Relations of Deterministics and Associated Stochastics in the Sense of an Ensemble Theory lead to many Solutions in Theoretical Physics

Rolf Warnemünde
E-Mail: rolf.warnemuende@t-online.de

Abstract

With the method of establishing a clear connection between deterministics and associated stochastics in terms of an ensemble theory Maxwell's equations are theoretically derived and a Geometrodynamics of collective turbulent motions is developed. This in turn leads to a unification of Maxwell's and Gravitational field as well as the explanation and emergence of photons.

Keywords— **Stochastics and deterministics, Maxwell's equations, unification of Maxwell's field and gravitational field, explanation of the photon**

Contents

1 Introduction

Basically deterministic processes can at best be represented stochastically in terms of an ensemble theory.This is the case, for example, in fluid-dynamic considerations of turbulence, whose motions are partly composed of molecular collective motions and additionally stochastic molecular motions.

The electrodynamic equations of the vacuum were developed in the 19th century after successful unification of electric and magnetic field. They are derived with the presented ensemble method solely from the assumption of continuously differentiable field quantities and a constant propagation velocity. The generalized Maxwell equations developed in this way correspond to equations of motion of general continuously differentiable vector fields and in the special case of elastic deformation to a generalization of non-linear phenomena.

In the context of general relativity, the fluctuations of the Einstein hypersurface can be interpreted as deformation fluctuations of a corresponding Euclidean observation space. For the deformation fluctuation equations to be applied then, a constant propagation velocity, the speed of light, must be assumed. Thus, alternative gravitational waves are found immediately. By using Einstein's equations, one is led to a unification of maxwell field and gravitational field. This connection is quantitative and shows the correspondence of space deformation fluctuation and electromagnetic wave.

Based on these considerations, the emergence of photon and gamma quanta can be understood, which is not possible within the framework of quantum field theory.

2 Stochastic and Deterministic General Vector Fields

$$\boxed{\begin{gathered} f_{t_\epsilon}(\vec{\mathbf{x}},t,\vec{\mathbf{E}},\vec{\mathbf{B}}) = \int\limits_{\vec{\mathbf{B}}}\int\limits_{\vec{\mathbf{E}}} W_{t_\epsilon}(\vec{\mathbf{x}},t,\vec{\mathbf{E}},\vec{\mathbf{B}},\vec{\mathbf{E}}',\vec{\mathbf{B}}')\cdot f_{t_\epsilon}(\vec{\mathbf{x}}-\Delta\vec{\mathbf{x}},t-t_\epsilon,\vec{\mathbf{E}}',\vec{\mathbf{B}}')d\vec{\mathbf{E}}'d\vec{\mathbf{B}}' \\ \Updownarrow \\ \frac{\partial}{\partial t}\vec{\mathbf{B}}-\vec{\nabla}\times\vec{\mathbf{E}}=0 \\ \frac{\partial}{\partial t}\left(\frac{B^2}{E^2}\cdot\vec{\mathbf{E}}\right)+\vec{\nabla}\times\vec{\mathbf{B}}=0 \end{gathered}}$$

2.1 Introduction

Subsequently continuum fluctuations of general 3 dimensional vector fields $\vec{\mathbf{A}}(\vec{\mathbf{x}},t)$ with $\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{A}} \neq \mathbf{0}$ are analysed. They have to be sufficiently often continuously differentiable. Defining the vector fields $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ by

$$\begin{aligned} \vec{\mathbf{E}} &= \partial\vec{\mathbf{A}}/\partial t \neq 0 \\ \vec{\mathbf{B}} &= \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{A}} \neq 0 \end{aligned} \tag{1}$$

and owing to the exchangeability of the operators $\partial/\partial t$ und $\vec{\boldsymbol{\nabla}}\times$

$$\frac{\partial\vec{\mathbf{B}}}{\partial t} = \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{E}} \tag{2}$$

follows. This is a necessary consequence of the condition of the continuous differentiability of $\vec{\mathbf{A}}(\vec{\mathbf{x}},t)$. This relation is known according to the Maxwell Equations. The for this purpose dual equation is subsequently beeing looked for. In an analogous approach derivating the turbulence equations a stochastic continuum process in the frame of an ensemble theory is formulated such that according to a deterministic theory the already known as well as the related dual equation arise with fluctuating quantities $\vec{\mathbf{E}}$ und $\vec{\mathbf{B}}$.

2.2 The Transition: Stochastic Theory $\longleftrightarrow$ Deterministic Theory

In the following the accuracies of the considered motion quantities are determined by t_ε-measurement processes. t_ε characterising the accuracy. Every space-time-point$(\vec{\mathbf{x}},t)$ a continuously differentiable distribution density $f_{\boldsymbol{t_\epsilon}}$ is assigned to the motion quantities $\vec{\mathbf{E}}_{t_\epsilon} = \partial\vec{\mathbf{A}}_{t_\epsilon}/\partial t$ and $\vec{\mathbf{B}}_{t_\epsilon} = \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{A}}_{t_\epsilon}$ with

$$f_{\boldsymbol{t_\epsilon}} = f_{\boldsymbol{t_\epsilon}}(\vec{\mathbf{x}},t,\vec{\mathbf{E}},\vec{\mathbf{B}}). \tag{3}$$

In the with $\boldsymbol{t_\epsilon}$ or $\boldsymbol{\epsilon}$ indexed functions $f_{\boldsymbol{t_\epsilon}}$ it is automatically assumed that the included motion quantities $(\vec{\mathbf{E}},\vec{\mathbf{B}})$ are assigned to a $\boldsymbol{t_\epsilon}$-measurement accuracy. The indexing of the motion quantities may be omitted in functions appropriately indexed themselves.

After the execution of a $\lim \boldsymbol{t_\epsilon} \to 0$-process

$$\lim_{\boldsymbol{t_\epsilon}\to 0} f_{\boldsymbol{t_\epsilon}}(\vec{\mathbf{x}},t,\vec{\mathbf{E}},\vec{\mathbf{B}}) = f(\vec{\mathbf{x}},t,\vec{\mathbf{E}},\vec{\mathbf{B}}) \tag{4}$$

f and $(\vec{\boldsymbol{E}},\vec{\boldsymbol{B}})$ are understood in the sense of an exact measurement process.

The stochastic transport of the fluctuation quantities

$$\Big(\vec{\mathbf{E}}'_{t_\epsilon}(\vec{\boldsymbol{x}} - \Delta\vec{\boldsymbol{x}}, t - t_\epsilon), \vec{\mathbf{B}}'_{t_\epsilon}(\vec{\boldsymbol{x}} - \Delta\vec{\boldsymbol{x}}, t - t_\epsilon)\Big) \longrightarrow \Big(\vec{\mathbf{E}}_{t_\epsilon}(\vec{\boldsymbol{x}},t), \vec{\mathbf{B}}_{t_\epsilon}(\vec{\boldsymbol{x}},t)\Big)$$

happens by the transition probability density $W_{t_\epsilon} = W_{t_\epsilon}(\vec{\boldsymbol{x}},t,\vec{\mathbf{E}},\vec{\mathbf{B}},\vec{\mathbf{E}}',\vec{\mathbf{B}}')$ with

$$\boxed{\begin{aligned} \lim_{t_\epsilon\to 0} W_{t_\epsilon} &= \delta(\vec{\mathbf{E}},\vec{\mathbf{B}};\vec{\mathbf{E}}',\vec{\mathbf{B}}') \\ f_{t_\epsilon}(\vec{\mathbf{x}},t,\vec{\mathbf{E}},\vec{\mathbf{B}}) &= \int\limits_{\vec{\mathbf{B}}'}\int\limits_{\vec{\mathbf{E}}'} W_{t_\epsilon}(\vec{\mathbf{x}},t,\vec{\mathbf{E}},\vec{\mathbf{B}},\vec{\mathbf{E}}',\vec{\mathbf{B}}') \cdot f_{t_\epsilon}(\vec{\mathbf{x}} - \Delta\vec{\mathbf{x}}, t - t_\epsilon, \vec{\mathbf{E}}',\vec{\mathbf{B}}')d\vec{\mathbf{E}}'d\vec{\mathbf{B}}' \\ \Delta\vec{\mathbf{x}} &= t_\epsilon \cdot \vec{\mathbf{E}}' \times \frac{\vec{\mathbf{B}}'}{B'^2} \quad \text{and } \vec{\mathbf{E}}' \times \frac{\vec{\mathbf{B}}'}{B'^2} = \text{velocity of propagation.} \end{aligned}} \tag{5}$$

These equations define stochastic continuum fluctuations of the quantities $\vec{\mathbf{E}}$ und $\vec{\mathbf{B}}$ in the sense of an ensemble-theory and represent a Markov Process of natural causality.

[1]

The test-functions of distribution theory obtain by this formulation of a transition probability density W_{t_ϵ} an immediate physical meaning.
f_{t_ϵ} is developed until the 1st order about $(\vec{\mathbf{x}},t) \Longrightarrow$

$$\begin{gathered} f_{t_\epsilon}(t-t_\epsilon,\vec{\mathbf{x}}-\triangle\vec{\mathbf{x}},\vec{\mathbf{E}}',\vec{\mathbf{B}}') = f'_{t_\epsilon} - \frac{\partial f'_{t_\epsilon}}{\partial t}\cdot t_\epsilon - \triangle\vec{\mathbf{x}}\cdot\vec{\boldsymbol{\nabla}} f'_{t_\epsilon} + \boldsymbol{O}({t_\epsilon}^2) \\ f'_{t_\epsilon} = f_{t_\epsilon}(\vec{\mathbf{x}},t,\vec{\mathbf{E}}',\vec{\mathbf{B}}') \end{gathered} \tag{8}$$

und one gets

$$\int_{\vec{\mathbf{E}}}\int_{\vec{\mathbf{B}}} W_{t_\epsilon}\left[\frac{\partial f'_{t_\epsilon}}{\partial t} + \vec{\mathbf{E}}'\times\frac{\vec{\mathbf{B}}'}{B'^2}\cdot\vec{\boldsymbol{\nabla}} f'_{t_\epsilon}\right] d\vec{\mathbf{E}}' d\vec{\mathbf{B}}' + \boldsymbol{O}({t_\epsilon}^2) = \frac{\int_{\vec{\mathbf{B}}}\int_{\vec{\mathbf{E}}} W_{t_\epsilon} f'_{t_\epsilon} d\vec{\mathbf{E}}' d\vec{\mathbf{B}}' - f_{t_\epsilon}}{t_\epsilon}. \tag{9}$$

By the process $t_\epsilon \to 0$ W_{t_ϵ} degenerates to a δ-function:

$$\lim_{\boldsymbol{t_\epsilon \to 0}} W_{t_\epsilon} = \delta(\vec{\mathbf{E}},\vec{\mathbf{B}};\vec{\mathbf{E}}',\vec{\mathbf{B}}') \tag{10}$$

$\lim \boldsymbol{t_\epsilon} \to 0$ applied leads to

$$\frac{\partial f}{\partial t} + \vec{\mathbf{E}}\times\frac{\vec{\mathbf{B}}}{B^2}\cdot\vec{\boldsymbol{\nabla}} f = \lim_{t_\epsilon\to 0}\frac{\int_{\vec{\mathbf{E}}}\int_{\vec{\mathbf{B}}} W_{t_\epsilon} f'_{t_\epsilon} d\vec{\mathbf{E}}' d\vec{\mathbf{B}}' - f_{t_\epsilon}}{t_\epsilon}. \tag{11}$$

Recovering equation (2) after the transition to deterministic consideration the exchange term has to vanish, in this case.

$$\lim_{t_\epsilon\to 0}\frac{\int_{\vec{\mathbf{B}}}\int_{\vec{\mathbf{E}}} W_{t_\epsilon} f'_{t_\epsilon} d\vec{\mathbf{E}}' d\vec{\mathbf{B}}' - f_{t_\epsilon}}{t_\epsilon} = \mathbf{0}. \tag{12}$$

This link is an integral part of the considered stochastic process.
Limiting ourselves to one system ν of the ensemble the function $f(\vec{\mathbf{x}},t,\vec{\mathbf{E}},\vec{\mathbf{B}})$ in the space-time-point $(\vec{\boldsymbol{x}},t)$ degenerates to a $\delta-$function

$$f(\vec{\mathbf{x}},t,\vec{\mathbf{E}},\vec{\mathbf{B}}) \longrightarrow \delta(\vec{\mathbf{E}}_{(\vec{\mathbf{x}},t,\nu)},\vec{\mathbf{B}}_{(\vec{\mathbf{x}},t,\nu)};\vec{\mathbf{E}},\vec{\mathbf{B}})\text{-function}. \tag{13}$$

From equation (11) arises the key-equation

$$\boxed{\frac{\partial}{\partial t}\delta + \vec{\mathbf{E}}_{(\vec{\boldsymbol{x}},t,\nu)}\times\frac{\vec{\mathbf{B}}_{(\vec{\boldsymbol{x}},t,\nu)}}{B^2_{(\vec{\boldsymbol{x}},t,\nu)}}\cdot\vec{\boldsymbol{\nabla}}\delta = \mathbf{0}}. \tag{14}$$

Respectively subsection 3.2.2 the $\boldsymbol{\Xi}[...]$-operator is inserted as follows

$$\begin{aligned} \boldsymbol{\Xi}\left[\int_{\vec{\mathbf{E}}}\int_{\vec{\mathbf{B}}}\delta(\vec{\mathbf{B}}_{(\vec{\boldsymbol{x}},t,\nu)},\vec{\mathbf{E}}_{(\vec{\boldsymbol{x}},t,\nu)};\vec{\mathbf{B}},\vec{\mathbf{E}})\vec{\mathbf{B}} d\vec{\mathbf{B}} d\vec{\mathbf{E}}\right] &= \boldsymbol{\Xi}[\vec{\mathbf{B}}_{(\vec{\boldsymbol{x}},t,\nu)}] = \vec{\mathbf{B}}(\vec{\boldsymbol{x}},t) \\ \boldsymbol{\Xi}\left[\int_{\vec{\mathbf{E}}}\int_{\vec{\mathbf{B}}}\delta(\vec{\mathbf{B}}_{(\vec{\boldsymbol{x}},t,\nu)},\vec{\mathbf{E}}_{(\vec{\boldsymbol{x}},t,\nu)};\vec{\mathbf{B}},\vec{\mathbf{E}})\vec{\mathbf{E}} d\vec{\mathbf{B}} d\vec{\mathbf{E}}\right] &= \boldsymbol{\Xi}\left[\vec{\mathbf{E}}_{(\vec{\boldsymbol{x}},t,\nu)}\right] = \vec{\mathbf{E}}(\vec{\boldsymbol{x}},t) \end{aligned} \tag{15}$$

[1]The usual Markov process is used analogously to

$$\rho(\vec{\mathbf{x}},t) = \int_{V'} G(\vec{\mathbf{x}},t;\vec{\mathbf{x}}',t')\rho(\vec{\mathbf{x}}',t')d\vec{\mathbf{x}}' \tag{6}$$

and the Green function

$$G(\vec{\mathbf{x}},t;\vec{\mathbf{x}}',t') = \left(\frac{1}{4\pi D(t-t')}\right)^{\frac{3}{2}} \boldsymbol{e}^{-\frac{(\vec{\mathbf{x}}-\vec{\mathbf{x}}')^2}{4\pi D(t-t')}} \quad . \tag{7}$$

A movement process from $(\vec{\mathbf{x}}',t')$ to $(\vec{\mathbf{x}},t)$ is unknown and can therefore lead to inconsistencies.

or

$$\mathbf{\Xi}\left[\int_{\vec{\mathbf{E}}}\int_{\vec{\mathbf{B}}}\delta(\vec{\mathbf{B}}_{(\vec{x},t,\nu)},\vec{\mathbf{E}}_{(\vec{x},t,\nu)};\vec{\mathbf{b}},\vec{\mathbf{E}})\left(\frac{B^2}{E^2}\cdot\vec{\mathbf{E}}\right)d\vec{\mathbf{B}}d\vec{\mathbf{E}}\right]=\mathbf{\Xi}\left[\frac{B^2_{(\vec{x},t,\nu)}}{E^2_{(\vec{x},t,\nu)}}\cdot\vec{\mathbf{E}}_{(\vec{x},t,\nu)}\right]=\frac{B^2(\vec{x},t)}{E^2(\vec{x},t)}\cdot\vec{\mathbf{E}}(\vec{x},t), \tag{16}$$

developing the deterministic equations from the key equation.

2.3 The Deterministic Fluctuation-Equations

The key-equation (14) represents the interface for the transition of stochastic to deterministic consideration. From the perspective of statistics over the states of movement of the parallelly assumed deterministic processes in the respective point $(\vec{\mathbf{x}},t)$ one is confined to a single system and such to a single state of motion $(\vec{\mathbf{E}}_{(\vec{\mathbf{x}},t)},\vec{\mathbf{B}}_{(\vec{\mathbf{x}},t)})$. In this situation the vectors of the motion quantities may be pushed before and behind the differential operators

$$\begin{aligned}\vec{\mathbf{E}}_{(\vec{x},t,\nu)}\times\frac{\vec{\mathbf{B}}_{(\vec{x},t,\nu)}}{B^2_{(\vec{x},t,\nu)}}\cdot\vec{\boldsymbol{\nabla}}\delta &= -\frac{\vec{\mathbf{B}}_{(\vec{x},t,\nu)}}{B^2_{(\vec{x},t,\nu)}}\times\vec{\mathbf{E}}_{(\vec{x},t,\nu)}\cdot\vec{\boldsymbol{\nabla}}\delta\\ &= -\frac{\vec{\mathbf{B}}_{(\vec{x},t,\nu)}}{B^2_{(\vec{x},t,\nu)}}\cdot\vec{\boldsymbol{\nabla}}\times\vec{\mathbf{E}}_{(\vec{x},t,\nu)}\delta\end{aligned}$$

Further more there is

$$\begin{aligned}\frac{\partial}{\partial t}(\frac{\vec{\mathbf{B}}_{(\vec{x},t,\nu)}\cdot\vec{\mathbf{B}}_{(\vec{x},t,\nu)}}{B^2_{(\vec{x},t,\nu)}}\delta)-\frac{\vec{\mathbf{B}}_{(\vec{x},t,\nu)}}{B^2_{(\vec{x},t,\nu)}}\cdot\vec{\boldsymbol{\nabla}}\times(\vec{\mathbf{E}}_{(\vec{x},t,\nu)}\delta)=0&\\ \Longrightarrow\ \frac{\vec{\mathbf{B}}_{(\vec{x},t,\nu)}}{B^2_{(\vec{x},t,\nu)}}\cdot[\frac{\partial}{\partial t}(\vec{\mathbf{B}}_{(\vec{x},t,\nu)}\delta)-\vec{\boldsymbol{\nabla}}\times(\vec{\mathbf{E}}_{(\vec{x},t,\nu)}\delta)]=0&\\ \Longrightarrow\ \frac{\partial}{\partial t}(\vec{\mathbf{B}}_{(\vec{x},t,\nu)}\delta)-\vec{\boldsymbol{\nabla}}\times(\vec{\mathbf{E}}_{(\vec{x},t,\nu)}\delta)=0.&\end{aligned} \tag{17}$$

Now the vector fields of the motion quantities $(\vec{\mathbf{E}}_{(\vec{x},t,\nu)},\ \vec{\mathbf{B}}_{(\vec{x},t,\nu)})$ of the one determinstic system are created about the point $(\vec{x},t)$ and such the transition to the deterministic equations of the one system has succeeded.
One obtains

$$\mathbf{\Xi}\left[\int_{\vec{\mathbf{B}}}\int_{\vec{\mathbf{E}}}\left[\frac{\partial}{\partial t}(\vec{\mathbf{B}}_{(\vec{x},t,\nu)}\delta)-\vec{\boldsymbol{\nabla}}\times(\vec{\mathbf{E}}_{(\vec{x},t,\nu)}\delta)=0\right]d\vec{\mathbf{E}}d\vec{\mathbf{B}}\right]. \tag{18}$$

As integration and differentiation are exchangeable $\Longrightarrow$

$$\frac{\partial}{\partial t}\mathbf{\Xi}[\vec{\mathbf{B}}_{(\vec{x},t,\nu)}]-\vec{\boldsymbol{\nabla}}\times\mathbf{\Xi}[\vec{\mathbf{E}}_{(\vec{x},t,\nu)}]=0 \tag{19}$$

and it results in the 1.st of the dual fluctuation equations

$$\frac{\partial}{\partial t}\vec{\mathbf{B}}-\vec{\boldsymbol{\nabla}}\times\vec{\mathbf{E}}=0. \tag{20}$$

Hereby the stochastic-deterministic connection is established.
Back to the key-equation (14)

$$\frac{\partial}{\partial t}\delta+\vec{\mathbf{E}}_{(\vec{x},t,\nu)}\times\frac{\vec{\mathbf{B}}_{(\vec{x},t,\nu)}}{B^2_{(\vec{x},t,\nu)}}\cdot\vec{\boldsymbol{\nabla}}\delta=\mathbf{0}$$

one obtains by simple conversion

$$\begin{aligned}\frac{\partial}{\partial t}\left(\vec{\mathbf{E}}_{(\vec{x},t,\nu)}\cdot\frac{\vec{\mathbf{E}}_{(\vec{x},t,\nu)}}{E^2_{(\vec{x},t,\nu)}}\delta\right)+\vec{\mathbf{E}}_{(\vec{x},t,\nu)}\cdot\vec{\boldsymbol{\nabla}}\times\left(\frac{\vec{\mathbf{B}}_{(\vec{x},t,\nu)}}{B^2_{(\vec{x},t,\nu)}}\delta\right)=0&\\ \frac{\partial}{\partial t}\left(\frac{B^2_{(\vec{x},t,\nu)}}{E^2_{(\vec{x},t,\nu)}}\cdot\vec{\mathbf{E}}_{(\vec{x},t,\nu)}\delta\right)+\vec{\boldsymbol{\nabla}}\times(\vec{\mathbf{B}}_{(\vec{x},t,\nu)}\delta)=0&\end{aligned} \tag{21}$$

and

$$\boldsymbol{\Xi}\left[\int_{\vec{\mathbf{B}}}\int_{\vec{\mathbf{E}}}\left[\frac{\partial}{\partial t}\left(\frac{B^2_{(\vec{x},t,\nu)}}{E^2_{(\vec{x},t,\nu)}}\cdot\vec{\mathbf{E}}_{(\vec{x},t,\nu)}\delta\right)+\vec{\boldsymbol{\nabla}}\times(\vec{\mathbf{B}}_{(\vec{x},t,\nu)}\delta)=0\right]d\vec{\mathbf{E}}d\vec{\mathbf{B}}\right] \tag{22}$$

respectively

$$\frac{\partial}{\partial t}\boldsymbol{\Xi}\left[\frac{B^2_{(\vec{x},t,\nu)}}{E^2_{(\vec{x},t,\nu)}}\cdot\vec{\mathbf{E}}_{(\vec{x},t,\nu)}\right]+\vec{\boldsymbol{\nabla}}\times\boldsymbol{\Xi}[\vec{\mathbf{B}}_{(\vec{x},t,\nu)}]=0. \tag{23}$$

So we have the second of the two dual equations

$$\frac{\partial}{\partial t}(\frac{B^2}{E^2}\cdot\vec{\mathbf{E}})+\vec{\boldsymbol{\nabla}}\times(\vec{\mathbf{B}})=0. \tag{24}$$

The result is recapitulated by the following equation system:

$$\boxed{\begin{aligned}&\frac{\partial}{\partial t}\vec{\mathbf{B}}-\vec{\boldsymbol{\nabla}}\times\vec{\mathbf{E}}=0\\&\frac{\partial}{\partial t}\left(\frac{B^2}{E^2}\cdot\vec{\mathbf{E}}\right)+\vec{\boldsymbol{\nabla}}\times\vec{\mathbf{B}}=0\\&\vec{\mathbf{E}}\times\frac{\vec{\mathbf{B}}}{B^2}=\text{propagation speed}\end{aligned}} \tag{25}$$

with $|\vec{\mathbf{E}}\times\frac{\vec{\mathbf{B}}}{B^2}|\leq|\vec{\mathbf{E}}|\cdot|\frac{\vec{\mathbf{B}}}{B^2}|$. I.e. $\frac{E^2}{B^2}$ is not the quadratic propagation speed. Interestingly, this only becomes clear after the involvement of the stochastic ensemble theory.

