The stability and the size of the electron

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Abstract: The vacuum energy is uniform in space; therefore, the Casimir energy does not affected by the orientation of the parallel conducting plates. The surface of two hemispheres, forming a hollow conducting sphere, can be depicted as the assembly of small parallel plates with different orientations. Super positioning the effects of these parallel plates allows calculating the Casimir energy for the two hollow conducting hemispheres. The derived equation identically recovers the physical content of the fine structure constant, and reproduces its value reasonably well. This agreement indicates that the fine structure constant is a scaling factor between the photon energy with wavelength of the circumference of the sphere and the electrostatic repulsion energies for the same size of sphere. The Casimir energy for the two hollow conducting hemispheres is three times higher than the electrostatic repulsion energy of a unit charge. This energy ratio is independent from the size of the sphere. Thus the Casimir energy exceeds the repulsion energy of the electron regardless of its size, which makes a viable alternative explaining the stability of the electron. Assuming that Planck energy sets a limit on the maximum photon energy, allows calculating the diameter of the electron, which is equivalent with the Planck length.

1 Introduction

Based on quantum field theory the empty space is filled with fluctuating electromagnetic waves, with all possible wavelengths. The presence of the electromagnetic waves in empty space means that empty space contains a certain amount of energy. Pressure is induced between two parallel conducting mirror plates facing into each other because the waves longer than the distance between the plates creates pressure on the two faces of the mirror. Thus the pressure outside of the plates will be higher than inside, resulting in the attraction of the two mirror plates. This effect is known as the Casimir effect [1, 2], named after the Dutch physicist, who predicted the existence of vacuum pressure in 1948. Half a century later the predicted Casimir force between two surfaces has been experimentally verified [3, 4], and confirmed by many experiments since then [5].

One of the biggest unresolved questions in physics, why the electron containing the same charge is stable. In 1953 Casimir suggested that the electron shell might be suppressed by the vacuum field in the interior of the electron resulting in an inward pressure [6]. Boyer computed the zero-point energy on the sphere and concluded that the Casimir force is not attractive but rather repulsive [7]. Davies exposed errors in Boyer`s derivation, but concluded that despite these errors the outcome of the derivation is correct [8]. Based on Boyer`s results Casimir Shell Model I has been discredited. Casimir also suggested that the shell-like distribution of the charge might completely suppress the vacuum field in the interior of the shell [6]. Puthoff investigated this Shell Model II, and showed that the energy contributions of the coulomb and the vacuum fields vanish [9]. Pereira questioned this conclusion, and claimed that without specifying the cut off frequency in the model, which is a free parameter, conclusions should not have been drawn [10].

Criticisms concerning the Boyer`s results have been raised [ex. 11]. The credibility of the mathematical treatments, like renormalization, has been debated, and has been found unjustifiable. Recently the possibility of repulsive forces based on topology for a wide class of systems has been ruled out [12]. It has also been shown that the Casimir force between any two symmetrical bodies related by reflection is always attractive [13]. Based on these new researches, the Casimir effect on hollow conducting spheres remains a viable alternative explaining the stability of the electron. This possibility is investigated in this study.

2 Casimir energy for two hollow conducting hemispheres

The Casimir energy between two ideal conducting parallel plates (E_{CII}) at zero temperature is given [1, 2] as:

$$E_{CII}(a) \cong -\frac{\pi^2 \hbar c}{720 a^3} A \qquad (1)$$

where $\hbar$ is the reduced Planck constant, c is the speed of light in vacuum, a is the distance of separation of the parallel plates, and A is the surface area of the plates. The vacuum energy is homogeneous in space; therefore, the energy for the parallel plates does not depend on the orientation of the plates. Two conducting hollow hemispheres forming a sphere can be depicted as the assembly of small parallel plates, facing to each other on the opposite sides of the hemispheres. Super positioning the contributions of the small parallel plates of the surface of the two hemispheres gives the Casimir energy for two hemispheres $\left(E_{C_o}\right)$ as:

$$E_{C_o}(d) \cong -\frac{\pi^2 \hbar c}{720 a^3} 2\pi \left(\frac{d}{2}\right)^2 \quad where \quad d = a \qquad (2)$$

where d is the diameter of the sphere. The energy for the two hollow hemispheres then can be given as:

$$E_{C_o}(d) \cong -\frac{\pi^3 \hbar c}{1440 d} \qquad (3)$$

This relationship can also be derived from energy density considerations as it shown in Appendix 1.

