On the Riemann Tensor

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Abstract

In this article the Riemann tensor has been discussed in two sections. The first section considers two formulas to project results that are inconsistent in their nature. The next section considering tensor transformations and the linear homogeneous equations proves that the Riemann tensor is the rank four null tensor.

Keywords:Riemann Tensor,Covariant Tensor,Schwarzschild's Geometry,Linear homogeneous equations.

Introduction

In two sections the article brings out the following facts(1) based on General Relativistic concepts results indicating at inconsistencies in conventional theories are deduced.(2)The article proves, considering tensor transformations and the linear homogeneous equations, that the Riemann tensor is the rank four null tensor.

Section I

First we derive two formulas

1.

$$\frac{\partial\left(A^p B_p\right)}{\partial x^\alpha} = A^p \frac{\partial B_p}{\partial x^\alpha} + B_p \frac{\partial A^p}{\partial x^\alpha}$$

We consider the standard formula[1]

$$\nabla_\alpha A^p = \frac{\partial A^p}{\partial x^\alpha} + \Gamma^p{}_{\alpha s} A^s$$

$$\Rightarrow \frac{\partial A^p}{\partial x^\alpha} = \nabla_\alpha A^p - \Gamma^p{}_{\alpha s} A^s$$

Again we consider the standard formula[2]

$$\nabla_\alpha B_p = \frac{\partial B_p}{\partial x^\alpha} - \Gamma^s{}_{\alpha p} B_s$$

$$\Rightarrow \frac{\partial B_p}{\partial x^\alpha} = \nabla_\alpha B_p + \Gamma^s{}_{\alpha p} B_s$$

$$\frac{\partial\left(A^p B_p\right)}{\partial x^\alpha} = A^p\left(\nabla_\alpha B_p + \Gamma^s{}_{\alpha p} B_s\right) + B_p\left(\nabla_\alpha A^p - \Gamma^p{}_{\alpha s} A^s\right)$$

$$\frac{\partial\left(A^p B_p\right)}{\partial x^\alpha} = A^p \nabla_\alpha B_p + B_p \nabla_\alpha A^p + \Gamma^s{}_{\alpha p} B_s A^p - \Gamma^p{}_{\alpha s} B_p A^s$$

The last term to the right,by the interchange of the dummy indics:$s \leftrightarrow p$

$$\Gamma^{p}{}_{\alpha s} B_p A^s = \Gamma^{s}{}_{\alpha p} B_s A^p$$

=Therefore,

$$\frac{\partial(A^p B_p)}{\partial x^\alpha} = A^p \nabla_\alpha B_p + B_p \nabla_\alpha A^p + \Gamma^{s}{}_{\alpha p} B_s A^p - \Gamma^{s}{}_{\alpha p} B_s A^p$$

$$\frac{\partial(A^p B_p)}{\partial x^\alpha} = A^p \nabla_\alpha B_p + B_p \nabla_\alpha A^p = \nabla_\alpha (A^p B_p)$$

$$\frac{\partial(A^p B_p)}{\partial x^\alpha} = \nabla_\alpha (A^p B_p) \ (1)$$

2. Next we consider the result

$$\frac{\partial}{\partial x^\gamma}\left(B_{\alpha\beta} A^{\alpha\beta}\right) = A_{\alpha\beta} \nabla_\gamma B^{\alpha\beta} + B^{\alpha\beta} \nabla_\gamma A_{\alpha\beta}$$

Proof:

We consider the following relations

$$\nabla_\gamma A^{\alpha\beta} = A^{\alpha\beta};_\gamma = \frac{\partial A^{\alpha\beta}}{\partial x^\gamma} + \Gamma_{\gamma s}{}^{\alpha} A^{s\beta} + \Gamma_{\gamma s}{}^{\beta} A^{\alpha s}$$

$$\nabla_\gamma B_{\alpha\beta} = B_{\alpha\beta};_\gamma = \frac{\partial B_{\alpha\beta}}{\partial x^\gamma} - \Gamma^{s}{}_{\gamma\alpha} B_{s\beta} - \Gamma^{s}{}_{\gamma\beta} B_{\alpha s}$$

