The New Notation for Hyperoperation of a Sequence

Kyumin Nam

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Abstract

For a sequence $a_1, a_2, \dots, a_n$, we define the exponent, tetration and pentation of a sequence a_n as $\underset{k=1}{\overset{n}{\mathrm{E}}}(a_k) = a_1[3]a_2[3]\cdots[3]a_n$, $\underset{k=1}{\overset{n}{\mathrm{T}}}(a_k) = a_1[4]a_2[4]\cdots[4]a_n$, $\underset{k=1}{\overset{n}{\Phi}}(a_k) = a_1[5]a_2[5]\cdots[5]a_n$. Also, we define the i-th hyperoperation of a sequence a_n as $\underset{k=1}{\overset{n}{\mathrm{H}}}{}_i(a_k) = a_1[i]a_2[i]\cdots[i]a_n$.

Contents

Introduction

In this paper, we provide the notation for hyperoperations of a sequence. This notation will make it easier to write hyperoperations of sequence that are very long, and difficult to write.

Definition

The notation for summation and product of sequence a_n is already defined as

$$\underset{k=1}{\overset{n}{\Sigma}}(a_k) = a_1[1]a_2[1]\cdots[1]a_n = a_1 + a_2 + \cdots + a_n \text{ [1]}$$

$$\underset{k=1}{\overset{n}{\Pi}}(a_k) = a_1[2]a_2[2]\cdots[2]a_n = a_1 a_2 \cdots a_n \text{ [2]}$$

using the uppercase of σ, and π because the first letter of ‘summation’, and ‘product’ corresponds to σ, and π.

Now, we will define the new notations for exponent, tetration, pentation, and hyperoperation of sequence a_n by the same way as a Capital-sigma notation and Capital-pi notation.

For a sequence $a_1, a_2, \ldots, a_n$, we define the notation for exponent of a sequence a_n as

$$\underset{k=1}{\overset{n}{\mathrm{E}}}(a_k) = a_1[3]a_2[3]\cdots[3]a_n = a_1 \uparrow a_2 \uparrow \cdots \uparrow a_n$$

using the uppercase of ε because the first letter of ‘exponent’ corresponds to ε.

Also, we define the notation for tetration and pentation of a sequence a_n as

$$\underset{k=1}{\overset{n}{\mathrm{T}}}(a_k) = a_1[4]a_2[4]\cdots[4]a_n = a_1 \uparrow\uparrow a_2 \uparrow\uparrow \cdots \uparrow\uparrow a_n$$

$$\underset{k=1}{\overset{n}{\Phi}}(a_k) = a_1[5]a_2[5]\cdots[5]a_n = a_1 \uparrow^3 a_2 \uparrow^3 \cdots \uparrow^3 a_n$$

using the uppercase of τ, and ϕ because the first letter of ‘tetration’, and ‘pentation’ corresponds to τ, and ϕ.

Moreover, we define the notation for i-th hyperoperation of a sequence a_n as

$$\underset{k=1}{\overset{n}{\mathrm{H}}}{}_i\,(a_k) = a_1[i]a_2[i]\cdots[i]a_n$$

using the uppercase of η because the first letter of ‘hyperoperation’ corresponds to uppercase of η.

Examples

(1) $\overset{4}{\underset{k=1}{\mathrm{E}}}(2k) = 2 \uparrow 4 \uparrow 6 \uparrow 8 = 2^{4^{6^{8}}}$

(2) $\overset{3}{\underset{k=1}{\mathrm{T}}}(k-4)^2 = 9 \uparrow\uparrow 4 \uparrow\uparrow 1 = 9^{9^{9^{9}}}$

(3) $\overset{n}{\underset{k=1}{\mathrm{H}_1}}(k) = \sum_{k=1}^{n} k = 1 + 2 + ... + n = \frac{1}{2}n(n+1)$

(4) $\overset{n}{\underset{k=1}{\mathrm{H}_2}}(k) = \prod_{k=1}^{n} k = 1 \times 2 \times \cdots \times n = n!$

Conclusion

In this paper, we provided the new notation for hyperoperation of a sequence. We defined the exponent, tetration, and pentation of a sequence a_n as $\overset{n}{\underset{k=1}{\mathrm{E}}}(a_k) = a_1[3]a_2[3]\cdots[3]a_n$, $\overset{n}{\underset{k=1}{\mathrm{T}}}(a_k) = a_1[4]a_2[4]\cdots[4]a_n$, $\overset{n}{\underset{k=1}{\Phi}}(a_k) = a_1[5]a_2[5]\cdots[5]a_n$ for a sequence $a_1, a_2, \ldots, a_n$. Also, we defined the i-th hyperoperation of a sequence a_n as $\overset{n}{\underset{k=1}{\mathrm{H}_i}}(a_k) = a_1[i]a_2[i]\cdots[i]a_n$ for a sequence $a_1, a_2, \ldots, a_n$.

Reference

[1] "Summation". wikipedia.com. Last edited on 6 January 2022.
[2] "Multiplication". wikipedia.com. Last edited on 30 January 2022.