3 Stability requirement for the electron

Assuming that the charge of the electron is distributed uniformly on its surface gives the repulsion energy (E_{e_r}) induced by the same charge as:

$$E_{e_r} = \frac{1}{4\pi\varepsilon_o} \frac{e^2}{d_e} \qquad (4)$$

where ε_o is the permittivity of free space, e is the elementary charge, and d_e is the diameter of the electron. If the Casimir energy exceeds the repulsion energy of the unit charge then the Casimir energy can be a viable alternative explaining the stability of the electron. It is argued that the pressure balance, or the equality between the repulsion and Casimir energies, is not a sufficient condition for the stability of the electron as was suggested by Puthoff [9]. In case of pressure balance, the electron would be in an unstable equilibrium, and the smallest energy disturbance would destabilize the electron. The stability of the electron; therefore, requires that the Casimir energy should be sufficiently higher than the Coulomb repulsion energy as:

$$E_{C_o}(d) > E_{e_r}(d) \tag{5}$$

Based on equations 3-5 the condition for stability can be given as:

$$\frac{\pi^3 \hbar c}{1440}\frac{1}{d} > \frac{e^2}{4\pi\varepsilon_o}\frac{1}{d} \tag{6}$$

Rearranging this relationship recovers the physical content of the fine structure constant as:

$$\frac{\pi^3}{1440} > \frac{1}{4\pi\varepsilon_o}\frac{e^2}{\hbar c}. \tag{7}$$

The recommended value of the inverse fine structure constant [14] is $\alpha^{-1} = 137.035999084\ (21)$. Comparing this value to the constant multiplier of the Casimir energy (eq. 3) indicates that:

$$\frac{\pi^3}{1440} = 2.1532\times10^{-2} \approx 3\alpha = 2.1892\times10^{-2}. \tag{8}$$

The difference is 1.6%. Based on the identically reproduced physical content of the fine structure constant, and the good agreement with its value it is suggested that the Casimir energy for the two hollow conducting hemispheres should be given as:

$$E_{C_o}(d) = -3\alpha\hbar c\frac{1}{d}. \tag{9}$$

Assuming that this is the precise description of the Casimir energy for the two hollow conducting hemispheres, then the original Casimir expression given for parallel plates should be modified as:

$$E_{CII} = -\frac{6\alpha\hbar c}{\pi a^3}A = -\frac{\pi^2\hbar c}{708.16\, a^3}A \tag{10}$$

The Casimir energy for hollow conducting sphere is three times higher than the repulsion energy of the electron with the same size:

$$\frac{E_{C_o}}{E_{e_r}} = \frac{3\alpha\hbar c}{\frac{e^2}{4\pi\varepsilon_o}} = 3 \tag{11}$$

For the electron this equality gives the relationship between Casimir energy and repulsion energy as:

$$\alpha\hbar c\frac{1}{d_e} = \frac{1}{4\pi\varepsilon_o}\frac{e^2}{d_e}, \tag{12}$$

where d_e is the diameter of the electron. Substituting the frequency for the wavelength of the circumference of the electron $(2\pi d_e)$ gives the photon energy as:

$$\alpha E_\gamma(\lambda = 2\pi d_e) = E_{e_r} \tag{13}$$

This expression indicates that the fine structure constant is a scaling factor between the repulsion energy of a unit charge and the energy of the photon with wavelength of the circumference of the electron:

$$\alpha = \frac{E_{e_r}}{E_\gamma}. \tag{14}$$

4. Constrain on the size of the electron

The natural units can be derived by normalizing the fundamental physical constants as it was suggested by Stoney in 1881. Planck added the Planck constant [16] and normalized the following physical constants:

$$G = c = \hbar = k_B = 1 \tag{15}$$

where G is Newton`s gravitational constant, and k_B is Boltzmann’s constant. The derived normalized natural units for length, mass, and time are:

$$l_P = \sqrt{\frac{\hbar G}{c^3}} = 1.616\times10^{-35}m, \tag{16}$$

$$m_P = \sqrt{\frac{\hbar c}{G}} = 2.176\times10^{-8}kg, \tag{17}$$

$$t_P = \frac{l_P}{c} = \sqrt{\frac{\hbar G}{c^5}} = 5.391\times10^{-44}s, \tag{18}$$

where l_P, m_P, and t_P are the Planck length, mass, and time respectively. The definition of the Planck temperature is left out because it is not relevant to the current study. The Planck energy (E_P) can be either calculated from the Planck mass, or from the expression of the Heisenberg uncertainty principle as:

$$E_P = m_P c^2 = \frac{\hbar}{t_P} = \sqrt{\frac{\hbar c^5}{G}} = 1.956\times10^{9}J. \tag{19}$$

There is a general agreement among physicists that the ultraviolet cutoff of vacuum fluctuations occurs at the Planck length [16]. Assuming that the Planck energy sets a limit on the photon energy $\left(E_\gamma = E_P\right)$, allows calculating the size of the electron as:

$$d_e = \frac{1}{4\pi\alpha\varepsilon_o}\frac{e^2}{E_P}. \tag{20}$$

Substituting the fine structure constant, and the Planck energy, gives the size of the electron:

$$d_e = \sqrt{\frac{\hbar G}{c^3}} = l_P = 1.616\times10^{-35}m. \tag{21}$$

The expression recovers the Planck length for the size of the electron. The recovery of the Planck length indicates that the derived relationship for the Casimir energy for hollow hemispheres is a coherent description of the electron.