The equation system (25) is in such general terms that the physical significance depends on the interpretation of the starting field $\vec{\mathbf{A}}$, the boundary conditions as well as on the initial conditions. Hereunder, a deformation vector field, the velocity vector field of turbulence motions or the fluctuations of any other continuously differentiable vector field may be understood. These equations possess with boundary- and suitable initial conditions exactly one solution after the theorem of Cauchy-Kowalewskaja [6]. This statement is at first restricted to the calculation of the fields $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$. Calculating the field $\vec{\mathbf{A}}$ with the mere knowledge of

$$\frac{\partial\vec{\mathbf{A}}}{\partial t}=\vec{\mathbf{E}} \tag{26}$$

is not possible in all cases. A negative example is the calculation of $\vec{\mathbf{v}}$ with the knowledge of $\frac{\partial\vec{\mathbf{v}}}{\partial t}$ related to turbulent velocity fluctuations as shown in section 3.4. However, in this case these relations are applied completing the turbulence equations.

2.3.1 Surfacelike Deformation-Fluctuations in 3-Dimensional Space

Let $\vec{\mathbf{d}}$ be a continuously differentiable deformation vector field defining an area and $\vec{\mathbf{b}}$ und $\vec{\mathbf{e}}$ the derived fields

$$\vec{\mathbf{e}}=\frac{\partial}{\partial t}\vec{\mathbf{d}},\qquad\vec{\mathbf{b}}=\vec{\boldsymbol{\nabla}}\times\vec{\mathbf{d}} \tag{27}$$

with

$$\begin{aligned}\vec{\mathbf{d}}(x,y,t)&=\Big(\mathbf{d_x}(x,y,t),\mathbf{d_y}(x,y,t),\mathbf{d_z}(x,y,t)\Big)\\\vec{\mathbf{e}}(x,y,t)&=\Big(\mathbf{e_x}(x,y,t),\mathbf{e_y}(x,y,t),\mathbf{e_z}(x,y,t)\Big)\\\vec{\mathbf{b}}(x,y,t)&=\Big(\mathbf{b_x}(x,y,t),\mathbf{b_y}(x,y,t),\mathbf{b_z}(x,y,t)\Big).\end{aligned} \tag{28}$$

Then the deformation is without loss of generality seen as deformation of the $\mathbf{x}-\mathbf{y}$-area. The equations of motion formally equal the equations of 3-dimensional fluctuations

$$\begin{aligned}&\frac{\partial}{\partial t}\vec{\mathbf{b}}-\vec{\boldsymbol{\nabla}}\times\vec{\mathbf{e}}=0\\&\frac{\partial}{\partial t}\left(\frac{b^2}{e^2}\cdot\vec{\mathbf{e}}\right)+\vec{\boldsymbol{\nabla}}\times\vec{\mathbf{b}}=0\\&\vec{\mathbf{e}}\times\frac{\vec{\mathbf{b}}}{b^2}=\text{propagation speed,}\end{aligned}\tag{29}$$

only, the operator $\vec{\boldsymbol{\nabla}}\times$ corresponds to

$$\vec{\boldsymbol{\nabla}}\times\vec{\mathbf{d}}=\begin{pmatrix}\partial d_z/\partial y\\-\partial d_z/\partial x\\\partial d_y/\partial x-\partial d_x\partial y\end{pmatrix}.\tag{30}$$

The solution uniquely succeeds by the initial conditions $\vec{\mathbf{b}}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t_0})$ and $\vec{\mathbf{e}}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t_0})$ according to the theorem of Cauchy-Kowalewskaya [6]. The solution for this area corresponds to a partial solution of a 3-dimensional complete solution. Physical material properties are not explicitly included in these equations. They have to be implicitly considered by initial and boundary conditions. Sole precondition is that the appropriate materials act continuously. It also means that the physical process has to be clarified enabling the corresponding initial and border conditions.

2.3.2 1-Dimensional Deformation-Fluctuations in 3-Dimensional Space

Let $\vec{\mathbf{d}}$ be a continuously differentiable deformation vector field defining a trajectory and $\vec{\mathbf{b}}$ und $\vec{\mathbf{e}}$ the derived fields

$$\vec{\mathbf{e}}=\frac{\partial}{\partial t}\vec{\mathbf{d}},\qquad\vec{\mathbf{b}}=\vec{\boldsymbol{\nabla}}\times\vec{\mathbf{d}}\tag{31}$$

with

$$\begin{aligned}\vec{\mathbf{d}}(\boldsymbol{x},\boldsymbol{t})=&(\mathbf{d_x}(\boldsymbol{x},\boldsymbol{t}),\mathbf{d_y}(\boldsymbol{x},\boldsymbol{t}),\mathbf{d_z}(\boldsymbol{x},\boldsymbol{t}))\\\vec{\mathbf{e}}(\boldsymbol{x},\boldsymbol{t})=&(\mathbf{e_x}(\boldsymbol{x},\boldsymbol{t}),\mathbf{e_y}(\boldsymbol{x},\boldsymbol{t}),\mathbf{e_z}(\boldsymbol{x},\boldsymbol{t}))\\\vec{\mathbf{b}}(\boldsymbol{x},\boldsymbol{t})=&(\mathbf{b_x}(\boldsymbol{x},\boldsymbol{t}),\mathbf{b_y}(\boldsymbol{x},\boldsymbol{t}),\mathbf{b_z}(\boldsymbol{x},\boldsymbol{t})).\end{aligned}\tag{32}$$

Then the deformation is without loss of generality seen as deformation of the $\mathbf{x}$-coordinate. The equations of motion formally equal the equations of 3-dimensional fluctuations

$$\begin{aligned}&\frac{\partial}{\partial t}\vec{\mathbf{b}}-\vec{\boldsymbol{\nabla}}\times\vec{\mathbf{e}}=0\\&\frac{\partial}{\partial t}\left(\frac{b^2}{e^2}\cdot\vec{\mathbf{e}}\right)+\vec{\boldsymbol{\nabla}}\times\vec{\mathbf{b}}=0\\&\vec{\mathbf{e}}\times\frac{\vec{\mathbf{b}}}{b^2}=\text{propagation speed,}\end{aligned}\tag{33}$$

only, the operator $\vec{\boldsymbol{\nabla}}\times$ corresponds to

$$\vec{\boldsymbol{\nabla}}\times\vec{\mathbf{d}}=\begin{pmatrix}&0\\-&\partial d_z/\partial x\\&\partial d_y/\partial x\end{pmatrix}.\tag{34}$$

This leads to the component equations

$$\begin{aligned}\partial b_y/\partial t=&-\partial e_z/\partial x\\\partial b_z/\partial x=&\partial e_y/\partial x\\[2ex]\partial[(b^2/e^2)\cdot e_y)]\partial t=&-\partial b_z/\partial x\\\partial[(b^2/e^2)\cdot e_z)]\partial t=&\partial b_y/\partial x\\[2ex]\vec{\mathbf{e}}\times\vec{\mathbf{b}}/b^2=&\text{propagation speed.}\end{aligned}\tag{35}$$

The x-component remains constant. The solution uniquely results from the initial conditions $\vec{\mathbf{b}}(\boldsymbol{x}, \boldsymbol{t_0})$ and $\vec{\mathbf{e}}(\boldsymbol{x}, \boldsymbol{t_0})$ according to the theorem of Cauchy-Kowalewskaya [6]. The solution for this 1-dimensional trajectory corresponds to a partial solution of a 3-dimensional complete solution. Physical material properties are not explicitly included in these equations. They have to be implicitly considered by initial and boundary conditions. Sole precondition is that the appropriate materials act continuously. It also means that the physical process has to be clarified enabling the corresponding initial and border conditions.

2.4 Derivation of the Vacuum Maxwell Equation

The propagation speed having the constant amount of light velocity one obtains the known equations of vacuum-electrodynamics in the coordinate system of the observer:

$$\boxed{\begin{aligned} &\frac{\partial}{\partial t}\vec{\mathbf{B}} - \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{E}} = 0 \\ &\frac{1}{c^2}\frac{\partial}{\partial t}\vec{\mathbf{E}} + \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{B}} = 0 \qquad \text{mit} \qquad \vec{\mathbf{E}} \perp \vec{\mathbf{B}} \\ &\vec{\mathbf{E}} \times \frac{\vec{\mathbf{B}}}{B^2} = \vec{\mathbf{c}} = \text{propagation speed of light.} \end{aligned}} \tag{36}$$

So the electrodynamic equations of vacuum are generally derived, too. Usually, they are seen in the above equations with $-\vec{\mathbf{E}}$. It is more than pure supposition, that they describe properties of space-time without a unification of General Relativity and electromagnetic field in vacuum having succeeded, though many physicists not least Einstein [8], Jordan [15] and many others having endeavoured.

There is still the explanation of the associated initial field $\vec{\mathbf{A}}$ it generally happening in the frame of vector potential considerations, without recognizing $\vec{\mathbf{A}}$ as definite physical object. Only by a direct comprehension of the vector potential the electromagnetic field may be explained without means of mechanical quantities.[2]

2.5 Summary

The first application of the ensemble method cumulates in the derivation of the vacuum maxwell equations solely from the assumption of the continuous differentiability of the maxwell field and a constant propagation velocity. So far, they have only been experimentally proven, but with very successful applications in many fields of physics. The hypothesis-free equations, except for the assumption of a continuum field and its constant propagation velocity, make the maxwell equations the most reliable equations in physics at present.

3 Deterministic Turbulent Mass-Transport

3.1 Introduction

Here a probabilistic theory of turbulent particle transport is considered from a stochastic ensemble consideration of an unlimited number of parallelly existent, deterministic continuum fluctuations. In 2 the relation of partial differential equations of deterministic continuum fluctuations to the stochastic ensemble-counterpart is established. The causal Markov Process matters, essentially. Its local description leads to two vector fields with a dual pair of coupled partial, quasilinear differential vector equations distinguishing between mass transport and transport of pure motion quantities $\partial\vec{\mathbf{A}}/\partial t$ and $\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{A}}$ of fluctuating vector fields $\vec{\mathbf{A}}$.

section 3.2: Turbulent motions have the local velocities $\vec{\mathbf{v}} = \vec{\boldsymbol{\omega}} \times \vec{\mathbf{r}}$ resulting in a dual equation system of a vortex field $\vec{\boldsymbol{\omega}}$ and a curvature vector field $\vec{\mathbf{b}}$. Including the underlying momentum equations (not Navier-Stokes-equations) this system is not yet complete.

section 2: The deterministic transport of pure motion quantities of sufficiently often continuously differentiable fields $\partial\vec{\mathbf{A}}/\partial t$ and $\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{A}}$ is examined leading to a pair of dual coupled vector equations. Depending on interpretation they may be viewed as deformation fluctuation-equations, as generalisations of the Maxwell Equations of vacuum

[2]Electrodynamics is introduced in physics via mechanical effects.

or applied as equations of $\partial\vec{\mathbf{v}}/\partial t$ and $\vec{\boldsymbol{\nabla}}\times\vec{\mathbf{v}}$ of the turbulent velocity field $\vec{\mathbf{v}}$.

section 3.4: After a discussion of possible momentum equations as foundation for turbulence-calculations the results of section 3.2 and 2 are combined to a complete turbulence-equation system. This system consists of 12 equations with 12 unknowns.[3] From an initial velocity field $\vec{\mathbf{v}}(\vec{\boldsymbol{x}},t_0)$ and its partial, temporal derivation $\frac{\partial}{\partial t}\vec{\mathbf{v}}(\vec{\boldsymbol{x}},t)|_{t_0}$ the further evolution of the velocity field, its related vortex- and curvature fields as well as the accelleration field $\vec{\mathbf{q}}$ operating in the turbulence field may be calculated.[4] The accelleration field generally is not conservative meaning $\vec{\boldsymbol{\nabla}}\times\vec{\mathbf{q}}\neq\mathbf{0}$. Matter density distributions may be determined via the continuum-equation in the frame of a subsequent evaluation in consequence of thermodynamic state quantities beeing computable (as far as a local thermodynamic equilibrium is existent, which, however, is questionable in the case of turbulence).

3.2 The Connection of Deterministic Turbulence and its Stochastic Interpretation in Terms of an Ensemble Theory

$$\boxed{\begin{array}{c} f_{t_\epsilon}(t,\vec{\mathbf{x}},\vec{\omega},\vec{\mathbf{r}})=\displaystyle\int\limits_{\vec{\omega}'}\int\limits_{\vec{r}'} W_{t_\epsilon}(t,\vec{\mathbf{x}},\vec{\omega},\vec{\mathbf{r}},\vec{\omega}',\vec{r}')\cdot f_{t_\epsilon}(\vec{\mathbf{x}}-\Delta\vec{\mathbf{x}},t-t_\epsilon,\vec{\omega}',\vec{\mathbf{r}}')d\vec{\omega}'d\vec{\mathbf{r}}' \\ \Updownarrow \\ \dfrac{\partial}{\partial t}\vec{\omega}-\vec{\boldsymbol{\nabla}}\times\vec{\mathbf{a}}-\dfrac{1}{2}\vec{\boldsymbol{\nabla}}\times\vec{\mathbf{q}}=0 \\ \dfrac{\partial}{\partial t}\vec{\mathbf{b}}+\vec{\boldsymbol{\nabla}}\times\vec{\omega}-\dfrac{1}{2}\vec{\mathbf{b}}\left[\dfrac{\vec{\omega}}{\omega^2}\cdot\vec{\boldsymbol{\nabla}}\times\vec{\mathbf{q}}\right]=0 \end{array}}$$

3.2.1 Introduction

A stochastical ensemble-consideration of deterministic fields is understood as the examination of an unlimited number of comparable, parallelly existent systems, analogously to section 2.2. In this case turbulently moved fluids are examined considering statistical deliberations and its deterministic counterparts. That a linking of deterministic and stochastic theory may be available and further more that out of this connection additionally important (sometimes otherwise not known) relations arise for deterministic formulations, is shown in the following. This is discussed for a turbulent mass transport.

3.2.2 The Transition: Stochastic Theory $\longleftrightarrow$ Deterministic Theory

Every space-time-point $(\vec{\boldsymbol{x}},t)$ a continuously differentiable fluid element distribution over the motion quantities $\vec{\omega}_{t_\epsilon}$ and $\vec{\mathbf{r}}_{t_\epsilon}$ is assigned according to

$$f_{\boldsymbol{t_\epsilon}}=f_{\boldsymbol{t_\epsilon}}(\vec{\boldsymbol{x}},t,\vec{\boldsymbol{\omega}},\vec{\mathbf{r}}). \tag{37}$$

Indexing functions with $\boldsymbol{t_\epsilon}$ it is automatically assumed that the included motion quantities $(\vec{\boldsymbol{\omega}},\vec{\mathbf{r}})$ are assigned to a $\boldsymbol{t_\epsilon}$-measurement accuracy. The indexing of the motion quantities may be omitted in the functions if the functions are accordingly indexed.

After an execution of a $\lim \boldsymbol{t_\epsilon}\to 0$ process, such as

$$\lim_{\boldsymbol{t_\epsilon}\to 0} f_{\boldsymbol{t_\epsilon}}(\vec{\boldsymbol{x}},t,\vec{\boldsymbol{\omega}},\vec{\boldsymbol{r}})=f(\vec{\boldsymbol{x}},t,\vec{\boldsymbol{\omega}},\vec{\boldsymbol{r}}) \tag{38}$$

f and $(\vec{\boldsymbol{\omega}},\vec{\mathbf{r}})$ are understood as results of an exact measuring process.

The change of motion quantities in point $(\vec{\boldsymbol{x}},t)$

$$\Big(\vec{\boldsymbol{\omega}}'_{t_\epsilon}(\vec{\boldsymbol{x}}-\Delta\vec{\boldsymbol{x}},t-t_\epsilon),\vec{\boldsymbol{r}}'_{t_\epsilon}(\vec{\boldsymbol{x}}-\Delta\vec{\boldsymbol{x}},t-t_\epsilon)\Big)\longrightarrow\Big(\vec{\boldsymbol{\omega}}_{t_\epsilon}(\vec{\boldsymbol{x}},t),\vec{\boldsymbol{r}}_{t_\epsilon}(\vec{\boldsymbol{x}},t)\Big)$$

[3] The Einstein Equations of General Relativity consist of 10 equations. Suitable evolution equations with initial- and possibly boundary conditions remain troublesome in a 3+1-geometry.

[4] As there are only motion quantities in this equation system, it is successfully used for evolution problems in General Relativity.

is controlled by the transition probability density $W_{t_\epsilon} = W_{t_\epsilon}(\vec{\boldsymbol{x}}, t, \vec{\boldsymbol{\omega}}, \vec{\boldsymbol{r}}, \vec{\boldsymbol{\omega}}', \vec{\boldsymbol{r}}')$. [5] with

$$\boxed{\begin{aligned} \lim_{t_\epsilon \to 0} W_{t_\epsilon} &= \delta(\vec{\boldsymbol{\omega}}, \vec{\mathbf{r}}; \vec{\boldsymbol{\omega}}', \vec{\mathbf{r}}') \\ f_{t_\epsilon}(\vec{\mathbf{x}}, t, \vec{\boldsymbol{\omega}}, \vec{\mathbf{r}}) &= \int\limits_{\vec{\mathbf{r}}} \int\limits_{\vec{\boldsymbol{\omega}}} W_{t_\epsilon}(\vec{\mathbf{x}}, t, \vec{\boldsymbol{\omega}}, \vec{\mathbf{r}}, \vec{\boldsymbol{\omega}}', \vec{\mathbf{r}}') \cdot f_{t_\epsilon}(\vec{\mathbf{x}} - \Delta\vec{\mathbf{x}}, t - t_\epsilon, \vec{\boldsymbol{\omega}}', \vec{\mathbf{r}}') d\vec{\boldsymbol{\omega}}' d\vec{\mathbf{r}}' \\ \Delta\vec{\mathbf{x}} &= t_\epsilon \cdot \vec{\boldsymbol{\omega}}' \times \vec{\mathbf{r}}' \end{aligned}}\,. \tag{39}$$

These equations characterize stochastic turbulence of the continuum in the frame of an ensemble theory and represent a Markov Process with natural causality.
f_{t_ϵ} is developed in (39) until the 1st order around $(\vec{\mathbf{x}}, t) \Longrightarrow$

$$f_{t_\epsilon}(\vec{\mathbf{x}} - \triangle\vec{\mathbf{x}}, t - t_\epsilon, \vec{\boldsymbol{\omega}}', \vec{\mathbf{r}}') = f_{t_\epsilon}(\vec{\mathbf{x}}, t, \vec{\boldsymbol{\omega}}', \vec{\mathbf{r}}') - \frac{\partial f'_{t_\epsilon}}{\partial t} \cdot t_\epsilon - \triangle\vec{\mathbf{x}} \cdot \vec{\boldsymbol{\nabla}} f_{t_\epsilon}(\vec{\mathbf{x}}, t, \vec{\boldsymbol{\omega}}', \vec{\mathbf{r}}') + \boldsymbol{O}({t_\epsilon}^2) \tag{40}$$

with $f'_{t_\epsilon} = f_{t_\epsilon}(\vec{\mathbf{x}}, t, \vec{\boldsymbol{\omega}}', \vec{\mathbf{r}}')$ and one obtains

$$\int_{\vec{\mathbf{r}}} \int_{\vec{\boldsymbol{\omega}}} W_{t_\epsilon}[\frac{\partial f'_{t_\epsilon}}{\partial t} + \vec{\boldsymbol{\omega}}' \times \vec{\mathbf{r}}' \cdot \vec{\boldsymbol{\nabla}} f'_{t_\epsilon}] d\vec{\boldsymbol{\omega}}' d\vec{\mathbf{r}}' + \boldsymbol{O}({t_\epsilon}^2) = \frac{\int_{\vec{\mathbf{r}}} \int_{\vec{\boldsymbol{\omega}}} W_{t_\epsilon} f'_{t_\epsilon} d\vec{\boldsymbol{\omega}}' d\vec{\mathbf{r}}' - f_{t_\epsilon}}{t_\epsilon}. \tag{41}$$

$\lim \boldsymbol{t_\epsilon} \to 0$ applied to (41) leads to

$$\frac{\partial f}{\partial t} + \vec{\boldsymbol{\omega}} \times \vec{\mathbf{r}} \cdot \vec{\boldsymbol{\nabla}} f = \lim_{t_\epsilon \to 0} \frac{\int_{\vec{\mathbf{r}}} \int_{\vec{\boldsymbol{\omega}}} W_{t_\epsilon} f'_{t_\epsilon} d\vec{\boldsymbol{\omega}}' d\vec{\mathbf{r}}' - f_{t_\epsilon}}{t_\epsilon}. \tag{42}$$

The right side must contain the characteristics of the turbulent fluid.

$$\lim_{t_\epsilon \to 0} \frac{\int_{\vec{\mathbf{r}}} \int_{\vec{\boldsymbol{\omega}}} W_{t_\epsilon} f'_{t_\epsilon} d\vec{\boldsymbol{\omega}}' d\vec{\mathbf{r}}' - f_{t_\epsilon}}{t_\epsilon} = F \tag{43}$$

F has to be chosen such, that the deterministic vortex equations result under the influence of the assumed acceleration field. Further on the ansatz

$$F = \frac{1}{2} \left[\frac{\vec{\boldsymbol{\omega}}}{\omega^2} \cdot \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{q}} \right] f \tag{44}$$

is shown precisely fulfilling this condition. Thus one obtains

$$\frac{\partial f}{\partial t} + \vec{\boldsymbol{\omega}} \times \vec{\mathbf{r}} \cdot \vec{\boldsymbol{\nabla}} f = \frac{1}{2} \left[\frac{\vec{\boldsymbol{\omega}}}{\omega^2} \cdot \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{q}} \right] f. \tag{45}$$

Limiting ourselves to one system of the ensemble the distribution function f degenerates to a δ-function.

$$f \to \delta(\vec{\boldsymbol{\omega}}_{(\vec{\boldsymbol{x}}, t, \nu)}, \vec{\mathbf{r}}_{(\vec{\boldsymbol{x}}, t, \nu)}; \vec{\boldsymbol{\omega}}, \vec{\mathbf{r}}) \tag{46}$$

The indexing of quantities like $\vec{\boldsymbol{\omega}}_{(\vec{\boldsymbol{x}}, t, \nu)}$ by $(\vec{\boldsymbol{x}}, t)$ means the vector $\vec{\boldsymbol{\omega}}$ in the space-time point $(\vec{\boldsymbol{x}}, t)$ [6] whereas $\vec{\boldsymbol{\omega}}(\vec{\boldsymbol{x}}, t)$ represents the spatiotemporal field $\vec{\boldsymbol{\omega}}$ in dependence on $(\vec{\boldsymbol{x}}, t)$.
It results in the key equation for the transition stochastic-deterministic

$$\boxed{\frac{\partial}{\partial t}\delta + \vec{\boldsymbol{\omega}}_{(\vec{\boldsymbol{x}}, t, \nu)} \times \vec{\mathbf{r}}_{(\vec{\boldsymbol{x}}, t, \nu)} \cdot \vec{\boldsymbol{\nabla}}\delta = \frac{1}{2} \left[\frac{\vec{\boldsymbol{\omega}}_{(\vec{\boldsymbol{x}}, t, \nu)}}{\omega^2_{(\vec{\boldsymbol{x}}, t, \nu)}} \cdot \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{q}}_{(\vec{\boldsymbol{x}}, t, \nu)} \right] \delta}\,. \tag{47}$$

Definition of the operator $\boldsymbol{\Xi}[...]$:
From the vector $\vec{\mathbf{A}}_{(\vec{\boldsymbol{x}}, t, \nu)}$ respectively the scalar function value $\mathbf{f}_{(\vec{\boldsymbol{x}}, t, \nu)}$ existing in the space-time-point $(\vec{\boldsymbol{x}}, t)$ of the system ν a vector function respectively a scalar function arises by the operator $\boldsymbol{\Xi}$

$$\boldsymbol{\Xi} \left[\vec{\mathbf{A}}_{(\vec{\boldsymbol{x}}, t, \nu)} \right] = \vec{\mathbf{A}}(\vec{\boldsymbol{x}}, t) \tag{48}$$

