jN, 3]],
   gLONFROMr[gridN[[jN, 6]] ], gLATFROMr[gridN[[jN, 6]] ], gridN[[jN, 6]]}];
 j = j + 1]
For[jS = 1, jS ≤ Length[gridS], jS ++, AppendTo[grid, {j, gridS[[jS, 2]], gridS[[jS, 3]],
   gLONFROMr[gridS[[jS, 6]] ], gLATFROMr[gridS[[jS, 6]] ], gridS[[jS, 6]]}];
 j = j + 1]
```

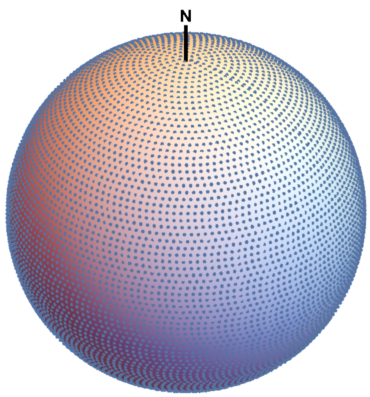
In[76]:=
```
(*For verification of the ranges of the gLON and gLAT for grid points: *)
ListPlot[{Sort[Table[grid[[j, 4]], {j, Length[grid]}]],
   Sort[Table[grid[[j, 5]], {j, Length[grid]}]]}];
```

In[77]:=
```
nGrid = Length[grid];
rHj[j_] := rHj[j] = rGrid[[j]] (*unit radial vector to H*)
gLONGrid = Table[grid[[j, 4]] , {j, Length[grid]}];
gLATGrid = Table[grid[[j, 5]] , {j, Length[grid]}];
rGrid = Table[grid[[j, 6]]  , {j, Length[grid]}];
```

In[82]:=
```
vHij[i_, j_] := vHij[i, j] = (rHj[j] - (rHj[j].ri[i]) ri[i]) /
   (√ ((rHj[j] - (rHj[j].ri[i]) ri[i]).(rHj[j] - (rHj[j].ri[i]) ri[i])))
(*unit vector tangent to the great circle connecting the ith source
 with Hj in tangent space of the ith source*)
nSxHij[i_, j_] := nSxHij[i, j] =
  Cross[ri[i], rHj[j]] / (√ ((Cross[ri[i], rHj[j]]).(Cross[ri[i], rHj[j]])))
(*unit vector perpendicular to the plane of the great circle containing the ith source and Hj*)
ηiHj[i_, j_] := ηiHj[i, j] = arccos[Abs[nSxψi[i].nSxHij[i, j]]]
```

In[85]:=
```
(*Plot the grid points*)
Show[{Graphics3D[{Sphere[{0, 0, 0}], Thick, Line[{{0, 0, -1.2}, {0, 0, 1.2}}],
    Text[Style["N", Bold], {0, 0, 1.25}]}, Boxed → False], ListPointPlot3D[
   Table[rHj[j], {j, nGrid}], PlotStyle → {PointSize[0.007]}]}, ImageSize → 72 × 4]
Print["Figure A2. The grid. There are ", nGrid, " grid points."]
```

Out[85]=




```
Figure A2. The grid. There are 10356 grid points.
```

A6. Setting up circular regions to analyze

In this section, we (a) Collect the sources in circular regions centered on the grid points, (b) drop regions with too few sources, (c) remove duplicates, keeping just one region when more than one region have the same sources.

For (b), the significance estimates are not considered valid with fewer than 7 sources in a region. The problem is common to all probability distributions, like Gaussians, that assign nonzero probability to non-viable angles. For us, samples with fewer than 7 sources, the probability of negative alignment angles is unacceptably high. See Ref. ### for more discussion.

We collect the 5830 sources in 5° radius overlapping circular regions each centered on a grid point. Initially there are 10356 regions, as many regions as there are grid points. However, the Hub Test statistics requires 7 or more sources in a sample, so underpopulated regions are ignored. That reduces the number of regions to 3558 sufficiently populated regions.

The average number of sources in a region is 28, an arithmetic mean. There are just as many regions that contain more than 8 sources as there are that contain fewer. So eight is the median number of sources. There are 284 regions with exactly 7 sources and just one region with the most sources, 314.

Definitions:

rgnPOPnumMIN = minimum number of sources needed for a region to be included in the analysis, “POP” - population
rgnCntrAndSrcId0 A region for every grid point is included.
rgnCntrAndSrcId2 This table has only sufficiently populated regions, $N \geq$ rgnPOPnumMIN (minimum)
sortRgnNsrc sort number of sources in each region
1. region ID 2. number of sources, smallest # first
NsrcMIN Fewest number of sources in any regions (must be at least rgnPOPnumMIN)
NsrcMAX Maximum number of sources in any region
rgnIDsWithnSrc[n]Id #s in rgnCntrAndSrcId2 of regions with exactly “n” sources
duplicatk Pairs of regions {k1,k2} that contain the same sources
dropDupk list of the second region in each pair in duplicatk. These are to be dropped.
rgnCntrAndSrcId No two regions have the same sources. Each region has at least the minimum number of sources.
1. sequential Id# 2. grid ID for region’s center point 3. Source data00 id#s for sources contained in the region
nSrck[k] number of sources in the kth region
nSrcTable list of the number of sources in the regions
srcIDrgnk[k] list of data00 ID #s for sources in the kth region
gLONSrcRgnk[k] galactic longitudes of the sources in the kth region
gLATSrcRgnk[k] gal. latitudes of the sources in the kth region
xyHSrcRgnk[k] Aitoff plot coordinates for the sources in the kth region
rAVEk[k] unit radial vector to average location of the sources in the kth region
gLONAVEk[k] Longitude at the average location of the sources in the kth region
gLATAVEk[k] Latitude at the average location of the sources in the kth region
ρSrcToAVEk[k] angles between the sources and the average location in the kth region
ρRMSk[k] root mean square radius of the kth region
ηBarHkj(k,j) average alignment angle at Hj for the sources in the kth region

kNjηMinjηMax ***angles in degrees***
1. region ID# in “rgnCntrAndSrcId” table 2. N = number of sources in the region 3. {j, ηmin} : j = grid point ID# where $\bar{\eta}$ is minimum ηmin 4. {j, ηmax} : j = grid point ID# where $\bar{\eta}$ is maximum ηmax

ηmink[k] In degrees. The smallest alignment angle ηmin determines how well the sources in the kth region align with any point Hj on the grid, i.e. anywhere on the sphere.
ηmaxk[k] In degrees. The largest avoidance angle ηmax gives a measure of avoidance from any point Hj on the sphere

```
In[87]:= rgnPOPnumMIN = 7. (*minimum number of sources*);

In[88]:= (*Identify sources in each region whose center is on the grid. Collect results. *)
rgnCntrAndSrcId0 = {};
For[j = 1, j ≤ Length[grid], j++, rgnCntr = grid[[j, 6]];
 ρrgn = rgnRadius;  (* region radius in degrees*)
 rgnSrcId = {};
 For[i = 1, i ≤ Length[data00], i++, If[ri[i].rgnCntr ≥ cos[ρrgn], AppendTo[rgnSrcId, i]]];
 AppendTo[rgnCntrAndSrcId0, {j, rgnSrcId}]  ]

In[90]:= Print["Initially, there are ",
  Length[rgnCntrAndSrcId0], " regions, one for each grid point."]

Initially, there are 10356 regions, one for each grid point.
```

In[91]:=
```
(*Get a table with only sufficiently populous regions.*)
rgnCntrAndSrcId2 = {};
j = 0;
For[igrid = 1, igrid ≤ Length[grid], igrid++,
 If[Length[ rgnCntrAndSrcId0[[igrid, 2]] ] ≥ rgnPOPnumMIN, (j = j + 1;
   AppendTo[rgnCntrAndSrcId2, {j, rgnCntrAndSrcId0[[igrid, 1]], rgnCntrAndSrcId0[[igrid, 2]]}])]]
```

In[94]:=
```
Print["There are ", Length[rgnCntrAndSrcId2], " regions with 7 or more stars."]
```

```
There are 3664 regions with 7 or more stars.
```

In[95]:=
```
sortRgnNsrc = Sort[Table[{k, Length[ rgnCntrAndSrcId2[[k, 3]] ]},
     {k, Length[rgnCntrAndSrcId2]}], #1[[2]] < #2[[2]] &];
{sortRgnNsrc[[1]], sortRgnNsrc[[-1]]};
NsrcMIN = sortRgnNsrc[[1, 2]];
NsrcMAX = sortRgnNsrc[[-1, 2]];
```

In[99]:=
```
For[n = 1, n ≤ 2 NsrcMAX, n++, rgnIDsWithnSrc0[n] = {}]
rgnIDsWithnSrc0[NsrcMAX];
(*Collect the IDs*)
For[k = 1, k <= Length[rgnCntrAndSrcId2], k++,
 AppendTo[rgnIDsWithnSrc0[ Length[rgnCntrAndSrcId2[[k, 3]]] ], k]]
```

In[102]:=
```
duplicatk = {};
For[n = NsrcMIN, n ≤ NsrcMAX, n++,  For[k1 = 1, k1 ≤ Length[rgnIDsWithnSrc0[n]] - 1,
  k1++ ,  For[k3 = k1 + 1, k3 ≤ Length[rgnIDsWithnSrc0[n]], k3++,
   If[ Length[Union[rgnCntrAndSrcId2[[ rgnIDsWithnSrc0[n][[k1]], 3 ]] -
        rgnCntrAndSrcId2[[rgnIDsWithnSrc0[n][[k3]], 3]]]] == 1, AppendTo[duplicatk,
      {rgnIDsWithnSrc0[n][[k1]], rgnIDsWithnSrc0[n][[k3]]}]    ]          ]       ]          ]
```

In[103]:=
```
Print["For example, region ", duplicatk[[2, 1]], " and region ",
 duplicatk[[2, 2]], " have the same sources. The stars' data00 IDs are ",
 rgnCntrAndSrcId2[[ duplicatk[[2, 1]], 3 ]], " and ",
 rgnCntrAndSrcId2[[ duplicatk[[2, 2]], 3 ]], "." ]
```

```
For example, region 846 and region 880 have the same sources. The stars' data00 IDs are
 {1952, 1956, 2027, 2032, 2072, 2098, 2111} and {1952, 1956, 2027, 2032, 2072, 2098, 2111}.
```

In[104]:=
```
(*Get the second region in each pair in duplicatk. These will be dropped. *)
dropDupk = Union[Table[duplicatk[[d2, 2]], {d2, Length[duplicatk]}]];
```

In[105]:=
```
Print["We drop ", Length[dropDupk], " regions for having duplicate populations, leaving ",
 Length[rgnCntrAndSrcId2] - Length[dropDupk], " regions."]
```

```
We drop 106 regions for having duplicate populations, leaving 3558 regions.
```

Remove duplicate populations.

In[106]:=
```
rgnCntrAndSrcId = {}; k = 1;
For[ ka = 1, ka ≤ Length[rgnCntrAndSrcId2], ka++,   If[ Not[MemberQ[dropDupk, ka]] ,
  (AppendTo[ rgnCntrAndSrcId, {k, rgnCntrAndSrcId2[[ka, 2]], rgnCntrAndSrcId2[[ka, 3]]}];
   k = k + 1)          ]                    ]
```

In[108]:=
```
For[n = 1, n ≤ 2 NsrcMAX, n++, rgnIDsWithnSrc[n] = {}]
rgnIDsWithnSrc[NsrcMAX];
(*Collect the IDs*)
For[k = 1, k <= Length[rgnCntrAndSrcId], k++,
 AppendTo[rgnIDsWithnSrc[ Length[rgnCntrAndSrcId[[k, 3]]] ], k]]
```

In[111]:=
```
ListPlot[Table[{n, Length[rgnIDsWithnSrc[n] ]}, {n, 1, NsrcMAX + 5}],
 PlotRange → {{0, NsrcMAX + 5}, All}, PlotLabel → "Number of regions with n sources ",
 GridLines → Automatic, Frame → True, FrameLabel → {"n", "Number"}, ImageSize → 72 × 4]
Print["Figure A3. There are ", Length[rgnIDsWithnSrc[NsrcMIN]],
 " regions with ", NsrcMIN, " sources, the minimum number. There is(are) ",
 Length[rgnIDsWithnSrc[NsrcMAX]], " region(s) with the maximum number of sources, ",
 NsrcMAX, ". There are a total of ", Length[ rgnCntrAndSrcId], " regions."]
```

Out[111]=

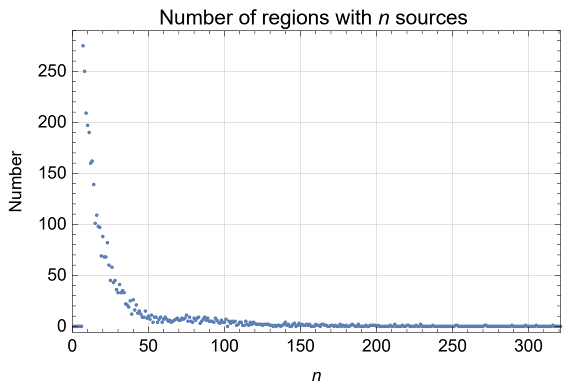


Figure A3. There are 275 regions with 7 sources, the minimum number. There is(are) 1 region(s) with the maximum number of sources, 316. There are a total of 3558 regions.

In[113]:=
```
nSrck[k_] := nSrck[k] = Length[ rgnCntrAndSrcId[[k, 3]] ]
(*number of sources in the kth region*)
nSrcTable = Sort[Table[nSrck[k], {k, Length[ rgnCntrAndSrcId]}]];
srcIDrgnk[k_] := srcIDrgnk[k] = rgnCntrAndSrcId[[k, 3]]
(* data00 id numbers of the sources in the kth region*)
gLONSrcRgnk[k_] :=
  gLONSrcRgnk[k] = Table[data00[[  id08, 2  ]] * (360. / (2. π)), {id08, srcIDrgnk[k]}];
(* RAs in degrees for the sources in the kth region*)
gLATSrcRgnk[k_] := gLATSrcRgnk[k] =
   Table[data00[[  id08 , 3  ]] * (360. / (2. π)), {id08, srcIDrgnk[k]}]; (* decs  *)
xyHSrcRgnk[k_] := xyHSrcRgnk[k] = Table[{  xHGal[ gLONSrcRgnk[k][[i]], gLATSrcRgnk[k][[i]] ],
    yHGal[ gLONSrcRgnk[k][[i]], gLATSrcRgnk[k][[i]] ] }, {i, Length[ gLONSrcRgnk[k] ] } ]
  (*Aitoff coordinates for the locations of the sources in the kth region*)
```

In[119]:=
```
rAVEk0[k_] := rAVEk0[k] =
  Sum[ri[ rgnCntrAndSrcId[[k, 3, n1]] ], {n1, Length[ rgnCntrAndSrcId[[k, 3]] ]}] / nSrck[k]
rAVEk[k_] := rAVEk[k] = rAVEk0[k] / (rAVEk0[k].rAVEk0[k])^(1/2)
gLONAVEk[k_] := gLONAVEk[k] = gLONFROMr[rAVEk[k]]
gLATAVEk[k_] := gLATAVEk[k] = gLATFROMr[rAVEk[k]]
```

In[123]:=
```
(*We need the RMS radius of the k^th region to determine significances.*)
ρSrcToAVEk[k_] :=
 ρSrcToAVEk[k] = Table[arccos[ ri[ rgnCntrAndSrcId[[k, 3, n1]] ].rAVEk[k] ],
   {n1, Length[ rgnCntrAndSrcId[[k, 3]] ]}]
ρRMSk[k_] := ρRMSk[k] = ((1 / Length[rgnCntrAndSrcId[[k, 3]] ])
      Sum[ρSrcToAVEk[k][[i]]^2, {i, Length[ ρSrcToAVEk[k] ]}])^(1/2)
```

In[125]:=
```
(*ηBarHkj: average alignment angle at Hj for the sources in the kth region, Eq. (1).*)
ηBarHkj[k_, j_] :=
 ηBarHkj[k, j] = Sum[ηiHj[i, j], {i, srcIDrgnk[k]}] / Length[srcIDrgnk[k]]
```

kNjηMinjηMax ***angles in degrees***

1. region ID# in "rgnCntrAndSrcId" table 2. N = number of sources in the region 3. {j, ηmin} : j = grid point ID# where $\overline{\eta}$ is minimum ηmin 4. {j, ηmax} : j = grid point ID# where $\overline{\eta}$ is maximum ηmax

The following cell has the kNjηMinjηMax table. It is very large and, therefore, it is hidden from view. To see it go to "Cell Properties" and click "Open".

In[127]:=
```
(*KEEP THIS CELL to generate the kNjηMinjηMax table.*)
(*t1=TimeUsed[]
  kNjηMinjηMax={};
For[k=1,k≤Length[rgnCntrAndSrcId],k++,ηBark=Table[{j,ηBarHkj[k,j]},{j,Length[grid]}];
   sortηBark=Sort[ηBark,#1[[2]]<#2[[2]]&];
   jηMin=sortηBark[[1]];
   jηMax=sortηBark[[-1]];
   AppendTo[kNjηMinjηMax,{k,nSrck[k],jηMin,jηMax}]]
  t2=TimeUsed[]
   t2-t1*)
(*This cell takes some time. On Feb. 11, 2022, it took 5648.547` seconds.*)
```

In[128]:=
```
(*Save kNjηMinjηMax*)
(*SetDirectory[homeDirectory]
 Put[kNjηMinjηMax,"20220211kNjEtaMinjEtaMaxHBstars.dat"]
*)
```

In[129]:=
```
(*Get kNjηMinjηMax, if you've got it.*)
(*SetDirectory[homeDirectory] ;
kNjηMinjηMax=Get["20220211kNjEtaMinjEtaMaxHBstars.dat"];*)
```

In[130]:=
```
Print["The number of regions in the table kNjηMinjηMax is ", Length[kNjηMinjηMax],
 ", which should be the same as the number in rgnCntrAndSrcId, i.e. ",
 Length[rgnCntrAndSrcId], "."]
```

