Finding regions of correlated polarization directions of the 1450 QSO radio sources in the JVAS1450 catalog

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Abstract

This article presents a way to survey catalogued data, here the JVAS1450 catalog of polarized radio QSOs that has been measured, collected, catalogued, and made available by others. The polarization directions are spread out haphazard over the Northern Equatorial hemisphere. We find five degree radius regions whose sources' polarization directions converge significantly at points on the Celestial Sphere or diverge significantly. Samples are collected for further study. The appendix consists of a computer software program that performs the needed calculations. The computer program can be adapted to other choices of region radii and to other sets of transverse vectors.

Keywords: Alignment; Hub Test; Transverse Vectors; Polarization; Jets; Survey

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0. Preface

The pdf version of this notebook is available online from the viXra archive. Search by title and author.
To find the ready-to-run notebook follow one of the links in Ref. 1.
The notebooks in this series were created using Wolfram Mathematica, Version Number: 12.1, Ref. 2.

Note(s):

(1) Some numerical quantities in the pdf version may differ from the ready-to-run version in Ref. 1 because the ready-to-run version may have been run after the pdf was produced. The ready-to-run version and the pdf version may be updated independently of one another.

(2) The notation is undergoing a change from "S" indicating significance to "*p*" standing for significance. Some of the "S" labels have most likely survived.

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1. Introduction

Observations of an astronomical object may include quantities such as polarization and jets that can be represented as transverse vectors, vectors that are perpendicular to the direction to the object itself. If a set of transverse vectors from many objects are correlated, this information may reflect on the objects themselves or of the intervening medium through which the observations are made or otherwise.

Given a set of transverse vectors on the sky, one may ask if their directions are correlated. Possible transverse vectors include the polarization direction of electromagnetic radiation and the direction of asymmetries such as jets. Reducing a jumble of transverse vectors to regions with interesting correlations is the goal of the present article.

The data for the QSOs studied in this report are taken from the JVAS1450, Ref. 3,4, a catalog of 1450 QSOs that was kindly communicated to me by one of the authors of Ref. 3. Details of the dataset can be found in Ref. 3. As explained in Ref. 3, the JVAS1450 catalog includes data from the earlier large JVAS/CLASS 8.4-GHz catalog, Ref. 4. See Fig. 1 for a display of the data treated in this article.

The test of alignment used in this article, the Hub Test, extends the polarization directions, making Great Circle geodesics on the Celestial Sphere. The polarization directions are perfectly aligned if they intersect at some point H on the sphere. The directions are well-aligned when they converge in a small area near some point H_{min} . The Hub Test can find correlations for samples with hubs H_{min} that are near the sources as well as the distant Hubs of other alignment tests.

The Hub Test is equally capable of finding avoidance hubs, $H_{\max}$, places where the density of the Great Circle geodesics is low. Part I the article focusses on alignment, while Part II the Appendix treats both avoidance and alignment on equal terms.

The survey discussed herein seeks interesting samples to study. The QSO sources are assigned to 5° radius circular regions centered on the grid points of a 2° mesh. To be evaluated, a minimum of seven sources per region is required. The regions are sorted by the significance of their alignments according to the Hub Test. Previous articles, Ref. 5 and 6, have evaluated a couple of interesting samples of QSOs that are identified in this survey.

The Hub Test is briefly presented in Sec. 2. The catalog of QSOs and mapping the data occurs in Sec. 3. Then, Sec. 4 analyzes the alignment of 5° radius samples and locates the significantly aligned samples. Maps of the results can be found in Sec. 4. Locating neighboring significantly aligned sets of sources is the goal and that is accomplished in Sec. 4. Sec. 5 completes Part I the Article with some concluding remarks.