[5]The otherwise in distribution theory used test functions in this connection have an immediate physical meaning with the formulation of the transition probability density.
[6]That is the situation considering stochastically.

respectively

$$\Xi\Big[\mathbf{f}_{(\vec{x},t,\nu)}\Big] = \mathbf{f}(\vec{x},t) \tag{49}$$

an appropriate field existing around the point $(\vec{x},t)$. The Operator $\Xi[...]$ evokes this functionality to "life". Accordingly the following relationships are noted:

$$\begin{aligned}
&\Xi\left[\int_{\vec{\mathbf{r}}}\int_{\vec{\boldsymbol{\omega}}} \delta(\vec{\boldsymbol{\omega}}_{(\vec{x},t,\nu)},\vec{\mathbf{r}}_{(\vec{x},t,\nu)};\vec{\boldsymbol{\omega}},\vec{\mathbf{r}})d\vec{\boldsymbol{\omega}}d\vec{\mathbf{r}}\right] = \mathbf{1}\\
&\Xi\left[\int_{\vec{\mathbf{r}}}\int_{\vec{\boldsymbol{\omega}}} \delta(\vec{\boldsymbol{\omega}}_{(\vec{x},t,\nu)},\vec{\mathbf{r}}_{(\vec{x},t,\nu)};\vec{\boldsymbol{\omega}},\vec{\mathbf{r}})\vec{\boldsymbol{\omega}}d\vec{\boldsymbol{\omega}}d\vec{\mathbf{r}}\right] = \Xi\Big[\vec{\boldsymbol{\omega}}_{(\vec{x},t,\nu)}\Big] = \vec{\boldsymbol{\omega}}(\vec{x},t)\\
&\Xi\left[\int_{\vec{\mathbf{r}}}\int_{\vec{\boldsymbol{\omega}}} \delta(\vec{\boldsymbol{\omega}}_{(\vec{x},t,\nu)},\vec{\mathbf{r}}_{(\vec{x},t,\nu)};\vec{\boldsymbol{\omega}},\vec{\mathbf{r}})\vec{\mathbf{r}}d\vec{\boldsymbol{\omega}}d\vec{\mathbf{r}}\right] = \Xi\Big[\vec{\mathbf{r}}_{(\vec{x},t,\nu)}\Big] = \vec{\mathbf{r}}(\vec{x},t)
\end{aligned} \tag{50}$$

or

$$\Xi\left[\int_{\vec{\mathbf{r}}}\int_{\vec{\boldsymbol{\omega}}} \delta(\vec{\boldsymbol{\omega}}_{(\vec{x},t,\nu)},\vec{\mathbf{r}}_{(\vec{x},t,\nu)};\vec{\boldsymbol{\omega}},\vec{\mathbf{r}})\omega^2\vec{\mathbf{r}}d\vec{\boldsymbol{\omega}}d\vec{\mathbf{r}}\right] = \Xi\Big[\omega^2_{(\vec{\mathbf{x}},t,\nu)}\vec{\mathbf{r}}_{(\vec{\mathbf{x}},t,\nu)}\Big] = \omega^2(\vec{\mathbf{x}},t)\vec{\mathbf{r}}(\vec{\mathbf{x}},t). \tag{51}$$

3.3 Deterministic Equations of Turbulence

From the general momentum equation

$$\frac{\partial\vec{\mathbf{v}}}{\partial t} + (\vec{\mathbf{v}}\cdot\vec{\boldsymbol{\nabla}})\vec{\mathbf{v}} = \vec{\boldsymbol{q}} \tag{52}$$

the vortex equation may be developed using the $\vec{\boldsymbol{\nabla}}\times$-operator

$$\frac{\partial}{\partial t}\vec{\boldsymbol{\omega}} - \vec{\boldsymbol{\nabla}}\times(\vec{\mathbf{v}}\times\vec{\boldsymbol{\omega}}) - \frac{1}{2}\vec{\boldsymbol{\nabla}}\times\vec{\boldsymbol{q}} = 0. \tag{53}$$

The relations of deterministic and stochastic description are established the same vortex equation opening up from the above key equation. In the following the method is presented designing the dual pair of deterministic vector equations from the key equation

$$\frac{\partial}{\partial t}\delta + \vec{\boldsymbol{\omega}}_{(\vec{x},t,\nu)}\times\vec{\mathbf{r}}_{(\vec{x},t,\nu)}\cdot\vec{\boldsymbol{\nabla}}\delta = \frac{1}{2}\left[\frac{\vec{\boldsymbol{\omega}}_{(\vec{x},t,\nu)}}{\omega^2_{(\vec{x},t,\nu)}}\cdot\vec{\boldsymbol{\nabla}}\times\vec{\mathbf{q}}_{(\vec{x},t,\nu)}\right]\delta. \tag{54}$$

In this situation the vectors of the motion quantities may be pushed before and behind the differential operators. The Term

$$\frac{1}{2}\left[\frac{\vec{\boldsymbol{\omega}}_{(\vec{x},t,\nu)}}{\omega^2_{(\vec{x},t,\nu)}}\cdot\vec{\boldsymbol{\nabla}}\times\vec{\mathbf{q}}_{(\vec{x},t,\nu)}\right]\delta \tag{55}$$

guarantees the finding of equation (53) and its dual one. It is

$$\vec{\mathbf{v}}\perp\vec{\boldsymbol{\omega}}\perp\vec{\mathbf{r}}. \tag{56}$$

and setting

$$\vec{\mathbf{a}} = \vec{\mathbf{v}}\times\vec{\boldsymbol{\omega}} \tag{57}$$

this results in

$$\vec{\mathbf{r}}\parallel\vec{\mathbf{a}}. \tag{58}$$

Such $\vec{\mathbf{a}}$ and $\vec{\mathbf{r}}$ are linked as follows[7]

$$\vec{\mathbf{r}} = \frac{\vec{\mathbf{a}}}{\omega^2}. \tag{59}$$

$\Longrightarrow$

$$\text{with} \qquad \delta = \delta(\vec{\boldsymbol{\omega}}_{(\vec{x},t,\nu)},\vec{\mathbf{r}}_{(\vec{x},t,\nu)};\vec{\boldsymbol{\omega}},\vec{\mathbf{r}})$$

[7] Symbols as ω, r, a, v etc. always mean amounts of the corresponding vectors.

$$\begin{aligned}\vec{\boldsymbol{\omega}}_{(\vec{\boldsymbol{x}},t,\nu)} \times \vec{\mathbf{r}}_{(\vec{\boldsymbol{x}},t,\nu)} \cdot \vec{\boldsymbol{\nabla}}\delta &= -\vec{\mathbf{r}}_{(\vec{\boldsymbol{x}},t,\nu)} \times \vec{\boldsymbol{\omega}}_{(\vec{\boldsymbol{x}},t,\nu)} \cdot \vec{\boldsymbol{\nabla}}\delta \\ &= -\vec{\boldsymbol{\omega}}_{(\vec{\boldsymbol{x}},t,\nu)} \cdot \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{r}}_{(\vec{\boldsymbol{x}},t,\nu)}\delta \\ &= -\frac{\vec{\boldsymbol{\omega}}_{(\vec{\boldsymbol{x}},t,\nu)}}{\omega^2_{(\vec{\boldsymbol{x}},t,\nu)}} \cdot \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{a}}_{(\vec{\boldsymbol{x}},t,\nu)}\delta.\end{aligned}$$

Inserting in (54) gives

$$\begin{aligned}\frac{\partial}{\partial t}\left(\frac{\vec{\boldsymbol{\omega}}_{(\vec{\boldsymbol{x}},t,\nu)} \cdot \vec{\boldsymbol{\omega}}_{(\vec{\boldsymbol{x}},t,\nu)}}{\omega^2_{(\vec{\boldsymbol{x}},t,\nu)}}\delta\right) - \frac{\vec{\boldsymbol{\omega}}_{(\vec{\boldsymbol{x}},t,\nu)}}{\omega^2_{(\vec{\boldsymbol{x}},t,\nu)}} \cdot \vec{\boldsymbol{\nabla}} \times (\vec{\mathbf{a}}_{(\vec{\boldsymbol{x}},t,\nu)}\delta) - \frac{1}{2}\left[\frac{\vec{\boldsymbol{\omega}}_{(\vec{\boldsymbol{x}},t,\nu)}}{\omega^2_{(\vec{\boldsymbol{x}},t,\nu)}} \cdot \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{q}}_{(\vec{\boldsymbol{x}},t,\nu)}\right]\delta = 0 \\ \Longrightarrow \frac{\vec{\boldsymbol{\omega}}_{(\vec{\boldsymbol{x}},t,\nu)}}{\omega^2_{(\vec{\boldsymbol{x}},t,\nu)}} \cdot \left[\frac{\partial}{\partial t}(\vec{\boldsymbol{\omega}}_{(\vec{\boldsymbol{x}},t,\nu)}\delta) - \vec{\boldsymbol{\nabla}} \times (\vec{\mathbf{a}}_{(\vec{\boldsymbol{x}},t,\nu)}\delta) - \frac{1}{2}\left[\cdot\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{q}}_{(\vec{\boldsymbol{x}},t,\nu)}\right]\delta\right] = 0 \\ \Longrightarrow \frac{\partial}{\partial t}(\vec{\boldsymbol{\omega}}_{(\vec{\boldsymbol{x}},t,\nu)}\delta) - \vec{\boldsymbol{\nabla}} \times (\vec{\mathbf{a}}_{(\vec{\boldsymbol{x}},t,\nu)}\delta) - \frac{1}{2}\left[\cdot\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{q}}_{(\vec{\boldsymbol{x}},t,\nu)}\right]\delta = 0\end{aligned} \tag{60}$$

One obtains

$$\boldsymbol{\Xi}\left[\int_{\vec{\mathbf{r}}}\int_{\vec{\boldsymbol{\omega}}}\left[\frac{\partial}{\partial t}(\vec{\boldsymbol{\omega}}_{(\vec{\boldsymbol{x}},t,\nu)}\delta) - \vec{\boldsymbol{\nabla}} \times (\vec{\mathbf{a}}_{(\vec{\boldsymbol{x}},t,\nu)}\delta) - \frac{1}{2}\left[\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{q}}_{(\vec{\boldsymbol{x}},t,\nu)}\right]\delta = 0\right]d\vec{\boldsymbol{\omega}}d\vec{\mathbf{r}}\right] \tag{61}$$

because integration and differentiation are interchangeable, it follows that

$$\left[\frac{\partial}{\partial t}\boldsymbol{\Xi}\left[\vec{\boldsymbol{\omega}}_{(\vec{\boldsymbol{x}},t,\nu)}\right] - \vec{\boldsymbol{\nabla}} \times \boldsymbol{\Xi}\left[\vec{\mathbf{a}}_{(\vec{\boldsymbol{x}},t,\nu)}\right] - \frac{1}{2}\vec{\boldsymbol{\nabla}} \times \boldsymbol{\Xi}\left[\vec{\mathbf{q}}_{(\vec{\boldsymbol{x}},t,\nu)}\right] = 0 \tag{62}$$

and we have the first of the dual turbulence equations

$$\frac{\partial}{\partial t}\vec{\boldsymbol{\omega}} - \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{a}} - \frac{1}{2}\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{q}} = 0 \tag{63}$$

accordingly

$$\frac{\partial}{\partial t}\vec{\boldsymbol{\omega}} - \vec{\boldsymbol{\nabla}} \times (\vec{\mathbf{v}} \times \vec{\boldsymbol{\omega}}) - \frac{1}{2}\vec{\boldsymbol{\nabla}} \times \vec{\boldsymbol{q}} = 0.$$

Hereby the connection of stochastics and deterministics is achieved. From the key-equation above a second equation, the dual one, may be derived.
Back to the initial equation (54)

$$\frac{\partial}{\partial t}\delta + \vec{\boldsymbol{\omega}}_{(\vec{\boldsymbol{x}},t,\nu)} \times \vec{\mathbf{r}}_{(\vec{\boldsymbol{x}},t,\nu)} \cdot \vec{\boldsymbol{\nabla}}\delta = \frac{1}{2}\left[\frac{\vec{\boldsymbol{\omega}}_{(\vec{\boldsymbol{x}},t,\nu)}}{\omega^2_{(\vec{\boldsymbol{x}},t,\nu)}} \cdot \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{q}}_{(\vec{\boldsymbol{x}},t,\nu)}\right]\delta$$

Simple conversions give

$$\begin{aligned}\frac{\partial}{\partial t}\left(\vec{\mathbf{r}}_{(\vec{\boldsymbol{x}},t,\nu)} \cdot \frac{\vec{\mathbf{r}}_{(\vec{\boldsymbol{x}},t,\nu)}}{r^2_{(\vec{\boldsymbol{x}},t,\nu)}}\delta\right) + \vec{\mathbf{r}}_{(\vec{\boldsymbol{x}},t,\nu)} \cdot \vec{\boldsymbol{\nabla}} \times (\vec{\boldsymbol{\omega}}_{(\vec{\boldsymbol{x}},t,\nu)}\delta) - \frac{\vec{\mathbf{r}}_{(\vec{\boldsymbol{x}},t,\nu)} \cdot \vec{\mathbf{r}}_{(\vec{\boldsymbol{x}},t,\nu)}}{r^2_{(\vec{\boldsymbol{x}},t,\nu)}}\frac{1}{2}\left[\frac{\vec{\boldsymbol{\omega}}_{(\vec{\boldsymbol{x}},t,\nu)}}{\omega^2_{(\vec{\boldsymbol{x}},t,\nu)}} \cdot \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{q}}_{(\vec{\boldsymbol{x}},t,\nu)}\right]\delta = 0 \\ \longrightarrow \vec{\boldsymbol{r}}_{(\vec{\boldsymbol{x}},t,\nu)}\left[\frac{\partial}{\partial t}\frac{\vec{\mathbf{r}}_{(\vec{\boldsymbol{x}},t,\nu)}}{r^2_{(\vec{\boldsymbol{x}},t,\nu)}}\delta + \vec{\boldsymbol{\nabla}} \times (\vec{\boldsymbol{\omega}}_{(\vec{\boldsymbol{x}},t,\nu)}\delta) - \frac{\vec{\mathbf{r}}_{(\vec{\boldsymbol{x}},t,\nu)}}{r^2_{(\vec{\boldsymbol{x}},t,\nu)}}\frac{1}{2}\left[\frac{\vec{\boldsymbol{\omega}}_{(\vec{\boldsymbol{x}},t,\nu)}}{\omega^2_{(\vec{\boldsymbol{x}},t,\nu)}} \cdot \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{q}}_{(\vec{\boldsymbol{x}},t,\nu)}\right]\delta\right] = 0\end{aligned} \tag{64}$$

Using the curvature vector field of the fluid trajectories $\vec{\mathbf{b}} = \frac{\vec{\mathbf{r}}}{r^2}$ the equation is written

$$\frac{\partial}{\partial t}(\vec{\mathbf{b}}_{(\vec{\boldsymbol{x}},t,\nu)}\delta) + \vec{\boldsymbol{\nabla}} \times (\vec{\boldsymbol{\omega}}_{(\vec{\boldsymbol{x}},t,\nu)}\delta) - \frac{1}{2}\vec{\mathbf{b}}_{(\vec{\boldsymbol{x}},t,\nu)}\frac{\vec{\boldsymbol{\omega}}_{(\vec{\boldsymbol{x}},t,\nu)}}{\omega^2_{(\vec{\boldsymbol{x}},t,\nu)}} \cdot \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{q}}_{(\vec{\boldsymbol{x}},t,\nu)}\delta = 0 \tag{65}$$

and applying the operators $\boldsymbol{\Xi}$ arises

$$\boldsymbol{\Xi}\left[\int_{\vec{\mathbf{r}}}\int_{\vec{\boldsymbol{\omega}}}\left[\frac{\partial}{\partial t}(\vec{\mathbf{b}}_{(\vec{\boldsymbol{x}},t,\nu)}\delta) + \vec{\boldsymbol{\nabla}} \times (\vec{\boldsymbol{\omega}}_{(\vec{\boldsymbol{x}},t,\nu)}\delta) - \frac{1}{2}\vec{\mathbf{b}}_{(\vec{\boldsymbol{x}},t,\nu)}\frac{\vec{\boldsymbol{\omega}}_{(\vec{\boldsymbol{x}},t,\nu)}}{\omega^2_{(\vec{\boldsymbol{x}},t,\nu)}} \cdot \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{q}}_{(\vec{\boldsymbol{x}},t,\nu)}\delta = 0\right]d\vec{\boldsymbol{\omega}}d\vec{\mathbf{r}}\right] \tag{66}$$

respectively

$$\frac{\partial}{\partial t}\Xi[\vec{\mathbf{b}}_{(\vec{x},t,\nu)}] + \vec{\boldsymbol{\nabla}} \times \Xi[\vec{\boldsymbol{\omega}}_{(\vec{x},t,\nu)}] - \frac{1}{2}\Xi\left[\left(\vec{\mathbf{b}}\frac{\vec{\boldsymbol{\omega}}}{\omega^2} \cdot \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{q}}\right)_{(\vec{x},t,\nu)}\right] = 0. \tag{67}$$

Such the second of the dual turbulence equations is approached

$$\frac{\partial}{\partial t}\vec{\mathbf{b}} + \vec{\boldsymbol{\nabla}} \times \vec{\boldsymbol{\omega}} - \frac{1}{2}\vec{\mathbf{b}}\left[\frac{\vec{\boldsymbol{\omega}}}{\omega^2} \cdot \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{q}}\right] = 0. \tag{68}$$

Closing this dual equation system

$$\boxed{\begin{array}{l} \frac{\partial}{\partial t}\vec{\boldsymbol{\omega}} - \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{a}} - \frac{1}{2}\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{q}} = 0 \\ \frac{\partial}{\partial t}\vec{\mathbf{b}} + \vec{\boldsymbol{\nabla}} \times \vec{\boldsymbol{\omega}} - \frac{1}{2}\vec{\mathbf{b}}\left[\frac{\vec{\boldsymbol{\omega}}}{\omega^2} \cdot \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{q}}\right] = 0 \\ \vec{\mathbf{v}} = \vec{\boldsymbol{\omega}} \times \frac{\vec{\mathbf{b}}}{b^2}, \ \vec{\mathbf{a}} = \vec{\mathbf{v}} \times \vec{\boldsymbol{\omega}} \end{array}} \tag{69}$$

further equations are necessary besides the momentum equations. In the case of the Navier-Stokes-equations

$$\frac{\partial \vec{\mathbf{v}}}{\partial t} + (\vec{\mathbf{v}} \cdot \vec{\boldsymbol{\nabla}})\vec{\mathbf{v}} = -\frac{1}{\rho}\vec{\boldsymbol{\nabla}}\mathbf{p} + \vec{\mathbf{g}} + \nu\Delta\vec{\mathbf{v}} + (\zeta + \frac{\nu}{3})\vec{\boldsymbol{\nabla}}\ (\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{v}})$$

i.e.

$$\vec{\mathbf{q}} = -\frac{1}{\rho}\vec{\boldsymbol{\nabla}}\mathbf{p} + \vec{\mathbf{g}} + \nu\Delta\vec{\mathbf{v}} + (\zeta + \frac{\nu}{3})\vec{\boldsymbol{\nabla}}\ (\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{v}})$$

this could happen by simultaneously using the known continuity, energy as well as state equation. But this proves not to be expedient. In section 3.4 the complete equation system is presented and it is shown that the usual Navier-Stokes-equations are not warranting the correct momentum balancing in turbulence.
The term

$$-\frac{1}{2}\vec{\mathbf{b}}\left[\frac{\vec{\boldsymbol{\omega}}}{\omega^2} \cdot \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{q}}\right]$$

may lead to removable singularities in space-time-points $(\vec{\mathbf{x}}, t)$ when turning points occur in the fluid element trajectories $\vec{\boldsymbol{\omega}} = 0$ and $\vec{\mathbf{b}} = \mathbf{0}$ arising simultaneously. In this case the whole term is calculated from its surroundings. The same shall apply for the calculation of the velocity $\vec{\mathbf{v}}$. In such cases there is an alternative way shown in section 3.4, too.

3.4 Geometrodynamics of Turbulence

$$\boxed{\begin{array}{l} \vec{\mathbf{E}} + \frac{1}{2}\vec{\boldsymbol{\nabla}}\vec{\mathbf{v}}^2 - 2\vec{\mathbf{v}} \times \vec{\boldsymbol{\omega}} = \vec{\mathbf{q}} \\ \hline \frac{\partial}{\partial t}\vec{\boldsymbol{\omega}} - \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{a}} = \frac{1}{2}\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{q}} \\ \frac{\partial}{\partial t}\vec{\mathbf{b}} + \vec{\boldsymbol{\nabla}} \times \vec{\boldsymbol{\omega}} = \frac{1}{2}\vec{\mathbf{b}}\left[\frac{\vec{\boldsymbol{\omega}}}{\boldsymbol{\omega}^2} \cdot \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{q}}\right] \\ \frac{\partial}{\partial t}\vec{\mathbf{F}} = -2\vec{\boldsymbol{\nabla}} \times \vec{\boldsymbol{\omega}} \text{ mit } \vec{\mathbf{F}} = \frac{4\omega^2}{E^2} \cdot \vec{\mathbf{E}} \end{array}}$$

3.4.1 Introduction

For a fluctuating continuum field

$$\frac{d}{dt}\vec{\mathbf{v}}(\vec{\mathbf{x}},t) = \frac{\partial \vec{\mathbf{v}}}{\partial t} + (\vec{\mathbf{v}} \cdot \vec{\boldsymbol{\nabla}})\vec{\mathbf{v}} = \vec{\mathbf{q}}(\vec{\mathbf{x}},t) \tag{70}$$

may be formally comprehended as a momentum equation. As soon as hydrodynamics is involved where a local thermodynamic balance is assumed, the Eulerian equations

$$\vec{\mathbf{q}} \stackrel{?}{=} -\frac{1}{\rho}\vec{\boldsymbol{\nabla}}\mathbf{p} \tag{71}$$

are noted with the indication of the 2nd Newtonian law. They are only justified under restrictive rules like incompressibility of fluids or $\frac{1}{\rho}\vec{\boldsymbol{\nabla}}\mathbf{p} = \vec{\boldsymbol{\nabla}}\mathbf{h}$ (h=spec. enthalpy) and or negligible rubbing viscosity. So only limiting cases of fluid dynamics are characterized.

