FORMULA FOR THE GOLDBACH FUNCTION

V. Barbera

Abstract
This paper presents an exact elementary formula for the Goldbach function.

Goldbach function *g(2n)*[1]

Given a positive integer $m>3$ *we have*[2]:

$$1+\sum_{p=2}^{\lfloor\sqrt{m}\rfloor}\left(\left\lfloor\frac{m}{p}\right\rfloor-\left\lfloor\frac{m-1}{p}\right\rfloor\right)=\begin{cases}1 & \text{if } m \text{ is prime}\\ i>1 & \text{if } m \text{ is composite}\end{cases}$$

therefore:

$$g(2n)=\left\lfloor\left[1+\sum_{p=2}^{\lfloor\sqrt{2n-3}\rfloor}\left(\left\lfloor\frac{2n-3}{p}\right\rfloor-\left\lfloor\frac{2n-4}{p}\right\rfloor\right)\right]^{-1}\right\rfloor+\sum_{m=5}^{n}\left\lfloor\left[1+\sum_{p=2}^{\lfloor\sqrt{m}\rfloor}\left(\left\lfloor\frac{m}{p}\right\rfloor-\left\lfloor\frac{m-1}{p}\right\rfloor\right)+\sum_{p=2}^{\lfloor\sqrt{2n-m}\rfloor}\left(\left\lfloor\frac{2n-m}{p}\right\rfloor-\left\lfloor\frac{2n-m-1}{p}\right\rfloor\right)\right]^{-1}\right\rfloor$$

given that the prime numbers except *2* and *3* are congruent to $\pm1(mod\,6)$ eliminating the values of *m* and *p* multiples of *2* and *3* we have $m=\pm1+6\cdot k$ and $p=\pm1+6\cdot a$ three cases can be distinguished:

$$n=3\cdot k\equiv 0\,mod(3)$$

$2\cdot n=6\cdot k=(-1+6\cdot k_1)+(1+6\cdot k_2)$ with k_1 *from 1 to* $(k-1)$ and $k_2=k-k_1$

$$g(2\cdot n)=g(6\cdot k)=\sum_{k_1=1}^{k-1}\left\lfloor\left[1+\sum_{a=1}^{\left\lfloor\frac{1+\sqrt{-1+6\cdot k_1}}{6}\right\rfloor}\left(\left\lfloor\frac{-1+6\cdot k_1}{-1+6\cdot a}\right\rfloor-\left\lfloor\frac{-2+6\cdot k_1}{-1+6\cdot a}\right\rfloor\right)+\sum_{a=1}^{\left\lfloor\frac{-1+\sqrt{-1+6\cdot k_1}}{6}\right\rfloor}\left(\left\lfloor\frac{-1+6\cdot k_1}{1+6\cdot a}\right\rfloor-\left\lfloor\frac{-2+6\cdot k_1}{1+6\cdot a}\right\rfloor\right)\right.\right.$$
$$\left.\left.+\sum_{a=1}^{\left\lfloor\frac{1+\sqrt{1+6\cdot(k-k_1)}}{6}\right\rfloor}\left(\left\lfloor\frac{1+6\cdot(k-k_1)}{-1+6\cdot a}\right\rfloor-\left\lfloor\frac{6\cdot(k-k_1)}{-1+6\cdot a}\right\rfloor\right)+\sum_{a=1}^{\left\lfloor\frac{-1+\sqrt{1+6\cdot(k-k_1)}}{6}\right\rfloor}\left(\left\lfloor\frac{1+6\cdot(k-k_1)}{1+6\cdot a}\right\rfloor-\left\lfloor\frac{6\cdot(k-k_1)}{1+6\cdot a}\right\rfloor\right)\right]^{-1}\right\rfloor$$

$$n=1+3\cdot k\equiv 1\,mod(3)$$

$2\cdot n=2+6\cdot k=(1+6\cdot k_1)+(1+6\cdot k_2)$ with k_1 *from 1 to* $\lfloor k/2\rfloor$ and $k_2=k-k_1$