```
The number of regions in the table kNjηMinjηMax is 3558
 , which should be the same as the number in rgnCntrAndSrcId, i.e. 3558.
```

```
In[131]:= kNjηMinjηMax[[1]];
Table[kNjηMinjηMax[[i]], {i, 10}];
Table[kNjηMinjηMax[[i, 3, 2]], {i, 10}];

In[134]:= ηmink[k_] := ηmink[k] = kNjηMinjηMax[[k, 3, 2]]
(*In degrees. The smallest alignment angle ηmin determines how well the sources in
 the kth region align with any point Hj on the grid, i.e. anywhere on the sphere.*)
ηmaxk[k_] := ηmaxk[k] = kNjηMinjηMax[[k, 4, 2]] (*In degrees. The largest avoidance
   angle ηmax gives a measure of avoidance from any point Hj on the sphere*)

In[136]:= Print["Initially, a total of ", nGrid,
  " regions are created, each centered on one of the ", nGrid, " grid points which are ",
  dθ1, " degrees apart. The regions are circular, each with a radius of ",
  rgnRadius, " degrees."]
Print["Regions with duplicate lists of sources are dropped. Regions must
   have a minimum number of sources, at least ", rgnPOPnumMIN, " sources."]
Print["There are ", Length[rgnCntrAndSrcId],
  " regions with sufficient populations after the duplicates are dropped."]
```

```
Initially, a total of 10 356
  regions are created, each centered on one of the 10 356 grid points which are
 2 degrees apart. The regions are circular, each with a radius of 5. degrees.

Regions with duplicate lists of sources are dropped.
   Regions must have a minimum number of sources, at least 7. sources.

There are 3558 regions with sufficient populations after the duplicates are dropped.
```

A7. Probability Distributions and Significance of the Regions

The problem of "significance" is to determine the likelihood that random polarization directions would have better alignment or avoidance than the observed polarization directions. Suppose we are given a region with a smallest alignment angle $\overline{\eta}_{\min}$ and a largest avoidance angle $\overline{\eta}_{\max}$. The most reliable method of finding the significance of either value is to creates many copies of the region but assign the sources randomly directed polarizations. Collect the angles $\overline{\eta}_{\min}$ and $\overline{\eta}_{\max}$ for each randomly directed copy and make a probability distribution for the collection of $\overline{\eta}_{\min}$ and a probability distribution for the collection of $\overline{\eta}_{\max}$. Fit the distributions with suitable functions and integrate to find the significances. This process is "Direct Method A". It takes a lot of time and effort and would not be practical for a survey with hundreds of regions.

To avoid Direct Method A, we apply a combination of Interpolation Method B and Function Method C. Both are based on a "Library" of random run data. One finds that the probability distributions for smallest alignment angle $\overline{\eta}_{\min}$ with random runs can be fit by a function with just two free parameters, called the location $\eta 0$ of the peak and the half-width σ.

The probability distributions for alignment and avoidance of samples with randomly oriented polarization directions involve two parameters η_0 and σ, one set for alignment, $\eta_0^{\min}$ and $\sigma^{\min}$, and one set for avoidance, $\eta_0^{\max}$ and $\sigma^{\max}$. We choose

$$P_{\min}(\eta) = \left(\frac{\text{norm}}{\sigma\,(2\,\pi)^{1/2}}\right)\left(1 + e^{4\,\frac{(\eta-\eta 0-\sigma)}{\sigma}}\right)^{-1} e^{-\frac{1}{2}\left(\frac{\eta-\eta 0}{\sigma}\right)^2}\,, \qquad \text{(A1)}$$

and

$$P_{\max}(\eta) = \left(\frac{\text{norm}}{\sigma\,(2\,\pi)^{1/2}}\right)\left(1 + e^{-4\,\frac{(\eta-\eta 0+\sigma)}{\sigma}}\right)^{-1} e^{-\frac{1}{2}\left(\frac{\eta-\eta 0}{\sigma}\right)^2}\,, \qquad \text{(A2)}$$

where norm is the normalization constant, norm = 1.22029 , so that the definite integral over all η of the probability distributions is equal to one. For details see Ref. 17.

We assume that just two properties of a region determine its significance, the number of sources nSrc and the root-mean-square distance ρRMS of the sources from their mean location. Thus the parameters are taken to be functions η0(nSrc, ρRMS) and σ(nSrc, ρRMS). The Library has tables of the distribution parameters η0 and σ for many discrete sets of nSrc and ρRMS.

Interpolating the Library data to get η0 and σ is herein called ‘Interpolation Method B’. As an alternative, with some loss of accuracy, the Library data can be fit with suitable functions. Substituting nSrc and ρRMS in those functions to get η0 and σ is called ‘Function Method C’. Again, for details, see Ref. 17.

Definitions:

norm a constant used to normalize the distribution so the integral of probability is 1.
probMIN0, probMAX0 probability distributions for alignment (MIN) and avoidance (MAX), functions of η, η_0, σ

probMIN0[η, η0, σ], probMAX0 probability distributions for probability of η using parameters η_0, σ
signiMIN0[η, η0, σ], signiMAX0 significance of η using given parameters η_0, σ

In[139]:=
```
(* The normalization factor "norm" is needed for the probability density *)
norm = ((1/(2 π)^(1/2)) NIntegrate[(1 + e^(4 (y-1)))^-1 e^(-y^2/2), {y, -∞, ∞}])^-1;
(*, i.e. y = ((η - η0)/σ); dy = dη/σ *)
Print["The normalization would be unity for a Gaussian. With the step function it is ",
 norm, "."]
```

The normalization would be unity for a Gaussian. With the step function it is 1.22029.

In[141]:=
```
probMIN0[η_, η0_, σ_] := probMIN0[η, η0, σ] = (norm/(σ (2 π)^(1/2))) (1 + e^(4 (η-η0-σ)/σ))^-1 e^(-1/2 ((η - η0)/σ)^2)
signiMIN0[η_, η0_, σ_] := signiMIN0[η, η0, σ] = NIntegrate[probMIN0[η1, η0, σ], {η1, -∞, η}]
```

In[143]:=
```
probMAX0[η_, η0_, σ_] := probMAX0[η, η0, σ] = (norm/(σ (2 π)^(1/2))) (1 + e^(-4 (η-η0+σ)/σ))^-1 e^(-1/2 ((η - η0)/σ)^2)
signiMAX0[η_, η0_, σ_] := signiMAX0[η, η0, σ] = NIntegrate[probMAX0[η1, η0, σ], {η1, η, ∞}]
```

In[145]:=
```
Print["The significance signiMIN0[η,η0,σ] is
  the integral of probMIN0, i.e. signiMIN0 = ∫_-∞^η P_MIN^0(η_i) dη_i: "]
Print["The significance signiMAX0[η,η0,σ] is the integral of
  probMAX0, i.e. signiMAX0 = ∫_η^∞ P_MAX^0(η_i) dη_i: " ]
```

The significance signiMIN0[η,η0,σ] is
the integral of probMIN0, i.e. signiMIN0 = $\int_{-\infty}^{\eta} P_{MIN}{}^{0}(\eta_i)\, d\eta_i$:

The significance signiMAX0[η,η0,σ] is the integral of probMAX0, i.e. signiMAX0 = $\int_{\eta}^{\infty} P_{MAX}{}^{0}(\eta_i)\, d\eta_i$:

A7a. The Library data

Definitions:

fitData Parameters of the alignment (min) and avoidance (max) random run distributions. Originally in radians, converted to degrees after it is inputted below.

fitData:

1a. nSrci[i] Number of sources 1b. ρNomi[i] Nominal radius, deg. 1c. ρRMSi[i] RMS radius, deg.

2a. η0mini[i] peak alignment distribution 2b. dη0mini[i] standard error

3a. σmini[i] half-width alignment distr. 3b. dσmini[i] standard error

4a. η0maxi[i] peak alignment distribution 4b. dη0maxi[i] standard error

5a. σmaxi[i] half-width alignment distr. 5b. dσmaxi[i] standard error

wi[i] inverse square root of the number of sources, $w = 1/N^{1/2}$

τRMSi[i] inverse RMS radius, in deg.$^{-1}$

The following cell has the fitData table with the information about the probability distribution parameters. It is very large and, therefore, it is hidden from view. To see it go to "Cell Properties" and click "Open".

```
In[148]:= (*Identify the items in the fitData table with functions
           having recognizable names. Convert fitData in radians to DEGREES:*)
          nSrci[i_] := nSrci[i] = fitData[[i, 1, 1]]

In[149]:= ρNomi[i_] := ρNomi[i] = fitData[[i, 1, 2]] (360. / (2. π))
          (*The nominal radius in degrees*)
          ρRMSi[i_] := ρRMSi[i] = fitData[[i, 1, 3]] (360. / (2. π)) (*The RMS radius *)

In[151]:= η0mini[i_] := η0mini[i] = fitData[[i, 2, 1]] (360. / (2. π))
          dη0mini[i_] := dη0mini[i] = fitData[[i, 2, 2]] (360. / (2. π))

In[153]:= σmini[i_] := σmini[i] = fitData[[i, 3, 1]] (360. / (2. π))
          dσmini[i_] := dσmini[i] = fitData[[i, 3, 2]] (360. / (2. π))

In[155]:= η0maxi[i_] := η0maxi[i] = fitData[[i, 4, 1]] (360. / (2. π))
          dη0maxi[i_] := dη0maxi[i] = fitData[[i, 4, 2]] (360. / (2. π))

In[157]:= σmaxi[i_] := σmaxi[i] = fitData[[i, 5, 1]] (360. / (2. π))
          dσmaxi[i_] := dσmaxi[i] = fitData[[i, 5, 2]] (360. / (2. π))