[The above relations do not assume $A^{\alpha\beta}$ and $B_{\alpha\beta}$ as symmetric tensors]

We obtain,

$$\frac{\partial}{\partial x^\gamma}\left(B_{\alpha\beta} A^{\alpha\beta}\right) = B_{\alpha\beta}\left(\nabla_\gamma A^{\alpha\beta} - \Gamma_{\gamma s}{}^{\alpha} A^{s\beta} - \Gamma_{\gamma s}{}^{\beta} A^{\alpha s}\right) + A^{\alpha\beta}\left(\nabla_\gamma B_{\alpha\beta} + \Gamma^{s}{}_{\gamma\alpha} B_{s\beta} + \Gamma^{s}{}_{\gamma\beta} B_{\alpha s}\right)$$

$$\frac{\partial}{\partial x^\gamma}\left(B_{\alpha\beta} A^{\alpha\beta}\right) = B_{\alpha\beta}\left(-\Gamma_{\gamma s}{}^{\alpha} A^{s\beta} - \Gamma_{\gamma s}{}^{\beta} A^{\alpha s}\right) + A^{\alpha\beta}\left(\Gamma^{s}{}_{\gamma\alpha} B_{s\beta} + \Gamma^{s}{}_{\gamma\beta} B_{\alpha s}\right) + A^{\alpha\beta} \nabla_\gamma B_{\alpha\beta} + B^{\alpha\beta} \nabla_\gamma A_{\alpha\beta}$$

$$= -\Gamma_{\gamma s}{}^{\alpha} g^{s\beta} B_{\alpha\beta} - \Gamma_{\gamma s}{}^{\beta} g^{\alpha s} B_{\alpha\beta} + \Gamma^{s}{}_{\gamma\alpha} A^{\alpha\beta} T_{s\beta} + \Gamma^{s}{}_{\gamma\beta} A^{\alpha\beta} B_{s\alpha} + A^{\alpha\beta} \nabla_\gamma B_{\alpha\beta} + B^{\alpha\beta} \nabla_\gamma A_{\alpha\beta}$$

$$\frac{\partial}{\partial x^\gamma}\left(B_{\alpha\beta} A^{\alpha\beta}\right) = \left(-\Gamma_{\gamma s}{}^{\alpha} A^{s\beta} B_{\alpha\beta} + \Gamma^{s}{}_{\gamma\alpha} A^{\alpha\beta} B_{s\beta}\right) + \left(\Gamma^{s}{}_{\gamma\beta} A^{\alpha\beta} B_{\alpha s} - \Gamma_{\gamma s}{}^{\beta} A^{\alpha s} B_{\alpha\beta}\right) + A^{\alpha\beta} \nabla_\gamma B_{\alpha\beta} + B^{\alpha\beta} \nabla_\gamma A_{\alpha\beta}$$

[In the above α, s, β are dummy indices]

We work out the two parentheses separately.

With the second term in the first parenthesis to the right we interchange as follows

$$\alpha \leftrightarrow s$$

$$\left(-\Gamma_{\gamma s}{}^{\alpha}A^{s\beta}B_{\alpha\beta}+\Gamma^{s}{}_{\gamma\alpha}A^{\alpha\beta}B_{s\beta}\right)=\left(-\Gamma_{\gamma s}{}^{\alpha}A^{s\beta}T_{\alpha\beta}+\Gamma^{\alpha}{}_{\gamma s}A^{s\beta}B_{\alpha\beta}\right)=0$$

With the second term in the second parenthesis

$$\beta\leftrightarrow s$$

$$\left(\Gamma^{s}{}_{\gamma\beta}A^{\alpha\beta}B_{\alpha s}-\Gamma_{\gamma s}{}^{\beta}A^{\alpha s}B_{\alpha\beta}\right)=\left(\Gamma^{s}{}_{\gamma\beta}A^{\alpha\beta}B_{\alpha s}-\Gamma_{\gamma\beta}{}^{s}A^{\alpha\beta}B_{\alpha s}\right)=0$$

$$\frac{\partial}{\partial x^{\gamma}}\left(B_{\alpha\beta}A^{\alpha\beta}\right)=A^{\alpha\beta}\nabla_{\gamma}B_{\alpha\beta}+B^{\alpha\beta}\nabla_{\gamma}A_{\alpha\beta}=\nabla_{\gamma}\left(B_{\alpha\beta}A^{\alpha\beta}\right)$$

$$\frac{\partial}{\partial x^{\gamma}}\left(B_{\alpha\beta}A^{\alpha\beta}\right)=\nabla_{\gamma}\left(B_{\alpha\beta}A^{\alpha\beta}\right)\ (2)$$

