QUADRUPLET SUMS OF QUARK-LEPTON MASSES

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ABSTRACT. Adding the charm quark to the Koide triplet forms a quadruplet that approximates $\frac{2}{5}$. The precision of this result is accurate to $\mathcal{O}(10^{-5})$. We find that the charm mass sits at a minimum of a general quadruplet curve. Using this calculated charm mass and the heavy leptons which are directly measured, we predict the mass of the up, down, strange, and bottom quarks. Determining mass in this way avoids the inconsistency of mixing the running mass with the pole mass for the sums of these quark masses and serves as a prediction for more accurate techniques.

1. INTRODUCTION

The well-known Koide formula [Koi82] [Koi83] is an empirical formula for the sum of the lepton masses accurate to $\mathcal{O}(10^{-5})$.

$$K_l \equiv \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = \frac{2}{3} \tag{1}$$

The number, $\frac{2}{3}$, is either a coincidence or represents something structural about the empirical masses of the leptons. The search for the number, $\frac{2}{3}$, using the quark masses has not been as accurate. Rodejohann and Zhang [RZ11] sorted the quarks into two groups according to heavy and light by grouping the charm quark with the heavy quarks and the strange quark with the light quarks. This weighted grouping is closer to $\frac{2}{3}$ than the vertical (family) grouping or the horizontal grouping but still not as successful as the Koide formula for the leptons. Kartavtsev [Kar11] suggested summing all six quark masses.

$$K_q \equiv \frac{\sum m_q}{(\sum \sqrt{m_q})^2} \tag{2}$$

Gao & Li [GL16] pointed out that the quarks use different definitions of quark mass. The up,down, and strange quarks use the current quark mass, charm and bottom use the running mass at different energy scales, and the top quark is measured by its pole mass. They conclude that it is meaningless to combine the masses of these various quarks without distinguishing these differences.

Cao [Cao12] conjectured that since the heavy quark sum is only .3% different than the Koide heavy lepton sum, the light lepton sum of the neutrino masses are within similar tolerance of the light quark sum. Cao's conjecture suggests that sums different than the number, $\frac{2}{3}$, could be physically relevant.

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In this note, we find a novel relation in the quark-lepton mass data. We first find sums for the charm quark with accuracy to $\mathcal{O}(10^{-5})$ by only using directly measured quantities in its construction. Our approach generalizes the notion of a grouping to include any possible groupings instead of being limited to the usual weighted, vertical, or horizontal grouping. By relaxing the grouping constraint, we widen the search for structure or coincidence in the data. We use the masses referenced in Zyla [ZPDG20]. We do not consider the neutrinos.

We adopt the following notation. Equation (1) will be represented as

$$(e\mu\tau) = \tfrac{2}{3}$$

for example. Equation (2) with four parameters instead of six will be represented as

$$(q_1 q_2 q_3 q_4) = \lambda$$

for example.

We list the masses for the six quarks and three leptons in Table 1. The values of all of the triplet and quadruplet data we searched are listed in lexical order for the convenience of the reader in Table 2 of Appendix A. Only the lepton masses and the top quark mass are directly measurable. The other quark masses are usually determined by calculating the running mass at a particular energy scale. We take another approach. We find Koide-like relations using four parameter sums that derive these other masses, (up, down, strange, charm, bottom), from the directly measurable masses. Extending the Koide formula by adding a single quark to the lepton sum is fruitful. For example, the quadruplet $(e\mu\tau c) = \frac{2}{5}$ in Table 2 has a precision of $\mathcal{O}(10^{-5})$ similar to the Koide formula (1). Is $\frac{2}{5}$ any more or less structural than $\frac{2}{3}$? This is the kind of question that Cao's conjecture raises.

Table 1. Masses of the quark-lepton mass matrix in GeV/c^2.