In[159]:= wi[i_] := wi[i] = 1 / nSrci[i]^(1/2) (*w = 1/N^(1/2)*)
          τRMSi[i_] := τRMSi[i] = 1 / ρRMSi[i] (* inverse RMS radius in inverse degrees*)
```

A7b. Interpolation Method B

The Library introduced for Method B in Sec. A7a is essentially a table of the values of the four parameters, $\eta_0^{\min}$, $\sigma^{\min}$, $\eta_0^{\max}$, and $\sigma^{\max}$, needed to determine the probability distributions and significances in Eqs. (A1-A4).

Instead of the variables N and ρRMS, the number of sources and the root-mean-square radius, we choose to consider the four parameters as functions of w and τRMS, the inverse square root of N and the inverse of the radius ρRMS,

$$w = N^{-1/2} \quad \text{and} \quad \tau\mathrm{RMS} = \rho\mathrm{RMS}^{-1}, \qquad \text{(A5)}$$

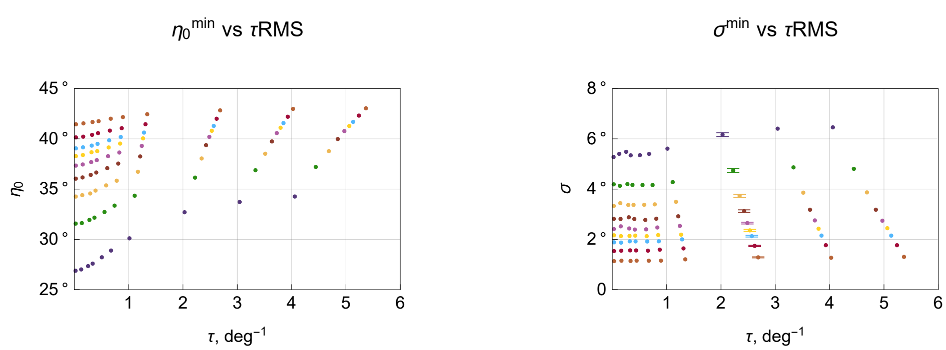
which changes variables from $(N, \rho\mathrm{RMS})$ to $(w, \tau\mathrm{RMS})$.

Definitions:

Tables: w$\tau\eta$0minLib, wτdη0minLib, w$\tau\eta$0maxLib, wτdη0maxLib, w$\tau\sigma$minLib, wτdσminLib, w$\tau\sigma$maxLib, wτdσmaxLib

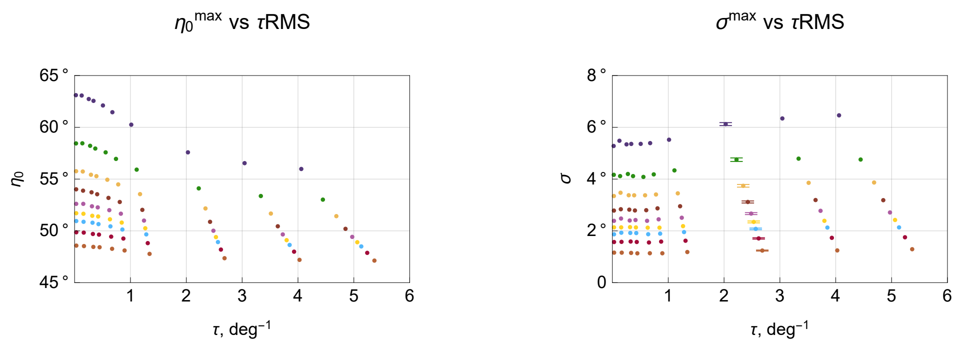
The tables w$\tau\eta$0minLib ... have Library data in the form [(w, τRMS), quantity] where "quantity" is one of the parameters or their standard errors: $\eta_0^{\min}$, d$\eta_0^{\min}$, $\sigma^{\min}$, d$\sigma^{\min}$, $\eta_0^{\max}$, d$\eta_0^{\max}$, $\sigma^{\max}$, d$\sigma^{\max}$

The associated interpolation functions are η0minBint, dη0minBint, η0maxBint, dη0maxBint, σminBint, dσminBint, σmaxBint, dσmaxBint



{N, color} = {{9, ■}, {16, ■}, {25, ■}, {36, ■}, {49, ■}, {64, ■}, {81, ■}, {121, ■}, {225, ■}}

Figure A4. The values of the parameters η_0 and σ in the Library for alignment (min $\overline{\eta}$(H)) distributions Eq. (A1). The functions are represented as functions of $\tau = 1/\rho$RMS, Eq. (A5). The uncertainties d$\eta_0^{\min}$ in the distribution peak values $\eta_0^{\min}$ are too small to plot. The uncertainties d$\sigma^{\min}$ in the half-widths $\sigma^{\min}$ are displayed once for each value of N. Other d$\sigma^{\min}$ are approximately the same as the one displayed.



$\{N, \text{color}\}$ =
{{9, }, {16, }, {25, }, {36, }, {49, }, {64, }, {81, }, {121, }, {225, }}

Figure A5. The values of the parameters η_0 and σ in the Library for avoidance (max $\overline{\eta}$(H)) distributions Eq. (A2), similar to Fig. A4 except here for avoidance. Note that the η_0 for avoidance here and alignment in Fig. A4 are symmetric about 45°. The σ for avoidance agree quite well with the values of σ for alignment.

Setting up the interpolations takes two steps. First a tables of the data are constructed. Each table has the form $\{w, \tau\text{RMS}, \text{parameter}\}$. Second, the interpolation for each parameter is defined. There are four parameters $\eta_0{}^{min}$, σ^{min}, $\eta_0{}^{max}$, and σ^{max} and each one has a standard error $d\eta_0{}^{min}$, $d\sigma^{min}$, $d\eta_0{}^{max}$, and $d\sigma^{max}$ developed in the fitting process that gives fitData from random run data.

In[161]:=
```
wτη0minLib = Table[{{wi[i], τRMSi[i]}, η0mini[i]}, {i, Length[fitData]}];
wτdη0minLib = Table[{wi[i], τRMSi[i], dη0mini[i]}, {i, Length[fitData]}];
```

In[163]:=
```
wτη0maxLib = Table[{wi[i], τRMSi[i], η0maxi[i]}, {i, Length[fitData]}];
wτdη0maxLib = Table[{wi[i], τRMSi[i], dη0maxi[i]}, {i, Length[fitData]}];
```

In[165]:=
```
wτσminLib = Table[{wi[i], τRMSi[i], σmini[i]}, {i, Length[fitData]}];
wτdσminLib = Table[{wi[i], τRMSi[i], dσmini[i]}, {i, Length[fitData]}];
```

In[167]:=
```
wτσmaxLib = Table[{wi[i], τRMSi[i], σmaxi[i]}, {i, Length[fitData]}];
wτdσmaxLib = Table[{wi[i], τRMSi[i], dσmaxi[i]}, {i, Length[fitData]}];
```

In[169]:=
```
η0minBint = Interpolation[wτη0minLib];(* int - interpolation function*)
dη0minBint = Interpolation[wτdη0minLib];
```

Interpolation: Interpolation on unstructured grids is currently only supported for InterpolationOrder->1 or InterpolationOrder->All. Order will be reduced to 1.

Interpolation: Interpolation on unstructured grids is currently only supported for InterpolationOrder->1 or InterpolationOrder->All. Order will be reduced to 1.

In[171]:=
```
η0maxBint = Interpolation[wτη0maxLib];
dη0maxBint = Interpolation[wτdη0maxLib];
```

Interpolation: Interpolation on unstructured grids is currently only supported for InterpolationOrder->1 or InterpolationOrder->All. Order will be reduced to 1.

Interpolation: Interpolation on unstructured grids is currently only supported for InterpolationOrder->1 or InterpolationOrder->All. Order will be reduced to 1.

In[173]:=
```
σminBint = Interpolation[wτσminLib];
dσminBint = Interpolation[wτdσminLib];
```

Interpolation: Interpolation on unstructured grids is currently only supported for InterpolationOrder->1 or InterpolationOrder->All. Order will be reduced to 1.

Interpolation: Interpolation on unstructured grids is currently only supported for InterpolationOrder->1 or InterpolationOrder->All. Order will be reduced to 1.

In[175]:=
```
σmaxBint = Interpolation[wτσmaxLib];
dσmaxBint = Interpolation[wτdσmaxLib];
```

Interpolation: Interpolation on unstructured grids is currently only supported for InterpolationOrder->1 or InterpolationOrder->All. Order will be reduced to 1.

Interpolation: Interpolation on unstructured grids is currently only supported for InterpolationOrder->1 or InterpolationOrder->All. Order will be reduced to 1.

By the rules of interpolations, when the variables w and τ are in the range of the Library data, then Mathematica finds an average value from the surrounding Library data points. In terms of the variables w and τRMS, the ranges are

$$\frac{1}{15} \leq w \leq \frac{1}{3} \quad \text{and} \quad 0.024\ \text{deg}^{-1} \lesssim \tau\text{RMS} \lesssim 4\ \text{deg}^{-1} \qquad \text{(Ranges of Interpolation Variables)} \qquad \text{(A6)}$$

Note: The limiting values shown for τRMS are only approximate because the limits shown are values of $1/\rho$Nominal, not τRMS, and the nominal values ρNominal only approximate the root-mean-square values ρRMS, $\rho\text{Nominal} \approx \rho\text{RMS}$.

A7c. Fit the Library Data with Functions, Function Method C

Applying Interpolation Method B when one or both sample's variables are outside the Library data set, results in extrapolation, not interpolation. Instead of interpolating Library data points that surround the sample's variables, Mathematica guesses what lies beyond the Library's boundaries. In such a case or for other situations that arise, one can apply the following alternative Formula Method C to find the distribution parameters $\eta_0{}^{\text{min}}$, σ^{min}, $\eta_0{}^{\text{max}}$, and σ^{max} and, with Eqs. (A4, A5), the significances desired.

Formula Method C finds formulas to fit the four distribution parameters $\eta_0{}^{\text{min}}(w,\ \tau\text{RMS})$, $\sigma^{\text{min}}(w,\ \tau\text{RMS})$, $\eta_0{}^{\text{max}}(w,\ \tau\text{RMS})$, and $\sigma^{\text{max}}(w,\ \tau\text{RMS})$. Best, Big and Small are just copied from 20211112InterpolateAndFormula2a which are copied from 20211116AlternateRandomRunStatsDegrees.nb

Definitions:

See definitions at start of Sec. A7a for quantities eta0, sigma, ...

Alignment:

Library data fitting functions:

eta0minFit[w,τ], eta0minFitbig[w,τ], eta0minFitsmall[w,τ], deta0minFit[w,τ]

sigmaminFit[w,τ], sigminFitBig[w,τ], sigminFitSmall[w,τ], dsigmaminFit[w,τ]

Plots of the Library data fitting function for the $\mathrm{iN}^{\mathrm{th}}$ value of w:

plotTauEtamin[iN], plotTauEtaminbig[iN], plotTauEtaminsmall[iN],

plotTausigmamin[iN], ...

Display of the fitting functions for all values of w:

eta0MinVSTauFit (Big, Best, Small) and eta0MinVSTauFit0 (Best only)

sortPercentDiffEta0minfit	percent differences between the Library data and the relevant fitting function, here for $\eta_0^{\min}$.
sortPercentDiffSigmaminfit	Same, but for $\sigma^{\min}$

Avoidance:

REPEAT ALL OF THE ABOVE AGAIN, BUT THIS TIME WITH “MAX”, NOT “MIN”, *i.e.* eta0maxFit[w,τ], ..., sortPercentDif-fSigmamaxfit

In[177]:=
```
(*Equation (A7), 20211112InterpolateAndFormula2a.nb  *)
eta0minFit[w_, τ_] := eta0minFit[w, τ] =
  45.0269 - w (47.386 + 7.32 w - 17.789 Tanh[(0.7096 - 0.3488 w) (-0.5348 + τ)])
```

In[178]:=
```
(*Equation (A8)*)eta0minFitbig[w_, τ_] := eta0minFitbig[w, τ] =
  45.0434 - w (47.031 + 6.83 w + (-17.789 + 0.302 Sign[(-0.7096 + 0.3488 w) (-0.5348 + τ)])
       Tanh[(-0.5348 + τ + 0.0254 Sign[0.7096 - 0.3488 w])
          (0.7096 + w (-0.3488 + 0.0321 Sign[-0.5348 + τ]) + 0.0137 Sign[-0.5348 + τ])])
```

In[179]:=
```
(*Equation (A9)*)eta0minFitsmall[w_, τ_] := eta0minFitsmall[w, τ] =
  45.0103 - w (47.741 + 7.81 w + (-17.789 - 0.302 Sign[(-0.7096 + 0.3488 w) (-0.5348 + τ)])
       Tanh[(-0.5348 + τ - 0.0254 Sign[0.7096 - 0.3488 w])
          (0.7096 + w (-0.3488 - 0.0321 Sign[-0.5348 + τ]) - 0.0137 Sign[-0.5348 + τ])])
```

In[180]:=
```
deta0minFit[w_, τ_] := deta0minFit[w, τ] = eta0minFitbig[w, τ] - eta0minFit[w, τ]
```

In[181]:=
```
(*Equation (A10)*)
sigmaminFit[w_, τ_] :=
 sigmaminFit[w, τ] = 0.25 w (73.570 - 8.29 w + (3.093 + 10.658 w) Tanh[1.2161 (-1.6072 + τ)])
```

In[182]:=
```
(*Equation (A11)*)
sigminFitBig[w_, τ_] := sigminFitBig[w, τ] = 0.25 w
   (73.679 - 7.86 w + (3.093 + w (10.658 + 0.508 Sign[-1.6072 + τ]) + 0.126 Sign[-1.6072 + τ])
      Tanh[(-1.6072 + τ + 0.0202 Sign[3.093 + 10.658 w]) (1.2161 + 0.0441 Sign[-1.6072 + τ])])
```

```
In[183]:= (*Equation (A12)*)
sigminFitSmall[w_, τ_] := sigminFitSmall[w, τ] = 0.25 w
    (73.460 - 8.73 w + (3.093 + w (10.658 - 0.508 Sign[-1.6072 + τ]) - 0.126 Sign[-1.6072 + τ])
       Tanh[(-1.6072 + τ - 0.0202 Sign[3.093 + 10.658 w]) (1.2161 - 0.0441 Sign[-1.6072 + τ])])

In[184]:= dsigmaminFit[w_, τ_] := dsigmaminFit[w, τ] = sigminFitBig[w, τ] - sigmaminFit[w, τ]

In[185]:= (*Equation (A13)*)
eta0maxFit[w_, τ_] :=
 eta0maxFit[w, τ] = 45.1455 + w (44.230 + 8.35 w - 14.632 Tanh[0.6808 (-0.8608 + τ)])

In[186]:= (*Equation (A14)*)eta0maxFitbig[w_, τ_] :=
 eta0maxFitbig[w, τ] = 45.1632 + w (44.483 + 8.85 w + (-14.632 + 0.179 Sign[-0.8608 + τ])
         Tanh[(-0.8777 + τ) (0.6808 + 0.0106 Sign[0.8608 - τ])])

In[187]:= (*Equation (A15)*)eta0maxFitsmall[w_, τ_] :=
 eta0maxFitsmall[w, τ] = 45.1279 + w (43.977 + 7.85 w + (-14.632 - 0.179 Sign[-0.8608 + τ])
         Tanh[(-0.8439 + τ) (0.6808 - 0.0106 Sign[0.8608 - τ])])

In[188]:= deta0maxFit[w_, τ_] := deta0maxFit[w, τ] = eta0maxFitbig[w, τ] - eta0maxFit[w, τ]

In[189]:= (*Equation (A16)*)
sigmamaxFit[w_, τ_] :=
 sigmamaxFit[w, τ] = 0.25 w (73.287 - 8.11 w + (2.773 + 11.126 w) Tanh[1.2850 (-1.6242 + τ)])

In[190]:= (*Equation (A17)*)
sigmaxFitBig[w_, τ_] := sigmaxFitBig[w, τ] = 0.25 w
    (73.400 - 7.66 w + (2.773 + w (11.126 + 0.521 Sign[-1.6242 + τ]) + 0.129 Sign[-1.6242 + τ])
       Tanh[(-1.6242 + τ + 0.0210 Sign[2.773 + 11.126 w]) (1.2850 + 0.0494 Sign[-1.6242 + τ])])

In[191]:= (*Equation (A18)*)
sigmaxFitSmall[w_, τ_] := sigmaxFitSmall[w, τ] = 0.25 w
    (73.174 - 8.567 w + (2.773 + w (11.126 - 0.521 Sign[-1.6242 + τ]) - 0.129 Sign[-1.6242 + τ])
       Tanh[(-1.6242 + τ - 0.0210 Sign[2.773 + 11.126 w]) (1.2850 - 0.0494 Sign[-1.6242 + τ])])