But generally, $\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{q}} \neq \mathbf{0}$ is to be presumed. $\vec{\mathbf{q}}$ is in contrast to Newtonian mechanics a non-conservative acceleration field. $\vec{\mathbf{q}}$ has transversal and longitudinal parts

$$\vec{\mathbf{q}} = \vec{\mathbf{q}}_{\perp} + \vec{\mathbf{q}}_{||}. \tag{72}$$

The same applies for the velocity field $\vec{\mathbf{v}}$

$$\vec{\mathbf{v}} = \vec{\mathbf{v}}_{\perp} + \vec{\mathbf{v}}_{||} = \vec{\boldsymbol{\omega}} \times \vec{\mathbf{R}}. \tag{73}$$

The disassembly of the velocity field is adequately taken into account by the development of the dual turbulence equation system. In the momentum equation (70) 12 unknowns are “hiddenly“ contained and with the turbulence equation only 9 coupled equations are available. For the field $\boldsymbol{\rho}\vec{\mathbf{q}}$ a disassembly in longitudinal und transversal part has to be considered, too.

$$\boldsymbol{\rho}\frac{d}{dt}\vec{\mathbf{v}}(\vec{\mathbf{x}},t) = \boldsymbol{\rho}\vec{\mathbf{q}} = (\boldsymbol{\rho}\vec{\mathbf{q}})_{\perp} + (\boldsymbol{\rho}\vec{\mathbf{q}})_{||} \tag{74}$$

Using the Navier-Stokes-equations leads to

$$\boldsymbol{\rho}\vec{\mathbf{q}} = (\boldsymbol{\rho}\vec{\mathbf{q}})_{\perp} + (\boldsymbol{\rho}\vec{\mathbf{q}})_{||} \stackrel{?}{=} -\vec{\boldsymbol{\nabla}}\mathbf{p} + \boldsymbol{\rho}\cdot\vec{\mathbf{g}} + \eta\Delta\vec{\mathbf{v}} + (\xi + \frac{\eta}{3})\vec{\boldsymbol{\nabla}}\ (\vec{\boldsymbol{\nabla}}\cdot\vec{\mathbf{v}})$$

$\Longrightarrow$ [8]

$$(\boldsymbol{\rho}\vec{\mathbf{q}})_{\perp} \stackrel{?}{=} -\eta\vec{\boldsymbol{\nabla}} \times \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{v}} \tag{75}$$

and

$$(\boldsymbol{\rho}\vec{\mathbf{q}})_{||} \stackrel{?}{=} -\vec{\boldsymbol{\nabla}}\mathbf{p} + \boldsymbol{\rho}\cdot\vec{\mathbf{g}} + (\xi + \eta\frac{4}{3})\vec{\boldsymbol{\nabla}}\ (\vec{\boldsymbol{\nabla}}\cdot\vec{\mathbf{v}}). \tag{76}$$

$\vec{\mathbf{g}}$ =earth acceleration

As turbulent motions of sufficiently high reynolds number create negligible viscosity effects and on the other hand $\vec{\mathbf{q}}_{\perp}$ represents the decisive propulsion of the vortex motions turbulences are not correctly calculated by the usual equation system consisting of Navier-Stokes-equations, equation of continuity and energy equation. Equation (75) can not be correct. $\vec{\mathbf{q}}_{||}$ contributes nothing to the propulsion of the vortex motions. The turbulent dissipation can not be attributed to viscosity but to the matter exchange of the fluid elements and involved thermodynamic changes of state, if a local thermodynamic state is possible. Then the turbulent dissipation decisively decomposes the kinetic energy. $\Longrightarrow$

$$\boldsymbol{\rho}\vec{\mathbf{q}} = (\boldsymbol{\rho}\vec{\mathbf{q}})_{\perp} + (\boldsymbol{\rho}\vec{\mathbf{q}})_{||} \neq -\vec{\boldsymbol{\nabla}}\mathbf{p} + \boldsymbol{\rho}\cdot\vec{\mathbf{g}} + \eta\Delta\vec{\mathbf{v}} + (\xi + \frac{\eta}{3})\vec{\boldsymbol{\nabla}}\ (\vec{\boldsymbol{\nabla}}\cdot\vec{\mathbf{v}}) \tag{77}$$

The equations, often called conservation laws [3](Navier-Stokes-equations, equation of continuity and energy equation), do not meet these requirements for turbulence with the exception of the equation of continuity.

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[8] $\Delta\vec{\mathbf{v}} = \vec{\boldsymbol{\nabla}}\ (\vec{\boldsymbol{\nabla}}\cdot\vec{\mathbf{v}}) - \vec{\boldsymbol{\nabla}} \times \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{v}}$

3.4.2 The Complete Set of Turbulence-Equations

In the turbulence equations (69) the viscous terms according to high reynolds numbers may be omitted whereas for sufficienly small reynolds numbers (laminar motions) they obtain significance.
The equation system

$$\frac{\partial \vec{\mathbf{v}}}{\partial t} + \frac{1}{2}\vec{\boldsymbol{\nabla}}\vec{\mathbf{v}}^2 - 2\vec{\mathbf{v}} \times \vec{\boldsymbol{\omega}} = \vec{\mathbf{q}} \tag{78}$$

$$\frac{\partial}{\partial t}\vec{\boldsymbol{\omega}} - \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{a}} = \frac{1}{2}\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{q}} \tag{79}$$

$$\frac{\partial}{\partial t}\vec{\mathbf{b}} + \vec{\boldsymbol{\nabla}} \times \vec{\boldsymbol{\omega}} = \frac{1}{2}\vec{\mathbf{b}}\left[\frac{\vec{\boldsymbol{\omega}}}{\boldsymbol{\omega}^2} \cdot \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{q}}\right] \tag{80}$$

with

$$\vec{\mathbf{v}} = \vec{\boldsymbol{\omega}} \times \frac{\vec{\mathbf{b}}}{\mathbf{b}^2}, \; \vec{\mathbf{a}} = \vec{\mathbf{v}} \times \vec{\boldsymbol{\omega}}, \; \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{v}} \perp \vec{\mathbf{v}} \tag{81}$$

is not complete and as the Navier-Stokes-equations as momentum balance are refuted, the usual energy equation, derived from Navier-Stokes-equations and equation of continuity, is rejected, too. So the customarily for completion used energy equation, equation of continuity and state equation can not fill this gap.

There is the possibility observing the evolution of the velocity field not only by mass transport via the equations (78), (79) and (80) but via the progress of their fluctuation quantities $\frac{\partial \vec{\mathbf{v}}}{\partial t}$ and $\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{v}}$, too. Assuming the equation system (25)

$$\frac{\partial}{\partial t}\vec{\mathbf{B}} - \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{E}} = 0 \tag{82}$$

$$\frac{\partial}{\partial t}\left(\frac{B^2}{E^2} \cdot \vec{\mathbf{E}}\right) + \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{B}} = 0 \tag{83}$$

$$\vec{\mathbf{E}} \times \frac{\vec{\mathbf{B}}}{B^2} = propagationspeed \tag{84}$$

with

$$|\vec{\mathbf{E}} \times \frac{\vec{\mathbf{B}}}{B^2}| \leq |\vec{\mathbf{E}}| \cdot |\frac{\vec{\mathbf{B}}}{B^2}|$$

and

$$\vec{\mathbf{E}} = \frac{\partial \vec{\mathbf{v}}}{\partial t} \text{ and } \vec{\mathbf{B}} = \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{v}}, \text{as well as} \vec{\mathbf{F}} = \frac{B^2}{E^2} \cdot \vec{\mathbf{E}},$$

one obtains the further equation

$$\boxed{\frac{\partial}{\partial t}\vec{\mathbf{F}} + 2\vec{\boldsymbol{\nabla}} \times \vec{\boldsymbol{\omega}} = 0}. \tag{85}$$

Equation 82 with $\vec{\mathbf{B}} = \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{v}} = 2\vec{\boldsymbol{\omega}}$ results in

$$\frac{\partial}{\partial t}2\vec{\boldsymbol{\omega}} - \vec{\boldsymbol{\nabla}} \times \frac{\partial \vec{\mathbf{v}}}{\partial t} = 0.$$

It corresponds to (79) on account of

$$\vec{\boldsymbol{\nabla}} \times \frac{\partial \vec{\mathbf{v}}}{\partial t} = 2 \cdot \left(\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{a}} + \frac{1}{2}\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{q}}\right) = 2 \cdot \frac{\partial \vec{\boldsymbol{\omega}}}{\partial t}$$

with

$$\begin{aligned}
\vec{\mathbf{v}} &= \vec{\boldsymbol{\omega}} \times \frac{\vec{\mathbf{b}}}{b^2}, \\
\vec{\mathbf{a}} &= \vec{\mathbf{v}} \times \vec{\boldsymbol{\omega}}, \\
\vec{\mathbf{v}} &\perp \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{v}} \\
\vec{\mathbf{E}} &= \frac{\partial \vec{\mathbf{v}}}{\partial t} \\
\vec{\mathbf{E}} &= 4\omega^2 \vec{\mathbf{F}}^{-1}.
\end{aligned}$$

The invers vector respectively the scalar product means $\vec{\mathbf{F}}^{-1} = \vec{\mathbf{F}}/\vec{\mathbf{F}}^2 \Longrightarrow \vec{\mathbf{F}}^{-1} \cdot \vec{\mathbf{F}} = \mathbf{1}$.
This corresponds to the relation of a curvature vector $\vec{\mathbf{b}}$ and its associated radius vector $\vec{\mathbf{r}}$ of a continuously differentiable trajectory in one point $(\vec{\mathbf{x}}, t)$ with $\vec{\mathbf{b}} \cdot \vec{\mathbf{r}} = \mathbf{1}$.

The motion of a turbulence field is characterised by a vortex field $\vec{\omega}(\vec{\mathbf{x}}, t)$ and a curvature vector field[9] $\vec{\mathbf{b}}(\vec{\mathbf{x}}, t)$.

So one obtains the complete equation system

$$\boxed{\begin{array}{l} \vec{\mathbf{E}} + \frac{1}{2}\vec{\nabla}\vec{\mathbf{v}}^2 - 2\vec{\mathbf{v}} \times \vec{\omega} = \vec{\mathbf{q}} \\ \hline \frac{\partial}{\partial t}\vec{\omega} - \vec{\nabla} \times \vec{\mathbf{a}} = \frac{1}{2}\vec{\nabla} \times \vec{\mathbf{q}} \\ \frac{\partial}{\partial t}\vec{\mathbf{b}} + \vec{\nabla} \times \vec{\omega} = \frac{1}{2}\vec{\mathbf{b}}\left[\frac{\vec{\omega}}{\omega^2} \cdot \vec{\nabla} \times \vec{\mathbf{q}}\right] \\ \frac{\partial}{\partial t}\vec{\mathbf{F}} = -2\vec{\nabla} \times \vec{\omega} \text{ with } \vec{\mathbf{F}} = \frac{4\omega^2}{E^2} \cdot \vec{\mathbf{E}} \end{array}} . \tag{86}$$

At this a pairwise orthogonality of the vectors $(\vec{\mathbf{v}}, \vec{\omega}, \vec{\mathbf{b}})$ i.e.: $\vec{\mathbf{v}} \perp \vec{\omega}$, $\vec{\mathbf{v}} \perp \vec{\mathbf{b}}$, $\vec{\mathbf{b}} \perp \vec{\omega}$ exists. **Pursueing the trajectory of a fluid element beeing possible only after the calculation of the deterministic turbulence field the trajectory is accompanied by a frame of $\vec{\mathbf{v}}, \vec{\omega}$ and $\vec{\mathbf{b}}$** except in points where $\vec{\omega} = \mathbf{0}$ and $\vec{\mathbf{b}} = \mathbf{0}$ (turning points). This is the situation in physics, where the trajectory is sensibly considered in dependence of the time as a path parameter. In mathematics (differential geometry) the path length is chosen as parameter, which leads to another accompanying triplet.Nevertheless, in this case $\vec{\mathbf{v}} \neq \mathbf{0}$ has to be otherwise the turbulence has come to an end.

3.4.3 Comments on the Application of the Complete Equation System

On account of the theorem of Cauchy-Kowalewskaja [6] a unique solution is existing. The equation system may be numerically solved for the fields $\vec{\omega}$, $\vec{\mathbf{b}}$, $\vec{\mathbf{q}}$ and $\vec{\mathbf{E}} = \frac{\partial \vec{\mathbf{v}}}{\partial t}$ (this is treated as an independent field as well as $\vec{\omega}$,$\mathbf{b}$ und $\vec{\mathbf{q}}$) simultaneously obtaining the fields $\vec{\mathbf{a}}$ and $\vec{\mathbf{v}}$. The special approach of [29] enables 2 times continuously differentiable solutions not meaning analytical results. The order of differentiability may be principally driven forward. This particularly goes at the expense of the calculation effort.
Numerically solving this equation system [29] inflexible difference schemes are forbidden as beeing usual according to DNS-calculations (Direct Numerical Simulations related to Navier-Stokes-, continuum- and energy equation), as in the above equation system from the field environment removable singularities of $\vec{\mathbf{v}} = \vec{\omega} \times \frac{\vec{\mathbf{b}}}{\mathbf{b}^2}$, $\frac{1}{2}\vec{\mathbf{b}}\left[\frac{\vec{\omega}}{\omega^2} \cdot \vec{\nabla} \times \vec{\mathbf{q}}\right]$ and $(2\vec{\omega})^2\vec{\mathbf{F}}^{-1} = \frac{\partial \vec{\mathbf{v}}}{\partial t}$ in different space-time-points $(\vec{\mathbf{x}}, t)$ are to be recognized. This outcome is a result of possible turning points of the fluid element trajectories leading to simultaneous values of $\vec{\omega} = 0$ and $\vec{b} = 0$. Die fineness of the time discretisations is determined by the vortex field $\vec{\omega}$.
The in some turbulence models mentioned space- and time-scaling in this theory is led back to the fluctuations of curvature fields $\vec{\mathbf{b}}$ and vortex fields $\vec{\omega}$. Quantitative dependencies become accessible through numerical calculations. Though friction losses according to heavy turbulent motions (high reynolds numbers) may be omitted the kinetic energy density may significantly decrease. Thus a part has to be converted into inner energy of thermodynamics if a local thermodynamic balance is existent. It is recalled, that turbulent fluid motions are characterized the surroundings of fluid elements continuously exchanging their matter and thus their thermodynamic state quantities, too, if they exist. But this can be doubted.
The equation system (86) stands out only consisting of motion quantities, i.e. velocities and their temporal and spatial differentiations, a vector curvature field, its assigned vortex field and an abstract accelleration field $\vec{\mathbf{q}}$. Mass distributions respectively densities and thermodynamic quantities as pressure and inner energy do not appear. This fact finds its application in the general-relativistic considerations, too. The density distributions may be calculated

[9]Generally, one meets in physics curvature tensor fields at least of 2nd degree as in deformation theory or General Relativity.

by subsequent evaluation via the known velocity fields and the equation of continuity

$$\frac{\partial}{\partial t}\boldsymbol{\rho} = -\vec{\boldsymbol{\nabla}} \cdot (\boldsymbol{\rho}\vec{\mathbf{v}}). \tag{87}$$

The complete turbulence equation system may be solved even if no local thermodynamics is existent. Then the subsequent evaluation is limited to density calculations. One obtains the thermodynamic pressure distribution if existent by the subsequently calculated density field $\boldsymbol{\rho}$ and the accelleration field $\vec{\mathbf{q}}$ assuming

$$(\boldsymbol{\rho}\vec{\mathbf{q}})_{||} = -\vec{\boldsymbol{\nabla}}\mathbf{p} + \boldsymbol{\rho}\vec{\mathbf{g}} + (\xi + \eta\frac{4}{3})\vec{\boldsymbol{\nabla}}\ (\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{v}}). \tag{88}$$

via Poisson-equation [10] :

$$\Delta\mathbf{p} = -\vec{\boldsymbol{\nabla}} \cdot (\boldsymbol{\rho}\vec{\mathbf{q}}) + \vec{\boldsymbol{\nabla}} \cdot \boldsymbol{\rho}\vec{\mathbf{g}} + \vec{\boldsymbol{\nabla}} \cdot (\xi + \eta\frac{4}{3})\vec{\boldsymbol{\nabla}}\ (\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{v}}). \tag{89}$$

At high reynolds numbers

$$\Delta\mathbf{p} = -\vec{\boldsymbol{\nabla}} \cdot (\boldsymbol{\rho}\vec{\mathbf{q}}) + \vec{\boldsymbol{\nabla}} \cdot \boldsymbol{\rho}\vec{\mathbf{g}} \tag{90}$$

is certainly sufficient. But it is not obvious, whether $(\boldsymbol{\rho}\vec{\mathbf{q}})_{||}$ may be represented this way. Upon positive comparison density- and pressure evolution are determined without knowledge of a related state equation. Knowing the state equation all desired thermodynamic state quantities of a single-phase system result. On the other hand a physical process is to be found to create the used inital conditions.

The Turbulence depends on an initially assumed motion field

$$\left(\vec{\boldsymbol{\omega}}(\vec{\mathbf{x}}, t_0), \vec{\mathbf{b}}(\vec{\mathbf{x}}, t_0), \frac{\partial\vec{\mathbf{v}}}{\partial t}|_{t_0}\right) \Longrightarrow \vec{\mathbf{q}}(\vec{\mathbf{x}}, t_0),^{11} \tag{91}$$

determining the further course, alone. Evaluating $\vec{\mathbf{q}}(\vec{\mathbf{x}}, t_0)$ happens by summation of the terms in the momentum equation. An interaction of geometrodynamics and thermodynamics, maybe assumed in accordance with the Navier-Stokes-equations, does not apply.

3.4.4 The Impossibility of Calculating Turbulence Fields Only Knowing $\frac{\partial\vec{\mathbf{v}}}{\partial t}(\vec{\mathbf{x}}, t)$

The impression may arise applying turbulence calculations that it is sufficient to use the equation system

$$\begin{gathered}
\frac{\partial}{\partial t}\vec{\mathbf{B}} - \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{E}} = 0 \\
\frac{\partial}{\partial t}\left(\frac{B^2}{E^2} \cdot \vec{\mathbf{E}}\right) + \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{B}} = 0 \\
\vec{\mathbf{E}} \times \frac{\vec{\mathbf{B}}}{B^2} = \text{propagation speed,} \\
|\vec{\mathbf{E}} \times \frac{\vec{\mathbf{B}}}{B^2}| \leq |\vec{\mathbf{E}}| \cdot |\frac{\vec{\mathbf{B}}}{B^2}|
\end{gathered}$$

and

$$\vec{\mathbf{E}} = \frac{\partial\vec{\mathbf{v}}}{\partial t} \text{ und } \vec{\mathbf{B}} = \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{v}}, \text{ sowie } \vec{\mathbf{F}} = \frac{B^2}{E^2} \cdot \vec{\mathbf{E}}$$

to determinate the velocity field numerically from the knowledge of $\frac{\partial\vec{\mathbf{v}}}{\partial t}|_i$ by

$$\vec{\mathbf{v}}(\vec{\mathbf{x}}, t)_{i+1} = \frac{\partial\vec{\mathbf{v}}(\vec{\mathbf{x}}, t)}{\partial t}|_i\Delta t_i + \vec{\mathbf{v}}(\vec{\mathbf{x}}, t_0)_i. \tag{92}$$

Usually numerical time-integrations via $\Delta\vec{\mathbf{v}} = \frac{\partial\vec{\mathbf{v}}}{\partial t} \cdot \Delta t$ lead in relation to turbulence calculations firstly to

[10] The transversal part $(\boldsymbol{\rho}\vec{\mathbf{q}})_\perp$ disappears with divergence formation
[11] Inserting in equation (78)

error accumulation for a $\vec{\mathbf{v}}(\vec{\mathbf{x}},t)$ evaluation (refined methods of numerical mathematics integrating such vector functions do not help) and secondly achieve $\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{v}} \not\perp \vec{\mathbf{v}}$ with progressing time evolution. 1st is one reason why weather forecasts at meteorology are difficult (besides the principally faults of the used momentum equations). The forcasts are limited to few days. The choice of shorter time steps does not help. This difficulty does not exist regarding laminar fluid dynamics. The reason for this fundamental problem of turbulence is explained as follows:

Solving the equation system (86) numerically the pairwise orthogonality of the vectors $\vec{\mathbf{v}},\vec{\boldsymbol{\omega}},\vec{\mathbf{b}}$ ($\vec{\mathbf{v}} \perp \vec{\boldsymbol{\omega}}$, $\vec{\mathbf{v}} \perp \vec{\mathbf{b}}$, $\vec{\mathbf{b}} \perp \vec{\boldsymbol{\omega}}$) has to be considered as constraint. For analytic solutions, which cannot be formulated, these conditions should be fulfilled by the initial values.
Calculating $\vec{\mathbf{v}}(\vec{\mathbf{x}},t)$ by $\vec{\boldsymbol{\omega}}$ and $\vec{\mathbf{r}}$

$$\vec{\mathbf{v}}(\vec{\mathbf{x}},t) = \vec{\boldsymbol{\omega}}(\vec{\mathbf{x}},t) \times \vec{\mathbf{r}}(\vec{\mathbf{x}},t) \text{ with } \vec{\mathbf{r}}(\vec{\mathbf{x}},t) = \frac{\vec{\mathbf{b}}(\vec{\mathbf{x}},t)}{b^2}$$

there is a time integration of the velocity field of higher accuracy. It holds

$$\frac{\partial \vec{\mathbf{v}}}{\partial t} = \frac{\partial \vec{\boldsymbol{\omega}}}{\partial t} \times \vec{\mathbf{r}} + \vec{\boldsymbol{\omega}} \times \frac{\partial \vec{\mathbf{r}}}{\partial t}.$$

The numerical time evolution of $\vec{\mathbf{v}}_{\boldsymbol{i}} \Longrightarrow \vec{\mathbf{v}}_{\boldsymbol{i+1}}$ arises calculating $\vec{\mathbf{v}}_{\boldsymbol{i}} = \vec{\boldsymbol{\omega}}_i \times \vec{\mathbf{r}}_i$ by means of

$$\vec{\boldsymbol{\omega}}_{i+1} = \frac{\partial \vec{\boldsymbol{\omega}}}{\partial t}|_i \cdot \Delta t_i + \vec{\boldsymbol{\omega}}_i + ...$$

and

$$\vec{\mathbf{r}}_{i+1} = \frac{\partial \vec{\mathbf{r}}}{\partial t}|_i \cdot \Delta t_i + \vec{\mathbf{r}}_i + ...$$

to

$$\vec{\mathbf{v}}_{\boldsymbol{i+1}} = \left(\frac{\partial \vec{\boldsymbol{\omega}}}{\partial t}|_i \cdot \Delta t_i + \vec{\boldsymbol{\omega}}_i\right) \times \left(\frac{\partial \vec{\mathbf{r}}}{\partial t}|_i \cdot \Delta t_i + \vec{\mathbf{r}}_i\right) + ...$$

i.e.

$$\vec{\mathbf{v}}_{\boldsymbol{i+1}} = \left(\vec{\boldsymbol{\omega}}_i \times \vec{\mathbf{r}}_i\right) + \left(\frac{\partial \vec{\boldsymbol{\omega}}}{\partial t}|_i \cdot \Delta t_i \times \vec{\mathbf{r}}_i + \vec{\boldsymbol{\omega}}_i \times \frac{\partial \vec{\mathbf{r}}}{\partial t}|_i \cdot \Delta t_i\right) + \left(\frac{\partial \vec{\boldsymbol{\omega}}}{\partial t}|_i \cdot \Delta t_i \times \frac{\partial \vec{\mathbf{r}}}{\partial t}|_i \cdot \Delta t_i\right) + ...$$

respectively

$$\boxed{\vec{\mathbf{v}}_{\boldsymbol{i+1}} = \vec{\mathbf{v}}_i + \frac{\partial \vec{\mathbf{v}}}{\partial t}|_i \cdot \Delta t_i + \left(\frac{\partial \vec{\boldsymbol{\omega}}}{\partial t}|_i \times \frac{\partial \vec{\mathbf{r}}}{\partial t}|_i\right) \cdot (\Delta t_i)^2 + \boldsymbol{...}} \tag{93}$$

$\frac{\partial \vec{\mathbf{r}}}{\partial t}$ is derived as follows:

$$\vec{\mathbf{b}} = \vec{\mathbf{r}} \cdot (\vec{\mathbf{b}} \cdot \vec{\mathbf{b}}) \tag{94}$$

$\Longrightarrow$

$$\frac{\partial \vec{\mathbf{b}}}{\partial t} = \mathbf{b^2} \frac{\partial \vec{\mathbf{r}}}{\partial t} + 2\vec{\mathbf{r}}\left(\frac{\partial \vec{\mathbf{b}}}{\partial t} \cdot \vec{\mathbf{b}}\right)$$

$\Longrightarrow$

$$\frac{\partial \vec{\mathbf{r}}}{\partial t} = \left[\frac{\partial \vec{\mathbf{b}}}{\partial t} - 2\frac{\vec{\mathbf{b}}}{b^2}\left(\frac{\partial \vec{\mathbf{b}}}{\partial t} \cdot \vec{\mathbf{b}}\right)\right]/b^2.$$

In particular space-time points $(\vec{\mathbf{x}},t)$ fluid elements may be in the proximity or direct in a turning point, in which $\vec{\boldsymbol{\omega}}(\vec{\mathbf{x}},t) = \mathbf{0}$ as well as $\vec{\mathbf{b}}(\vec{\mathbf{x}},t) = \mathbf{0}$ and such $\vec{\mathbf{r}}(\vec{\mathbf{x}},t) = \vec{\mathbf{b}}/b^2 = \infty$ holds. So the temporal evolution term of 2nd order is vital for turbulence calculations. That is why the with (92) mentioned velocity integration is not expedient. Considering the complete turbulence equation set the temporal velocity integration automatically results in the desired order.