$B_{\alpha\beta}A^{\alpha\beta}$may be of the form $g_{\alpha\beta}P^{\alpha}Q^{\beta}$ $(2')$

If f is a scalar then $\frac{\partial f}{\partial x^{j}}$is a rank on covariant tensor

$$\nabla_{i}\frac{\partial f}{\partial x^{j}}=\frac{\partial^{2}f}{\partial x^{i}\partial x^{j}}-\Gamma^{s}{}_{ij}\frac{\partial f}{\partial x^{s}}$$

$$=\frac{\partial^{2}f}{\partial x^{j}\partial x^{i}}-\Gamma^{s}{}_{ji}\frac{\partial f}{\partial x^{s}}=\nabla_{j}\frac{\partial f}{\partial x^{i}}$$

$$\nabla_{i}\frac{\partial f}{\partial x^{j}}=\nabla_{j}\frac{\partial f}{\partial x^{i}}\ (3)$$

where

$$g^{\alpha\beta}P_{\alpha}Q_{\beta}=f\qquad(4)$$

$$\nabla_{i}\frac{\partial\left(g^{\alpha\beta}P_{\alpha}Q_{\beta}\right)}{\partial x^{j}}=\nabla_{j}\frac{\partial\left(g^{\alpha\beta}P_{\alpha}Q_{\beta}\right)}{\partial x^{i}}\ \ (5.1)$$

Keeping in mind $\nabla_{i}g^{\alpha\beta}=0$ and considering (3) and (5))we have

$$\nabla_{i}\frac{\partial\left(g^{\alpha\beta}P_{\alpha}Q_{\beta}\right)}{\partial x^{j}}=\nabla_{i}\left(g^{\alpha\beta}\nabla_{j}\left(P_{\alpha}Q_{\beta}\right)\right)=g^{\alpha\beta}\nabla_{i}\nabla_{j}\left(P_{\alpha}Q_{\beta}\right)\ (5.2)$$

Similarly

$$\nabla_{j}\frac{\partial\left(g^{\alpha\beta}P_{\alpha}Q_{\beta}\right)}{\partial x^{i}}=\nabla_{j}\left(g^{\alpha\beta}\nabla_{i}\left(P_{\alpha}Q_{\beta}\right)\right)=g^{\alpha\beta}\nabla_{j}\nabla_{i}\left(P_{\alpha}Q_{\beta}\right)(5.3)$$

From (5.1),(5.2) and (5.3) we have

$$g^{\alpha\beta}\nabla_i\nabla_j(P_\alpha Q_\beta) = g^{\alpha\beta}\nabla_j\nabla_i(P_\alpha Q_\beta)$$

$$\Rightarrow g^{\alpha\beta}\nabla_i\nabla_j(P_\alpha Q_\beta) - g^{\alpha\beta}\nabla_j\nabla_i(P_\alpha Q_\beta) = 0$$

$$\Rightarrow g^{\alpha\beta}[\nabla_i\nabla_j - \nabla_j\nabla_i](P_\alpha Q_\beta) = 0\, for\ arbitrary P_\alpha\ and\ Q_\beta\ (6)$$