In[192]:= dsigmamaxFit[w_, τ_] := dsigmamaxFit[w, τ] = sigmaxFitBig[w, τ] - sigmamaxFit[w, τ]
```

A7d. Combine Interpolation Method B and Function Method C

Apply Interpolation Method B when both sample's variables are within the range of the Library data set. Otherwise, one can apply alternative Formula Method C to find the distribution parameters $\eta_0^{\min}$, $\sigma^{\min}$, $\eta_0^{\max}$, and $\sigma^{\max}$.

Definitions:

Alignment:

Methods B,C combination parameter functions:

η0min[w,τ], dη0min[w,τ] peak alignment distribution, standard error

σmin[w,τ], dσmin[w,τ]   half-width alignment distr., standard error
η0max[w,τ], dη0max[w,τ]   peak avoidance distribution, standard error
σmax[w,τ], dσmax[w,τ]   half-width avoidance distr., standard error

probMIN[η, nSrc, ρRMS], probMAX   probability distributions for probability of η using a sample's values of *N*, ρRMS

signiMIN[η,nSrc,ρRMS], signiMAX   significance of η using a sample's values of *N*, ρRMS

```
In[193]:= η0min[w_, τ_] :=
   If[(1/15. ≤ w ≤ 1/3.) && (0.025 ≤ τ ≤ 3.9), η0minBint[w, τ], eta0minFit[w, τ]];
  dη0min[w_, τ_] := If[(1/15. ≤ w ≤ 1/3.) && (0.025 ≤ τ ≤ 3.9),
    dη0minBint[w, τ], deta0minFit[w, τ]];
  η0max[w_, τ_] := If[(1/15. ≤ w ≤ 1/3.) && (0.025 ≤ τ ≤ 3.9),
    η0maxBint[w, τ], eta0maxFit[w, τ]];
  dη0max[w_, τ_] := If[(1/15. ≤ w ≤ 1/3.) && (0.025 ≤ τ ≤ 3.9),
    dη0maxBint[w, τ], deta0maxFit[w, τ]];
  σmin[w_, τ_] := If[(1/15. ≤ w ≤ 1/3.) && (0.025 ≤ τ ≤ 3.9),
    σminBint[w, τ], sigmaminFit[w, τ]];
  dσmin[w_, τ_] := If[(1/15. ≤ w ≤ 1/3.) && (0.025 ≤ τ ≤ 3.9),
    dσminBint[w, τ], dsigmaminFit[w, τ]];
  σmax[w_, τ_] := If[(1/15. ≤ w ≤ 1/3.) && (0.025 ≤ τ ≤ 3.9),
    σmaxBint[w, τ], sigmamaxFit[w, τ]];
  dσmax[w_, τ_] := If[(1/15. ≤ w ≤ 1/3.) && (0.025 ≤ τ ≤ 3.9),
    dσmaxBint[w, τ], dsigmamaxFit[w, τ]];

In[201]:= probMIN[η_, nSrc_, ρRMS_] :=
   probMIN[η, nSrc, ρRMS] = probMIN0[η, η0min[nSrc^(-1/2), ρRMS^-1], σmin[nSrc^(-1/2), ρRMS^-1]]
  signiMIN[η_, nSrc_, ρRMS_] := signiMIN[η, nSrc, ρRMS] =
   signiMIN0[η, η0min[nSrc^(-1/2), ρRMS^-1], σmin[nSrc^(-1/2), ρRMS^-1]]
  probMAX[η_, nSrc_, ρRMS_] := probMAX[η, nSrc, ρRMS] =
   probMAX0[η, η0max[nSrc^(-1/2), ρRMS^-1], σmax[nSrc^(-1/2), ρRMS^-1]]
  signiMAX[η_, nSrc_, ρRMS_] := signiMAX[η, nSrc, ρRMS] =
   signiMAX0[η, η0max[nSrc^(-1/2), ρRMS^-1], σmax[nSrc^(-1/2), ρRMS^-1]]
```

A7e. Significance of alignment and avoidance for the regions

To get significance formulas for each region, we use the number of sources and the ρRMS for each region, *i.e.* nSrck[k] and ρRMSk[k] for the k^{th}region.

Definitions:

sigMINk[k]   significance of η0min for the kth region in kNjηMinjηMax sigMAXk[k]   significance of η0max for the kth region in kNjηMinjηMax

ηMINVerySigkList   k for Regions with very significant alignment
sortηMINVerySigkList   sort ηMINVerySigkList, smallest sig. p first (smallest p is most significant)

ηMAXVerySigkList   k for Regions with very significant avoidance
sortηMAXVerySigkList   sort ηMAXVerySigkList, smallest sig. p first (smallest p is most significant)

negLogSigηminBest[k] $a = -\log_{10} p$ exponent a in significance $p = 10^{-a}$, for the kth region in kNjηMinjηMax, alignment
negLogSigηminBig[k], negLogSigηminSmall[k]   one σ range of a
...First2   Best 2 significance regions for alignment

negLogSigηmaxBest[k] $a = -\log_{10} p$ exponent a in significance $p = 10^{-a}$, for the kth region in kNjηMinjηMax, avoidance
negLogSigηmaxBig[k], negLogSigηmaxSmall[k]   one σ range of a
...First2   Best 2 significance regions for avoidance

In[205]:=
```
sigMINk[k_] := sigMINk[k] = signiMIN[ ηmink[k], nSrck[k], ρRMSk[k]  ]
sigMAXk[k_] := sigMAXk[k] = signiMAX[ηmaxk[k], nSrck[k], ρRMSk[k] ]
```

In[207]:=
```
ηMINVerySigkList = {};
For[k = 1, k ≤ Length[kNjηMinjηMax], k++,
 If[0.01 ≥ sigMINk[k], AppendTo[ηMINVerySigkList, {k, sigMINk[k]}]]]
ηMINVerySigkList;
Length[ηMINVerySigkList];
```

In[211]:=
```
sortηMINVerySigkList = Sort[ηMINVerySigkList, #1[[2]] < #2[[2]] &];
Table[sortηMINVerySigkList[[i]], {i, 10}];
Print["The number of very significantly aligned regions is ",
 Length[sortηMINVerySigkList], "."]
```

```
The number of very significantly aligned regions is 2806.
```

In[214]:=
```
(*Get the ID#s k for Regions with very significant avoidance.*)
ηMAXVerySigkList = {};
For[k = 1, k ≤ Length[kNjηMinjηMax], k++,
 If[0.01 ≥ sigMAXk[k], AppendTo[ηMAXVerySigkList, {k, sigMAXk[k]}]]]
ηMAXVerySigkList;
Length[ηMAXVerySigkList];
```

In[218]:=
```
sortηMAXVerySigkList = Sort[ηMAXVerySigkList, #1[[2]] < #2[[2]] &];
Table[sortηMAXVerySigkList[[i]], {i, 10}];
```

In[220]:=
```
Print["The number of regions with very significant avoidance is ",
 Length[sortηMAXVerySigkList], "."]
```

```
The number of regions with very significant avoidance is 2805.
```

In[221]:=
```
(*Uncertainty of a =
 -Log10p for ηmink with uncertainty σηmink = ±1° . Note p = 10^-a. *)
negLogSigηminBest[k_] := negLogSigηminBest[k] = -Log[10,
   signiMIN0[ ηmink[k], η0min[nSrck[k]^-1/2, ρRMSk[k]^-1], σmin[nSrck[k]^-1/2, ρRMSk[k]^-1]]]
```

In[222]:=
```
negLogSigηminBig[k_] := negLogSigηminBig[k] =
  -Log[10, signiMIN0[ ηmink[k] - (1), η0min[nSrck[k]^-1/2, ρRMSk[k]^-1] + dη0min[nSrck[k]^-1/2,
      ρRMSk[k]^-1], σmin[nSrck[k]^-1/2, ρRMSk[k]^-1] -  dσmin[nSrck[k]^-1/2, ρRMSk[k]^-1]]]
```

In[223]:=
```
negLogSigηminSmall[k_] := negLogSigηminSmall[k] =
  -Log[10, signiMIN0[ ηmink[k] + (1), η0min[nSrck[k]^-1/2, ρRMSk[k]^-1] - dη0min[nSrck[k]^-1/2,
      ρRMSk[k]^-1], σmin[nSrck[k]^-1/2, ρRMSk[k]^-1] +  dσmin[nSrck[k]^-1/2, ρRMSk[k]^-1]]]
```

In[224]:=
```
(*Uncertainty of -Log10p for ηmaxk with uncertainty σηmaxk = ±1° . Note p = 10^-a. *)
negLogSigηmaxBest[k_] := negLogSigηmaxBest[k] = -Log[10,
   signiMAX0[ ηmaxk[k], η0max[nSrck[k]^-1/2, ρRMSk[k]^-1], σmax[nSrck[k]^-1/2, ρRMSk[k]^-1]]]
negLogSigηmaxBig[k_] := negLogSigηmaxBig[k] = -Log[10, signiMAX0[ ηmaxk[k] + (1),
    η0max[nSrck[k]^-1/2, ρRMSk[k]^-1] - dη0max[nSrck[k]^-1/2, ρRMSk[k]^-1],
    σmax[nSrck[k]^-1/2, ρRMSk[k]^-1] -  dσmax[nSrck[k]^-1/2, ρRMSk[k]^-1]]]
negLogSigηmaxSmall[k_] := negLogSigηmaxSmall[k] = -Log[10, signiMAX0[ ηmaxk[k] - (1),
    η0max[nSrck[k]^-1/2, ρRMSk[k]^-1] + dη0max[nSrck[k]^-1/2, ρRMSk[k]^-1],
    σmax[nSrck[k]^-1/2, ρRMSk[k]^-1] +  dσmax[nSrck[k]^-1/2, ρRMSk[k]^-1]]]
```

In[227]:=
```
negLogVerySigηminFirst2 =
  Table[Around[negLogSigηminBest[k], {negLogSigηminBest[k] - negLogSigηminSmall[k],
     negLogSigηminBig[k] - negLogSigηminBest[k]}],
   {k, Table[sortηMINVerySigkList[[i, 1]], {i, 2}]}];
```

In[228]:=
```
negLogVerySigηminLast2 =
  Table[Around[negLogSigηminBest[k], {negLogSigηminBest[k] - negLogSigηminSmall[k],
     negLogSigηminBig[k] - negLogSigηminBest[k]}],
   {k, Table[sortηMINVerySigkList[[-i, 1]], {i, 2}]}];
```

Place Mark A: For the 2 most significantly aligned regions, $p = 10^{-a} \leqslant 10^{-2}$, the $a = -\mathrm{Log}_{10}p$ values are $\{294.^{+20.}_{-20.}, 285.^{+20.}_{-19.}\}$.

For the last 2 very significantly aligned regions, $p \leqslant 10^{-2}$, the $-\mathrm{Log}_{10}p$ values are $\{2.00^{+0.26}_{-0.24}, 2.01^{+0.31}_{-0.28}\}$.

In[231]:=
```
negLogVerySigηmaxFirst2 =
  Table[Around[negLogSigηmaxBest[k], {negLogSigηmaxBest[k] - negLogSigηmaxSmall[k],
     negLogSigηmaxBig[k] - negLogSigηmaxBest[k]}],
   {k, Table[sortηMAXVerySigkList[[i, 1]], {i, 2}]}];
```

In[232]:=
```
negLogVerySigηmaxLast2 =
  Table[Around[negLogSigηmaxBest[k], {negLogSigηmaxBest[k] - negLogSigηmaxSmall[k],
     negLogSigηmaxBig[k] - negLogSigηmaxBest[k]}],
   {k, Table[sortηMAXVerySigkList[[-i, 1]], {i, 2}]}];
```

For the first 2 regions with very significant avoidance, $p \leqslant 10^{-2}$, the $-\mathrm{Log}_{10}p$ values are $\{297.^{+20.}_{-20.}, 287.^{+20.}_{-19.}\}$.

For the last 2 regions with very significant avoidance, $p \leqslant 10^{-2}$, the $-\mathrm{Log}_{10}p$ values are $\{2.00^{+0.4}_{-0.34}, 2.00^{+0.4}_{-0.32}\}$.

In[235]:=
```
negLogVerySigηmax =
  Table[Around[negLogSigηmaxBest[k], {negLogSigηmaxBest[k] - negLogSigηmaxSmall[k],
     negLogSigηmaxBig[k] - negLogSigηmaxBest[k]}],
   {k, Table[sortηMAXVerySigkList[[i, 1]], {i, Length[sortηMAXVerySigkList]}]}];
```

For the regions with very significant avoidance, $S = p \leqslant 10^{-2}$, some of the $-\mathrm{Log}_{10}S$ values are

Out[237]= $\{297.^{+20.}_{-20.}, 287.^{+20.}_{-19.}, 281.^{+19.}_{-18.}, 267.^{+19.}_{-18.}, 253.^{+18.}_{-17.}, 243.^{+17.}_{-16.}, 228.^{+16.}_{-15.}, 226.^{+16.}_{-15.}, 222.^{+15.}_{-15.}, 213.^{+15.}_{-14.}, 201.^{+14.}_{-14.}, 201.^{+15.}_{-14.}, 193.^{+14.}_{-13.}, 186.^{+14.}_{-13.}, 178.^{+13.}_{-12.}, 176.^{+13.}_{-13.}, 174.^{+13.}_{-12.}, 170.^{+13.}_{-12.}, 169.^{+13.}_{-12.}, 169.^{+12.}_{-12.}\}$

In[238]:=
```
(*lpNegLogVerySigAlign=ListPlot[negLogVerySigηmin,
   PlotRange→{{0,20},{0,5.5}},PlotLabel→" -Log10p, Alignment  ",
   GridLines→Automatic,Frame→True,FrameLabel→{"Rank","-Logp"},ImageSize→72 4];*)
```

In[239]:=
```
(*lpNegLogVerySigAvoid=
  ListPlot[negLogVerySigηmax,PlotRange→All,PlotLabel→" -Log10p, Avoidance  ",
   GridLines→Automatic,Frame→True,FrameLabel→{"Rank","-Logp"},ImageSize→72 4];*)
```

In[240]:=
```
(*GraphicsRow[{lpNegLogVerySigAlign,lpNegLogVerySigAvoid}]
 Print[
  "Figure Not Numbered. The negative log of the significance p for regions with very
    significant alignment (left) and avoidance (right). The most significant region
    has rank 1, the next most significant has rank 2, etc. Most of the uncertainty
    is due to the experimental uncertainty in the polarization position angles ψ."]*)
```

In[241]:=
```
Print["The number of regions with very significant alignment is ",
 Length[ηMINVerySigkList], " regions, i.e. S = p ≤ 10^-2 = 0.01 ."]
Print["The number of regions with very significant avoidance is ",
 Length[ηMAXVerySigkList], " regions, i.e. S = p ≤ 10^-2 = 0.01 ."]
Print[""]
Print["The region with the most significant alignment is region number ",
 sortηMINVerySigkList[[1, 1]], ", which has S = p = ", sortηMINVerySigkList[[1, 2]], "."]
Print["The region with the most significant avoidance is region number ",
 sortηMAXVerySigkList[[1, 1]], ", which has S = p = ", sortηMAXVerySigkList[[1, 2]], "."]
```

The number of regions with very significant alignment is 2806 regions, i.e. $S = p \leq 10^{-2} = 0.01$.

The number of regions with very significant avoidance is 2805 regions, i.e. $S = p \leq 10^{-2} = 0.01$.

The region with the most significant alignment is region number 2385, which has $S = p = 7.29568\times10^{-295}$.

The region with the most significant avoidance is region number 2385, which has $S = p = 7.18621\times10^{-298}$.

A8. Mapping the significance of the regions

Definitions:

allMinIDs ID k for very sig. aligned regions, more significant first
allMaxIDs ID k for very sig. avoidance regions, more significant first

ALIGNMENT:
gLONj, gLATj longitude and latitude of the regions in allMinIDs
gLONHgLATHlogSigForAllMin { x,y, a } for regions in allMinIDs, where x,y are Aitoff coordinates and $a = -\log_{10} p$, is the exponent defined previously
plotxyHGalLogSig0, plotxyHGalLogSig Aitoff plot of {x,y,a}

meridianPoints, parallelsPoints auxiliary, for plotting constant lat and long

AVOIDANCE: add “Max” to the following quantities

gLONj, gLATj longitude and latitude of the regions in allMinIDs
gLONHgLATHlogSigForAllMin { x,y, a } for regions in allMinIDs, where x,y are Aitoff coordinates and $a = -\log_{10} p$, is the exponent defined previously
plotxyHGalLogSig0, plotxyHGalLogSig Aitoff plot of {x,y,a}

ALIGNMENT:
gLONgLATlogSigForAllMin { long., lat., a } for regions in allMinIDs
sortgLONgLATlogSigForAllMin as above, largest a first
lp1, lp2 Aitoff plot of $a = -\log_{10} p$, color coded a

AVOIDANCE:
gLONgLATlogSigForAllMax { long., lat., a } for regions in allMaxIDs
sortgLONgLATlogSigForAllMax as above, largest a first
lp1max, lp2max Aitoff plot of $a = -\log_{10} p$, color coded a

ψGAL0To180i[i] polarization directions ψ as given in data00, 0<ψ<180°
ψGAL90To270i[i] polarization directions ψ, but with 90°<ψ<270°

meanψ0To180k[k] average ψ for kth region in kNjηMinjηMax, standard range for ψ, 0<ψ<180°
stanDevψ0To180k[k] standard deviation of above
meanψ90To270k[k] average ψ for kth region in kNjηMinjηMax, range for ψ, 90°<ψ<270°
stanDevψ90To270k[k] standard deviation of above

meanψk[k_], stanDevψk[k_] choose a range, give mean, standard dev. of polarization directions ψ in the kth region
gLONgLATmeanψForAllMin {Long.,Lat., mean ψ} for very sig aligned regions
gLONgLATmeanψForAllMin {Long.,Lat., stan. dev. ψ}

lp2B plot of color-coded mean polarization directions, Aitoff

