Square equations represented with gnomon.

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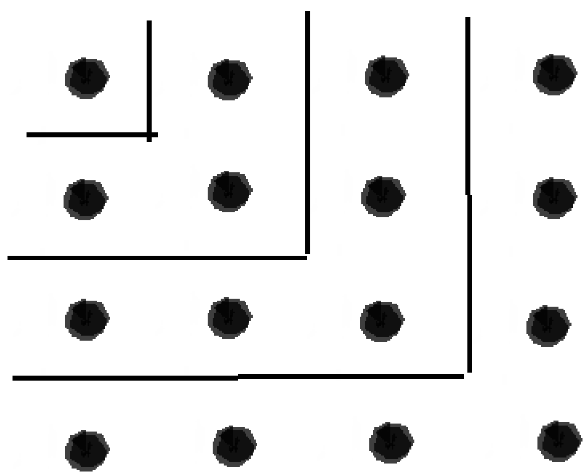
0- Abstract:

In this paper I am going to present a soft extension of gnomon theory in geometry. The well known case for n^2 can be adjusted to (2n+1)^2 and (2n)^2. I show it with simple graphs and an algebraic explanation.

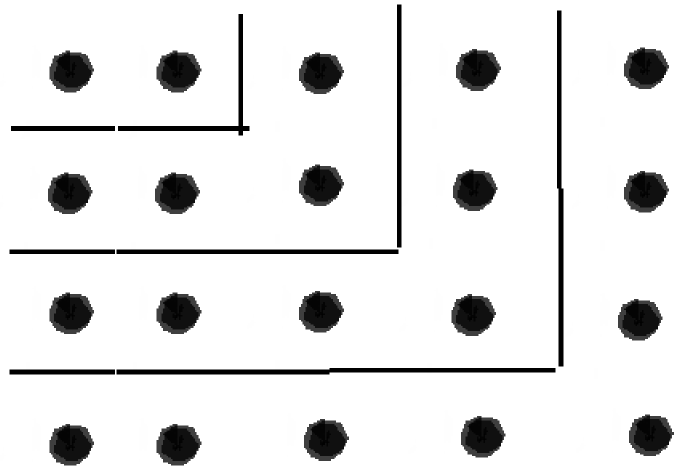
1- Introduction:

First, I am going to define the concept of gnomon, it is a figure that, added to a given figure, makes a larger figure of the same shape. (1)
According to the pre-Parmenidean Phytagoreanism that I saw in *The presocratic phylosophers* (2), we can do an abstraction of even and odd number following the construction of a graph. We can make a serial constitution of the odd numbers (2n+1) simply advancing diagonally in the 2-dimensional pyramid. (figure 1),



(figure 1)

On the other hand, we can also do a construction of the even numbers (2n) starting with the number two and advancing in this new type of gnomon's graph (figure 2).

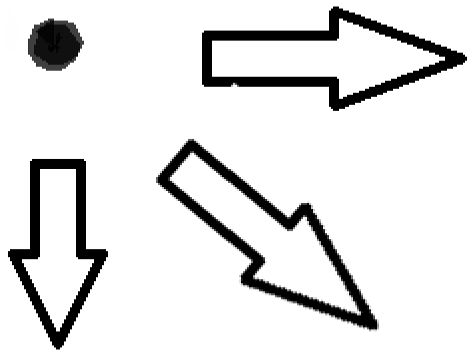


(figure 2)

Now we can start with the interesting part, as you see in figure 1, the sum of every individual level is $(n+1)^2$ this is because:

$$(1) \quad \sum_{n \in \mathbb{N}} (2n+1) = (n+1)^2$$

For example if we sum the first three levels we have: $1+3+5 = 9 = 3^2$. It is logical a simplification of this in the next graph (figure 3):

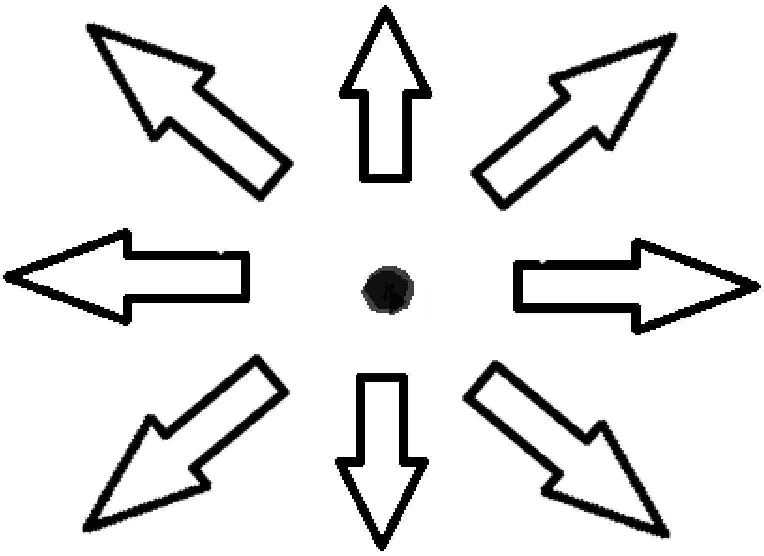


(figure 3)

In the next section you will see how it is possible to extend this to squares of even numbers and squares of odd numbers.