We have,

$$\nabla_\mathrm{i}\nabla_j(P_\alpha Q_\beta) = \nabla_\mathrm{i}(P_\alpha \nabla_j Q_\beta + Q_\beta \nabla_j P_\alpha)$$

$$\nabla_\mathrm{i}\nabla_j(P_\alpha Q_\beta) = P_\alpha\nabla_\mathrm{i}\nabla_j Q_\beta + (\nabla_\mathrm{i}P_\alpha)(\nabla_j Q_\beta) + (\nabla_i Q_\beta)(\nabla_\mathrm{j}P_\alpha) + Q_\beta\nabla_\mathrm{i}\nabla_j P_\alpha\ (7)$$

$$\nabla_\mathrm{j}\nabla_i(P_\alpha Q_\beta) = \nabla_\mathrm{j}(P_\alpha \nabla_i Q_\beta + Q_\beta \nabla_i P_\alpha)$$

$$\nabla_\mathrm{j}\nabla_i(P_\alpha Q_\beta) = P_\alpha\nabla_\mathrm{j}\nabla_i Q_\beta + (\nabla_\mathrm{j}P_\alpha)(\nabla_i Q_\beta) + (\nabla_j Q_\beta)(\nabla_\mathrm{i}P_\alpha) + Q_\beta\nabla_\mathrm{j}\nabla_i P_\alpha\ \ (8)$$

From (7) and (8)

$$[\nabla_\mathrm{i}\nabla_j - \nabla_\mathrm{j}\nabla_i](P_\alpha Q_\beta) = P_\alpha(\nabla_\mathrm{i}\nabla_j - \nabla_\mathrm{j}\nabla_i)Q_\beta + Q_\beta(\nabla_\mathrm{i}\nabla_j - \nabla_\mathrm{j}\nabla_i)P_\alpha (9)$$

Using (6) we obtain,

$$g^{\alpha\beta}[\nabla_\mathrm{i}\nabla_j - \nabla_\mathrm{j}\nabla_i](P_\alpha Q_\beta) = g^{\alpha\beta}P_\alpha(\nabla_\mathrm{i}\nabla_j - \nabla_\mathrm{j}\nabla_i)Q_\beta + g^{\alpha\beta}Q_\beta(\nabla_\mathrm{i}\nabla_j - \nabla_\mathrm{j}\nabla_i)P_\alpha = 0$$

$$g^{\alpha\beta}P_\alpha(\nabla_\mathrm{i}\nabla_j - \nabla_\mathrm{j}\nabla_i)Q_\beta + g^{\alpha\beta}Q_\beta(\nabla_\mathrm{i}\nabla_j - \nabla_\mathrm{j}\nabla_i)P_\alpha = 0\ \ (10)$$

$$g^{\alpha\beta}P_\alpha(\nabla_\mathrm{j}\nabla_i - \nabla_\mathrm{i}\nabla_j)Q_\beta + g^{\alpha\beta}Q_\beta(\nabla_\mathrm{j}\nabla_i - \nabla_\mathrm{i}\nabla_j)P_\alpha = 0\ \ (11)$$

Next we apply the following formula[3] on (10)or equivalently on (11)

$$[\nabla_\mathrm{j}\nabla_i - \nabla_\mathrm{i}\nabla_j]A_p = R^n{}_{pij}A_n (12)$$

Applying (8)(12) on the left side of (7) (10)we have,

$$g^{\alpha\beta}P_\alpha R^n{}_{\beta ij}Q_n + g^{\alpha\beta}Q_\beta R^n{}_{\alpha ij}P_n = 0\, for\ arbitray P_\alpha, Q_\beta\ (13)$$

Partial differentiating (13) with respect to $P_{n=k}, k$ being a free index and not a dummy we obtain

$$g^{k\beta}P_k R^n{}_{\beta ij}Q_n + g^{\alpha\beta}Q_\beta R^k{}_{\alpha ij}P_k = 0[no\ summation\ on\ k](14)$$

$$g^{k\beta}R^n{}_{\beta ij}Q_n + g^{\alpha\beta}Q_\beta R^k{}_{\alpha ij} = 0$$

Partial differentiating with respect to $Q_{n=s}$

$$g^{k\beta}R^{s}{}_{\beta ij}+g^{\alpha s}R^{k}{}_{\alpha ij}=0[summation\ on\ \alpha\ and\ \beta](15)$$

For orthogonal systems

$$g^{kk}R^{s}{}_{kij}+g^{ss}R^{k}{}_{sij}=0\ [no\ summation\ on\ k\ or\ on\ s](16)$$