Particle	*Mass*	*Particle*	*Mass*	*Particle*	*Mass*
up	$0.00216^{+.00049}_{-.00026}$	*charm*	$1.27 \pm .02$	*top*	$172.76 \pm .30$
down	$0.00467^{+.00048}_{-.00017}$	*strange*	$0.093^{+.011}_{-.005}$	*bottom*	$4.18^{+.03}_{-.02}$
electron	0.0005109989461 $\pm$0.0000000000031	*muon*	0.1056583745 $\pm$0.0000000024	*tau*	$1.77686 \pm .00012$

2. Calculation of the quark masses

The Koide triplet $(e\mu\tau)$ has a value of $0.66666 \approx 2/3$, correct to $\mathcal{O}(10^{-5})$. We find that the charm quark sum constructed by adding a quark to the Koide triplet preserves this level of precision. Let

$$(e\mu\tau q) = \frac{m_e + m_\mu + m_\tau + m_q}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau} + \sqrt{m_q})^2} = \lambda \tag{3}$$

for some quark q. The precision of the $(e\mu\tau c)$ λ-value of $0.4 \approx 2/5$ is correct to $\mathcal{O}(10^{-5})$.

The mass of the charm quark can be determined based on the lepton masses alone by examining features of the

$$(e\mu\tau q) = \lambda$$

curve for quark mass q. We find the minimum λ value of this curve to be

$$\lambda = .4$$

$$\text{at zero slope and } q = 1.25534$$

which can be identified with the mass of the charm quark.

Next we use the lepton masses and the calculated charm quark mass to determine the mass of the strange quark. We solve

$$(e\mu\tau c) = \frac{\sqrt{(qce)}}{2}$$

for q. We find that

$$q = .0964735$$

which can be identified with the mass of the strange quark.

Similarly, the mass of the down quark can be determined based on the lepton masses along with the calculated charm quark mass. We solve

$$(e\mu\tau c) = \frac{\sqrt{(q\mu e)}}{2}$$

for q. We find that

$$q = .0047$$

which can be identified with the mass of the down quark.

The mass of the up quark can be determined using the calculated charm and strange masses along with the lepton masses. We solve

$$(e\mu\tau c) = \frac{\sqrt{(\tau sqe)}}{2}$$

for q. We find that

$$q = .002105$$

which can be identified with the mass of the up quark.

Using the calculated strange and down quark masses along with the calculated charm mass and the empirical lepton masses, we can determine in a similar way the bottom mass.