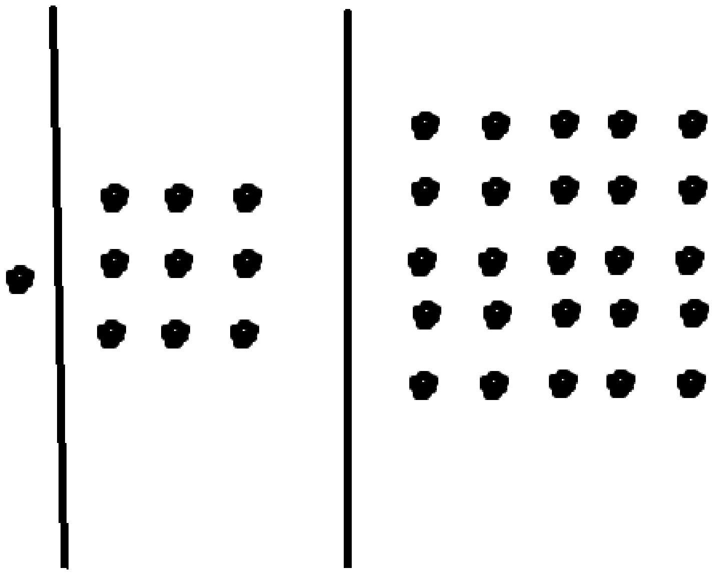
2- Gnomon for square even numbers and square odd numbers.

Following the logic of the figure 3, we can imagine an 8-arrow succession of dots. (figure 4)



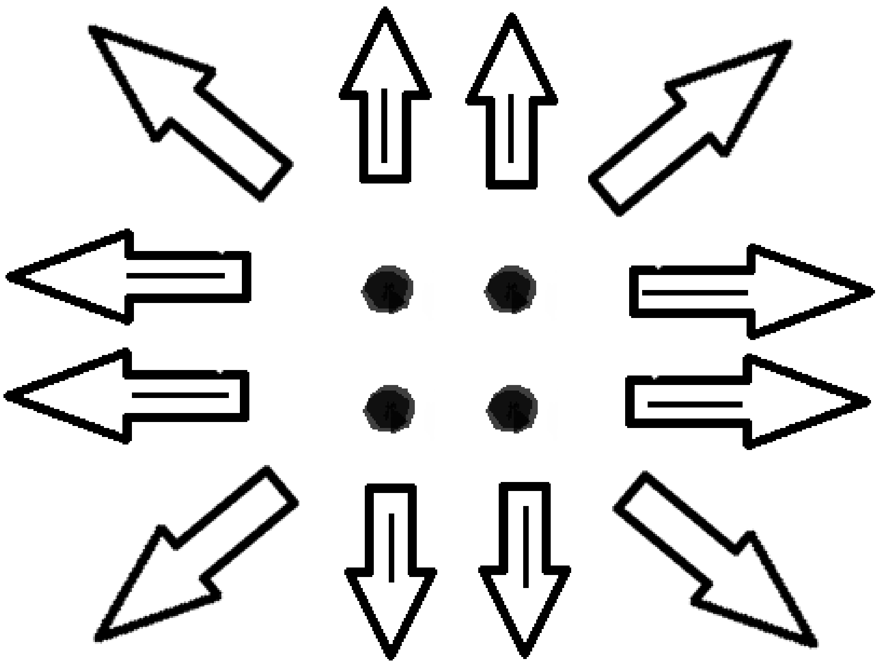
(figure 4)

I will show you the first three steps in figure 5, which represents 1, 9, 25. It is not very difficult to see that the next number which this gnomon will form is 49. It follows the $(2n+1)^2$ succession.



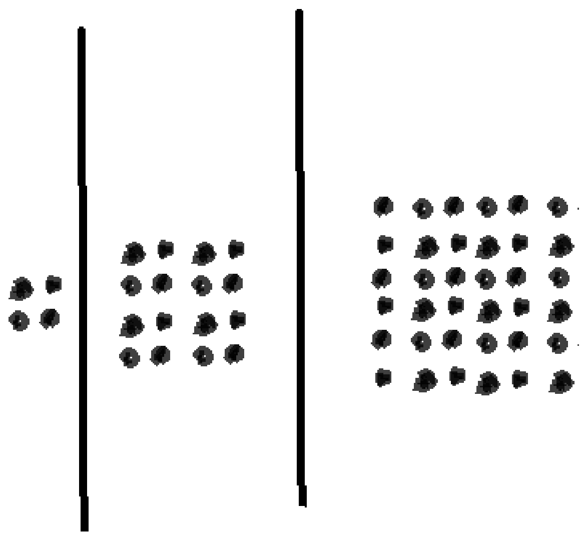
(figure 5)

I can also show how it is possible do the square of an even number but in this case is something a little more difficult, here we must distinguish between edges and sides. The edges (external part of the gnomon which are in a corner) follow he normal arithmetic succession but the sides (the rest of the external part) are accumulative. Lets see graphically, in the figure 6 we can see the process of accumulation of the sides represented with an inside line.



(figure 6)

As you can imagine, the dots grow in every directions. However in top, below, right and left directions it has accumulative properties. Lets see the first three examples in the last figure.



(figure 7)

As you can see if you count, in figure 7 we have $(2n)^2$.

3- Conclusions:

You have seen in this paper the approach of 2-dimensional geometry to the algebraic properties in an ancient method. Both graph representations for squares can follow until infinity.

4- References:

(1) https://en.wikipedia.org/wiki/Gnomon_(figure)
(2) G. S Kirk & J. E. Reven: *The presocratic phylosophers.*