This also disproves all so-called DNS methods (direct numerical simulation), regardless of whether one accepts the Navier Stokes equations or not. .

3.5 Summary

With the installation of the equation system (86) a geometrodynamics of turbulence is expressed only obtaining motion quantities i.e. it only consists of velocities and their time and space derivatives. A corresponding statement is made for their initial- and boundary conditions. This is a theory of turbulent collective motions in which the stochastic part of the molecular motions is not included. In the usual fluid dynamics, it is assumed that this part is sufficiently taken into account by a local thermodynamic equilibrium, which, if it should ever exist, is repeatedly disturbed by fluctuating collective motions. Thus, also corresponding measurements of thermodynamic state variables (apart from density) prove to be questionable.
Laminar fluid dynamics consists of a velocity field of fluid dynamic collective motions whose accelerations are determined by thermodynamic state variables and a linear strain stress tensor of the velocity field. However, even the equations of motion of laminar fluid dynamics are inaccurate, as a check of the experimental coefficients of this tensor shows. This is in contrast to the linear elasticity theory with the same tensor acting on a deformation vector field and well proven coefficients in the engineering field. This difference is to be understood physically as follows: In linear elasticity theory, external deformations are not only largely reversible macroscopically, but also the molecular composition of previously identified molecules. In fluid dynamics, previously identified molecules shift against each other, which can be interpreted approximately as a distortion of the fluid as a whole only in very simple cases. Otherwise, it is assumed that the molecules behave in the sense of a local thermodynamic equilibrium. In a gas, this would mean that the equipartition law is to be assumed for the velocity distribution. However, this can never be exactly the case in a fluid.
Thus, the presented turbulence theory of the fluid consists of a continuum of deterministically turbulent moving collective motions, in which the stochastic molecular motions are not explicitly included. In this sense, the theory can also be seen as exact, since it does not contain any hypotheses The subsequent calculation of the thermodynamic state variables, with the exception of density, is questionable because they obviously cannot exist. Measurement results of these state variables can do nothing prove. The situation here is that there must first be clear ideas about the quantities that can be measured and not vice versa.

Turbulence cannot occur with conservative acceleration fields.

4 Unification of Maxwell Field and Gravitational Field

$$\boxed{\mathbf{R}_{\mu\nu} = 8\pi \cdot G_N \left(\mathbf{T}_{\mu\nu} - \frac{1}{2}\mathbf{g}_{\mu\nu}\mathbf{T} \right)}$$

Electrodynamics with its Maxwell Equations is the only field theory of classical physics students of physics are generally faced with in the frame of theoretical physics (at least in Germany). The Maxwell Equations above are shown formally beeing a limiting case of classical continuum physics. Because of the constant velocity of light they were the reason for setting up the Einsteinian Special Relativity. The adjustment of the electrodynamic field to Space-Time caused many physicists including Albert Einstein to try an identification of these fields with Space-Time fluctuations. Obviously, electromagnetic fluctuations are properties of Space-Time itself, though a prove is missing.

In section 2 continuum fluctuations of general vector fields are discussed. Now we consider deformation vector fields $\vec{\mathbf{d}}(\vec{\mathbf{x}}, t)$ with $\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{d}} \neq \mathbf{0}$. They are sufficiently often continuously differentiable. Defining $\vec{\mathbf{e}}$ und $\vec{\mathbf{b}}$ by

$$\begin{aligned} \vec{\mathbf{e}} &= \partial\vec{\mathbf{d}}/\partial t \neq 0 \\ \vec{\mathbf{b}} &= \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{d}} \neq 0 \end{aligned} \tag{95}$$

and interchanging the sequence of the operators $\partial/\partial t$ and $\vec{\boldsymbol{\nabla}}\times$

$$\frac{\partial\vec{\mathbf{b}}}{\partial t} = \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{e}} \tag{96}$$

directly follows. So this equation is a necessary consequence of the continuous differentiability of $\vec{\mathbf{d}}(\vec{\mathbf{x}}, t)$. The hereto dual equation is found according to section 2 with

$$\begin{aligned}
&\frac{\partial}{\partial t}\vec{\mathbf{b}} - \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{e}} = 0 \\
&\frac{\partial}{\partial t}\left(\frac{b^2}{e^2} \cdot \vec{\mathbf{e}}\right) + \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{b}} = 0 \\
&\vec{\mathbf{e}} \times \frac{\vec{\mathbf{b}}}{b^2} = \text{propagation speed}
\end{aligned} \tag{97}$$

Assuming the constant speed of light the Maxwell Equations of vacuum[12] are obtained:

$$\boxed{\begin{aligned}
&\frac{\partial}{\partial t}\vec{\mathbf{b}} - \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{e}} = 0 \\
&\frac{1}{c^2}\frac{\partial}{\partial t}\vec{\mathbf{e}} + \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{b}} = 0 \\
&\vec{\mathbf{e}} \times \frac{\vec{\mathbf{b}}}{b^2} = \vec{\mathbf{c}} = \text{propagation speed of light.}
\end{aligned}} \tag{98}$$

4.1 Space-Time of General Relativity and its Riemannian Hypersurface

First, the Riemannian hypersurface of Space-Time is considered as deformation of an Euclidian space. For a precise mathematical definition of the Riemannian space [24] is noted.

The Riemannian space is generally defined by a manifold, which consists of a point set, charts or coordinate systems and a symmetrical metric tensor field. Riemannian space and a suitable Euclidian space are one to one linked by the coordinate system. The according mapping is in mathematics not explicitly used as all considerations are abstractly concerned with the connections of the Riemannian space itself not interesting what kind of picture succeeds in the observational coordinate space. The metric tensor arises in the point $P(\vec{\boldsymbol{x}}) \in \boldsymbol{M}$ with $\vec{\boldsymbol{x}} \in \boldsymbol{E}$ (Euclidian space) by scalar products of the tangential vectors $\vec{\boldsymbol{g}}_{\boldsymbol{i}}$.

$$\boldsymbol{g}_{\boldsymbol{ij}}(P(\vec{\boldsymbol{x}})) = \vec{\boldsymbol{g}}_{\boldsymbol{i}}(P(\vec{\boldsymbol{x}})) \cdot \vec{\boldsymbol{g}}_{\boldsymbol{j}}(P(\vec{\boldsymbol{x}})) \tag{99}$$

By free choice of the coordinate system $\boldsymbol{g}_{\boldsymbol{ij}}(P(\vec{\boldsymbol{x}}))$ may be determined in one point $(P(\vec{\boldsymbol{x}}))$. But this does not simultaneously hold for the neighborhood of this point.

The isomorphic mapping from Euclidian space into the Riemannian hypersurface is brought to physical life when interpreted as deformation of the Euclidian space, both spaces, Euclidian and Riemannian space, tangentially merging in one point. Here the deformation vector field $\vec{\boldsymbol{d}} = \vec{\boldsymbol{d}}(\vec{\boldsymbol{x}}, t)$ vanishes. These time dependent mappings can be interpreted as gravitational waves. The Riemannian hypersurface arises from

$$\vec{\boldsymbol{y}}(\vec{\boldsymbol{x}}, t) = \vec{\boldsymbol{d}}(\vec{\boldsymbol{x}}, t) + \vec{\boldsymbol{x}} \quad . \tag{100}$$

The gradient on the deformed field is described by

$$\text{covariant Tensor Elements}\left(\vec{\boldsymbol{\nabla}}\vec{\boldsymbol{y}}\right) = \left(\boldsymbol{\partial}_i \boldsymbol{y}_j\right) \tag{101}$$

and detailed

$$\left(\boldsymbol{\partial}_i \boldsymbol{y}_j\right) = \begin{pmatrix} \boldsymbol{\partial}_1 \boldsymbol{y}_1 & \boldsymbol{\partial}_1 \boldsymbol{y}_2 & \boldsymbol{\partial}_1 \boldsymbol{y}_3 \\ \boldsymbol{\partial}_2 \boldsymbol{y}_1 & \boldsymbol{\partial}_2 \boldsymbol{y}_2 & \boldsymbol{\partial}_2 \boldsymbol{y}_3 \\ \boldsymbol{\partial}_3 \boldsymbol{y}_1 & \boldsymbol{\partial}_3 \boldsymbol{y}_2 & \boldsymbol{\partial}_3 \boldsymbol{y}_3 \end{pmatrix} \qquad i, j = 1, 2, 3. \tag{102}$$

Defining the spatially tangential vector $\vec{\boldsymbol{t}}_i$ with

$$\vec{\boldsymbol{t}}_i = \boldsymbol{\partial}_i \vec{\boldsymbol{y}} = (\boldsymbol{\partial}_i \boldsymbol{y}_1, \boldsymbol{\partial}_i \boldsymbol{y}_2, \boldsymbol{\partial}_i \boldsymbol{y}_3)\,, \tag{103}$$

[12]The Maxwell Equations are usually presented by $\vec{\mathbf{e}} \to -\vec{\mathbf{e}}$

one obtains the spatial metric tensor $\boldsymbol{t}_{ij} = \vec{\boldsymbol{t}}_i \cdot \vec{\boldsymbol{t}}_j$ by

$$\Big(\boldsymbol{t_{ij}}\Big) = \Big(\boldsymbol{\partial}_i \boldsymbol{y}_j\Big) \cdot \Big(\boldsymbol{\partial}_i \boldsymbol{y}_j\Big)^{\boldsymbol{T}} \tag{104}$$

and

$$\boldsymbol{t_{ij}} = \boldsymbol{\partial}_i \boldsymbol{y}_1 \cdot \boldsymbol{\partial}_j \boldsymbol{y}_1 + \boldsymbol{\partial}_i \boldsymbol{y}_2 \cdot \boldsymbol{\partial}_j \boldsymbol{y}_2 + \boldsymbol{\partial}_i \boldsymbol{y}_3 \cdot \boldsymbol{\partial}_j \boldsymbol{y}_3 \tag{105}$$

as part of the metric tensors of Space-Time

$$\Big(\mathbf{g}_{\mu\nu}\Big) = \begin{pmatrix} \mathbf{g_{00}} & \mathbf{g_{01}} & \mathbf{g_{02}} & \mathbf{g_{03}} \\ \mathbf{g_{10}} & \mathbf{t_{11}} & \mathbf{t_{12}} & \mathbf{t_{13}} \\ \mathbf{g_{20}} & \mathbf{t_{21}} & \mathbf{t_{22}} & \mathbf{t_{23}} \\ \mathbf{g_{30}} & \mathbf{t_{31}} & \mathbf{t_{32}} & \mathbf{t_{33}} \end{pmatrix} \qquad \mu, \nu = 0, 1, 2, 3\ . \tag{106}$$

The metric-tensor elements $\boldsymbol{t}_{ij}$ of the spatial hypersurface are components of the metric-tensor element set $\boldsymbol{g}_{\mu\nu}$ of Space-Time. The corresponding statement does not hold for the Ricci Curvature Tensor. The Ricci Tensor elements $\boldsymbol{r}_{ij}$ of the Riemannian hypersurface as subspace of Space-Time are not part of the Ricci Tensor element set $\boldsymbol{R}_{\mu\nu}$ of the overall space.

$$\Big(\mathbf{R}_{\mu\nu}\Big) = \begin{pmatrix} \mathbf{R_{00}} & \mathbf{R_{01}} & \mathbf{R_{02}} & \mathbf{R_{03}} \\ \mathbf{R_{10}} & \mathbf{R_{11}} & \mathbf{R_{12}} & \mathbf{R_{13}} \\ \mathbf{R_{20}} & \mathbf{R_{21}} & \mathbf{R_{22}} & \mathbf{R_{23}} \\ \mathbf{R_{30}} & \mathbf{R_{31}} & \mathbf{R_{32}} & \mathbf{R_{33}} \end{pmatrix} \neq \begin{pmatrix} \mathbf{R_{00}} & \mathbf{R_{01}} & \mathbf{R_{02}} & \mathbf{R_{03}} \\ \mathbf{R_{10}} & \mathbf{r_{11}} & \mathbf{r_{12}} & \mathbf{r_{13}} \\ \mathbf{R_{20}} & \mathbf{r_{21}} & \mathbf{r_{22}} & \mathbf{r_{23}} \\ \mathbf{R_{30}} & \mathbf{r_{31}} & \mathbf{r_{32}} & \mathbf{r_{33}} \end{pmatrix} \tag{107}$$

$$\text{i.e.} \qquad \boldsymbol{r}_{ij} \neq \boldsymbol{R}_{ij} \qquad i, j = 1, 2, 3$$

Initially, it is the plan to express the Ricci Tensor of Space Time by the Ricci Tensor of the spatial hypersurface and its time dependent metric tensor

$$\boldsymbol{R}_{ij} = \boldsymbol{R}_{ij}(\boldsymbol{r}_{ij}, \boldsymbol{t}_{ij}) \qquad i, j = 1, 2, 3. \tag{108}$$

Formulating the energy momentum tensor of the right side of the Einstein equations

$$\mathbf{R}_{\boldsymbol{\mu\nu}} - \frac{1}{2}\mathbf{g}_{\boldsymbol{\mu\nu}}\mathbf{R} = 8\pi \cdot G_N \mathbf{T}_{\boldsymbol{\mu\nu}} \qquad \mu, \nu = 0, 1, 2, 3$$

by the related deformation fluctuations using its electromagnetic interpretation the unification of gravitational and electromagnetic field is outlined in the following section.
Originating from the Einstein equations

$$\mathbf{R}_{\boldsymbol{\mu\nu}} - \frac{1}{2}\mathbf{g}_{\boldsymbol{\mu\nu}}\mathbf{R} = 8\pi \cdot G_N \mathbf{T}_{\boldsymbol{\mu\nu}} \tag{109}$$

one obtains by contraction

$$\mathbf{trace}\Big(\mathbf{R}_{\boldsymbol{\mu\nu}} - \frac{1}{2}\mathbf{g}_{\boldsymbol{\mu\nu}}\mathbf{R}\Big) = \mathbf{g}^{\boldsymbol{\mu\mu}}\Big(\mathbf{R}_{\boldsymbol{\mu\mu}} - \frac{1}{2}\mathbf{g}_{\boldsymbol{\mu\mu}}\mathbf{R}\Big) = -\mathbf{R} = 8\pi \cdot G_N \mathbf{T}^{\mu}_{\mu} = 8\pi \cdot G_N \mathbf{T} \tag{110}$$

an alternative form of the Einstein Equations

$$\mathbf{R}_{\boldsymbol{\mu\nu}} = 8\pi \cdot G_N \Big(\mathbf{T}_{\boldsymbol{\mu\nu}} - \frac{1}{2}\mathbf{g}_{\boldsymbol{\mu\nu}}\mathbf{T}\Big). \tag{111}$$

4.2 The Ricci Tensor in the Origin of a Local Inertial-System

The Riemannian curvature tensor $\mathbf{R}^{\mu}_{.\nu\alpha\beta}$ is described in any coordinate system by the Christoffel symbols

$$\mathbf{\Gamma}^{\boldsymbol{\mu}}_{\boldsymbol{\nu\alpha}} = \left\{ \begin{matrix} \mu \\ \nu\ \alpha \end{matrix} \right\} = \frac{1}{2}\mathbf{g}^{\mu\lambda}\Big[\partial_\nu \mathbf{g}_{\alpha\lambda} + \partial_\alpha \mathbf{g}_{\lambda\nu} - \partial_\lambda \mathbf{g}_{\nu\alpha}\Big] \tag{112}$$

$$\mathbf{R}^{\mu}_{.\nu\alpha\beta} = \frac{\partial \mathbf{\Gamma}^{\mu}_{\nu\beta}}{\partial \mathbf{x}^{\alpha}} - \frac{\partial \mathbf{\Gamma}^{\mu}_{\nu\alpha}}{\partial \mathbf{x}^{\beta}} + \mathbf{\Gamma}^{\mu}_{\rho\alpha}\mathbf{\Gamma}^{\rho}_{\nu\beta} - \mathbf{\Gamma}^{\mu}_{\rho\beta}\mathbf{\Gamma}^{\rho}_{\nu\alpha}. \tag{113}$$

In the origin $\vec{\mathbf{x}_0}$ of a local inertial system [1] the partial derivatives with respect to coordinates of the metric tensor $\mathbf{g}_{\lambda\nu}$ vanish such that

$$\boldsymbol{\Gamma}^{\boldsymbol{\mu}}_{\boldsymbol{\nu\alpha}}(\vec{\mathbf{x}_0}) = \mathbf{0} \tag{114}$$

and

$$\mathbf{R}^{\mu}_{.\nu\alpha\beta}(\vec{\mathbf{x}_0}) = \frac{\partial \boldsymbol{\Gamma}^{\mu}_{\nu\beta}}{\partial \mathbf{x}^{\alpha}} - \frac{\partial \boldsymbol{\Gamma}^{\mu}_{\nu\alpha}}{\partial \mathbf{x}^{\beta}}. \tag{115}$$

In the origin of the coordinate system the metric tensor itself equals the Minkowski tensor.

$$\mathbf{g}_{\mu\nu}(\vec{\mathbf{x}_0}) = \eta_{\mu\nu} = \begin{pmatrix} -\mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{pmatrix} \tag{116}$$

Written out one obtains

$$\mathbf{R}^{\mu}_{.\nu\alpha\beta}(\vec{\mathbf{x}_0}) = \frac{1}{2}\eta^{\mu\lambda}\frac{\partial}{\partial x^{\alpha}}\left[\partial_\nu \mathbf{g}_{\beta\lambda} + \partial_\beta \mathbf{g}_{\lambda\nu} - \partial_\lambda \mathbf{g}_{\nu\beta}\right] - \frac{1}{2}\eta^{\mu\lambda}\frac{\partial}{\partial x^{\beta}}\left[\partial_\nu \mathbf{g}_{\alpha\lambda} + \partial_\alpha \mathbf{g}_{\lambda\nu} - \partial_\lambda \mathbf{g}_{\nu\alpha}\right] \tag{117}$$

$\Longrightarrow$

$$\mathbf{R}^{\mu}_{.\nu\alpha\beta}(\vec{\mathbf{x}_0}) = \frac{1}{2}\eta^{\mu\lambda}\left[\partial_\alpha\partial_\nu \mathbf{g}_{\beta\lambda} + \partial_\alpha\partial_\beta \mathbf{g}_{\lambda\nu} - \partial_\alpha\partial_\lambda \mathbf{g}_{\nu\beta}\right] - \frac{1}{2}\eta^{\mu\lambda}\left[\partial_\beta\partial_\nu \mathbf{g}_{\alpha\lambda} + \partial_\beta\partial_\alpha \mathbf{g}_{\lambda\nu} - \partial_\beta\partial_\lambda \mathbf{g}_{\nu\alpha}\right] \tag{118}$$

$\Longrightarrow$

$$\mathbf{R}^{\mu}_{.\nu\alpha\beta}(\vec{\mathbf{x}_0}) = \frac{1}{2}\eta^{\mu\lambda}\left[\partial_\alpha\partial_\nu \mathbf{g}_{\beta\lambda} + \partial_\beta\partial_\lambda \mathbf{g}_{\nu\alpha} - \partial_\alpha\partial_\lambda \mathbf{g}_{\nu\beta} - \partial_\beta\partial_\nu \mathbf{g}_{\alpha\lambda}\right] \tag{119}$$

$\Longrightarrow$

$$\mathbf{R}_{\mu\nu\alpha\beta}(\vec{\mathbf{x}_0}) = \frac{1}{2}\left[\partial_\alpha\partial_\nu \mathbf{g}_{\beta\lambda} + \partial_\beta\partial_\lambda \mathbf{g}_{\nu\alpha} - \partial_\alpha\partial_\lambda \mathbf{g}_{\nu\beta} - \partial_\beta\partial_\nu \mathbf{g}_{\alpha\lambda}\right]. \tag{120}$$

After contraction there is the associated Ricci Tensor

$$\mathbf{R}_{\mu\nu}(\vec{\mathbf{x}_0}) = \frac{1}{2}\left[\partial_\mu\partial_\alpha \mathbf{g}^{\alpha}_{\nu} + \partial_\nu\partial^\alpha \mathbf{g}_{\mu\alpha} - \partial_\alpha\partial^\alpha \mathbf{g}_{\mu\nu} - \partial_\nu\partial_\mu \mathbf{g}^{\alpha}_{\alpha}\right] \tag{121}$$

and as $\partial_\alpha\partial^\alpha = \Box$ means the D'Alembert-Operator $\Longrightarrow$

$$\boxed{\mathbf{R}_{\mu\nu}(\vec{\mathbf{x}_0}) = \frac{1}{2}\left[\partial_\mu\partial_\alpha \mathbf{g}^{\alpha}_{\nu} + \partial_\nu\partial_\alpha \mathbf{g}^{\alpha}_{\mu} - \Box \mathbf{g}_{\mu\nu} - \partial_\nu\partial_\mu \mathbf{g}\right]}\,. \tag{122}$$

This result may be obtained by linearization of the Riemannian curvature tensor, too. Choosing point $(\vec{\mathbf{x}_0})$ as the origin of a local inertial system, linearization is not necessary.

4.3 The Ricci Tensor of the Einstein Space in Dependence of Temporal Fluctuations of its Riemannian Hypersurface

The following relations correspond to [18] Landau Lifschitz volume 2 page.308-309. A time orthogonal coordinate system is always possible. In contrary to [18], we do not equate the velocity of light with 1.

$$\text{Def:} \qquad \varkappa_{\mathbf{ij}} = \frac{\partial \mathbf{g}_{\mathbf{ij}}}{\partial (c\boldsymbol{t})} \tag{123}$$

$\mathbf{r}_{\mathbf{ij}}$ means the Ricci Tensor of the Riemannian hypersurface.
$\Longrightarrow$

$$\begin{aligned}
\mathbf{R_{00}} =& -\frac{1}{2}\frac{\partial \varkappa_i^i}{\partial(ct)} - \frac{1}{4}\varkappa_i^j \varkappa_j^i \\
\mathbf{R_{0i}} =& \frac{1}{2}\left(\varkappa_{\mathrm{i};j}^j - \varkappa_{\mathrm{j};i}^j\right) \\
\mathbf{R_{ij}} =& \mathbf{r_{ij}} + \frac{1}{2}\frac{\partial \varkappa_{ij}}{\partial(ct)} + \frac{1}{4}\left(\varkappa_{\mathrm{ij}}\varkappa_{\mathrm{k}}^k - 2\varkappa_i^k \varkappa_{jk}\right)
\end{aligned} \tag{124}$$

i, j, k pass through $1, 2, 3$. ";" means partial derivation, here.