The mass of the bottom quark can be determined by solving

$$(sd\tau c) = \frac{\sqrt{(qs\mu)}}{2}$$

for q. We find that

$$m_q = 4.17994$$

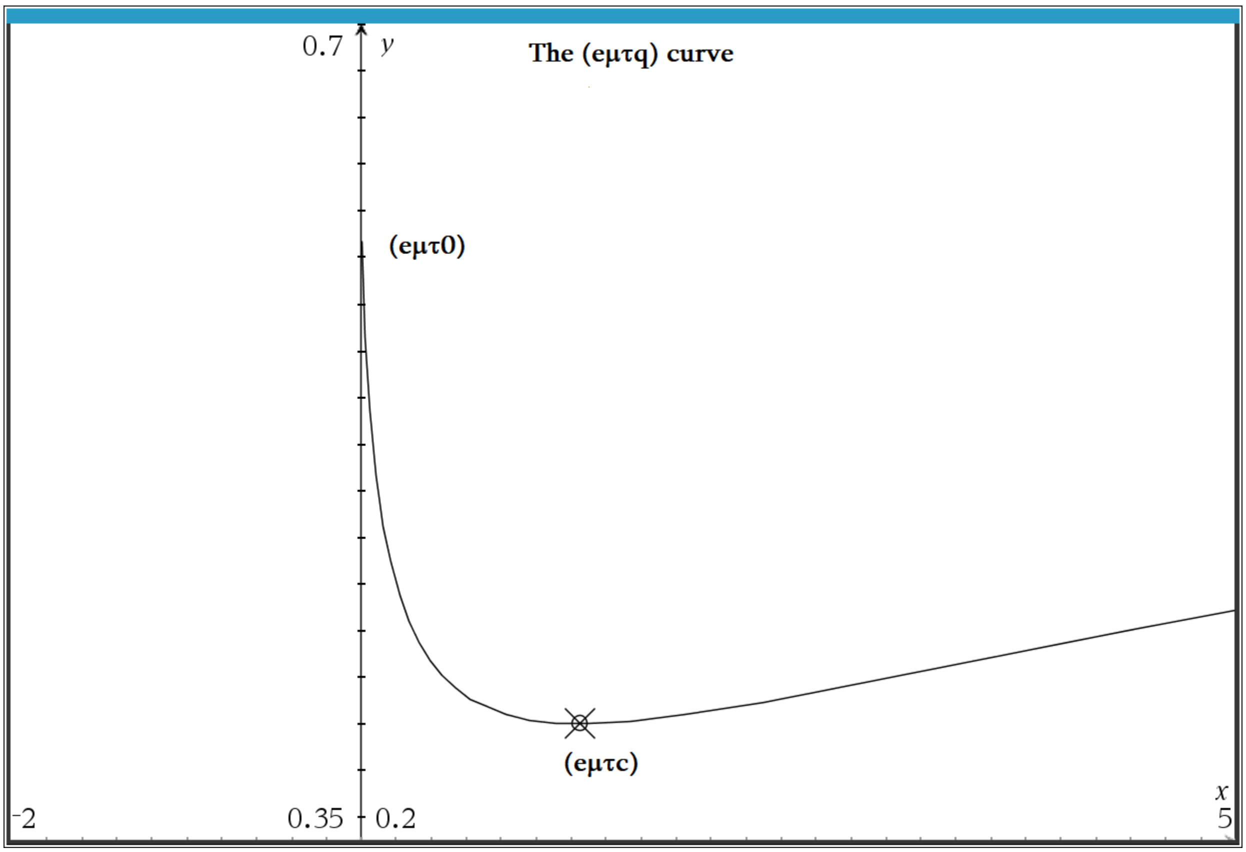
which can be identified with the mass of the bottom quark.

Extending the Koide formula by adding a single quark to the lepton sum is fruitful. There are oddities. For example, if we use the directly measured mass of the top quark in the following expression we have

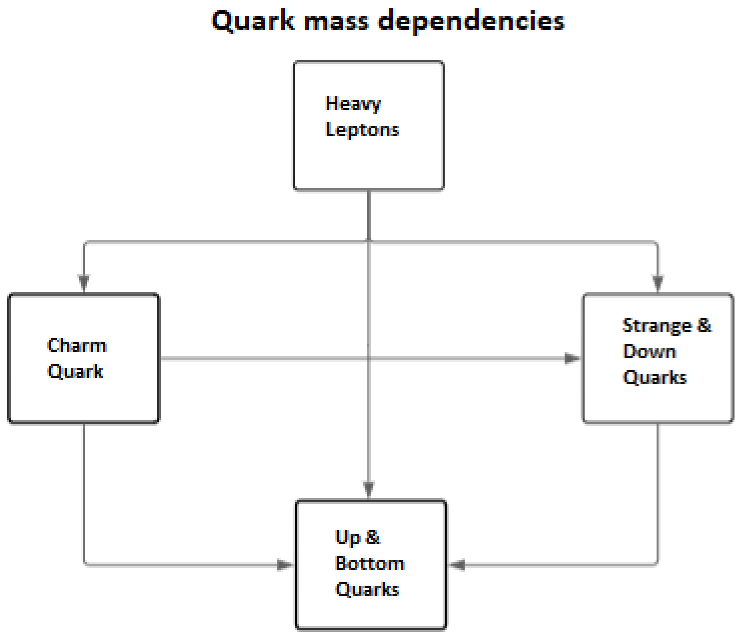
$$(e\mu ts) = \frac{m_e + m_\mu + m_t + m_s}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_t} + \sqrt{m_s})^2} = \frac{m_s}{m_\mu}$$

3. Summary

The charm mass prediction can be seen as a minimum of the $(e\mu\tau q)$ curve.



The precision for all the predictions comes from the precision of the τ lepton, viz. $\mathcal{O}(10^{-5})$. First, the strange and down masses can be predicted from the charm mass and the heavy leptons. Once the strange and down masses are in hand, then they along with the charm mass and heavy leptons are used to predict the up and bottom masses.