5. Conclusions

The Casimir energy for two hollow hemispheres is derived from the expression given for parallel plates. The derived relationship recovers the fine structure constant. The Casimir energy for the two hollow hemispheres is three times higher than the electrostatic repulsion energy of the unit charge. The energy ratio, Casimir and electrostatic repulsion, is independent of the size of the sphere. Assuming that the Planck energy sets a limit on the photon energy allows calculating the size of the electron. The calculated diameter of the electron is equal to the Planck length.

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Appendix 1

The Casimir energy [1] between two ideal conducting parallel plates at zero temperature (E_{CII}) is given in Eq. 1. as:

$$E_{CII} \cong -\frac{\pi^2 \hbar c}{720 a^3} A \tag{1}$$

Rearranging the equation it can be shown that at a given distance between the plates, the energy can also be defined as the function of the volume:

$$E_{C_{II}}(a) \cong -\frac{\pi^2 \hbar c}{720 a^4} V(a), \tag{22}$$

where V(a) is the enclosed volume of cube between the parallel plates distanced by a. Introducing the Casimir energy density $\left(\rho_{E_{CII}}\right)$ and rearranging the equation gives:

$$\rho_{E_{CII}}(a) = \frac{E_{CII}(a)}{V(a)} \cong -\frac{\pi^2 \hbar c}{720 a^4} \tag{23}$$

The Casimir energy density between the two parallel plates is independent of the size of the volume, since the surface area cancels out. Based on this independence it can be concluded that the Casimir energy density between the plates is uniform. This conclusion holds regardless of the orientation of the parallel plates.

The hollow conducting sphere is the assembly of small parallel plates, facing each other on the opposite sides of the sphere. The overall Casimir energy of the hollow conducting sphere is the sum of the individual parallel plates building up the surface of the sphere. The Casimir energy of the hollow conducting sphere ($E_{C_o}^{1D}$) can be written then as:

$$E_{C_o}^{1D} = \rho_{E_{CII}} V_{sphere} \tag{24}$$

where 1D refers to one dimension. The constant value of the Casimir energy density between the uniformly distanced parallel plates gives the following equality:

$$\frac{E_{CII_{plates}}}{V_{IIplates}} = \frac{E_{C_o}^{1D}}{V_{sphere}} \tag{25}$$

where the diameter of the sphere is equal with the separation of the parallel plates.

The Casimir energy of a hollow conducting sphere can be expressed then as:

$$E_{C_o}^{1D} = E_{CII_{plat\square s}} \frac{V_{sphere}}{V_{IIplates}} \;, \quad where \quad a = d \tag{26}$$

The volume ratio between a sphere and cylinder, where the diameter of the sphere and the cylinder is the same, can be given as:

$$\frac{\frac{1}{2}\frac{4\pi}{3}\left(\frac{d}{2}\right)^3}{\pi\left(\frac{d}{2}\right)^3} = \frac{2}{3}, \qquad where \quad d = d_{\ sphere} = d_{cylinder} \tag{27}$$

Modifying the energy calculated for parallel plates with the volume ratios gives the Casimir energy for hollow conducting sphere as:

$$E_{C_o}^{1D} \cong -\frac{2}{3}E_{C_{II}} = -\frac{\pi^2\hbar c}{1080 d^3}\, A. \tag{28}$$

The surface area between the two hemispheric shells is

$$A = \pi\left(\frac{d}{2}\right)^2, \tag{29}$$

giving the Casimir energy for hollow conducting sphere:

$$E_{C_o}^{1D}(d) \cong -\frac{\pi^3\hbar c}{4320 d}. \tag{30}$$

The Casimir energy in equation 30 calculated only for one dimension. In terms of a spherical object the contribution of the additional dimensions should also be counted. Thus the energy for a hollow conducting sphere can be given as:

$$E_{C_o} \cong -3E_{C_o}^{1D} = -2E_{C_{II}} = -\frac{\pi^3\hbar c}{1440 d}. \tag{31}$$

The derived expression (Eq. 31) recovers equation 3, which was derived from different assumptions. The identical outcome of the two derivations for the Casimir energy from different assumptions indicates that the derived Casimir energy relationship for hollow hemispheres is correct.