```
In[246]:= allMinIDs = Table[sortηMINVerySigkList [[j, 1]], {j, Length[sortηMINVerySigkList]}];
allMaxIDs = Table[sortηMAXVerySigkList [[j, 1]], {j, Length[sortηMAXVerySigkList]}];

In[248]:= gLONj = Table[gLONGrid[[rgnCntrAndSrcId[[sortηMINVerySigkList [[k, 1]], 2]]]],
    {k, Length[sortηMINVerySigkList]}];
Clear[gLATj]
gLATj = Table[gLATGrid[[rgnCntrAndSrcId[[sortηMINVerySigkList [[k, 1]], 2]]]],
    {k, Length[sortηMINVerySigkList]}];
gLONHgLATHlogSigForAllMin = Table[{xHGal[ gLONj[[j]], gLATj[[j]] ],
     yHGal[ gLONj[[j]], gLATj[[j]] ], -Log[10, sortηMINVerySigkList [[j, 2]]]},
    {j, Length[sortηMINVerySigkList]}];
plotxyHGalLogSig0 = ListPlot3D[{gLONHgLATHlogSigForAllMin,
     Table[{xHGal[-179.9, gLAT], yHGal[-179.9, gLAT], 0}, {gLAT, -90, 90, 5}],
     Table[{xHGal[179.9, gLAT], yHGal[179.9, gLAT], 0}, {gLAT, -90, 90, 5}]},
    PlotRange → All, Mesh → None, AxesLabel → {"gLON", "gLAT", "a "},
    PlotStyle → {Automatic, Gray, Gray},
    PlotLabel → "Alignment, Significance exponent a = -log10p ",
    Ticks → {Automatic, Automatic, Automatic}, Boxed → False, Axes → {False, False, True}];

In[253]:= meridianPoints = Table[Table[Point[{xHGal[ gLon, gLat ], yHGal[ gLon, gLat ], 3.5}],
     {gLat, -89, 89, 1}], {gLon, -180, 180, 30}];
parallelsPoints = Table[Table[Point[{xHGal[ gLon, gLat ], yHGal[ gLon, gLat ], 3.5}],
     {gLon, -180, 180, 1}], {gLat, -60, 60, 30}];