or

$2\cdot n=2+6\cdot k=3+(-1+6\cdot k)$

$$g(2\cdot n)=g(2+6\cdot k)=\left\lfloor\left[1+\sum_{a=1}^{\left\lfloor\frac{1+\sqrt{-1+6\cdot k}}{6}\right\rfloor}\left(\left\lfloor\frac{-1+6\cdot k}{-1+6\cdot a}\right\rfloor-\left\lfloor\frac{-2+6\cdot k}{-1+6\cdot a}\right\rfloor\right)+\sum_{a=1}^{\left\lfloor\frac{-1+\sqrt{-1+6\cdot k}}{6}\right\rfloor}\left(\left\lfloor\frac{-1+6\cdot k}{1+6\cdot a}\right\rfloor-\left\lfloor\frac{-2+6\cdot k}{1+6\cdot a}\right\rfloor\right)\right]^{-1}\right\rfloor$$
$$+\sum_{k_1=1}^{\left\lfloor\frac{k}{2}\right\rfloor}\left\lfloor\left[1+\sum_{a=1}^{\left\lfloor\frac{1+\sqrt{1+6\cdot k_1}}{6}\right\rfloor}\left(\left\lfloor\frac{1+6\cdot k_1}{-1+6\cdot a}\right\rfloor-\left\lfloor\frac{6\cdot k_1}{-1+6\cdot a}\right\rfloor\right)+\sum_{a=1}^{\left\lfloor\frac{-1+\sqrt{1+6\cdot k_1}}{6}\right\rfloor}\left(\left\lfloor\frac{1+6\cdot k_1}{1+6\cdot a}\right\rfloor-\left\lfloor\frac{6\cdot k_1}{1+6\cdot a}\right\rfloor\right)\right.\right.$$
$$\left.\left.+\sum_{a=1}^{\left\lfloor\frac{1+\sqrt{1+6\cdot(k-k_1)}}{6}\right\rfloor}\left(\left\lfloor\frac{1+6\cdot(k-k_1)}{-1+6\cdot a}\right\rfloor-\left\lfloor\frac{6\cdot(k-k_1)}{-1+6\cdot a}\right\rfloor\right)+\sum_{a=1}^{\left\lfloor\frac{-1+\sqrt{1+6\cdot(k-k_1)}}{6}\right\rfloor}\left(\left\lfloor\frac{1+6\cdot(k-k_1)}{1+6\cdot a}\right\rfloor-\left\lfloor\frac{6\cdot(k-k_1)}{1+6\cdot a}\right\rfloor\right)\right]^{-1}\right\rfloor$$

$$n=2+3\cdot k\equiv 2\, mod(3)$$

$2\cdot n=4+6\cdot k=-2+6\cdot(k+1)=(-1+6\cdot k_1)+(-1+6\cdot k_2)$ with k_1 *from 1 to* $\lfloor (k+1)/2\rfloor$ and $k_2=k+1-k_1$

or

$2\cdot n=4+6\cdot k=3+(1+6\cdot k)$

$$
\begin{aligned}
g(2\cdot n)=g(4+6\cdot k)=&\left\lfloor\left[1+\sum_{a=1}^{\left\lfloor\frac{1+\sqrt{1+6\cdot k}}{6}\right\rfloor}\left(\left\lfloor\frac{1+6\cdot k}{-1+6\cdot a}\right\rfloor-\left\lfloor\frac{6\cdot k}{-1+6\cdot a}\right\rfloor\right)+\sum_{a=1}^{\left\lfloor\frac{-1+\sqrt{1+6\cdot k}}{6}\right\rfloor}\left(\left\lfloor\frac{1+6\cdot k}{1+6\cdot a}\right\rfloor-\left\lfloor\frac{6\cdot k}{1+6\cdot a}\right\rfloor\right)\right]^{-1}\right\rfloor\\
&+\sum_{k_1=1}^{\left\lfloor\frac{k+1}{2}\right\rfloor}\left\lfloor\left[1+\sum_{a=1}^{\left\lfloor\frac{1+\sqrt{-1+6\cdot k_1}}{6}\right\rfloor}\left(\left\lfloor\frac{-1+6\cdot k_1}{-1+6\cdot a}\right\rfloor-\left\lfloor\frac{-2+6\cdot k_1}{-1+6\cdot a}\right\rfloor\right)+\sum_{a=1}^{\left\lfloor\frac{-1+\sqrt{-1+6\cdot k_1}}{6}\right\rfloor}\left(\left\lfloor\frac{-1+6\cdot k_1}{1+6\cdot a}\right\rfloor-\left\lfloor\frac{-2+6\cdot k_1}{1+6\cdot a}\right\rfloor\right)\right.\right.\\
&\qquad+\sum_{a=1}^{\left\lfloor\frac{1+\sqrt{-1+6\cdot(k+1-k_1)}}{6}\right\rfloor}\left(\left\lfloor\frac{-1+6\cdot(k+1-k_1)}{-1+6\cdot a}\right\rfloor-\left\lfloor\frac{-2+6\cdot(k+1-k_1)}{-1+6\cdot a}\right\rfloor\right)\\
&\qquad\left.\left.+\sum_{a=1}^{\left\lfloor\frac{-1+\sqrt{-1+6\cdot(k+1-k_1)}}{6}\right\rfloor}\left(\left\lfloor\frac{-1+6\cdot(k+1-k_1)}{1+6\cdot a}\right\rfloor-\left\lfloor\frac{-2+6\cdot(k+1-k_1)}{1+6\cdot a}\right\rfloor\right)\right]^{-1}\right\rfloor
\end{aligned}
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References

[1] https://en.wikipedia.org/wiki/Goldbach%27s_comet

[2] V. Barbera, Formula for the Prime-Counting Function, viXra:2112.0050