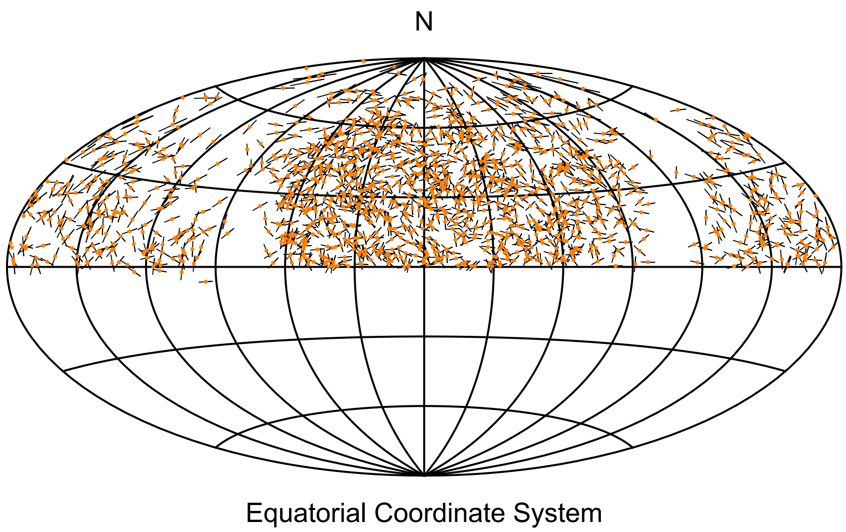

Figure 1. A whole-sphere map of the sources and polarization directions of the JVAS1450 catalog. The plot is centered on (RA,dec) = (180°,0). East is to the right.

2. The Hub Test

The Hub Test, Ref. 7, judges the alignment of transverse vectors with the directions to a point on the Celestial Sphere. By involving the direction to another point, the Hub test is indirect. For a single source, the basic quantities are illustrated in Fig. 2. The "alignment angle" η is the acute angle η between two great circles at S, $0° \leq \eta \leq 90°$. The alignment angle η measures how well the polarization direction $\hat{v}_{\psi}$ matches the direction $\hat{v}_H$ toward the point H. Perfect alignment occurs when $\eta = 0°$ and the two great circles overlap. When these two great circles are perpendicular, $\eta = 90°$, that indicates maximum "avoidance" of the polarization direction $\hat{v}_{\psi}$ with the point H on the sphere. The halfway value, $\eta = 45°$, favors neither alignment nor avoidance.

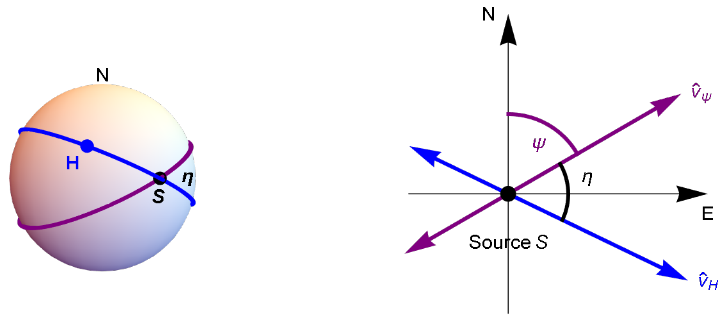

Figure 2: The Celestial sphere is pictured on the left and on the right is the plane tangent to the sphere at the source *S*. The linear polarization direction $\hat{v}_\psi$ lies in the tangent plane and determines the purple great circle on the sphere. A point *H* on the sphere together with the point *S* determine a second great circle, the blue circle drawn on the sphere. Clearly, *H* and *S* must be distinct in order to determine a unique great circle. The acute angle η measures the alignment of the polarization direction ψ with the point *H*.

With *N* sources S_i, $i = 1, ..., N$, there are *N* alignment angles η_{iH} at each point *H* . One can calculate an average alignment angle $\overline{\eta}$ at *H*,

$$\overline{\eta}(\mathrm{H}) = \frac{1}{N} \sum_{i=1}^{N} \eta_{\mathrm{iH}} , \qquad (1)$$

where

$$\cos(\eta_{\mathrm{iH}}) = | \hat{v}_\psi . \hat{v}_H | . \qquad (2)$$

Given a positive numerical value for the absolute value of the dot product on the right in (2), the solution for the angle η_{iH} is taken to be the positive acute angle with $0° \le \eta_{\mathrm{iH}} \le 90°$. Clearly, the average alignment angle $\overline{\eta}(\mathrm{H})$ at the point H must also be acute. An example of the function $\overline{\eta}(\mathrm{H})$ is presented in Figs. 3 and 4.

The alignment angle $\overline{\eta}(\mathrm{H})$ is a function of position *H* on the sphere. In general, the function $\overline{\eta}(\mathrm{H})$ is symmetric across diameters, $\overline{\eta}(\mathrm{H}) = \overline{\eta}(\mathrm{-H})$, because great circles are symmetric across diameters.

For random polarization directions, the average $\overline{\eta}(\mathrm{H})$ should be near 45°, since each alignment angle η_{iH} is acute, $0° \le \eta_{\mathrm{iH}} \le 90°$, and random polarization directions should not favor large values or small values of η_{iH}, and, therefore, average to about 45°.

Points *H* where the average alignment angle $\overline{\eta}(\mathrm{H})$ is smaller than 45°, the great circles tend to converge and where the angle $\overline{\eta}(\mathrm{H})$ is larger than 45°, the great circles can be said to diverge. The extremes of the function $\overline{\eta}(\mathrm{H})$ measure extreme convergence and extreme divergence of the great circles determined by the polarization directions. We use the term “alignment” for convergence and “avoidance” for divergence.