In[255]:= plotxyHGalLogSig = Show[{plotxyHGalLogSig0,
     Graphics3D[{PointSize[0.005], meridianPoints, parallelsPoints}]}, ImageSize → 72 × 6];
```

Out[256]=

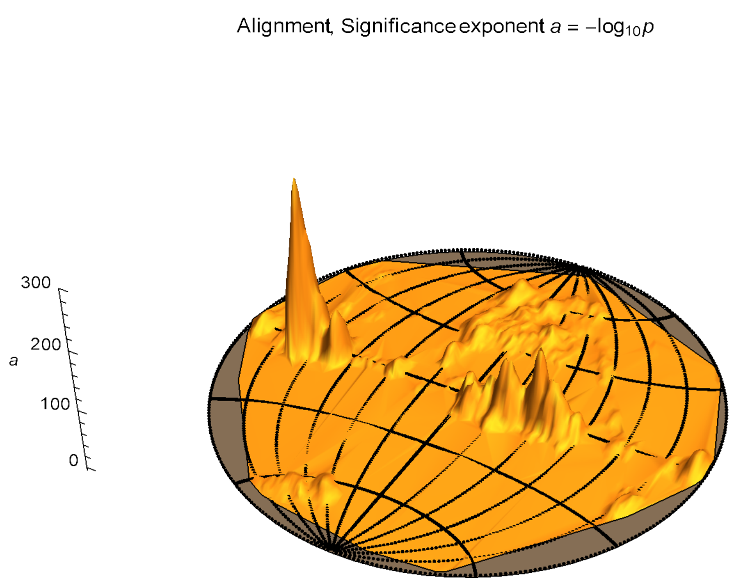


Figure A6. A raised relief map of the negative exponent a of the significance p, *i.e.* the logarithm $a = -\log_{10} p$ with $p = 10^{-a}$. Note the scale. The level plain represents 'very significant' regions, $p \approx 10^{-2} = 1\%$. Any hill that stands noticeably above the plain indicates exceptional significance.

In[257]:=
```
gLONjmax = Table[gLONGrid[[rgnCntrAndSrcId[[sortηMAXVerySigkList [[j, 1]], 2]]]],
   {j, Length[sortηMAXVerySigkList]}];
Clear[gLATj]
gLATjmax = Table[gLATGrid[[rgnCntrAndSrcId[[sortηMAXVerySigkList [[j, 1]], 2]]]],
   {j, Length[sortηMAXVerySigkList]}];
gLONHgLATHlogSigForAllMax = Table[{xHGal[ gLONjmax[[j]], gLATjmax[[j]] ],
    yHGal[ gLONjmax[[j]], gLATjmax[[j]] ], -Log[10, sortηMAXVerySigkList [[j, 2]]]},
   {j, Length[sortηMAXVerySigkList]}];
plotxyHGalLogSigmax0 = ListPlot3D[{gLONHgLATHlogSigForAllMax,
    Table[{xHGal[-179.9, gLAT], yHGal[-179.9, gLAT], 0}, {gLAT, -90, 90, 5}],
    Table[{xHGal[179.9, gLAT], yHGal[179.9, gLAT], 0}, {gLAT, -90, 90, 5}]},
   PlotRange → All, Mesh → None, AxesLabel → {"gLON", "gLAT", "a "},
   PlotStyle → {Automatic, Gray, Gray},
   PlotLabel → "Avoidance, Significance exponent a = -log₁₀p",
   Ticks → {Automatic, Automatic, Automatic}, Boxed → False, Axes → {False, False, True}];
```

In[262]:=
```
plotxyHGalLogSigmax = Show[{plotxyHGalLogSigmax0,
   Graphics3D[{PointSize[0.005], meridianPoints, parallelsPoints}]}, ImageSize → 72 × 6]
```

Out[262]=

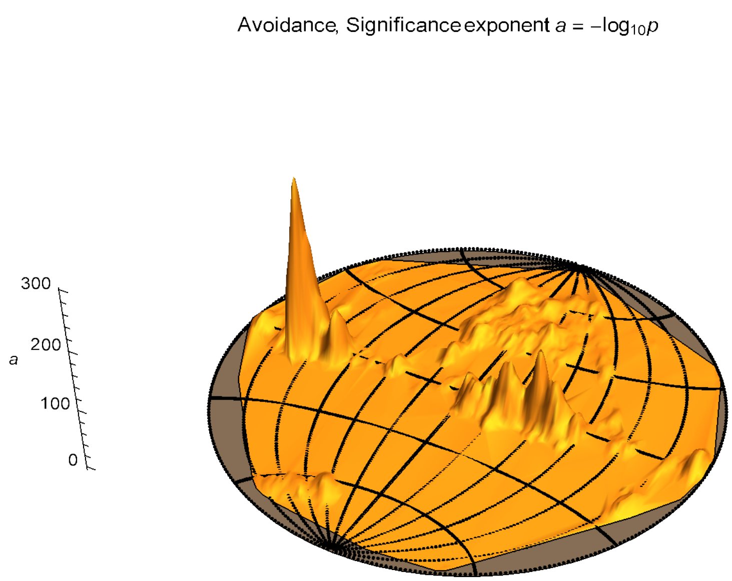


Figure A7. The avoidance version of the raised relief map in Fig. A6. This map shows the significance of avoidance, how likely it is that random polarization directions would give the same or larger maximum of the alignment angle function $\overline{\eta}(H)$. As in Fig. A6, the heights a are the negative exponent a of the significance p, with $p = 10^{-a}$. Thus greater altitude indicates more significance. Note that the terrain in this figure closely matches the terrain in Fig. A6, implying that the significance of alignment matches the significance of avoidance for most regions. That makes the regions with one highly significant, but the other not, special.

In[263]:=
```
gLONgLATlogSigForAllMin =
  Table[{gLONGrid[[rgnCntrAndSrcId[[sortηMINVerySigkList[[j, 1]], 2]]]],
    gLATGrid[[rgnCntrAndSrcId[[sortηMINVerySigkList[[j, 1]], 2]]]],
    -Log[10, sortηMINVerySigkList[[j, 2]]]}, {j, Length[sortηMINVerySigkList]}];
sortgLONgLATlogSigForAllMin = Sort[gLONgLATlogSigForAllMin, #1[[3]] > #2[[3]] &];
```

In[265]:=
```
lp1 = ListPlot[
   {Table[Style[{xHGal[ gLONAVEk[k], gLATAVEk[k] ], yHGal[ gLONAVEk[k], gLATAVEk[k] ]},
      LightGray], {k, Length[rgnCntrAndSrcId]}],
    Table[Style[{xHGal[sortgLONgLATlogSigForAllMin[[-j]][[1]],
        sortgLONgLATlogSigForAllMin[[-j]][[2]]], yHGal[sortgLONgLATlogSigForAllMin[[
          -j]][[1]], sortgLONgLATlogSigForAllMin[[-j]][[2]]]},
      ColorData["Rainbow"][(sortgLONgLATlogSigForAllMin[[-j]][[3]] - 2.) / 150.]],
     {j, Length[sortgLONgLATlogSigForAllMin]}]},
   PlotRange → {{-4., 4.}, {-2.2, 2.2}}, PlotStyle → PointSize[Medium],
   PlotLegends → BarLegend[{"Rainbow", {2.0, 150.}}, LegendLabel → "a=-Log10p"],
   Axes → False];
```

In[266]:=
```
lp2 = Show[{lp1, Table[ParametricPlot[{xHGal[α, δ], yHGal[α, δ]}, {δ, -90, 90},
      PlotStyle → {Black, Thickness[0.002]}, PlotPoints → 60], {α, -180, 180, 30}],
   Table[ParametricPlot[{xHGal[α, δ], yHGal[α, δ]}, {α, -179, 179},
      PlotStyle → {Black, Thickness[0.002]}, PlotPoints → 60],
     {δ, -60, 60, 30}], Graphics[{PointSize[0.004],
      Text[StyleForm["N", FontSize -> 14, FontWeight -> "Plain"], {0, 1.85}],
      Text[StyleForm["Alignment", FontSize -> 14, FontWeight -> "Plain"], {2.4, 1.5}],
      Text[StyleForm["Selected", FontSize → 12, FontWeight -> "Bold"], {-3.3, 1.0}],
      {Arrow[BezierCurve[{{-3.3, 0.8}, {-4.3, -1.5}, {xHGal[140, -5], yHGal[140, -5]}}]]},
      Text[StyleForm["Selected", FontSize → 12, FontWeight -> "Bold"], {-3.3, 1.0}],
      {Arrow[BezierCurve[{{-3.1, 1.2}, {-1.3, 2.5},
          {xHGal[38, 38], yHGal[38, 38]}}]]}}]}, ImageSize → 72 × 6]
```

Out[266]=

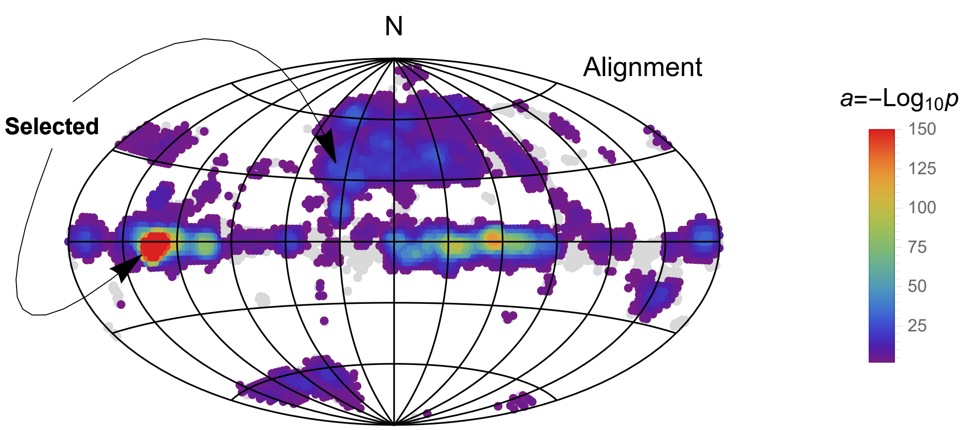


Figure A8. Color-coded alignment significance power parameter a, with significance $p = 10^{-a}$. The avoidance map is Fig. A8 below. The most significantly aligned regions hug the Galactic disk, light green through red. As in Fig. A6, it is evident that the selected area on the disk is exceptional.

In[267]:=
```
gLONgLATlogSigForAllMax =
  Table[{gLONGrid[[rgnCntrAndSrcId[[sortηMAXVerySigkList[[j, 1]], 2]]]],
    gLATGrid[[rgnCntrAndSrcId[[sortηMAXVerySigkList[[j, 1]], 2]]]],
    -Log[10, sortηMAXVerySigkList[[j, 2]]]}, {j, Length[sortηMAXVerySigkList]}];
sortgLONgLATlogSigForAllMax = Sort[gLONgLATlogSigForAllMax, #1[[3]] > #2[[3]] &];
```

In[269]:=
```
lp1max = ListPlot[
   {Table[Style[{xHGal[ gLONAVEk[k], gLATAVEk[k] ], yHGal[ gLONAVEk[k], gLATAVEk[k] ]},
      LightGray], {k, Length[rgnCntrAndSrcId]}],
    Table[Style[{xHGal[sortgLONgLATlogSigForAllMax[[-j]][[1]],
        sortgLONgLATlogSigForAllMax[[-j]][[2]]], yHGal[sortgLONgLATlogSigForAllMax[[
          -j]][[1]], sortgLONgLATlogSigForAllMax[[-j]][[2]]]},
       ColorData["Rainbow"][(sortgLONgLATlogSigForAllMax[[-j]][[3]] - 2.) / 150.]],
     {j, Length[sortgLONgLATlogSigForAllMax]}]},
   PlotRange → {{-4., 4.}, {-2.2, 2.2}}, PlotStyle → PointSize[Medium],
   PlotLegends → BarLegend[{"Rainbow", {2.0, 150.}}, LegendLabel → "-Log10p"],
   Axes → False];
```

In[270]:=
```
lp2max = Show[{lp1max, Table[ParametricPlot[{xHGal[α, δ], yHGal[α, δ]},
      {δ, -90, 90}, PlotStyle → {Black, Thickness[0.002]}, (*Mesh→{11,5,0}
      (*{23,11,0}*),MeshStyle→Thick,*)PlotPoints → 60], {α, -180, 180, 30}],
   Table[ParametricPlot[{xHGal[α, δ], yHGal[α, δ]}, {α, -179, 179},
     PlotStyle → {Black, Thickness[0.002]}, (*Mesh→{11,5,0}(*{23,11,0}*),
     MeshStyle→Thick,*)PlotPoints → 60], {δ, -60, 60, 30}], Graphics[{PointSize[0.004],
     Text[StyleForm["N", FontSize -> 14, FontWeight -> "Plain"], {0, 1.85}],
     Text[StyleForm["Avoidance", FontSize -> 14, FontWeight -> "Plain"], {2.4, 1.5}],
     Text[StyleForm["Galactic Coordinate System", FontSize -> 14, FontWeight -> "Plain"],
      {0, -1.85}]}]}], ImageSize → 72 × 6]
```

Out[270]=

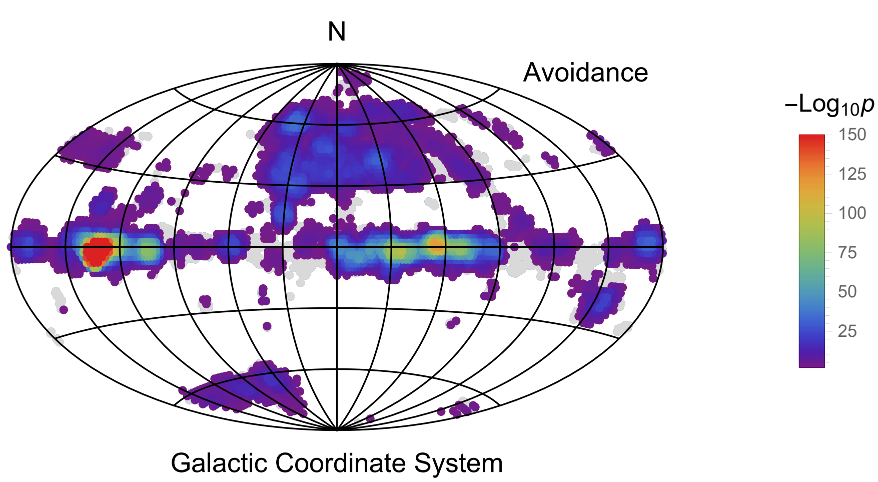


Figure A9. Comparing this avoidance version with the alignment version Fig. A8, one can locate those few regions with different significances for alignment and avoidance. But, on the whole, the maps match quite remarkably.

Mapping the average polarization direction introduces two problems:

(1) ψ is “non-oriented” meaning polarization vectors run back and forth, so ψ and $\psi \pm 180°$ are the same directions. Thus since ψ runs from 0° to 180° with 0° and 180° being the same direction, a bunch of similarly directed polarizations at 180° will be split.

Mitigation: For each star i , we (a) add 180° to ψ for $0 < \psi < 90°$ making ψ range from 90° to 270° (b) calculate the standard deviation for the ψ in both ranges (c) take the mean ψ for the range with the smaller standard deviation.

(2) *The polarizations may focus down on a hub H_{min} that is near the sources, then the mean polarization direction is misleading.*

Mitigation: Color-code only regions with standard deviation lower than some max value, say $\Delta\psi < 40°$ perhaps, plot bigger standard deviations as black dots.

In[271]:=
```
(*Set two ranges for polarization direction ψ:*)
ψGAL0To180i[i_] := ψGAL0To180i[i] = ψGALi[i]
ψGAL90To270i[i_] := If[ψGALi[i] < 90., ψGALi[i] + 180, ψGALi[i]]
```

In[273]:=
```
meanψ0To180k[k_] := meanψ0To180k[k] = mean[Table[ψGAL0To180i[i], {i, srcIDrgnk[k]}]]
stanDevψ0To180k[k_] :=
 stanDevψ0To180k[k] = stanDev[Table[ψGAL0To180i[i], {i, srcIDrgnk[k]}]]
meanψ90To270k[k_] := meanψ90To270k[k] = mean[Table[ψGAL90To270i[i], {i, srcIDrgnk[k]}]]
stanDevψ90To270k[k_] :=
 stanDevψ90To270k[k] = stanDev[Table[ψGAL90To270i[i], {i, srcIDrgnk[k]}]]
```

In[277]:=
```
(*The mean polarization direction ψ in the kth region*)
(*The standard deviation of the polarization directions ψ in the kth region*)
meanψk[k_] :=
 If[stanDevψ0To180k[k] ≤ stanDevψ90To270k[k], meanψ0To180k[k], meanψ90To270k[k]]
stanDevψk[k_] := If[stanDevψ0To180k[k] ≤ stanDevψ90To270k[k],
  stanDevψ0To180k[k], stanDevψ90To270k[k]]
```

In[279]:=
```
gLONgLATmeanψForAllMin =
  Table[{gLONGrid[[rgnCntrAndSrcId[[ηMINVerySigkList [[j, 1]], 2]]]],
    gLATGrid[[rgnCntrAndSrcId[[ηMINVerySigkList [[j, 1]], 2]]]],
    meanψk[ηMINVerySigkList [[j, 1]]]}, {j, Length[ηMINVerySigkList]}];
```

In[280]:=
```
gLONgLATstanDevψForAllMin =
  Table[{gLONGrid[[rgnCntrAndSrcId[[ηMINVerySigkList [[j, 1]], 2]]]],
    gLATGrid[[rgnCntrAndSrcId[[ηMINVerySigkList [[j, 1]], 2]]]],
    stanDevψk[ηMINVerySigkList [[j, 1]]]}, {j, Length[ηMINVerySigkList]}];
```

In[281]:=
```
lpB = ListPlot[
   {Table[Style[{xHGal[ gLONAVEk[k], gLATAVEk[k] ], yHGal[ gLONAVEk[k], gLATAVEk[k] ]},
      LightGray], {k, Length[rgnCntrAndSrcId]}], Table[Style[
      {xHGal[gLONgLATmeanψForAllMin[[-j]][[1]], gLONgLATmeanψForAllMin[[-j]][[2]]],
       yHGal[gLONgLATmeanψForAllMin[[-j]][[1]], gLONgLATmeanψForAllMin[[-j]][[2]]]},
      If[ (gLONgLATstanDevψForAllMin[[-j, 3]] < 40.),
       ColorData["Rainbow"][ (gLONgLATmeanψForAllMin[[-j]][[3]]) / 180.], Black]],
     {j, Length[gLONgLATmeanψForAllMin]}]}, PlotRange →
    {{-4., 4.}, {-2.2, 2.2}}, PlotStyle → PointSize[Medium],
   PlotLegends → BarLegend[{"Rainbow", {0, 180.}}, LegendLabel → "ψ, deg."],
   Axes → False];
```

In[282]:=

```
lp2B = Show[{lpB, Table[ParametricPlot[{xHGal[α, δ], yHGal[α, δ]}, {δ, -90, 90},
    PlotStyle → {Black, Thickness[0.002]}, PlotPoints → 60], {α, -180, 180, 30}],
   Table[ParametricPlot[{xHGal[α, δ], yHGal[α, δ]}, {α, -179, 179},
    PlotStyle → {Black, Thickness[0.002]}, PlotPoints → 60], {δ, -60, 60, 30}],
   Graphics[{PointSize[0.004], Text[StyleForm["N", FontSize -> 14, FontWeight -> "Plain"],
      {0, 1.85}], Text[StyleForm["Galactic Coordinate System", FontSize -> 14,
       FontWeight -> "Plain"], {0, -1.85}]}]}, ImageSize → 72 × 6];
```

Out[283]=

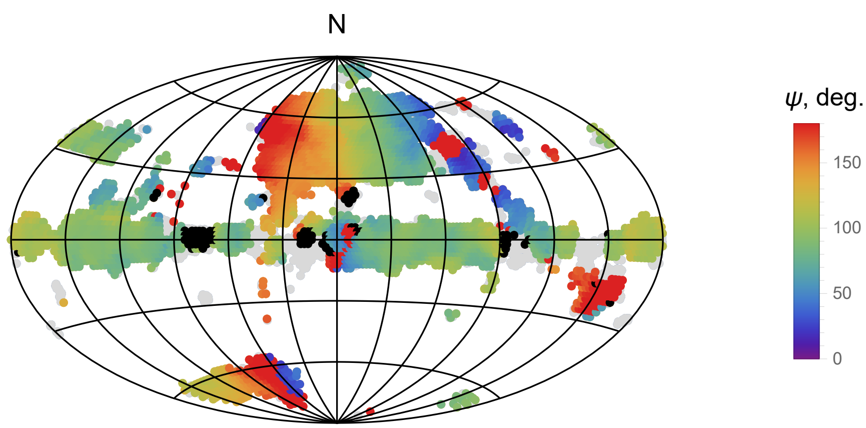


Figure A10. Color coded polarization directions. The average polarization direction $\overline{\psi}$ for the very significantly aligned 5° radius regions. Regions with wide-ranging ψ, $\Delta\psi > 40°$, are shaded Black, ■ . Since polarization is not oriented, the red 180° direction is equivalent to the blue 0° direction, so red wraps around into blue, ■ = ■ or South = North. Note the alternately Blue, Red, Blue, ..., stripe crossing the -90° meridian at latitude 30°. Conversely, where the regions on the Disk favor green, ■ or $\psi = 90°$, it means that $\overline{\psi}$ points along the Disk.

A9. Selecting sources for study

Definitions:

plotgLONgLATlogSigForAllMin {long., lat., a} for all very significantly aligned regions, table for list plots

lp3Dclump1a, showlp3Dclump1a plot of a in area of Clump 1, where $a = -\log_{10} p$ is the exponent for significance $p = 10^{-a}$.
firstClumpjsForSortηMINVerySigkList list of IDs i for
"gLONgLATlogSigForAllMin" table for regions selected for Clump 1
firstClumpksForRgnCntrAndSrcId list of IDs k for
" kNjηMinjηMax" table for regions selected for Clump 1
firstClumpStarsIDinData00 List of record # in table "data00" for the stars in Clump 1

firstClumpStarsIDinCatalog Identifying information for various star catalogs, HD,BD, ...

gLONgLATstarsFirstClump {long., lat.} for stars in Clump 1
plotgLONgLATstarsFirstClump plot of {long., lat.} for stars in Clump 1

ψClump1 polarization directions ψ for stars in Clump 1
hist1 histogram of ψ for Clump 1

sortDistanceClump1 distances to stars in Clump 1, not all stars have distances in data00
hlDistC1 histogram of distances to Clump 1 and related tables
nlmDistanceClump1 fit the distance histogram
histDistClump1, showhistDistClump1 plot the histogram

Clump 6 NE and SW have the same quantities as Clump 1, but with their names changed to maintain validity

lp3Db, showlp3Dp, sixthSWClumpjsForSortηMINVerySigkList, ...

```
In[284]:= plotgLONgLATlogSigForAllMin =
  Table[{-gLONGrid[[rgnCntrAndSrcId[[sortηMINVerySigkList [[j, 1]], 2]]]],
    gLATGrid[[rgnCntrAndSrcId[[sortηMINVerySigkList [[j, 1]], 2]]]],
    -Log[10, sortηMINVerySigkList [[j, 2]]]}, {j, Length[sortηMINVerySigkList]}];

In[285]:=
 (*logSig6=ListPlot3D[plotgLONgLATlogSigForAllMin,
  PlotRange→{{-180,180},{-90,90},{0,40}},AxesLabel→{"gLON","gLAT","-Log"},
  PlotLabel→"-Log10p",Ticks→{Table[{i,-i},{i,-180,180,60}],Automatic,Automatic}]*)

In[286]:= (*logSig9=ListPlot3D[plotgLONgLATlogSigForAllMin,
  PlotRange→{{-180,180},{-90,90},{0,9}},AxesLabel→{"gLON,°","gLAT,°","-Log"},
  PlotLabel→"-Log10p",Ticks→{Table[{i,-i},{i,-180,180,60}],Automatic,Automatic}]*)

In[287]:= (*logSig270=ListPlot3D[plotgLONgLATlogSigForAllMin,
  PlotRange→{{-180,180},{-90,90},{0,270}},AxesLabel→{"gLON, °","gLAT, °","-Log"},
  PlotLabel→"-Log10p",Ticks→{Table[{i,-i},{i,-180,180,60}],Automatic,Automatic}]*)
```

Select objects near (gLON,gLAT) = (130°, 0°), making Clump 1.