Thus the geometry of Space-Time may be opened up from geometrodynamics of space. Gravitational waves existing the energy momentum tensor $\mathbf{T}_{\mu\nu} \neq \mathbf{0}$ is given in the considered Space-Time area even if there is no matter. [13]

4.4 Gravitational Waves Corresponding to Electromagnetic Fluctuations

The deformation fluctuations of space and its as electromagnetic fluctuations noticed phenomena are subsequently faced to each other in a limited volume area as fourier developments . The considerations are performed based on treatments of natural vibrations of the electomagnetic field in vacuum in accordance to [18]. The usual electric field $\vec{\mathbf{E}}$ is replaced by $-\vec{\mathbf{E}}$, without loss of generality. An explicit dependency of the viewed overall volume in the canonical variables and such in the resulting energy density and the electromagnetic fields is avoided by modified normalisation of the canonical variables, in contrast to [18].
In pure field theories energy densities and accellerations should occur as **primary** quantities not energies and forces. The energy in one point $(\vec{\mathbf{x}}, t)$ is always zero but not the energy density. Analogically, the same is true for the relation of accelleration and force.

deformation fluctuations	**electromagnetic fluctuations**
From	
$\vec{\mathbf{d}} =$ deformation vectorfield	$\vec{\mathbf{A}} =$ vector potential
$\frac{\partial}{\partial t}\vec{\mathbf{b}} - \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{e}} = 0$	$\frac{\partial}{\partial t}\vec{\mathbf{B}} - \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{E}} = 0$
$\frac{1}{c^2}\frac{\partial}{\partial t}\vec{\mathbf{e}} + \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{b}} = 0$	$\frac{1}{c^2}\frac{\partial}{\partial t}\vec{\mathbf{E}} + \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{B}} = 0$
and	
$\vec{\mathbf{e}} = \partial\vec{\mathbf{d}}/\partial t \neq 0$	$\vec{\mathbf{E}} = \partial\vec{\mathbf{A}}/\partial t \neq 0$
$\vec{\mathbf{b}} = \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{d}} \neq 0$	$\vec{\mathbf{B}} = \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{A}} \neq 0$
one obtains	
$\frac{1}{\mathbf{c^2}}\frac{\partial^2\vec{\mathbf{d}}}{\partial t^2} = \boldsymbol{\Delta}\vec{\mathbf{d}}$	$\frac{1}{\mathbf{c^2}}\frac{\partial^2\vec{\mathbf{A}}}{\partial t^2} = \boldsymbol{\Delta}\vec{\mathbf{A}}$
Deformation field and according vector potential field are formally described by	
$\vec{\mathbf{d}} = \sum_{\vec{\mathbf{k}}} \vec{\mathbf{d}}_{\vec{\mathbf{k}}} = \sum_{\vec{\mathbf{k}}} \vec{\mathbf{a}}_{\vec{\mathbf{k}}} e^{i\vec{\mathbf{k}}\vec{\mathbf{r}}} + \vec{\mathbf{a}^*}_{\vec{\mathbf{k}}} e^{-i\vec{\mathbf{k}}\vec{\mathbf{r}}}$	$\vec{\mathbf{A}} = \sum_{\vec{\mathbf{k}}} \vec{\mathbf{A}}_{\vec{\mathbf{k}}} = \sum_{\vec{\mathbf{k}}} \vec{\mathfrak{A}}_{\vec{\mathbf{k}}} e^{i\vec{\mathbf{k}}\vec{\mathbf{r}}} + \vec{\mathfrak{A}}^*_{\vec{\mathbf{k}}} e^{-i\vec{\mathbf{k}}\vec{\mathbf{r}}}$
and it follows	
$\ddot{\vec{\mathbf{d}}}_{\vec{\mathbf{k}}} + c^2k^2\vec{\mathbf{d}}_{\vec{\mathbf{k}}} = \mathbf{0}$	$\ddot{\vec{\mathbf{A}}}_{\vec{\mathbf{k}}} + c^2k^2\vec{\mathbf{A}}_{\vec{\mathbf{k}}} = \mathbf{0}$
with	

[13] in contrary to Penrose [?] page 467 "The energy-momentum tensor in empty space is zero."

deformation fluctuations

$\vec{e} = \dot{\vec{d}} = \sum_{\vec{k}} \dot{\mathbf{d}}_{\vec{k}} = \sum_{\vec{k}} \left(\dot{\mathbf{a}}_{\vec{k}} e^{i\vec{k}\vec{r}} + \dot{\mathbf{a}}^*_{\vec{k}} e^{-i\vec{k}\vec{r}} \right)$

and

$\vec{b} = -i \sum_{\vec{k}} \vec{k} \times \left(\vec{\mathbf{a}}_{\vec{k}} e^{i\vec{k}\vec{r}} + \mathbf{a}^*{}_{\vec{k}} e^{-i\vec{k}\vec{r}} \right)$

$\mathbf{k_1} = \frac{2\pi \cdot n_x}{L_x}$, $\mathbf{k_2} = \frac{2\pi \cdot n_y}{L_y}$, $\mathbf{k_3} = \frac{2\pi \cdot n_z}{L_z}$; $\quad \vec{\mathbf{k}} = (\mathbf{k_1}, \mathbf{k_2}, \mathbf{k_3})$

$\mathbf{a}_{\mathbf{k}_i} \sim e^{-i\omega_{\mathbf{k}_i} t}, \qquad \omega_{\mathbf{k}_i} = ck_i$

electromagnetic fluctuations

$\vec{E} = \dot{\vec{A}} = \sum_{\vec{k}} \dot{\mathbf{A}}_{\vec{k}} = \sum_{\vec{k}} \left(\dot{\vec{\mathfrak{A}}}_{\vec{k}} e^{i\vec{k}\vec{r}} + \dot{\vec{\mathfrak{A}}}^*_{\vec{k}} e^{-i\vec{k}\vec{r}} \right)$

$\vec{B} = -i \sum_{\vec{k}} \vec{k} \times \left(\vec{\mathfrak{A}}_{\vec{k}} e^{i\vec{k}\vec{r}} + \vec{\mathfrak{A}}^*_{\vec{k}} e^{-i\vec{k}\vec{r}} \right)$

$\mathfrak{A}_{\vec{\mathbf{k}}_i} \sim e^{-i\omega_{\mathbf{k}_i} t}, \qquad \omega_{\mathbf{k}_i} = ck_i$

The wave vectors are calculated in a sufficiently great volume $V = L_x \cdot L_y \cdot L_z$.

$\mathcal{E} = \frac{1}{8\pi} \int_{V_0} (E^2/c^2 + B^2) dV$ means the energy of the field in volume V_0.

The energy density of the field is $\quad \mathfrak{E} = \frac{1}{8\pi} \sum_{\vec{k}} (E^2_{\vec{k}}/c^2 + B^2_{\vec{k}})$

Now, the following vectorial quantities (canonical variables) are defined:

$\vec{\mathbf{q}}_{\vec{k}} = \sqrt{\frac{1}{4\pi c^2}} (\vec{\mathbf{a}}_{\vec{k}} + \vec{\mathbf{a}}^*_{\vec{k}})$

$\vec{\mathbf{p}}_{\vec{k}} = -i\omega_{\vec{k}} \sqrt{\frac{1}{4\pi c^2}} (\vec{\mathbf{a}}_{\vec{k}} - \vec{\mathbf{a}}^*_{\vec{k}}) = \dot{\vec{\mathbf{q}}}_{\vec{k}}$

$\vec{\mathbf{q}}_{\mathbf{k}_i} \sim \cos(\omega_{\mathbf{k}_i} t), \qquad \vec{\mathbf{p}}_{\mathbf{k}_i} \sim \sin(\omega_{\mathbf{k}_i} t)$

$\vec{\mathbf{Q}}_{\vec{k}} = \sqrt{\frac{1}{4\pi c^2}} (\vec{\mathfrak{A}}_{\vec{k}} + \vec{\mathfrak{A}}^*_{\vec{k}})$

$\vec{\mathbf{P}}_{\vec{k}} = -i\omega_{\vec{k}} \sqrt{\frac{1}{4\pi c^2}} (\vec{\mathfrak{A}}_{\vec{k}} - \vec{\mathfrak{A}}^*_{\vec{k}}) = \dot{\vec{\mathbf{Q}}}_{\vec{k}}$

$\vec{\mathbf{Q}}_{\mathbf{k}_i} \sim \cos(\omega_{\mathbf{k}_i} t), \qquad \vec{P}_{\mathbf{k}_i} \sim \sin(\omega_{\mathbf{k}_i} t)$

Obviously, they are real and resolved according to complex quantities they give

$\vec{\mathbf{a}}_{\mathbf{k_j}} = \frac{i}{k_j} \sqrt{\pi} (\vec{\mathbf{p}}_{\mathbf{k_j}} - i\omega_{\vec{\mathbf{k}}_j} \vec{\mathbf{q}}_{\mathbf{k_j}})$
$\vec{\mathbf{a}}^*_{\mathbf{k_j}} = -\frac{i}{k_j} \sqrt{\pi} (\vec{\mathbf{p}}_{\mathbf{k_j}} + i\omega_{\mathbf{k_j}} \vec{\mathbf{q}}_{\mathbf{k_j}})$

$\vec{\mathfrak{A}}_{\mathbf{k_j}} = \frac{i}{k_j} \sqrt{\pi} (\vec{\mathbf{P}}_{\mathbf{k_j}} - i\omega_{\mathbf{k_j}} \vec{\mathbf{Q}}_{\mathbf{k_j}})$
$\vec{\mathfrak{A}}^*_{\mathbf{k_j}} = -\frac{i}{k_j} \sqrt{\pi} (\vec{\mathbf{P}}_{\mathbf{k_j}} + i\omega_{\mathbf{k_j}} \vec{\mathbf{Q}}_{\mathbf{k_j}})$.

Thus one obtains as expansion by characteristic vibrations (in concise presentation):

$\vec{\mathbf{d}} = \sqrt{4\pi} \sum_{\vec{k}} \frac{1}{k} \left(ck\vec{\mathbf{q}}_{\vec{k}} \cos(\vec{\mathbf{k}} \cdot \vec{\mathbf{r}}) - \vec{\mathbf{p}}_{\vec{k}} \sin(\vec{\mathbf{k}} \cdot \vec{\mathbf{r}}) \right)$
$\vec{\mathbf{e}} = \sqrt{4\pi} \sum_{\vec{k}} \mathbf{c} \left(ck\vec{\mathbf{q}}_{\vec{k}} \sin(\vec{\mathbf{k}} \cdot \vec{\mathbf{r}}) + \vec{\mathbf{p}}_{\vec{k}} \cos(\vec{\mathbf{k}} \cdot \vec{\mathbf{r}}) \right)$
$\vec{\mathbf{b}} = -\sqrt{4\pi} \sum_{\vec{k}} \frac{1}{k} \vec{\mathbf{k}} \times [ck\vec{\mathbf{q}}_{\vec{k}} \sin(\vec{\mathbf{k}} \cdot \vec{\mathbf{r}}) + \vec{\mathbf{p}}_{\vec{k}} \cos(\vec{\mathbf{k}} \cdot \vec{\mathbf{r}})]$

$\vec{\mathbf{A}} = \sqrt{4\pi} \sum_{\vec{k}} \frac{1}{k} \left(ck\vec{\mathbf{Q}}_{\vec{k}} \cos(\vec{\mathbf{k}} \cdot \vec{\mathbf{r}}) - \vec{\mathbf{P}}_{\vec{k}} \sin(\vec{\mathbf{k}} \cdot \vec{\mathbf{r}}) \right)$
$\vec{\mathbf{E}} = \sqrt{4\pi} \sum_{\vec{k}} \mathbf{c} \left(ck\vec{\mathbf{Q}}_{\vec{k}} \sin(\vec{\mathbf{k}} \cdot \vec{\mathbf{r}}) + \vec{\mathbf{P}}_{\vec{k}} \cos(\vec{\mathbf{k}} \cdot \vec{\mathbf{r}}) \right)$
$\vec{\mathbf{B}} = -\sqrt{4\pi} \sum_{\vec{k}} \frac{1}{k} \vec{\mathbf{k}} \times [ck\vec{\mathbf{Q}}_{\vec{k}} \sin(\vec{\mathbf{k}} \cdot \vec{\mathbf{r}}) + \vec{\mathbf{P}}_{\vec{k}} \cos(\vec{\mathbf{k}} \cdot \vec{\mathbf{r}})]$

respectively noted for the single modes:

$\vec{\mathbf{d}}_{k_j} = \sqrt{4\pi} \frac{1}{k_j} \left(ck_j \vec{\mathbf{q}}_{\mathbf{k_j}} \cos(\vec{\mathbf{k_j}} \cdot \vec{\mathbf{r}}) - \vec{\mathbf{p}}_{\mathbf{k_j}} \sin(\vec{\mathbf{k_j}} \cdot \vec{\mathbf{r}}) \right)$
$\vec{\mathbf{e}}_{k_j} = \sqrt{4\pi} \mathbf{c} \left(ck_j \vec{\mathbf{q}}_{\mathbf{k_j}} \sin(\vec{\mathbf{k_j}} \cdot \vec{\mathbf{r}}) + \vec{\mathbf{p}}_{\mathbf{k_j}} \cos(\vec{\mathbf{k_j}} \cdot \vec{\mathbf{r}}) \right)$
$\vec{\mathbf{b}}_{k_j} = -\sqrt{4\pi} \frac{1}{k_j} \vec{\mathbf{k}_j} \times [ck_j \vec{\mathbf{q}}_{\mathbf{k_j}} \sin(\vec{\mathbf{k_j}} \cdot \vec{\mathbf{r}}) + \vec{\mathbf{p}}_{\mathbf{k_j}} \cos(\vec{\mathbf{k_j}} \cdot \vec{\mathbf{r}})]$

$\vec{\mathbf{A}}_{k_j} = \sqrt{4\pi} \frac{1}{k_j} \left(ck_j \vec{\mathbf{Q}}_{\mathbf{k_j}} \cos(\vec{\mathbf{k_j}} \cdot \vec{\mathbf{r}}) - \vec{\mathbf{P}}_{\mathbf{k_j}} \sin(\vec{\mathbf{k_j}} \cdot \vec{\mathbf{r}}) \right)$
$\vec{\mathbf{E}}_{k_j} = \sqrt{4\pi} \mathbf{c} \left(ck_j \vec{\mathbf{Q}}_{\mathbf{k_j}} \sin(\vec{\mathbf{k_j}} \cdot \vec{\mathbf{r}}) + \vec{\mathbf{P}}_{\mathbf{k_j}} \cos(\vec{\mathbf{k_j}} \cdot \vec{\mathbf{r}}) \right)$
$\vec{\mathbf{B}}_{k_j} = -\sqrt{4\pi} \frac{1}{k_j} \vec{\mathbf{k}_j} \times [ck_j \vec{\mathbf{Q}}_{\mathbf{k_j}} \sin(\vec{\mathbf{k_j}} \cdot \vec{\mathbf{r}}) + \vec{\mathbf{P}}_{\mathbf{k_j}} \cos(\vec{\mathbf{k_j}} \cdot \vec{\mathbf{r}})]$

with $\mathfrak{E} = \sum_{\vec{k}} \mathfrak{E}_{\vec{k}} = \frac{1}{2} \sum_{\vec{k}} (E^2_{\vec{k}}/\mathbf{c^2} + B^2_{\vec{k}})$ and $\mathcal{E} = \sum_{\vec{k}} \mathcal{E}_{\vec{k}} = \frac{1}{2} \sum_{\vec{k}} \int_{V_0} (E^2_{\vec{k}}/\mathbf{c^2} + B^2_{\vec{k}}) dV$.
respectively $\mathfrak{E}_{\vec{\mathbf{k}}_j} = \frac{1}{2} (E^2_{\vec{\mathbf{k}}_j}/\mathbf{c^2} + B^2_{\vec{\mathbf{k}}_j})$ and $\mathcal{E}_{\vec{\mathbf{k}}_j} = \frac{1}{2} \int_{V_0} (E^2_{\vec{\mathbf{k}}_j}/\mathbf{c^2} + B^2_{\vec{\mathbf{k}}_j}) dV$.

They may formally considered as running waves moving discrete quantities of harmonic oscillators with the Hamilton Functions

$$\mathbf{H} = \sum_{\vec{k}} \mathbf{H}_{\vec{k}} = \sum_{\vec{k}} \frac{1}{2} (\mathbf{p}^2_{\vec{k}} + \omega^2_{\vec{k}} \mathbf{q}^2_{\vec{k}}), \qquad \mathcal{H} = \sum_{\vec{k}} \mathcal{H}_{\vec{k}} = \sum_{\vec{k}} \frac{1}{2} (\mathbf{P}^2_{\vec{k}} + \omega^2_{\vec{k}} \mathbf{Q}^2_{\vec{k}}) \tag{125}$$

and the oscillator equations

$$\ddot{\vec{\mathbf{q}}}_{\vec{k}} + \omega^2_{\vec{k}} \vec{\mathbf{q}}_{\vec{k}} = \mathbf{0}, \qquad \ddot{\vec{\mathbf{Q}}}_{\vec{k}} + \omega^2_{\vec{k}} \vec{\mathbf{Q}}_{\vec{k}} = \mathbf{0} \tag{126}$$

.

$$\mathbf{H} = \sum_{\vec{\mathbf{k}}} \mathbf{H}_{\vec{\mathbf{k}}} \qquad \mathbf{H}_{\vec{\mathbf{k}}} = \frac{1}{2}(\mathbf{p}^2_{\vec{\mathbf{k}}} + \omega^2_{\vec{\mathbf{k}}}\mathbf{q}^2_{\vec{\mathbf{k}}}), \qquad \mathcal{H} = \sum_{\vec{\mathbf{k}}} \mathcal{H}_{\vec{\mathbf{k}}} \qquad \mathcal{H}_{\vec{\mathbf{k}}} = \frac{1}{2}(\mathbf{P}^2_{\vec{\mathbf{k}}} + \omega^2_{\vec{\mathbf{k}}}\mathbf{Q}^2_{\vec{\mathbf{k}}}) \tag{127}$$

4.5 The Energy-Momentum-Tensor of the Electromagnetic Field

The energy momentum density tensor for the electromagnetic field (generally called Energy momentum tensor) in covariant components [27] is written with the choosen signature $(-1, 1, 1, 1)$

$$\mathbf{T}_{\boldsymbol{\mu\nu}} = \frac{1}{4\pi}\left(\mathbf{F}^{\boldsymbol{\alpha}}_{\boldsymbol{\mu}}\mathbf{F}_{\boldsymbol{\alpha\nu}} - \frac{1}{4}\mathbf{g}_{\boldsymbol{\mu\nu}}\mathbf{F}_{\boldsymbol{\alpha\beta}}\mathbf{F}^{\boldsymbol{\alpha\beta}}\right) \tag{128}$$

It is symmetric: $\mathbf{T}_{\boldsymbol{\mu\nu}} = \mathbf{T}_{\boldsymbol{\nu\mu}}$.
One obtains the Faraday-tensor of the electromagnetic field from

$$\mathbf{F}_{\mu\nu} = \partial_\mu \mathbf{A}_\nu - \partial_\nu \mathbf{A}_{\boldsymbol{\mu}} \qquad \boldsymbol{\mu}, \boldsymbol{\nu} = \mathbf{0}, \mathbf{1}, \mathbf{2}, \mathbf{3} \tag{129}$$

and detailed (they are chosen respectively the form of the above Maxwell Equations)

$$\begin{aligned}
\mathbf{F_{0i}} &= \partial_0 \mathbf{A}_i - \partial_i \mathbf{A_0} = \mathbf{E_i}/\boldsymbol{c}, \qquad \boldsymbol{i} = \mathbf{1}, \mathbf{2}, \mathbf{3} \\
\mathbf{F_{i0}} &= \partial_i \mathbf{A_0} - \partial_0 \mathbf{A_i} = -\mathbf{E_i}/\boldsymbol{c}, \qquad \boldsymbol{i} = \mathbf{1}, \mathbf{2}, \mathbf{3} \\
\mathbf{F_{12}} &= \partial_1 \mathbf{A_2} - \partial_2 \mathbf{A_1} = \mathbf{B_3} \\
\mathbf{F_{13}} &= \partial_1 \mathbf{A_3} - \partial_3 \mathbf{A_1} = -\mathbf{B_2} \\
\mathbf{F_{23}} &= \partial_2 \mathbf{A_3} - \partial_3 \mathbf{A_2} = \mathbf{B_1}
\end{aligned} \tag{130}$$

$$\Longrightarrow \mathbf{F}_{\mu\nu} = -\mathbf{F}_{\nu\mu}$$

$$\partial_{\boldsymbol{\rho}} \mathbf{F}_{\mu\nu} + \partial_{\boldsymbol{\mu}} \mathbf{F}_{\nu\rho} + \partial_{\boldsymbol{\nu}} \mathbf{F}_{\rho\mu} = \mathbf{0}$$

and in greater detail

$$\begin{aligned}
\partial_1 \mathbf{F_{23}} + \partial_3 \mathbf{F_{12}} + \partial_2 \mathbf{F_{31}} &= \mathbf{0} \\
\partial_2 \mathbf{F_{30}} + \partial_0 \mathbf{F_{23}} + \partial_3 \mathbf{F_{02}} &= \mathbf{0} \\
\partial_3 \mathbf{F_{01}} + \partial_1 \mathbf{F_{30}} + \partial_0 \mathbf{F_{13}} &= \mathbf{0} . \\
\partial_0 \mathbf{F_{12}} + \partial_2 \mathbf{F_{01}} + \partial_1 \mathbf{F_{20}} &= \mathbf{0}
\end{aligned}$$

The indices correspond to $0 \to ct$, $1 \to x$, $2 \to y$, $3 \to z$ complying with the following electrodynamic equations of vacuum[14]

$$\boldsymbol{div\vec{B} = 0} \qquad \text{and} \qquad \frac{\partial}{\partial t}\vec{\mathbf{B}} - \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{E}} = 0.$$

The expressions of the covariant and contravariant Faraday-tensors considering the minkowski tensor

$$\boldsymbol{\eta}_{\boldsymbol{\mu\nu}} = \begin{pmatrix} -\mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{pmatrix} \tag{131}$$

lead to

$$\mathbf{F}_{\mu\nu} = \begin{pmatrix} \mathbf{0} & \mathbf{E_1}/\boldsymbol{c} & \mathbf{E_2}/\boldsymbol{c} & \mathbf{E_3}/\boldsymbol{c} \\ -\mathbf{E_1}/\boldsymbol{c} & \mathbf{0} & \mathbf{B_3} & -\mathbf{B_2} \\ -\mathbf{E_2}/\boldsymbol{c} & -\mathbf{B_3} & \mathbf{0} & \mathbf{B_1} \\ -\mathbf{E_3}/\boldsymbol{c} & \mathbf{B_2} & -\mathbf{B_1} & \mathbf{0} \end{pmatrix} \qquad \mathbf{F}^{\mu\nu} = \begin{pmatrix} \mathbf{0} & -\mathbf{E_1}/\boldsymbol{c} & -\mathbf{E_2}/\boldsymbol{c} & -\mathbf{E_3}/\boldsymbol{c} \\ \mathbf{E_1}/\boldsymbol{c} & \mathbf{0} & \mathbf{B_3} & -\mathbf{B_2} \\ \mathbf{E_2}/\boldsymbol{c} & -\mathbf{B_3} & \mathbf{0} & \mathbf{B_1} \\ \mathbf{E_3}/\boldsymbol{c} & \mathbf{B_2} & -\mathbf{B_1} & \mathbf{0} \end{pmatrix} \tag{132}$$

[14] the polarity reversal $\vec{\boldsymbol{E}} \longrightarrow -\vec{\boldsymbol{E}}$ recognised

$$\mathbf{F}^{\mu}_{\nu}=\begin{pmatrix} \mathbf{0} & -\mathbf{E_1}/c & -\mathbf{E_2}/c & -\mathbf{E_3}/c \\ -\mathbf{E_1}/c & \mathbf{0} & \mathbf{B_3} & -\mathbf{B_2} \\ -\mathbf{E_2}/c & -\mathbf{B_3} & \mathbf{0} & \mathbf{B_1} \\ -\mathbf{E_3}/c & \mathbf{B_2} & -\mathbf{B_1} & \mathbf{0} \end{pmatrix} \quad . \tag{133}$$

Thus the covariant components of the electromagnetic energy momentum tensor are written

$$\mathbf{T}_{\mu\nu}=\frac{1}{4\pi}\begin{pmatrix} \boldsymbol{Q} & (\frac{\vec{\boldsymbol{E}}}{\boldsymbol{c}}\times\vec{\boldsymbol{B}})_1 & (\frac{\vec{\boldsymbol{E}}}{\boldsymbol{c}}\times\vec{\boldsymbol{B}})_2 & (\frac{\vec{\boldsymbol{E}}}{\boldsymbol{c}}\times\vec{\boldsymbol{B}})_3 \\ (\frac{\vec{\boldsymbol{E}}}{\boldsymbol{c}}\times\vec{\boldsymbol{B}})_1 & -[\frac{\boldsymbol{E}_1^2}{\mathbf{c^2}}+\boldsymbol{B}_1^2-\boldsymbol{Q}] & -\frac{\mathbf{E_1E_2}}{\mathbf{c^2}}-\mathbf{B_1B_2} & -\frac{\mathbf{E_1E_3}}{\mathbf{c^2}}-\mathbf{B_1B_3} \\ (\frac{\vec{\boldsymbol{E}}}{\boldsymbol{c}}\times\vec{\boldsymbol{B}})_2 & -\frac{\mathbf{E_1E_2}}{\mathbf{c^2}}-\mathbf{B_1B_2} & -[\frac{\boldsymbol{E}_2^2}{\mathbf{c^2}}+\boldsymbol{B}_2^2-\boldsymbol{Q}] & -\frac{\mathbf{E_2E_3}}{\mathbf{c^2}}-\mathbf{B_2B_3} \\ (\frac{\vec{\boldsymbol{E}}}{\boldsymbol{c}}\times\vec{\boldsymbol{B}})_3 & -\frac{\mathbf{E_1E_3}}{\mathbf{c^2}}-\mathbf{B_1B_3} & -\frac{\mathbf{E_2E_3}}{\mathbf{c^2}}-\mathbf{B_2B_3} & -[\frac{\boldsymbol{E}_3^2}{\mathbf{c^2}}+\boldsymbol{B}_3^2-\boldsymbol{Q}] \end{pmatrix} \tag{134}$$

$$\text{with} \qquad \boldsymbol{Q}=\frac{\mathbf{1}}{\mathbf{2}}(\frac{\boldsymbol{E}^2}{\boldsymbol{c}^2}+\boldsymbol{B}^2)$$