In general, a quark-lepton quadruplet can be equated to another quark-lepton triplet or quadruplet in the following way:

$$2 \cdot \frac{(q_1 + q_2 + q_3 + q_4)}{(\sqrt{q_1} + \sqrt{q_2} + \sqrt{q_3} + \sqrt{q_4})^2} = \sqrt{\frac{(q + q_5 + q_6 + q_7)}{(\sqrt{q} + \sqrt{q_5} + \sqrt{q_6} + \sqrt{q_7})^2}}$$

When we solve the quadratic for the general quark mass, q, we get

$$q = \frac{(\sqrt{k_2^2 k_1 - k_3(k_1 - 1)} \pm k_1 k_2)^2}{(k_1 - 1)^2} \tag{4}$$

where

$$k_1 = (2 \cdot \frac{(q_1 + q_2 + q_3 + q_4)}{(\sqrt{q_1} + \sqrt{q_2} + \sqrt{q_3} + \sqrt{q_4})^2})^2$$
$$k_2 = \sqrt{q_5} + \sqrt{q_6} + \sqrt{q_7}$$
$$k_3 = q_5 + q_6 + q_7$$

The charm mass point on the $(e\mu\tau q)$ curve acts like a dividing point between the light quarks and the heavy quarks. The strange, up, and down mass predictions are less than the charm mass and use the negative solution to equation (4). The bottom mass prediction is greater than the charm mass and uses the positive solution to equation (4).

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Appendix A.

Table 2. Triplet and Quadruplet K_q sums of the quark-lepton mass matrix.