```
In[288]:= lp3Dclump1a =
  ListPlot3D[plotgLONgLATlogSigForAllMin, PlotRange → {{-165, -85}, {-20, 15}, {2, 9}},
   AxesLabel → {"gLON, °", "gLAT, °", "α"}, PlotLabel → "Clump 1",
   Mesh → {Table[i1, {i1, -180, -60, 30}], Table[i2, {i2, -30, 30, 10}]},
   Ticks → {Table[{i, -i}, {i, -170, -80, 10}], Automatic, Automatic}];
```

```
In[289]:= showlp3Dclump1a = Show[{lp3Dclump1a, Graphics3D[{{Thick, Purple,
            Line[{{-155, -15, 8}, {-155, 10, 8}, {-95, 10, 8}, {-95, -15, 8}, {-155, -15, 8}}]},
           Black, Text[Style["a = -Log10p > 9", Medium, Bold, Black], {-132, 7, 5}]}]}]
```

Out[289]=

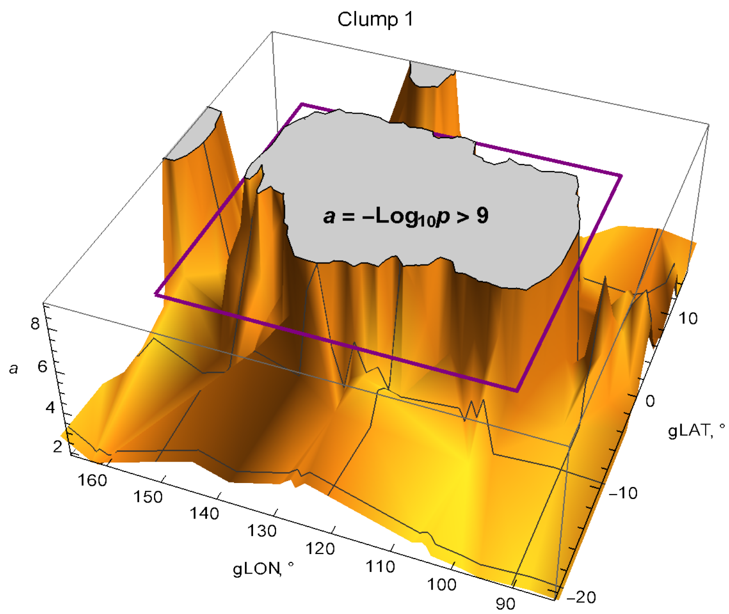


Figure A11. Selecting the first clump of stars, Clump 1. The center points of the regions chosen run from longitude 95° to 155° and latitude −15 ° to 10°, outlined in purple. The significance exponent a , where sig. = $p = 10^{-a}$, for a selected region must exceed nine, $a > 9$, meaning the significance p is at most 10^{-9}. See Figs. A6 and A8.

```
In[290]:= firstClumpjsForSortηMINVerySigkList = {};
Table[If[(95 ≤ gLONgLATlogSigForAllMin[[i, 1]] ≤ 155) &&
      (-15 ≤ gLONgLATlogSigForAllMin[[i, 2]] ≤ 10) &&
      (9 ≤ gLONgLATlogSigForAllMin[[i, 3]] ≤ 1000),
     AppendTo[firstClumpjsForSortηMINVerySigkList, i]],
   {i, Length[gLONgLATlogSigForAllMin]}];
firstClumpksForRgnCntrAndSrcId = Table[sortηMINVerySigkList [[j, 1]],
    {j, firstClumpjsForSortηMINVerySigkList}];
firstClumpStarsIDinData00 = Union[Flatten[Table[rgnCntrAndSrcId[[
       sortηMINVerySigkList [[j, 1]], 3]], {j, firstClumpjsForSortηMINVerySigkList}]]];
firstClumpStarsIDinCatalog = Table[data00[[firstClumpStarsIDinData00[[k]], 1]],
    {k, Length[ firstClumpStarsIDinData00 ]}];
Length[firstClumpjsForSortηMINVerySigkList];
Length[firstClumpksForRgnCntrAndSrcId];
Length[firstClumpStarsIDinData00];
```

In[298]:=
```
gLONgLATstarsFirstClump =
  Table[{data00[[i, 2]], data00[[i, 3]]} (360. / (2. π)), {i, firstClumpStarsIDinData00}];
plotgLONgLATstarsFirstClump = Table[{-data00[[i, 2]], data00[[i, 3]]} (360. / (2. π)),
   {i, firstClumpStarsIDinData00}];
ListPlot[plotgLONgLATstarsFirstClump, PlotRange → {{-180, 180}, {-90, 90}},
  Axes → False, PlotStyle → Blue,
  FrameTicks → {Table[{i, -i}, {i, -180, 180, 60}], Table[j, {j, -90, 90, 30}]},
  PlotLabel → "locations of stars in the first clump",
  GridLines → {Table[i, {i, -180, 180, 60}], Table[j, {j, -90, 90, 30}]},
  Frame → True, FrameLabel → {"gLON, deg.", "gLAT,deg."}, ImageSize → 72 × 4];
```

In[301]:=
```
ψClump1 = Table[ψGALi[i], {i, firstClumpStarsIDinData00}];
Mean[ψClump1];
hist1a = Histogram[ψClump1, {7}, PlotRange → {{0, 180}, Automatic}, Axes → False,
   FrameTicks → Automatic, PlotLabel → "PPA ψ in Clump 1", GridLines → Automatic,
   Frame → True, FrameLabel → {"ψ, deg.", "Number per bin"}, ImageSize → 72 × 4];
hist1 = Show[{hist1a, Graphics[
    {{Dashed, Line[{{Mean[ψClump1], 0}, {Mean[ψClump1], 250}}]},
     Text[StyleForm["ψ̄ = 90.°", FontSize -> 14, FontWeight -> "Plain"], {140, 200}]}]}]
```

Out[304]=

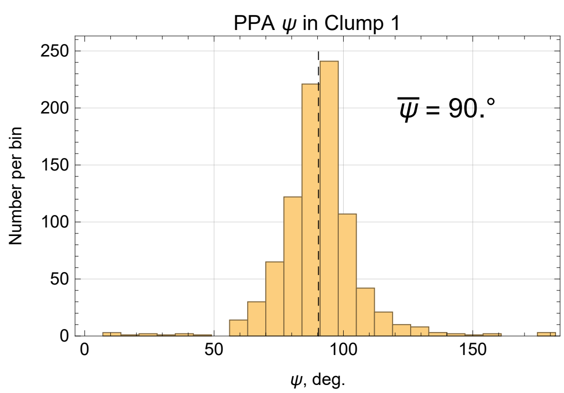


Figure A12. The polarization position angles ψ for stars in Clump 1 bunch up around the mean $\overline{\psi} = 90°$, as shown by this histogram.

In[305]:=
```
sortDistanceClump1 = Sort[Table[disti[i], {i, firstClumpStarsIDinData00}]];
hl0DistC1 = HistogramList[sortDistanceClump1, {0, 7500, 181}];
hlDistC1 = Table[{(1/2) (hl0DistC1[[1, i1]] + hl0DistC1[[1, i1 + 1]]),
    hl0DistC1[[2, i1]]}, {i1, Length[ hl0DistC1[[2]] ]}];
```

In[308]:=
```
nlmDistanceClump1 = NonlinearModelFit[hlDistC1,
   {a Exp[-(1/2.) ((x - 0) / b)^2] + c Exp[-(1/2.) ((x - x0) / d)^2]},
   {{a, 80.}, {b, 100.}, (*{x0,70.},*){c, 60.}, {d, 600.}, {x0, 2200.}}, x];
```

In[309]:=
```
Normal[nlmDistanceClump1];
nlmDistanceClump1["BestFitParameters"];
nlmDistanceClump1["ParameterErrors"];
nlmDistanceClump1["ParameterTable"]
nlmDistanceClump1["EstimatedVariance"];
```

Out[312]=

	Estimate	Standard Error	t-Statistic	P-Value
a	75.1394	3.73279	20.1296	3.53402×10^{-21}
b	467.443	39.489	11.8373	5.70612×10^{-14}
c	48.2711	1.83619	26.2888	4.32466×10^{-25}
d	956.704	63.4707	15.0732	3.97384×10^{-17}
x0	2151.43	59.042	36.4389	5.17905×10^{-30}

In[314]:=
```
histDistClump1 =
  Histogram[Sort[Table[disti[i], {i, firstClumpStarsIDinData00}]], {181}, Axes → False,
   FrameTicks → Automatic(*{Table[{i,-i},{i,-180,180,60}],Table[j,{j,-90,90,30}]}*),
   PlotRange → {{0., 7000.}, {0., 80.}}, PlotLabel → "Distances to the stars in Clump 1",
   GridLines → {Table[i, {i, 0, 8000, 1000}], Table[j, {j, 0, 90, 10}]}(*Automatic*),
   Frame → True, FrameLabel → {"distance, pc.", "# per interval"}, ImageSize → 72 × 4];
```

In[315]:=
```
showhistDistClump1 = Show[{histDistClump1, Plot[nlmDistanceClump1[x], {x, 0, 7000}]}]
```

Out[315]=

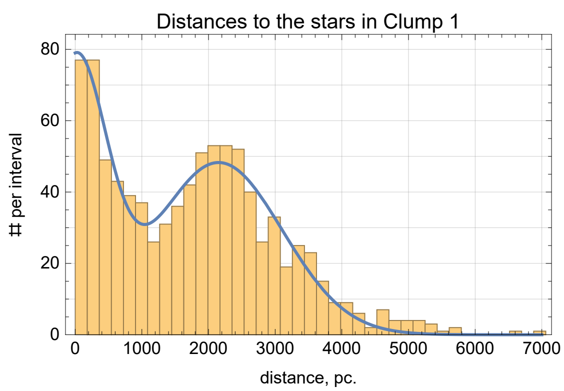


Figure A13. The distribution of the distances to the stars in Clump 1. The two Gaussian fit has one peak set by hand at the Origin at the Sun and the second peak allowed to vary, settling down at $D = 2150$ pc $= 7000$ light-years.

Clumps 6 Northeast (NE) and Southwest (SW)

Select regions near (gLON,gLAT) = (25°,30°), making Clump 6NE and Clump 6SW, northeast NE and southwest SW.

In[316]:=
```
(*Map the selection process:*)
lp3Db =
  ListPlot3D[plotgLONgLATlogSigForAllMin, PlotRange → {{-50, -14}, {18, 45}, {2, 21}},
   AxesLabel → {"gLON, °", "gLAT, °", "a"}, PlotLabel → "Clump 6 NE, SW",
   Mesh → {Table[i1, {i1, -180, +60, 15}], Table[i2, {i2, -30, 60, 10}]},
   Ticks → {Table[{i, -i}, {i, -45, -15, 5}], Automatic, Automatic}];
{x1, y1} = {36., 23.};
{x2, y2} = {28., 35.};
y6[x6_] := y6[x6] = y1 + ((x6 - x1) / (x2 - x1)) (y2 - y1)
showlp3Db =
 Show[{lp3Db, Graphics3D[{{Thick, Purple, Line[{{-36, 23, 20}, {-28, 35, 20}, {-17, 35, 20},
        {-17, 23, 20}, {-36, 23, 20}}], Line[{{-32, y6[32], 20}, {-28, 35, 20}, {-28, 42,
         20}, {-43, 42, 20}, {-43, y6[32], 20}, {-32, y6[32], 20}(*{-36,23,20}*)}]},
     Black, Text[Style[" NE ", Medium, Bold, Black], {-33, 35, 20}],
     Text[Style[(*"-Log10p > 21"*)"a > 21", Medium, Bold, Black], {-36, 37, 20}],
     Text[Style[" SW ", Medium, Bold, Black], {-27, 32., 20}],
     Text[Style[(*"-Log10p > 21"*)"a > 21", Medium, Bold, Black], {-25, 30, 20}],
     Point[{-x2, y2, 20}], Point[{-x1, y1, 20}], Line[{{-x1, y1, 20}, {-x2, y2, 20}}]}]}]
```

Out[320]=

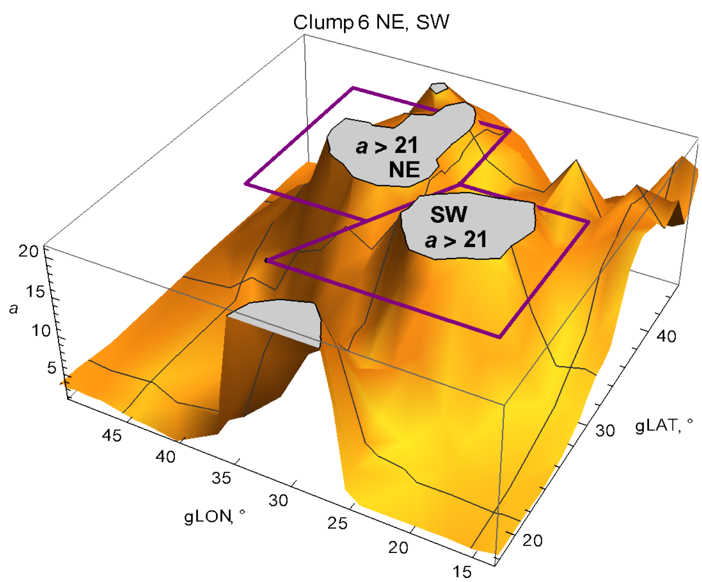


Figure A14. The process of separating the two significance hills is more complicated than what is needed to isolate Clump 1 in Fig. A11. We sent a straight line through the valley between the hills. Note that the significance exponent is very high, $a > 21$, which means the significances p for selected regions are better than $p = 10^{-21}$. The two hills top off a larger mound of exceptionally well aligned regions.