The trace of the electromagnetic energy momentum tensors vanishes

$$\mathbf{T}=\mathbf{0} \tag{135}$$

and the Einstein Equations simplify to

$$\mathbf{R}_{\boldsymbol{ij}}=8\pi\cdot G_N\mathbf{T}_{\boldsymbol{ij}}. \tag{136}$$

For further considerations the following eigenwave is choosen:

$$\mathbf{E_2}=\mathbf{E_3}=\mathbf{B_1}=\mathbf{B_3}=\mathbf{0}, \qquad \mathbf{E_1}\neq\mathbf{0}, \qquad \mathbf{B_2}\neq\mathbf{0}. \tag{137}$$

$\Longrightarrow$

$$\mathbf{T_{00}}=\frac{1}{\mathbf{8\pi}}\left(\frac{\mathbf{E_1^2}}{\mathbf{c^2}}+\mathbf{B_2^2}\right), \qquad \mathbf{T_{01}}=\mathbf{T_{02}}=\mathbf{0}, \qquad \mathbf{T_{03}}=\frac{1}{\mathbf{4\pi}}(\frac{\vec{\mathbf{E}}_\mathbf{1}}{\boldsymbol{c}}\times\vec{\mathbf{B}}_\mathbf{2}) \tag{138}$$

$$\mathbf{T_{ik}}=\mathbf{0} \qquad \text{für } \boldsymbol{i}\neq\boldsymbol{k} \qquad \boldsymbol{i},\boldsymbol{k}=\mathbf{1},\mathbf{2},\mathbf{3} \tag{139}$$

$$\mathbf{T_{11}}=\frac{-1}{\mathbf{8\pi}}\left(\frac{\mathbf{E_1^2}}{\mathbf{c^2}}-\mathbf{B_2^2}\right), \qquad \mathbf{T_{22}}=\frac{1}{\mathbf{8\pi}}\left(\frac{\mathbf{E_1^2}}{\mathbf{c^2}}-\mathbf{B_2^2}\right) \tag{140}$$

$$\mathbf{T_{33}}=\frac{1}{\mathbf{8\pi}}\left(\frac{\mathbf{E_1^2}}{\mathbf{c^2}}+\mathbf{B_2^2}\right) \tag{141}$$

4.6 The Quantitative Relation of Electromagnetic and Gravitational Waves

The quantitative connection is achieved via the Einstein Equations

$$\mathbf{R}_{\boldsymbol{\mu\nu}}=8\pi\cdot G_N\mathbf{T}_{\boldsymbol{\mu\nu}}.$$

The description of a natural oscillation takes place using deformation interpretation by

$$\boxed{\begin{aligned} \vec{\boldsymbol{d}}_{k_i} &=\sqrt{4\pi}\frac{1}{k_i}\left(ck_i\vec{\mathbf{q}}_{\boldsymbol{k_i}}cos(\vec{\boldsymbol{k_i}}\cdot\vec{\boldsymbol{r}})-\vec{\boldsymbol{p}}_{\boldsymbol{k_i}}sin(\vec{\boldsymbol{k_i}}\cdot\vec{\boldsymbol{r}})\right) \\ \vec{\boldsymbol{e}}_{k_i} &=\sqrt{4\pi}\boldsymbol{c}\left(ck_i\vec{\boldsymbol{q}}_{\vec{\boldsymbol{k_i}}}sin(\vec{\boldsymbol{k_i}}\cdot\vec{\boldsymbol{r}})+\vec{\boldsymbol{p}}_{\vec{\boldsymbol{k_i}}}cos(\vec{\boldsymbol{k_i}}\cdot\vec{\boldsymbol{r}})\right) \\ \vec{\boldsymbol{b}}_{k_i} &=-\sqrt{4\pi}\frac{1}{k_i}\vec{\boldsymbol{k_i}}\times\left[ck_i\vec{\boldsymbol{q}}_{\vec{\boldsymbol{k_i}}}sin(\vec{\boldsymbol{k_i}}\cdot\vec{\boldsymbol{r}})+\vec{\boldsymbol{p}}_{\vec{\boldsymbol{k_i}}}cos(\vec{\boldsymbol{k_i}}\cdot\vec{\boldsymbol{r}})\right], \end{aligned}} \tag{142}$$

and using the electromagnetic field interpretation by

$$
\boxed{\begin{aligned}
\vec{\boldsymbol{A}}_{k_i} &= \sqrt{4\pi}\frac{1}{k}\left(ck_i\vec{\boldsymbol{Q}}_{\vec{\boldsymbol{k_i}}}cos(\vec{\boldsymbol{k_i}}\cdot\vec{\ })-\vec{\boldsymbol{P}}_{\vec{\boldsymbol{k_i}}}sin(\vec{\boldsymbol{k_i}}\cdot\vec{\boldsymbol{r}})\right)\\
\vec{\boldsymbol{E}}_{k_i} &= \sqrt{4\pi}\boldsymbol{c}\left(ck_i\vec{\boldsymbol{Q}}_{\vec{\boldsymbol{k_i}}}sin(\vec{\boldsymbol{k_i}}\cdot\vec{\boldsymbol{r}})+\vec{\boldsymbol{P}}_{\vec{\boldsymbol{k_i}}}cos(\vec{\boldsymbol{k_i}}\cdot\vec{\boldsymbol{r}})\right)\\
\vec{\mathbf{B}}_{k_i} &= -\sqrt{4\pi}\frac{1}{k}\vec{\boldsymbol{k_i}}\times\left[ck_i\vec{\boldsymbol{Q}}_{\vec{\boldsymbol{k_i}}}sin(\vec{\boldsymbol{k_i}}\cdot\vec{\boldsymbol{r}})+\vec{\boldsymbol{P}}_{\vec{\boldsymbol{k_i}}}cos(\vec{\boldsymbol{k_i}}\cdot\vec{\boldsymbol{r}})\right]
\end{aligned}} \tag{143}
$$

with their corresponding energy density and energy in a volume surrounding the coordinate origin ($\vec{\mathbf{x_0}}$).

$$
\boxed{\begin{aligned}
\mathfrak{E}_{k_i} &= \frac{1}{2}\left(\frac{\boldsymbol{E}^2_{k_i}}{\boldsymbol{c^2}}+\boldsymbol{B}^2_{k_i}\right) && \text{Energiedichte}\\
\mathcal{E}_{k_i} &= \frac{1}{2}\int_{V_0}\left(\frac{\boldsymbol{E}^2_{k_i}}{\boldsymbol{c^2}}+\boldsymbol{B}^2_{k_i}\right)dV && \text{Energie}
\end{aligned}} \tag{144}
$$

.

The metric tensor of an elementary wave with $\vec{\mathbf{q}}_{\vec{\mathbf{k}}} \parallel \vec{\mathbf{e}}_x$, $\vec{\mathbf{p}}_{\vec{\mathbf{k}}} \parallel \vec{\mathbf{e}}_y$ and $\vec{\mathbf{k}} \parallel \vec{\mathbf{e}}_z$, $\vec{\mathbf{k}} \times \vec{\mathbf{q}}_{\vec{\mathbf{k}}} \parallel \vec{\mathbf{e}}_y$ is given by the tangential vectors:

$$
\vec{\boldsymbol{t}}_i = \boldsymbol{\partial}_i\vec{\boldsymbol{y}} = (\boldsymbol{\partial}_i\boldsymbol{y}_1, \boldsymbol{\partial}_i\boldsymbol{y}_2, \boldsymbol{\partial}_i\boldsymbol{y}_3)\,, \qquad \vec{\mathbf{y}} = \vec{\mathbf{d}} + \vec{\mathbf{x}}
$$

$\Longrightarrow$

$$
\vec{\boldsymbol{t}}_z = \boldsymbol{\partial_z}\vec{\boldsymbol{y}} = (\boldsymbol{\partial}_z\boldsymbol{d}_x, \mathbf{0}, \mathbf{1})\,.
$$

With $\vec{\mathbf{k}}\cdot\vec{\mathbf{r}} = \boldsymbol{k}\cdot\boldsymbol{z} = \boldsymbol{\omega}_k/c\cdot\boldsymbol{z}$ one obtains

$$
\vec{\boldsymbol{t}}_z = \left(-\sqrt{4\pi}\omega_k\vec{\mathbf{q}}_{\vec{\mathbf{k}}}sin(\boldsymbol{\omega}_k/c\cdot\boldsymbol{z}), -\sqrt{4\pi}\vec{\mathbf{p}}_{\vec{\mathbf{k}}}cos(\boldsymbol{\omega}_k/c\cdot\boldsymbol{z}), 1\right). \tag{145}
$$

As searched spatial metric tensor element remains

$$
\boldsymbol{t}_{zz} = 4\pi\left(\omega_k^2\mathbf{q}^2_{\mathbf{k}}sin^2(\boldsymbol{\omega}_k/c\cdot\boldsymbol{z}) + \mathbf{p}^2_{\mathbf{k}}cos^2(\boldsymbol{\omega}_k/c\cdot\boldsymbol{z})\right) + \mathbf{1} \tag{146}
$$

with

$$
\mathbf{q_k} = \mathbf{u_k}\boldsymbol{cos}(\omega_{\mathbf{k}}t), \qquad \mathbf{p_k} = \mathbf{v_k}\boldsymbol{sin}(\omega_{\mathbf{k}}t). \tag{147}
$$

The purpose is the evaluation of the equation

$$
\mathbf{R}_{\boldsymbol{zz}} = 8\pi\cdot G_N\mathbf{T}_{\boldsymbol{zz}}. \tag{148}
$$

It is appropriate to note, that

$$
\mathbf{T}_{zz} = \frac{1}{\boldsymbol{8\pi}}\left(\frac{\mathbf{E^2_x}}{c^2} + \mathbf{B^2_y}\right) = \frac{\mathfrak{E}_{\mathbf{k}}}{4\pi}. \tag{149}
$$

Starting from the Riemannian curvature tensor

$$
\mathbf{R}^{\sigma}_{.\nu\alpha\beta} = \partial_\alpha\mathbf{\Gamma}^{\sigma}_{\nu\beta} - \partial_\beta\mathbf{\Gamma}^{\sigma}_{\nu\alpha} + \mathbf{\Gamma}^{\sigma}_{\rho\alpha}\mathbf{\Gamma}^{\rho}_{\nu\beta} - \mathbf{\Gamma}^{\sigma}_{\rho\beta}\mathbf{\Gamma}^{\rho}_{\nu\alpha}. \tag{150}
$$

with

$$
\mathbf{\Gamma}^{\boldsymbol{\mu}}_{\boldsymbol{\nu\alpha}} = \frac{1}{2}\mathbf{g}^{\mu\lambda}\left[\partial_\nu\mathbf{g}_{\alpha\lambda} + \partial_\alpha\mathbf{g}_{\lambda\nu} - \partial_\lambda\mathbf{g}_{\nu\alpha}\right] \tag{151}
$$

leads by contraction to the Ricci tensor

$$
\mathbf{R}_{\mu\nu} = \mathbf{R}^{\sigma}_{.\mu\nu\sigma} = \partial_\nu\mathbf{\Gamma}^{\sigma}_{\mu\sigma} - \partial_\sigma\mathbf{\Gamma}^{\sigma}_{\mu\nu} + \mathbf{\Gamma}^{\sigma}_{\rho\nu}\mathbf{\Gamma}^{\rho}_{\mu\sigma} - \mathbf{\Gamma}^{\sigma}_{\rho\sigma}\mathbf{\Gamma}^{\rho}_{\mu\nu}. \tag{152}
$$

The metric tensor after the deformation by the above elementary wave is used in the time orthogonal coordinate system.

$$\eta_{\mu\nu} = \begin{pmatrix} \mathbf{-1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{pmatrix} \tag{153}$$

$$\mathbf{g}_{\mu\nu}(\vec{\mathbf{x}_0}) = \begin{pmatrix} \mathbf{-1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{t_{zz}} \end{pmatrix} \qquad \mathbf{g}^{\mu\nu}(\vec{\mathbf{x}_0}) = \begin{pmatrix} \mathbf{-1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1/t_{zz}} \end{pmatrix} \tag{154}$$

$$\begin{gathered} \mathbf{g}_{\mu\nu} \approx \eta_{\mu\nu} + \mathbf{h}_{\mu\nu}, \qquad \mathbf{g}^{\mu\nu} \approx \eta^{\mu\nu} - \mathbf{h}^{\mu\nu} \\ |\mathbf{h}_{\mu\nu}|,|\mathbf{h}^{\mu\nu}| \ll \mathbf{1} \end{gathered} \tag{155}$$

The Ricci tensor is typically written in a linear and non-linear proportion with respect to the Christoffel symbols stripped down.

$$\mathbf{R}^{(1)}_{\mu\nu}(\vec{\mathbf{x}_0}) = \partial_\nu \mathbf{\Gamma}^\sigma_{\mu\sigma} - \partial_\sigma \mathbf{\Gamma}^\sigma_{\mu\nu}, \qquad \mathbf{R}^{(2)}_{\mu\nu}(\vec{\mathbf{x}_0}) = \mathbf{\Gamma}^\sigma_{\rho\nu}\mathbf{\Gamma}^\rho_{\mu\sigma} - \mathbf{\Gamma}^\sigma_{\rho\sigma}\mathbf{\Gamma}^\rho_{\mu\nu} \tag{156}$$

Detailed examination of the Christoffel symbols

$$\mathbf{\Gamma}^{\boldsymbol{\sigma}}_{\boldsymbol{\mu\sigma}} = \frac{1}{2}\sum_\sigma\sum_\rho g^{\sigma\rho}\partial_\sigma g_{\mu\rho} + \frac{1}{2}\sum_\sigma\sum_\rho g^{\sigma\rho}\partial_\mu g_{\mu\rho} - \frac{1}{2}\sum_\sigma\sum_\rho g^{\sigma\rho}\partial_\rho g_{\mu\rho} \tag{157}$$

$$\begin{aligned} \frac{1}{2}\sum_\sigma\sum_\rho g^{\sigma\rho}\partial_\sigma g_{z\rho} &= \frac{1}{2}\underbrace{g^{00}\partial_0 g_{z0}}_{=0} + \frac{1}{2}g^{zz}\partial_z g_{zz} \\ \frac{1}{2}\sum_\sigma\sum_\rho g^{\sigma\rho}\partial_z g_{\rho\sigma} &= \frac{1}{2}\underbrace{g^{00}\partial_z g_{00}}_{=0} + \frac{1}{2}g^{zz}\partial_z g_{zz} \\ \frac{1}{2}\sum_\sigma\sum_\rho g^{\sigma\rho}\partial_\rho g_{\sigma z} &= \frac{1}{2}\underbrace{g^{00}\partial_0 g_{00}}_{=0} + \frac{1}{2}g^{zz}\partial_z g_{zz} \end{aligned} \tag{158}$$

$$\partial_z \mathbf{\Gamma}^\sigma_{\mathbf{z}\sigma} = \frac{1}{2}\partial_z \mathbf{g}^{zz}\partial_z \mathbf{g}_{zz} \tag{159}$$

$$\begin{aligned} \partial_\sigma \mathbf{\Gamma}^\sigma_{\mathbf{zz}} =& \frac{1}{2}\partial_0 \mathbf{g}^{00}[\partial_z \underbrace{\mathbf{g}_{z0}}_{=0} + \partial_z \underbrace{\mathbf{g}_{0z}}_{=0} - \partial_0 \mathbf{g}_{zz}] + \frac{1}{2}\partial_z \mathbf{g}^{zz}[\partial_z \mathbf{g}_{zz} + \partial_z \mathbf{g}_{zz} - \partial_z \mathbf{g}_{zz}] \\ =& + \frac{1}{2}\partial_0^2 \mathbf{g}_{zz} + \frac{1}{2}\partial_z \mathbf{g}^{zz}\partial_z \mathbf{g}_{zz} \end{aligned} \tag{160}$$

lead for the linear part to

$$\mathbf{R}^{(1)}_{zz}(\vec{\mathbf{x}_0}) = \partial_z \mathbf{\Gamma}^\sigma_{z\sigma} - \partial_\sigma \mathbf{\Gamma}^\sigma_{zz} = -\frac{1}{2}\partial_0^2 \mathbf{g}_{zz}. \tag{161}$$

The nonlinear part is determined for the considered elementary wave by

$$\mathbf{R}^{(2)}_{zz}(\vec{\mathbf{x}_0}) = \mathbf{\Gamma}^\sigma_{\rho z}\mathbf{\Gamma}^\rho_{z\sigma} - \mathbf{\Gamma}^\sigma_{\rho\sigma}\mathbf{\Gamma}^\rho_{zz} \tag{162}$$

with

$$\mathbf{\Gamma}^{\boldsymbol{\sigma}}_{\boldsymbol{\rho}\mathbf{z}} = \frac{1}{2}\mathbf{g}^{\sigma\sigma}\Big[\partial_\rho \mathbf{g}_{z\sigma} + \partial_z \mathbf{g}_{\sigma\rho} - \partial_\sigma \mathbf{g}_{\rho z}\Big] \tag{163}$$

$$\mathbf{\Gamma}^{\boldsymbol{\rho}}_{\mathbf{z}\boldsymbol{\sigma}} = \frac{1}{2}\mathbf{g}^{\rho\rho}\Big[\partial_z \mathbf{g}_{\sigma\rho} + \partial_z \mathbf{g}_{\rho z} - \partial_\rho \mathbf{g}_{z\sigma}\Big] \tag{164}$$

Considering the asumed elementary wave the single partial differentiaions $\partial_0,\ \partial_z$ of the metric tensor vanish in the space-time point $(0,0,0,0)$.

Thus one gets

$$\mathbf{R}_{zz}(\vec{\mathbf{x}_0}) = \mathbf{R}^{(1)}_{zz}(\vec{\mathbf{x}_0}) = -\frac{1}{2}\partial_0^2 \mathbf{g}_{zz}(\vec{\mathbf{x}_0}). \tag{165}$$

Now using

$$\mathbf{R}_{zz} = 8\pi \cdot G_N \mathbf{T}_{zz}.$$

and concerning

$$\partial_0 = \frac{1}{ic}\partial_t$$

the amplitude of the elementary gravitational wave (electromagnetic wave) gives the quantitative deformation of space by an electrodynamic elementary wave. Such the importance of the Einstein-Equations for microphysics is proved.

$$\boxed{\mathbf{d_k} = \frac{\mathbf{2}}{\omega_k^2}\sqrt{\pi\gamma\mathfrak{E}_\mathbf{k}}}. \tag{166}$$

with the constant of gravitation $\gamma = 6.67 \cdot 10^{-11} m^3 kg^{-1} s^{-2}$ and $\mathfrak{E}_\mathbf{k} =$ as energy density. In these considerations the light velocity c does not occur explicitly.

Setting $\mathfrak{E}_\mathbf{k} = 1Wsec/m^3$ and using $\omega_k^2 = (\mathbf{2}\pi \cdot \nu)^\mathbf{2}$ with $\nu = 50$ this results in $\mathbf{d_k} = 2.933 \cdot 10^{-10} m$. In comparison, the measured atomic radius of H^1 is given by $\approx 2.5 \cdot 10^{-11} m$. **Obviously, that effect has to be considered in practice.**

As Spin 1 is assigned to photons the same has to be assumed for the graviton. (A photon of giant wavelength from an other perspective, if it is existent.)
The Einstein Equations maybe achieve much more than describing cosmological processes!

4.7 Summary

Until today, electromagnetism is not directly understood. It is described with detours via mechanical effects, and appears to physicists after more than a century of successful handling as a matter of course. With the unification described, electromagnetism is directly attributed to basic concepts of physics, space and time. The commonly discussed gauge transformations are defined by the observation space or the coordinate space. The vector potential attains an absolute meaning.

5 Explanation of the Photon and the Creation of Photon Quanta in a Maxwell Field

5.1 Introduction

For quantum electrodynamics, photon and electron are central observation objects. But for both quantum particles there are no clear descriptions of their size and structure, including their states of motion. The uncertainty relation of quantum theory does not allow the simultaneous exact positioning of momentum and location.

However, they are at the centre of any discussion of quantum electrodynamics (in particular their interactions). Also the quantization of the electromagnetic field, which should produce photons, appears unsatisfactory. In the textbook of Landau-Lifschitz Volume IV [18], for example, the derivation of the quantization of the electromagnetic field results in inconsistencies, which are explained by remarks like "... we meet with one of the divergences which are due to the fact that the present theory is not logically complete and consistent". And Albert Einstein expressed it particularly drastically shortly before his death: "Jeder Hinz und Kunz meint heute, er habe verstanden, was ein Photon ist, aber sie irren sich."

On the whole, attempts are made to represent photons by spherical waves or even plane waves, which leads to contradictions. But photons propagate 1-dimensional, as it is not known in classical physics for elastic wave propagation in a 3-dimensional medium. In the following the photon turns out to be a "particle", which is defined in a point and due to the initial conditions of a 1-dimensional wave equation, unambiguously determins its detailed motion in space and time. The derivation avoids hypotheses and is based on a physics with natural causality.

5.2 Stochastic Fluctuation Movements with Presribed Velovity Direction

The following considerations comply with a special case of section 2.

Subsequently, continuum fluctuation vector fields of deformation $\vec{\mathbf{d}}(\mathbf{z},t)\perp\vec{\mathbf{i}}_z$ are assumed orthogonal to the z-direction of propagation without loss of generality

$$\vec{\mathbf{i}}_z\partial/\partial z \times \vec{\mathbf{d}} \neq \mathbf{0}. \tag{167}$$

The vector fields $\vec{\mathbf{e}}$ and $\vec{\mathbf{b}}$ defined by

$$\begin{aligned}\vec{\mathbf{e}} =& \partial\vec{\mathbf{d}}/\partial t \neq 0\\ \vec{\mathbf{b}} =& \vec{\mathbf{i}}_z\partial/\partial z \times \vec{\mathbf{d}} \neq 0, \qquad \vec{\mathbf{i}}_z = \text{unit vector in z-direction}\end{aligned} \tag{168}$$

are expected to be continuously differentiable, sufficiently often. According to interchangeability of the operators $\partial/\partial t$ und $\vec{\mathbf{i}}_z\partial/\partial z\times$ follows

$$\frac{\partial\vec{\mathbf{b}}}{\partial t} = \vec{\mathbf{i}}_z\partial/\partial z \times \vec{\mathbf{e}} \tag{169}$$

immediately. The dual equation for this is searched as follows:

In an analogous approach to the derivation of continuum flutuation equations of general 3-dimensional vector fields an stochastic ensemble theory is formulated leading over to the deterministic theory and resulting in a pair of dual deterministic equations for the fluctuation quantities $\vec{\mathbf{e}}$ und $\vec{\mathbf{b}}$.
A continuously differentiable distribution density

$$f_{\boldsymbol{t_\epsilon}} = f_{\boldsymbol{t_\epsilon}}(\mathbf{z},t,\vec{\mathbf{e}},\vec{\mathbf{b}}) \tag{170}$$

of the motion quantities $\vec{\mathbf{e}}_{t_\epsilon} = \partial\vec{\mathbf{d}}_{t_\epsilon}/\partial t$, $\vec{\mathbf{b}}_{t_\epsilon} = \vec{\mathbf{i}}_z\partial/\partial z \times \vec{\mathbf{d}}_{t_\epsilon}$ with $\vec{\mathbf{d}}_{t_\epsilon}\perp\vec{\mathbf{i}}_z$ as well as $\vec{\mathbf{e}}_{t_\epsilon}\perp\vec{\mathbf{i}}_z$ and $\vec{\mathbf{b}}_{t_\epsilon}\perp\vec{\mathbf{i}}_z$ is allocated every-space-time point $(\mathbf{z},t)$. For the with $\boldsymbol{t_\epsilon}$ or $\boldsymbol{\epsilon}$ indexed functions is automatically assumed, that the incorporated motion quantities $(\vec{\mathbf{e}},\vec{\mathbf{b}})$ are assigned to a $\boldsymbol{t_\epsilon}$ measurement accuracy. That is the indexing of the motion quantities may be omitted if the functions themselves are indexed.