Let us test equation (12)(16) using $R^{s}{}_{kij}$for Shwarzschild's geometry:k=r,s=t

$$g^{rr}R^{t}{}_{rij}+g^{tt}R^{r}{}_{tij}=0\ (17)[r\ and\ t\ are\ not\ a\ dummy\ indices\ ]$$

$$g^{rr}R^{t}{}_{rtr}+g^{tt}R^{r}{}_{ttr}=0\ (18.1)$$

We try verifying (14) using standard results for Schwarzschild's geometry

$$-\left(1-\frac{2Gm}{c^2r}\right)^{-1}\frac{2Gm}{c^2r^3-2Gmr^2}+\left(1-\frac{2Gm}{c^2r}\right)\frac{2(Gc^2mr-2G^2m^2)}{c^4r^4}=0\ (18.2)$$

$$-\left(1-\frac{2Gm}{c^2r}\right)^{-1}\frac{2Gm}{c^2r^3\left(1-\frac{2Gm}{c^2r}\right)}+\left(1-\frac{2Gm}{c^2r}\right)\frac{2Gmc^2r\left(1-\frac{2Gm}{c^2r}\right)}{c^4r^4}=0$$

If, $1-\frac{2Gm}{c^2r}\neq 0$

$$-\left(1-\frac{2Gm}{c^2r}\right)^{-2}\frac{1}{c^2r^3}+\left(1-\frac{2Gm}{c^2r}\right)^{2}\frac{1}{c^2r^3}=0$$

$$\frac{1}{c^2r^3}\left[\left(1-\frac{2Gm}{c^2r}\right)^{2}-\left(1-\frac{2Gm}{c^2r}\right)^{-2}\right]=0$$

$$\Rightarrow\left(1-\frac{2Gm}{c^2r}\right)^{2}-\left(1-\frac{2Gm}{c^2r}\right)^{-2}=0$$

$$\Rightarrow\left(1-\frac{2Gm}{c^2r}\right)^{2}=\left(1-\frac{2Gm}{c^2r}\right)^{-2}$$

$$\Rightarrow\left(1-\frac{2Gm}{c^2r}\right)^{4}=1\ (18.3)$$

Real solution, $1-\frac{2Gm}{c^2r}=\pm1\Rightarrow\frac{2Gm}{c^2r}=0\Rightarrow r\longrightarrow\infty$[flat space time where the Riemann tensor=rank four null tensor or $\frac{Gm}{c^2r}=1\Rightarrow$we are inside the event horizon

If

$$1-\frac{2Gm}{c^2r}=0,$$

the left side of (14')(18.2) becomes undefined[both terms become undefined]. If we consider $1 - \frac{2Gm}{c^2r} \longrightarrow \mathbf{0}$

we are as a matter of fact taking $1 - \frac{2Gm}{c^2r} \neq 0$,a case which has already been discussed.

One should keep in the mind that each point of the manifold is traversed by an infinitude of non geodesics asides geodesics.. For non geodesics four force is zero.Four vectors like P and Q my be identified from the stated non geodesics. Equation (8) utilizes a vector $A_p$which has to be consistent with the manifold being considered.To this end we may consider the four vector discussed

An Alternative Discrepancy:

$$R_{\alpha\beta\alpha\beta} = R_{\beta\alpha\beta\alpha} \ (19)$$

$$g_{\alpha k}R^k{}_{\beta\alpha\beta} = g_{\beta k}R^k{}_{\alpha\beta\alpha} \ (20)$$

With equation (16) we have considered a summation on k but not on α[by our choice

In the orthogonal system

$$g_{\alpha\alpha}R^\alpha{}_{\beta\alpha\beta} = g_{\beta\beta}R^\beta{}_{\alpha\beta\alpha}[no\ summation\ on\ alpha\ and\ beta](21.1)$$

$$g_{tt}R^t{}_{rtr} = g_{rr}R^r{}_{trt} \ (18)$$

$$\left(1 - \frac{2Gm}{c^2r}\right)\frac{2Gm}{c^2r^3 - 2Gmr^2} - \left(1 - \frac{2Gm}{c^2r}\right)^{-1}\left(-\frac{2(Gc^2mr - 2G^2m^2)}{c^4m^4}\right) = 0 \ \ (21.2)$$

$$\left(1 - \frac{2Gm}{c^2r}\right)\frac{2Gm}{c^2r^3 - 2Gmr^2} + \left(1 - \frac{2Gm}{c^2r}\right)^{-1}\left(\frac{2(Gc^2mr - 2G^2m^2)}{c^4m^4}\right) = 0$$