```
In[321]:= sixthSWClumpjsForSortηMINVerySigkList = {};
Table[If[(17 ≤ gLONgLATlogSigForAllMin[[i, 1]] ≤ 34) &&
      (23 ≤ gLONgLATlogSigForAllMin[[i, 2]] ≤ 35) && (21 ≤ gLONgLATlogSigForAllMin[[i, 3]]) &&
      (gLONgLATlogSigForAllMin[[i, 2]] < y6[gLONgLATlogSigForAllMin[[i, 1]]]),
    AppendTo[sixthSWClumpjsForSortηMINVerySigkList, i]],
   {i, Length[gLONgLATlogSigForAllMin]}];
sixthSWClumpksForRgnCntrAndSrcId = Table[sortηMINVerySigkList [[j, 1]],
    {j, sixthSWClumpjsForSortηMINVerySigkList}];
sixthSWClumpStarsIDinData00 = Union[Flatten[Table[rgnCntrAndSrcId[[
        sortηMINVerySigkList [[j, 1]], 3]], {j, sixthSWClumpjsForSortηMINVerySigkList}]]];
sixthSWClumpStarsIDinCatalog = Table[data00[[sixthSWClumpStarsIDinData00[[k]], 1]],
    {k, Length[ sixthSWClumpStarsIDinData00 ]}];
Length[sixthSWClumpjsForSortηMINVerySigkList];
Length[sixthSWClumpksForRgnCntrAndSrcId];
Length[sixthSWClumpStarsIDinData00];

In[329]:= gLONgLATstarssixthSWClump = Table[
    {data00[[i, 2]], data00[[i, 3]]} (360. / (2. π)), {i, sixthSWClumpStarsIDinData00}];
plotgLONgLATstarssixthSWClump = Table[{-data00[[i, 2]], data00[[i, 3]]} (360. / (2. π)),
    {i, sixthSWClumpStarsIDinData00}];
ListPlot[plotgLONgLATstarssixthSWClump, PlotRange → {{-180, 180}, {-90, 90}},
  AxesLabel → {"-gLON", "gLAT"}, PlotLabel → "locations of stars in the sixthSW clump",
  PlotStyle → Blue, Ticks → {Table[{i, -i}, {i, -180, 180, 60}], Automatic}];

In[332]:= sixthNEClumpjsForSortηMINVerySigkList = {};
Table[If[(28 ≤ gLONgLATlogSigForAllMin[[i, 1]] ≤ 43) &&
      (35 ≤ gLONgLATlogSigForAllMin[[i, 2]] ≤ 42) && (21 ≤ gLONgLATlogSigForAllMin[[i, 3]]) &&
      (gLONgLATlogSigForAllMin[[i, 2]] > y6[gLONgLATlogSigForAllMin[[i, 1]]]),
    AppendTo[sixthNEClumpjsForSortηMINVerySigkList, i]],
   {i, Length[gLONgLATlogSigForAllMin]}];
sixthNEClumpksForRgnCntrAndSrcId = Table[sortηMINVerySigkList [[j, 1]],
    {j, sixthNEClumpjsForSortηMINVerySigkList}];
sixthNEClumpStarsIDinData00 = Union[Flatten[Table[rgnCntrAndSrcId[[
        sortηMINVerySigkList [[j, 1]], 3]], {j, sixthNEClumpjsForSortηMINVerySigkList}]]];
sixthNEClumpStarsIDinCatalog = Table[data00[[sixthNEClumpStarsIDinData00[[k]], 1]],
    {k, Length[ sixthNEClumpStarsIDinData00 ]}];
Length[sixthNEClumpjsForSortηMINVerySigkList];
Length[sixthNEClumpksForRgnCntrAndSrcId];
Length[sixthNEClumpStarsIDinData00];

In[340]:= gLONgLATstarssixthNEClump = Table[
    {data00[[i, 2]], data00[[i, 3]]} (360. / (2. π)), {i, sixthNEClumpStarsIDinData00}];
plotgLONgLATstarssixthNEClump = Table[{-data00[[i, 2]], data00[[i, 3]]} (360. / (2. π)),
    {i, sixthNEClumpStarsIDinData00}];
ListPlot[{plotgLONgLATstarssixthSWClump, plotgLONgLATstarssixthNEClump},
  PlotRange → {{-180, 180}, {-90, 90}}, AxesLabel → {"gLON", "gLAT"},
  PlotLabel → "locations of stars in the sixth NE, SW clumps",
  PlotStyle → {Blue, Orange}, Ticks → {Table[{i, -i}, {i, -180, 180, 60}], Automatic}];
```

In[343]:=
```
Intersection[sixthSWClumpStarsIDinData00, sixthNEClumpStarsIDinData00];
Print["There are ", Length[%],
 " stars in both the NE and SW clumps 6, i.e. their intersection."]
```

```
There are 9 stars in both the NE and SW clumps 6, i.e. their intersection.
```

In order to see the differences between NE and SW, we remove the stars that are common to both.

In[345]:=
```
bothsixthSWsixthNEStarsIDinData00 =
  Intersection[sixthSWClumpStarsIDinData00, sixthNEClumpStarsIDinData00];
insixthSWNotinsixthNE = Complement[sixthSWClumpStarsIDinData00,
   bothsixthSWsixthNEStarsIDinData00];
{Length[bothsixthSWsixthNEStarsIDinData00], Length[insixthSWNotinsixthNE],
  Length[sixthSWClumpStarsIDinData00]};
insixthNENotinsixthSW = Complement[sixthNEClumpStarsIDinData00,
   bothsixthSWsixthNEStarsIDinData00];
{Length[bothsixthSWsixthNEStarsIDinData00], Length[insixthNENotinsixthSW],
  Length[sixthNEClumpStarsIDinData00]};
```

In[350]:=
```
ψClump6SWnotNE = Table[ψGALi[i], {i, insixthSWNotinsixthNE}];
Mean[ψClump6SWnotNE];
hist6SWa =
  Histogram[ψClump6SWnotNE, {7}, PlotRange → {{90, 270}, Automatic}, Axes → False,
   FrameTicks → Automatic(*{Table[{i,-i},{i,-180,180,60}],Table[j,{j,-90,90,30}]}*),
   PlotLabel → "Polarization Direction ψ, 6SW", GridLines → Automatic
   (*{Table[i,{i,-180,180,60}],Table[j,{j,-90,90,30}]}*), Frame → True,
   FrameLabel → {"ψ, deg.", "Number per bin"}, ImageSize → 72 × 4];
hist6SW = Show[{hist6SWa, Graphics[
     {{Dashed, Line[{{Mean[ψClump6SWnotNE], 0}, {Mean[ψClump6SWnotNE], 25}}]},
      Text[StyleForm["ψ̄ = 149°", FontSize -> 14, FontWeight -> "Plain"], {200, 22}]}]}];
```

In[354]:=
```
plotgLONgLATstarssixthSWNotNE =
  Table[{-data00[[i, 2]], data00[[i, 3]]} (360. / (2. π)), {i, insixthSWNotinsixthNE}];
plotgLONgLATstarssixthNENot6SW =
  Table[{-data00[[i, 2]], data00[[i, 3]]} (360. / (2. π)), {i, insixthNENotinsixthSW}];
plot6Distinct = ListPlot[{plotgLONgLATstarssixthNENot6SW, plotgLONgLATstarssixthSWNotNE},
   PlotRange → {{-180, 180}, {-90, 90}},
   PlotLabel → "Stars in either Clumps 6 NE or SW", PlotStyle → {Blue, Orange},
   (*Ticks→{Table[{i,-i},{i,-180,180,60}],Automatic},*)Axes → False, FrameTicks →
    {{Table[j, {j, -90, 90, 30}], None}, {Table[{i, -i}, {i, -180, 180, 60}], None}},
   GridLines → (*Automatic*){Table[i, {i, -180, 180, 60}], Table[j, {j, -90, 90, 30}]},
   Frame → True, FrameLabel → {"gLON, deg.", "gLAT, deg."}, ImageSize → 72 × 4];
```

In[357]:=
```
plot6Distinct
```

Out[357]=

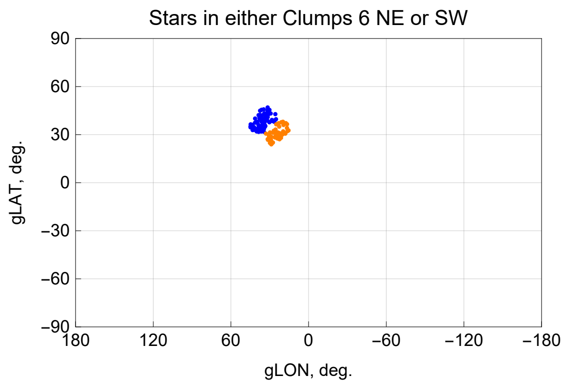


Figure A15. The locations on the sky of the stars in Clump 6 SW (Orange) and NE (Blue). Stars that are common to both SW and NE are not displayed.

In[358]:=
```
ψClump6NEnotSW =
  Table[If[ψGALi[i] < 90., ψGALi[i] + 180, ψGALi[i] ], {i, sixthNEClumpStarsIDinData00}];
Mean[ψClump6NEnotSW];
hist6NEa =
  Histogram[ψClump6NEnotSW, {7}, PlotRange → {{90, 270}, Automatic}, Axes → False,
   FrameTicks → Automatic(*{Table[{i,-i},{i,-180,180,60}],Table[j,{j,-90,90,30}]}*),
   PlotLabel → "Polarization direction ψ, 6NE", GridLines → Automatic
   (*{Table[i,{i,-180,180,60}],Table[j,{j,-90,90,30}]}*), Frame → True,
   FrameLabel → {"ψ, deg.", "Number per bin"}, ImageSize → 72 × 4];
hist6NE = Show[{hist6NEa, Graphics[
     {{Dashed, Line[{{Mean[ψClump6NEnotSW], 0}, {Mean[ψClump6NEnotSW], 25}}]},
      Text[StyleForm["ψ̄ = 163°", FontSize -> 14, FontWeight -> "Plain"], {220, 22}]}]}];
```

In[362]:=
```
GraphicsRow[{hist6SW, hist6NE}]
```

Out[362]=

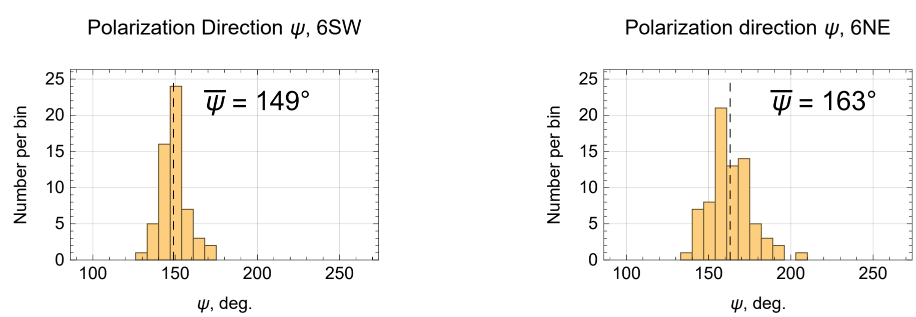


Figure A16. Comparing the polarization directions ψ for Clump 6 SW (Left) and Clump 6 NE (Right). Stars common to both clumps have been removed. The SW distribution is much narrower than the NE distribution. The NE distribution is shifted southward, toward 180°, by some 14° from the SW distribution.

In[363]:=
```
ListPlot[{Sort[Table[If[disti[i] > 0, disti[i]], {i, insixthNENotinsixthSW}]],
  Sort[Table[If[disti[i] > 0, disti[i]], {i, insixthSWNotinsixthNE}]]},
 PlotRange → {{0, 25}, Automatic}, Axes → False, FrameTicks → Automatic,
 PlotLabel → "Distance to stars in Clump 6 NE and SW",
 GridLines → Automatic, PlotLegends → {"NE", "SW"}, Frame → True,
 FrameLabel → {"star #, sorted by distance", "Distance, pc."}, ImageSize → 72 × 4]
```

Out[363]=

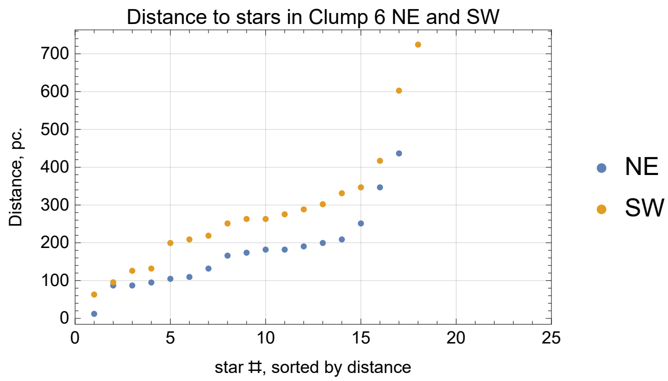


Figure A17. Comparing the distances to Clumps 6 SW and NE. The trend puts SW stars about 80 parsecs further away than the NE stars.

In[364]:=
```
ListPlot[{Sort[Table[Around[If[ψGALi[i] < 45, ψGALi[i] + 180, ψGALi[i]], σψi[i]],
    {i, insixthNENotinsixthSW}]],
  Sort[Table[Around[ψGALi[i], σψi[i]], {i, insixthSWNotinsixthNE}]]},
 PlotRange → {{0, 65}, {0, 270}}, Axes → False, FrameTicks → Automatic,
 PlotLabel → "Polarization directions ψ for stars in Clumps 6",
 GridLines → Automatic, PlotLegends → {"NE", "SW"}, Frame → True,
 FrameLabel → {"star #, sorted by ψ", "ψ, deg."}, ImageSize → 72 × 4]
{mean[Sort[Table[ψGALi[i], {i, insixthSWNotinsixthNE}]]],
  stanDev[Sort[Table[ψGALi[i], {i, insixthSWNotinsixthNE}]]]};
{mean[Sort[Table[ψGALi[i], {i, insixthNENotinsixthSW}]] ],
  stanDev[Sort[Table[ψGALi[i], {i, insixthNENotinsixthSW}]] ] };
```

Out[364]=

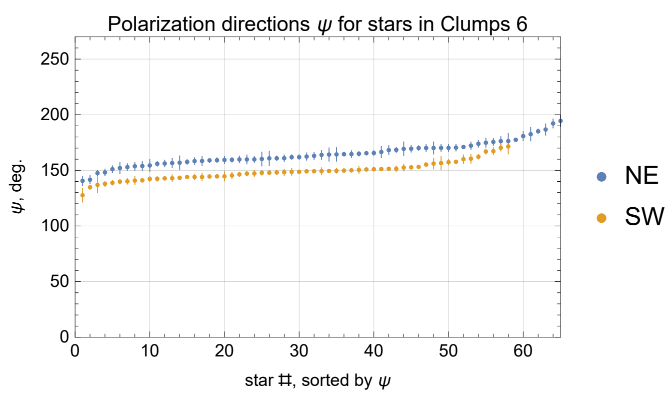


Figure A18. Polarization directions ψ for stars in Clump 6 NE and SW, but not in both. We take advantage of polarization vectors oscillatory nature and add 180° to some of the small ψ values for NE to make the plot continuous. The error bars displayed are the measurement uncertainties listed in the catalogs. The NE clump's polarization directions tend to be a bit steeper, *i.e.* more polar, than the SW ψ.