Only after execution of the limiting process

$$\lim_{\boldsymbol{t_\epsilon}\to 0} f_{\boldsymbol{t_\epsilon}}(\mathbf{z},t,\vec{\mathbf{e}},\vec{\mathbf{b}}) = f(\mathbf{z},t,\vec{\mathbf{e}},\vec{\mathbf{b}}) \tag{171}$$

f and $(\vec{\mathbf{e}},\vec{\mathbf{b}})$ are understood in the sense of an exact measuring process.
The stochastic transport of the fluctuation quantities

$$\Big(\vec{\mathbf{e}}'_{t_\epsilon}(\mathbf{z}-\Delta\mathbf{z},t-t_\epsilon)\vec{\mathbf{b}}'_{t_\epsilon}(\mathbf{z}-\Delta\mathbf{z},t-t_\epsilon)\Big) \longrightarrow \Big(\vec{\mathbf{e}}_{t_\epsilon}(\mathbf{z},t),\vec{\mathbf{b}}_{t_\epsilon}(\mathbf{z},t)\Big)$$

takes place by the transition probabillity density $W_{t_\epsilon} = W_{t_\epsilon}(\mathbf{z},t,\vec{\mathbf{e}},\vec{\mathbf{b}},\vec{\mathbf{e}}',\vec{\mathbf{b}}')$
with

$$\boxed{\begin{aligned}\lim_{t_\epsilon\to 0} W_{t_\epsilon} =& \delta(\vec{\mathbf{e}},\vec{\mathbf{b}};\vec{\mathbf{e}}',\vec{\mathbf{b}}')\\ f_{t_\epsilon}(\mathbf{z},t,\vec{\mathbf{e}},\vec{\mathbf{b}}) =& \int\limits_{\vec{\mathbf{b}}'}\int\limits_{\vec{\mathbf{e}}'} W_{t_\epsilon}(\mathbf{z},t,\vec{\mathbf{e}},\vec{\mathbf{b}},\vec{\mathbf{e}}',\vec{\mathbf{b}}')\cdot f_{t_\epsilon}(\mathbf{z}-\Delta\mathbf{z},t-t_\epsilon,\vec{\mathbf{e}}',\vec{\mathbf{b}}')d\vec{\mathbf{e}}'d\vec{\mathbf{b}}'\\ \Delta\mathbf{z} =& t_\epsilon\cdot\vec{\mathbf{e}}'\times\frac{\vec{\mathbf{b}}'}{b'^2}\cdot\vec{\mathbf{i}}_z \;\text{ und } \vec{\mathbf{e}}'\times\frac{\vec{\mathbf{b}}'}{b'^2}\cdot\vec{\mathbf{i}}_z = \text{Ausbreitungsgeschwindigkeit}\end{aligned}} \tag{172}$$

These equations define stochastic transport continuum fluctuations of the quantities $\vec{\mathbf{e}}$ and $\vec{\mathbf{b}}$ in the sense of an ensemble theory and represent a Markov process with natural causality.
f_{t_ϵ} is developed until the first order about $(\mathbf{z},t) \Longrightarrow$

$$f_{t_\epsilon}(\mathbf{z}-\triangle\mathbf{z},t-t_\epsilon,\vec{\mathbf{e}}',\vec{\mathbf{b}}') = f_{t_\epsilon} - \frac{\partial f'_{t_\epsilon}}{\partial t}\cdot t_\epsilon - \boldsymbol{\Delta}\mathbf{z}\cdot\frac{\boldsymbol{\partial}}{\boldsymbol{\partial z}}f'_{t_\epsilon} + \boldsymbol{O}({t_\epsilon}^2) \tag{173}$$

and one gets

$$\int\int W_{t_\epsilon}\left[\frac{\partial f'_{t_\epsilon}}{\partial t} + \vec{\mathbf{e}}'\times\frac{\vec{\mathbf{b}}'}{b'^2}\cdot\vec{\mathbf{i}}_z\frac{\boldsymbol{\partial}}{\boldsymbol{\partial z}}f'_{t_\epsilon}\right]d\vec{\mathbf{e}}'d\vec{\mathbf{b}}' + \boldsymbol{O}({t_\epsilon}^2) = \frac{\int\int W_{t_\epsilon}f'_{t_\epsilon}d\vec{\mathbf{e}}'d\vec{\mathbf{b}}' - f_{t_\epsilon}}{t_\epsilon}. \tag{174}$$

Executing the limiting process $t_\epsilon \to 0$ W_{t_ϵ} degenerates to a δ-function:

$$\lim_{\boldsymbol{t_\epsilon\to 0}} W_{t_\epsilon} = \delta(\vec{\mathbf{e}},\vec{\mathbf{b}};\vec{\mathbf{e}}',\vec{\mathbf{b}}') \tag{175}$$

and

$$\frac{\partial f}{\partial t} + \vec{\mathbf{e}}\times\frac{\vec{\mathbf{b}}}{b^2}\cdot\vec{\mathbf{i}}_z\frac{\boldsymbol{\partial}}{\boldsymbol{\partial z}}f = \lim_{t_\epsilon\to 0}\frac{\int_{\vec{\mathbf{r}}}\int_{\vec{\boldsymbol{\omega}}} W_{t_\epsilon}f'_{t_\epsilon}d\vec{\mathbf{e}}'d\vec{\mathbf{b}}' - f_{t_\epsilon}}{t_\epsilon} \qquad . \tag{176}$$

Rediscovering equation (169) the exchange term

$$\lim_{t_\epsilon\to 0}\frac{\int_{\vec{\mathbf{b}}}\int_{\vec{\mathbf{e}}} W_{t_\epsilon}f'_{t_\epsilon}d\vec{\mathbf{e}}'d\vec{\mathbf{b}}' - f_{t_\epsilon}}{t_\epsilon} = \mathbf{0}. \tag{177}$$

has to vanish after the transition to the deterministic consideration. This link is part of the viewed stochastic process.
Limiting ourselves to one system of the ensemble the function $f(\mathbf{z},t,\vec{\mathbf{e}},\vec{\mathbf{b}})$ degenerates in the space-time point $(\mathbf{z},t)$ to

$$f(\mathbf{z},t,\vec{\mathbf{e}},\vec{\mathbf{b}}) \longrightarrow \delta(\vec{\mathbf{e}}_{(\mathbf{z},t)},\vec{\mathbf{b}}_{(\mathbf{z},t)};\vec{\mathbf{e}}',\vec{\mathbf{b}}')\text{-function} \tag{178}$$

so that the key equation

$$\boxed{\frac{\partial}{\partial t}\delta + \vec{\mathbf{e}}_{(\boldsymbol{z},t)}\times\frac{\vec{\mathbf{b}}_{(\boldsymbol{z},t)}}{b^2_{(\boldsymbol{z},t)}}\cdot\vec{\mathbf{i}}_z\frac{\boldsymbol{\partial}}{\boldsymbol{\partial z}}\delta = \mathbf{0}} \tag{179}$$

develops from equation (176).

5.3 The Deterministic Fluctuation Equations for Fluctuations with Prescribed Velocity Direction

Equation (179) shows the interface for the transition from stochastic to deterministic consideration. From the view of the ensemble theory one is limited to the motion quantities $(\vec{\mathbf{e}}_{(\mathbf{z},t)},\vec{\mathbf{b}}_{(\mathbf{z},t)})$ of one deterministic system at the space-time-point $(\mathbf{z},t)$. In this situation the vectorial motion quantities may be shifted before and behind the differential operators They are seen as constant vectors.

$$\begin{aligned}
\vec{\mathbf{e}}_{(\boldsymbol{z},t)}\times\frac{\vec{\mathbf{b}}_{(\boldsymbol{z},t)}}{b^2_{(\boldsymbol{z},t)}}\cdot\vec{\mathbf{i}}_z\frac{\boldsymbol{\partial}}{\boldsymbol{\partial z}}\delta &= -\frac{\vec{\mathbf{b}}_{(\boldsymbol{z},t)}}{b^2_{(\boldsymbol{z},t)}}\times\vec{\mathbf{e}}_{(\boldsymbol{z},t)}\cdot\vec{\mathbf{i}}_z\frac{\boldsymbol{\partial}}{\boldsymbol{\partial z}}\delta \\
&= -\frac{\vec{\mathbf{b}}_{(\boldsymbol{z},t)}}{b^2_{(\boldsymbol{z},t)}}\cdot\vec{\mathbf{i}}_z\frac{\boldsymbol{\partial}}{\boldsymbol{\partial z}}\times\vec{\mathbf{e}}_{(\boldsymbol{z},t)}\delta.
\end{aligned}$$

It applies

$$\begin{aligned}
&\frac{\partial}{\partial t}(\frac{\vec{\mathbf{b}}_{(\boldsymbol{z},t)}\cdot\vec{\mathbf{b}}_{(\boldsymbol{z},t)}}{b^2_{(\boldsymbol{z},t)}}\delta) - \frac{\vec{\mathbf{b}}_{(\boldsymbol{z},t)}}{b^2_{(\boldsymbol{z},t)}}\cdot\vec{\mathbf{i}}_z\frac{\boldsymbol{\partial}}{\boldsymbol{\partial z}}\times(\vec{\mathbf{e}}_{(\boldsymbol{z},t)}\delta) = 0 \\
\Longrightarrow\ &\frac{\vec{\mathbf{b}}_{(\boldsymbol{z},t)}}{b^2_{(\boldsymbol{z},t)}}\cdot[\frac{\partial}{\partial t}(\vec{\mathbf{b}}_{(\boldsymbol{z},t)}\delta) - \vec{\mathbf{i}}_z\frac{\boldsymbol{\partial}}{\boldsymbol{\partial z}}\times(\vec{\mathbf{e}}_{(\boldsymbol{z},t)}\delta)] = 0 \\
\Longrightarrow\ &\frac{\partial}{\partial t}(\vec{\mathbf{b}}_{(\boldsymbol{z},t)}\delta) - \vec{\mathbf{i}}_z\frac{\boldsymbol{\partial}}{\boldsymbol{\partial z}}\times(\vec{\mathbf{e}}_{(\boldsymbol{z},t)}\delta) = 0 \qquad .
\end{aligned} \tag{180}$$

Using the following relations

$$\begin{aligned}\Xi\left[\int_{\vec{\mathbf{e}}}\int_{\vec{\mathbf{b}}}\delta(\vec{\mathbf{b}}_{(z,t)},\vec{\mathbf{e}}_{(z,t)};\vec{\mathbf{b}},\vec{\mathbf{e}})\vec{\mathbf{b}}d\vec{\mathbf{b}}d\vec{\mathbf{e}}\right]&=\Xi[\vec{\mathbf{b}}_{(z,t)}]=\vec{\mathbf{b}}(z,t)\\ \Xi\left[\int_{\vec{\mathbf{E}}}\int_{\vec{\mathbf{b}}}\delta(\vec{\mathbf{b}}_{(z,t)},\vec{\mathbf{e}}_{(z,t)};\vec{\mathbf{b}},\vec{\mathbf{e}}')\vec{\mathbf{e}}d\vec{\mathbf{b}}d\vec{\mathbf{e}}\right]&=\Xi\left[\vec{\mathbf{e}}_{(z,t)}\right]=\vec{\mathbf{e}}(z,t)\end{aligned}\tag{181}$$

or

$$\Xi\left[\int_{\vec{\mathbf{e}}}\int_{\vec{\mathbf{b}}}\delta(\vec{\mathbf{b}}_{(z,t)},\vec{\mathbf{e}}_{(z,t)};\vec{\mathbf{b}},\vec{\mathbf{e}})\left(\frac{b^2}{e^2}\cdot\vec{\mathbf{e}}\right)d\vec{\mathbf{b}}d\vec{\mathbf{e}}\right]=\Xi\left[\frac{b^2_{(z,t)}}{e^2_{(z,t)}}\cdot\vec{\mathbf{e}}_{(z,t)}\right]=\frac{b^2(z,t)}{e^2(z,t)}\cdot\vec{\mathbf{e}}(z,t)\tag{182}$$

the existing environments of the movement sizes ($\vec{\mathbf{e}}_{(\mathbf{z},t)}$, $\vec{\mathbf{b}}_{(\mathbf{z},t)}$) of the individual deterministic systems around the point $(\mathbf{z},t)$ are generated. This executes the transition to the deterministic system of equations:

$$\Xi\left[\int_{\vec{\mathbf{b}}}\int_{\vec{\mathbf{e}}}\left[\frac{\partial}{\partial t}(\vec{\mathbf{b}}\delta)-\vec{\mathbf{i}}_z\frac{\boldsymbol{\partial}}{\boldsymbol{\partial z}}\times(\vec{\mathbf{e}}\delta)=0\right]d\vec{\mathbf{e}}d\vec{\mathbf{b}}\right].\tag{183}$$

Integration and differentiation beeing exchangeable $\Longrightarrow$

$$\frac{\partial}{\partial t}\Xi[\vec{\mathbf{b}}_{(z,t)}]-\vec{\mathbf{i}}_z\frac{\boldsymbol{\partial}}{\boldsymbol{\partial z}}\times\Xi[\vec{\mathbf{e}}_{(z,t)}]=0\qquad.\tag{184}$$

So we have the first of the dual deterministic fluktuation equations

$$\frac{\partial}{\partial t}\vec{\mathbf{b}}-\vec{\mathbf{i}}_z\frac{\boldsymbol{\partial}}{\boldsymbol{\partial z}}\times\vec{\mathbf{e}}=0.\tag{185}$$

Back to the key equation (179)

$$\frac{\partial}{\partial t}\delta+\vec{\mathbf{e}}_{(z,t)}\times\frac{\vec{\mathbf{b}}_{(z,t)}}{b^2_{(z,t)}}\cdot\vec{\mathbf{i}}_z\frac{\boldsymbol{\partial}}{\boldsymbol{\partial z}}\delta=\mathbf{0}$$

one gets by simple transformations

$$\begin{aligned}\frac{\partial}{\partial t}\vec{\mathbf{e}}_{(z,t)}\cdot\frac{\vec{\mathbf{e}}_{(z,t)}}{e^2_{(z,t)}}\delta+\vec{\mathbf{e}}_{(z,t)}\cdot\vec{\mathbf{i}}_z\frac{\boldsymbol{\partial}}{\boldsymbol{\partial z}}\times\left(\frac{\vec{\mathbf{b}}_{(z,t)}}{b^2_{(z,t)}}\delta\right)&=0\\ \frac{\partial}{\partial t}\left(\frac{b^2_{(z,t)}}{e^2_{(z,t)}}\cdot\vec{\mathbf{e}}_{(z,t)}\delta\right)+\vec{\mathbf{i}}_z\frac{\boldsymbol{\partial}}{\boldsymbol{\partial z}}\times(\vec{\mathbf{b}}_{(z,t)}\delta)&=0\end{aligned}\tag{186}$$

$$\Xi\left[\int_{\vec{\mathbf{b}}}\int_{\vec{\mathbf{e}}}\left[\frac{\partial}{\partial t}\left(\frac{b^2_{(z,t)}}{e^2_{(z,t)}}\cdot\vec{\mathbf{e}}_{(z,t)}\delta\right)+\vec{\mathbf{i}}_z\frac{\boldsymbol{\partial}}{\boldsymbol{\partial z}}\times(\vec{\mathbf{b}}_{(z,t)}\delta)=0\right]d\vec{\mathbf{e}}d\vec{\mathbf{b}}\right]\tag{187}$$

or rather

$$\frac{\partial}{\partial t}\Xi\left[\frac{b^2_{(z,t)}}{e^2_{(z,t)}}\cdot\vec{\mathbf{e}}_{(z,t)}\right]+\vec{\mathbf{i}}_z\frac{\boldsymbol{\partial}}{\boldsymbol{\partial z}}\times\Xi[\vec{\mathbf{b}}_{(z,t)}]=0\tag{188}$$

a second, a dual equation

$$\frac{\partial}{\partial t}(\frac{b^2}{e^2}\cdot\vec{\mathbf{e}})+\vec{\mathbf{i}}_z\frac{\boldsymbol{\partial}}{\boldsymbol{\partial z}}\times\vec{\mathbf{b}}=0.\tag{189}$$

In sum the deterministic theory is represented by the following equation system:

$$\boxed{\begin{array}{l}\frac{\partial}{\partial t}\vec{\mathbf{b}}-\vec{\mathbf{i}}_z\frac{\boldsymbol{\partial}}{\boldsymbol{\partial z}}\times\vec{\mathbf{e}}=0\\ \frac{\partial}{\partial t}\left(\frac{b^2}{e^2}\cdot\vec{\mathbf{e}}\right)+\vec{\mathbf{i}}_z\frac{\boldsymbol{\partial}}{\boldsymbol{\partial z}}\times\vec{\mathbf{b}}=0\\ \vec{\mathbf{e}}\times\frac{\vec{\mathbf{b}}}{b^2}=\text{propagation speed}\end{array}} \tag{190}$$

with $|\vec{\mathbf{e}}\times\frac{\vec{\mathbf{b}}}{b^2}|\leq|\vec{\mathbf{e}}|\cdot|\frac{\vec{\mathbf{b}}}{b^2}|$. Viz. $\frac{e^2}{b^2}$ is not the square propagation speed. Interestingly, this becomes apparent after the enlistment of the stochastic ensemble theory. Stochastic and deterministic theory form one unit.

5.4 An Equation for a Photon or a Graviton from a Classical Viewpoint

If the deformation fluctuations are identified as space-time fluctuations, the constant propagation velocity c must be assumed. This results in the set of equations

$$\boxed{\begin{array}{l}\frac{\partial}{\partial t}\vec{\mathbf{b}}-\vec{\mathbf{i}}_z\frac{\boldsymbol{\partial}}{\boldsymbol{\partial z}}\times\vec{\mathbf{e}}=0\\ \frac{1}{c^2}\frac{\partial}{\partial t}\vec{\mathbf{e}}+\vec{\mathbf{i}}_z\frac{\boldsymbol{\partial}}{\boldsymbol{\partial z}}\times\vec{\mathbf{b}}=0\\ c=\text{propagation speed}\end{array}} \tag{191}$$

for the determined coordinate direction z. Obviously, one gets these equations at once making disappear the differentiation by coordinates in the equations (98), beeing perpendicular to the direction of propagation. It is, however, usefull to describe the related deformation process, in detail.

$$\boxed{\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{\mathbf{d}}(z,t)=\frac{\boldsymbol{\partial^2}}{\boldsymbol{\partial z^2}}\vec{\mathbf{d}}(z,t)} \tag{192}$$

may be understood as equation for gravitons or photons. From this results the above-mentioned elementary solution

$$\boxed{\begin{array}{rl}\vec{\mathbf{d}}_{k_i}=&\sqrt{4\pi}\frac{1}{k_i}\Big(ck_i\vec{\mathbf{q}}_{\mathbf{k_i}}cos(\mathbf{k_i}\cdot z)-\vec{\mathbf{p}}_{\mathbf{k_i}}sin(\mathbf{k_i}\cdot z)\Big)\\ \vec{\mathbf{e}}_{k_i}=&\sqrt{4\pi}\boldsymbol{c}\Big(ck\vec{\mathbf{q}}_{k_i}sin(\mathbf{k_i}\cdot z)+\vec{\mathbf{p}}_{k_i}cos(\mathbf{k_i}\cdot z)\Big)\\ \vec{\mathbf{b}}_{k_i}=&-\sqrt{4\pi}\frac{1}{k_i}\mathbf{k}_i\vec{\mathbf{i}}_z\times\Big[ck\vec{\mathbf{q}}_{k_i}sin(\mathbf{k_i}\cdot z)+\vec{\mathbf{p}}_{k_i}cos(\mathbf{k_i}\cdot z)\Big]\\ &\vec{\mathbf{q}}_{\mathbf{k_i}}\sim\boldsymbol{cos}(\omega_{\mathbf{k_i}}t)\qquad\vec{\mathbf{p}}_{\mathbf{k_i}}\sim\boldsymbol{sin}(\omega_{\mathbf{k_i}}t).\end{array}} \tag{193}$$

Graviton and Photon are different interpretations of one and the same object. This always applies on the assumption that a graviton exists at all, which is far from being self-evident. But it is conceivable that photons have a low energetic limit to their existence. They spiral in one direction through space.

5.5 The Quantization Process within Classical Physics

This process is activated by a deformation thrust that fulfils the appropriate initial conditions

$$\boxed{\begin{aligned}&\vec{\mathbf{d}}_{k_i0} = \vec{\mathbf{d}}_{k_i}(z_0,t_0) \perp \vec{\mathbf{i}}_z\\ &\vec{\mathbf{b}}_{k_i0} = \vec{\mathbf{b}}_{k_i}(z_0,t_0) = \vec{\mathbf{i}}_z\frac{\partial}{\partial z}\times\vec{\mathbf{d}}_{k_i}|_{(z_0,t_0)}\\ &\vec{\mathbf{e}}_{k_i0} = \vec{\mathbf{e}}_{k_i}(z_0,t_0) = \frac{\partial\vec{\mathbf{d}}_{k_i}}{\partial t}|_{(z_0,t_0)}\end{aligned}} \tag{194}$$

for the equations (191). These initial conditions define the photon in one point with all known properties of the quantum object **photon**. This process then presents the quantization of the electromagnetic field in more detail. This formal description is of course lacking the explanation of how such a detailed process can come about. [15]If the electromagnetic field in a vacuum is directly properties of space-time, this also applies to the photon. It has properties of deformation that are unimaginable in known elastic matter. The mathematical described propagation is 1-dimensional. The photon at point $(\vec{\mathbf{x}_0}, t_0)$ is represented by the initial conditions

$$\vec{\mathbf{b}}_{k_i}(z_0,t_0) \qquad \text{and} \qquad \vec{\mathbf{e}}_{k_i}(z_0,t_0)$$

for the equation set

$$\boxed{\begin{aligned}&\frac{\partial}{\partial t}\vec{\mathbf{b}} - \vec{\mathbf{i}}_z\frac{\boldsymbol{\partial}}{\boldsymbol{\partial z}}\times\vec{\mathbf{e}} = 0\\ &\frac{1}{c^2}\frac{\partial}{\partial t}\vec{\mathbf{e}} + \vec{\mathbf{i}}_z\frac{\boldsymbol{\partial}}{\boldsymbol{\partial z}}\times\vec{\mathbf{b}} = 0\\ &c = \text{propagation speed}\end{aligned}}$$

The deformation vector spirals like a helix with the experimentally determined frequency ν of the photon. The fluctuations posses the energy

$$\mathcal{E}_{Photon} = \hbar\cdot\omega \tag{195}$$

where the motion quantities $\vec{\mathbf{i}}_z\frac{\partial}{\partial z}\times\vec{\mathbf{d}}$ and $\frac{\partial\vec{\mathbf{d}}}{\partial t}$ propagate in the z-direction (without loss of generality) at the speed of light according to the above system of equations.

If the existence of a photon can be assumed at a space-time point, its momentum is also automatically known at this point! The next question would be: What generates the disturbances $\vec{\mathbf{e}}$ and $\vec{\mathbf{b}}$ or what creates spatial deformations? Deformation fluctuations of space are now explained by electromagnetic fluctuations.

5.6 Summary

The explanation of the photon is connected with a more detailed description of the quantization of electromagnetic fields. These are deformation impulses, which spread out in 1-dimensional space. Such deformation impulses cannot be realized in elastic bodies. Space proves to be a reality that can still hold some “surprises“, perhaps the explanation of what matter means. A more precise analysis of the electron could lead to corresponding advances.

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