If $1 - \frac{2Gm}{c^2r} \neq 0$

$$\left(1 - \frac{2Gm}{c^2r}\right)^2\frac{2Gm}{c^2r^3 - 2Gmr^2} + \frac{2(Gc^2mr - 2G^2m^2)}{c^4m^4} = 0$$

$$\left(1 - \frac{2Gm}{c^2r}\right)^2\frac{2Gm}{r^2(c^2r - 2Gm)} + \frac{2Gm(c^2r - 2Gm)}{c^4m^4} = 0$$

For $c^2r \neq 2Gm$

$$\left(1 - \frac{2Gm}{c^2r}\right)^2\frac{1}{r^2} + \frac{(c^2r - 2Gm)^2}{c^4m^4} = 0 \ (21.3)$$

The sum of two perfect squared quantities adding upto zero.For

$$1-\frac{2Gm}{c^2r}\neq 0,$$

we do have an impossibility

If

$$1-\frac{2Gm}{c^2r}=0,$$

the left side of (18') becomes undefined[both terms become undefined]. If we consider $1-\frac{2Gm}{c^2r}\longrightarrow \mathbf{0}$

we are actually taking $1-\frac{2Gm}{c^2r}\neq 0$,a case which has already been discussed.

Equations (14') or (19) point to some deep rooted fundamental problem with conventional theory

The non trivial components of the Riemann tensor are of a suspicious or of a controversial nature[as demonstrated y the Schwarzschild metric].

Section II

We considerhave in all frames of reference the formulas[4]

$$R_{\alpha\alpha\gamma\delta}=0, R_{\bar{\alpha}\bar{\alpha}\bar{\gamma}\bar{\delta}}=0, \dots\dots [24\ zero\ components\ in\ each\ reference\ frame]$$

Also

$$R_{\alpha\beta\gamma\gamma}=0, R_{\bar{\alpha}\bar{\beta}\bar{\gamma}\bar{\gamma}}=0, \dots\dots [24\ zero\ components\ in\ each\ reference\ frame]$$

The above results follow from

$$R_{\alpha\beta\gamma\delta}=-R_{\beta\alpha\gamma\delta}$$

$$\alpha=\beta \Longrightarrow R_{\alpha\alpha\gamma\delta}=0$$

$$R_{\alpha\beta\gamma\delta}=-R_{\alpha\beta\delta\gamma}$$

$$\gamma=\delta \Longrightarrow R_{\alpha\beta\gamma\gamma}=0$$

$$R_{\alpha\alpha\alpha\alpha}=0, R_{\bar{\alpha}\bar{\alpha}\bar{\alpha}\bar{\alpha}}=0[four\ zero\ compoments]$$

$$R_{\alpha\alpha\alpha\delta}=0, R_{\bar{\alpha}\bar{\alpha}\bar{\alpha}\bar{\delta}}=0[twelve\ zero\ compoments]$$

Again from the transformation of rank four covariant tensors we have,

$$R_{\bar{\alpha}\bar{\alpha}\bar{\gamma}\bar{\delta}} = \frac{\partial x^{\alpha}}{\partial \bar{x}^{\bar{\alpha}}}\frac{\partial x^{\beta}}{\partial \bar{x}^{\bar{\alpha}}}\frac{\partial x^{\gamma}}{\partial \bar{x}^{\bar{\gamma}}}\frac{\partial x^{\delta}}{\partial \bar{x}^{\bar{\delta}}} R_{\alpha\beta\gamma\delta}$$

$$0 = \frac{\partial x^{\alpha}}{\partial \bar{x}^{\bar{\alpha}}}\frac{\partial x^{\beta}}{\partial \bar{x}^{\bar{\alpha}}}\frac{\partial x^{\gamma}}{\partial \bar{x}^{\bar{\gamma}}}\frac{\partial x^{\delta}}{\partial \bar{x}^{\bar{\delta}}} R_{\alpha\beta\gamma\delta}(22)$$