	Triplet Sum		*Triplet Sum*		*Quad Sum*		*Quad Sum*		*Quad Sum*
$tb\tau$	0.65475	bse	0.7595	$tb\tau c$	0.57788	$tc\mu e$	0.81487	$bsud$	0.70477
tbc	0.66949	$b\mu u$	0.73457	$tb\tau s$	0.63156	$tcud$	0.84098	$bsue$	0.73096
tbs	0.73751	$b\mu d$	0.72187	$tb\tau\mu$	0.6301	$tcue$	0.84633	$bsde$	0.71835
$tb\mu$	0.73565	$b\mu e$	0.74901	$tb\tau u$	0.65109	$tcde$	0.84377	$b\mu ud$	0.69547
tbu	0.76236	bud	0.89795	$tb\tau d$	0.64939	$ts\mu u$	0.90555	$b\mu ue$	0.7211
tbd	0.76018	bue	0.9363	$tb\tau e$	0.65297	$ts\mu d$	0.9027	$b\mu de$	0.70875
tbe	0.76474	bde	0.91778	$tbcs$	0.64548	$ts\mu e$	0.90868	$bude$	0.87955
$t\tau c$	0.72207	τcs	0.41073	$tbc\mu$	0.64397	$tsud$	0.9396	$\tau cs\mu$	0.33993
$t\tau s$	0.79922	$\tau c\mu$	0.40645	$tbcu$	0.6657	$tsue$	0.94595	τcsu	0.39753
$t\tau\mu$	0.79711	τcu	0.48535	$tbcd$	0.66393	$tsde$	0.94291	τcsd	0.39174
$t\tau u$	0.82749	τcd	0.47739	$tbce$	0.66764	$t\mu ud$	0.93689	τcse	0.40416
$t\tau d$	0.82502	τce	0.49446	$tbs\mu$	0.70793	$t\mu ue$	0.94321	$\tau c\mu u$	0.39349
$t\tau e$	0.83021	$\tau s\mu$	0.51267	$tbsu$	0.73311	$t\mu de$	0.94019	$\tau c\mu d$	0.38779
tcs	0.81959	τsu	0.65979	$tbsd$	0.73107	$tude$	0.97946	$\tau c\mu e$	0.4
$tc\mu$	0.81739	τsd	0.64386	$tbse$	0.73536	$b\tau cs$	0.31646	τcud	0.46063
tcu	0.84901	τse	0.6783	$tb\mu u$	0.73127	$b\tau c\mu$	0.31438	τcue	0.47679
tcd	0.84643	$\tau\mu u$	0.64869	$tb\mu d$	0.72923	$b\tau cu$	0.34905	τcde	0.46904
tce	0.85184	$\tau\mu d$	0.6332	$tb\mu e$	0.73352	$b\tau cd$	0.34584	$\tau s\mu u$	0.48977
$ts\mu$	0.91166	$\tau\mu e$	0.66666	$tbud$	0.75558	$b\tau ce$	0.35266	$\tau s\mu d$	0.47989
tsu	0.94912	τud	0.85094	$tbue$	0.7601	$b\tau s\mu$	0.38328	$\tau s\mu e$	0.5012
tsd	0.94606	τue	0.90524	$tbde$	0.75794	$b\tau su$	0.43524	τsud	0.61087
tse	0.95248	τde	0.8789	$t\tau cs$	0.69502	$b\tau sd$	0.43036	τsue	0.64261
$t\mu u$	0.94637	$cs\mu$	0.47577	$t\tau c\mu$	0.69331	$b\tau se$	0.44075	τsde	0.6273
$t\mu d$	0.94333	csu	0.62462	$t\tau cu$	0.71779	$b\tau\mu u$	0.43149	$\tau\mu ud$	0.60113
$t\mu e$	0.94971	csd	0.60766	$t\tau cd$	0.7158	$b\tau\mu d$	0.42668	$\tau\mu ue$	0.63199
tud	0.98279	cse	0.64451	$t\tau ce$	0.71998	$b\tau\mu e$	0.43692	$\tau\mu de$	0.61711
tue	0.98959	$c\mu u$	0.61361	$t\tau s\mu$	0.76566	$b\tau ud$	0.48898	τude	0.82522
tde	0.98633	$c\mu d$	0.59718	$t\tau su$	0.79423	$b\tau ue$	0.50169	$cs\mu u$	0.45223
$b\tau c$	0.35618	$c\mu e$	0.63288	$t\tau sd$	0.7919	$b\tau de$	0.4956	$cs\mu d$	0.44222
$b\tau s$	0.44614	cud	0.82806	$t\tau se$	0.79678	$bcs\mu$	0.39088	$cs\mu e$	0.46392
$b\tau\mu$	0.44223	cue	0.88969	$t\tau\mu u$	0.79214	$bcsu$	0.4468	$csud$	0.57259
$b\tau u$	0.50829	cde	0.85972	$t\tau\mu d$	0.78982	$bcsd$	0.44151	$csue$	0.60617
$b\tau d$	0.50208	$s\mu u$	0.43882	$t\tau\mu e$	0.79468	$bcse$	0.45279	$csde$	0.58997
$b\tau e$	0.51531	$s\mu d$	0.41692	$t\tau ud$	0.81978	$bc\mu u$	0.44276	$c\mu ud$	0.56316
bcs	0.45865	$s\mu e$	0.46764	$t\tau ue$	0.82492	$bc\mu d$	0.43754	$c\mu ue$	0.59573
$bc\mu$	0.45443	sud	0.56654	$t\tau de$	0.82246	$bc\mu e$	0.44865	$c\mu de$	0.58003
bcu	0.52652	sue	0.68382	$tcs\mu$	0.7847	$bcud$	0.50528	$cude$	0.79903
bcd	0.51968	sde	0.6264	$tcsu$	0.8144	$bcue$	0.51925	$s\mu ud$	0.37041
bce	0.53426	μud	0.58139	$tcsd$	0.81198	$bcde$	0.51255	$s\mu ue$	0.41194
$bs\mu$	0.61214	μue	0.69737	$tcse$	0.81705	$bs\mu u$	0.5917	$s\mu de$	0.39217
bsu	0.74473	μde	0.6405	$tc\mu u$	0.81222	$bs\mu d$	0.58264	$sude$	0.51273
bsd	0.73176	ude	0.38875	$tc\mu d$	0.80981	$bs\mu e$	0.60199	μude	0.52833

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