On the right side of (22), $R_{\alpha\beta\gamma\delta}$ are the Riemann curvature components in a certain specified frame of reference and for some specified geometry. On the left side zero presents $R_{\alpha\alpha\gamma\delta} = 0$ in various other frames of reference[an infinitely many of them] against the arbitrary transformations $\frac{\partial x^{\alpha}}{\partial \bar{x}^{\bar{\alpha}}}\frac{\partial x^{\beta}}{\partial \bar{x}^{\bar{\alpha}}}\frac{\partial x^{\gamma}}{\partial \bar{x}^{\bar{\gamma}}}\frac{\partial x^{\delta}}{\partial \bar{x}^{\bar{\delta}}}$. These transformations being arbitrary the only option would be to have $R_{\bar{\alpha}\bar{\alpha}\bar{\gamma}\bar{\delta}}$for all components.

Equation ((20) represents an infinite set of linear homogeneous equations in a finite number of variables[unknowns] given by $R_{\alpha\beta\gamma\delta}$. If these equations are independent[even if subset of them greater than the number of variables are independent] then the variables $R_{\alpha\beta\gamma\delta}$ have to vanish

Further Analysis

We consider the following infinite number of homogeneous equations:

$$a_{\alpha\beta\gamma\delta} R_{\alpha\beta\gamma\delta} = 0 \ (23)$$

$$a_{\alpha\beta\gamma\delta} \equiv \frac{\partial x^{\alpha}}{\partial \bar{x}^{\bar{\alpha}}}\frac{\partial x^{\beta}}{\partial \bar{x}^{\bar{\alpha}}}\frac{\partial x^{\gamma}}{\partial \bar{x}^{\bar{\gamma}}}\frac{\partial x^{\delta}}{\partial \bar{x}^{\bar{\delta}}}$$

Assume there are k nontrivial $R_{\alpha\beta\gamma\delta}$in (23).For any specified k,$R_{\alpha\beta\gamma\delta}$we may think of an exercise where an infinite[buty not arbitrary] number of coefficient sets $\{a_{\alpha\beta\gamma\delta}\}$satisfying (A) are generated that is we speculate n infinite number of equations like (23)

Let us consider k equations from (23)out of the infinitude possible

We have k linear homogeneous equations. For non trivial $R_{\alpha\beta\gamma\delta}$,determinant of coefficient matrix has to be zero[for any k equations chosen]:det$[a_{\alpha\beta\gamma\delta}] = 0$. Thus the rank of the coefficient matrix is less than k.Now the dimension of row spa c=rank of matrix.Therefore the dimension of row space is less thank. At least one row is a linear combination of the others. Any k transformations chosen are not linearly independent if they satisfy (23)[k=number of non trivial $R_{\alpha\beta\gamma\delta}$]

But our linear transformation elements $a_{\alpha\beta\gamma\delta}$ are expected to be linearly independent row wise from the coefficient matrix.We must also keep in our mind that if two or more sets of equations[k equations and k unknowns] have the same solution set for all the sets then the equations have to be equivalent:each equation of any set should be expressible as a linear combination of the others.

For m linear homogeneous equations involving n variables[unknowns] in each line,if the number of independent equations exceeds the number of variables then each variable reduces to zero value.

Now from the definition of the Riemann tensor it follows that

$$R^{\alpha}{}_{\beta\gamma\delta} = \frac{\partial\Gamma^{\alpha}{}_{\beta\delta}}{\partial x^{\gamma}} - \frac{\partial\Gamma^{\alpha}{}_{\beta\gamma}}{\partial x^{\delta}} + \Gamma^{\alpha}{}_{\gamma\varepsilon}\Gamma^{\varepsilon}{}_{\beta\delta} - \Gamma^{\alpha}{}_{\delta\varepsilon}\Gamma^{\varepsilon}{}_{\beta\gamma} \; (24)$$

If α, β, γandδ are distinct then $R^{\alpha}{}_{\beta\gamma\delta} = 0$

Proof: We recall that for the orthogonal system $\Gamma^{\alpha}{}_{\beta\delta} = 0$ if $\alpha, \beta$and$\delta$are distnct

$$\Gamma^{\alpha}{}_{\beta\delta} = 0, \Gamma^{\alpha}{}_{\beta\gamma} = 0 \Longrightarrow \frac{\partial\Gamma^{\alpha}{}_{\beta\delta}}{\partial x^{\gamma}} = 0, \frac{\partial\Gamma^{\alpha}{}_{\beta\gamma}}{\partial x^{\gamma}} = 0$$

$$\Gamma^{\alpha}{}_{\gamma\varepsilon}\Gamma^{\varepsilon}{}_{\beta\delta} - \Gamma^{\alpha}{}_{\delta\varepsilon}\Gamma^{\varepsilon}{}_{\beta\gamma} = \Gamma^{\alpha}{}_{\gamma\gamma}\Gamma^{\gamma}{}_{\beta\delta} + \Gamma^{\alpha}{}_{\gamma\alpha}\Gamma^{\alpha}{}_{\beta\delta} - \Gamma^{\alpha}{}_{\delta\delta}\Gamma^{\delta}{}_{\beta\gamma} - \Gamma^{\alpha}{}_{\delta\alpha}\Gamma^{\alpha}{}_{\beta\gamma} = 0 \; (25)$$

Thus if If α, β, γandδ are distinct then $R^{\alpha}{}_{\beta\gamma\delta} = 0$

We transform $R^{\alpha}{}_{\beta\gamma\delta}$with If $\alpha, \beta, \gamma$and$\delta$from one coordinate system to an infinite number of orthogonal coordinate systems

$$R^{p}{}_{qrs} = \frac{\partial x^{q}}{\partial \bar{x}^{\alpha}} \frac{\partial \bar{x}^{\beta}}{\partial x^{q}} \frac{\partial \bar{x}^{\gamma}}{\partial x^{r}} \frac{\partial \bar{x}^{\delta}}{\partial x^{s}} \bar{R}^{\alpha}{}_{\beta\gamma\delta} \; (26)$$

On the left side if p,q,r and s are distinct then

$$0 = \frac{\partial x^{q}}{\partial \bar{x}^{\alpha}} \frac{\partial \bar{x}^{\beta}}{\partial x^{q}} \frac{\partial \bar{x}^{\gamma}}{\partial x^{r}} \frac{\partial \bar{x}^{\delta}}{\partial x^{s}} \bar{R}^{\alpha}{}_{\beta\gamma\delta} \; (27)$$

We have an infinite number of distinct linear homogeneous equations with a finite number of non trivial unknowns, $\bar{R}^{\alpha}{}_{\beta\gamma\delta}$. The indices may or may not be distinct with $\bar{R}^{\alpha}{}_{\beta\gamma\delta}$ on the right side. Therefore the Riemann tensor components ere zero in all systems. They have to be zero in all other frames of reference [the non orthogonal or orthogonal frames]

Our conclusion of zero valued Riemann tensor components has followed logically in a legitimate manner

The non trivial components of the Riemann tensor observed in conventional physics are of a suspicious or of a controversial nature[as demonstrated by the Schwarzschild metric].This observation has been already made in the last section[towards the end of it]

One should also keep in the mind that the null tensor[rank four ,covariant, follows the well known properties of the Riemann tensor listed below.

$$R_{\alpha\beta\gamma\delta} = -R_{\beta\alpha\gamma\delta}$$

$$R_{\alpha\beta\gamma\delta} = -R_{\alpha\beta\delta\gamma}$$

$$R_{\beta\alpha\delta\gamma} = R_{\alpha\beta\delta\gamma}$$

$$R_{\alpha\beta\delta\gamma} + R_{\alpha\delta\beta\gamma} + R_{\alpha\gamma\delta\beta} = 0$$

Conclusions

As claimed at the outset we have discovered a conflict between standard results and derived equations. Then considering linear homogeneous equations we have derived that the Riemann tensor is a rank four null tensor.

[Data sharing is not applicable to this article as no new data were created or analyzed in this study.]

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