In this article and notebook, we often use “min” to label the smallest alignment angle $\overline{\eta}_{\mathrm{min}}$, the minimum value of the function $\overline{\eta}(\mathrm{H})$, Eq. (1). The points on the Celestial Sphere where the minimum occurs are the “hubs” H_{min} and $-H_{\mathrm{min}}$. Thus “min” is associated with convergence of the polarization directions. For divergence, the hubs H_{max} and $-H_{\mathrm{max}}$ locate places where the polarization directions most avoid, as indicated by the largest alignment angle $\overline{\eta}_{\mathrm{max}}$, the maximum value of the function $\overline{\eta}(\mathrm{H})$. Thus, we very often label an avoidance related quantity with “max”.

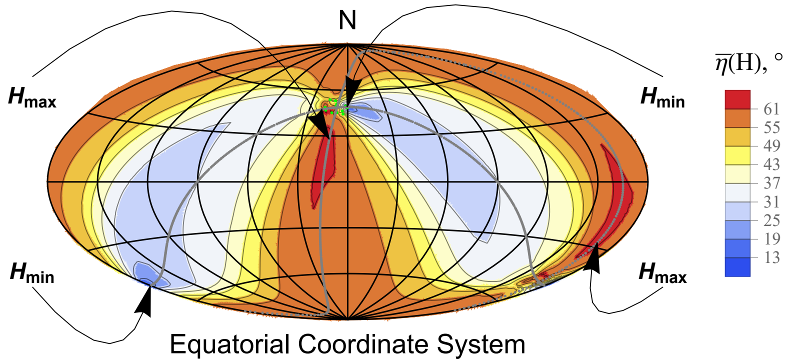

Figure 3: For the sample of 13 QSOs called Clump 2 in Fig. 6, the alignment angle function $\overline{\eta}$(H) in (1) is mapped on the Celestial Sphere (Aitoff plot, centered on $(\alpha,\delta) = (180°,0)$, East to the right). The QSOs are shaded green ■. The smallest alignment angle, $\overline{\eta}_{\min} =$ 10.9°, is located at the hubs $H_{\min}$ and $-H_{\min}$, where the polarization directions converge best. One alignment hub $H_{\min}$ is located very close to the QSOs. The largest alignment angle, $\overline{\eta}_{\max} =$ 62.7°, occurs at the avoidance hubs $H_{\max}$ and $-H_{\max}$. See Ref. 6.

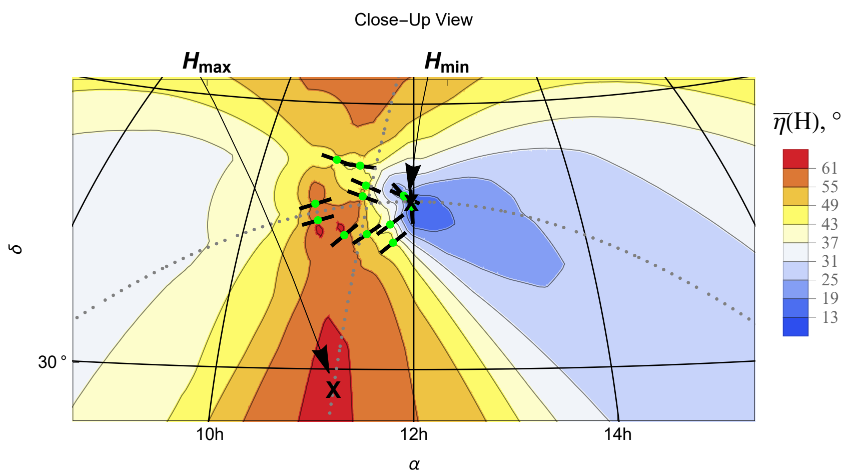

Figure 4: The region near the QSOs in Fig. 3. The QSOs are located at the green dots. The short black lines through the QSOs indicate the polarization directions. Measuring polarization directions ψ clockwise from North, one sees that the angles ψ range from more than $\psi = 90°$ for the northern-most QSOs to 45° or so for the southerly QSOs. Most are in the general direction of the alignment hub $H_{\min}$, but their directions depend on where they are located. The QSOs display parallax.

The significance, p or p-value and sometimes S, of the smallest alignment angle $\overline{\eta}_{\min}$ is defined as the likelihood that randomly directed polarization vectors would produce a smaller value of $\overline{\eta}_{\min}$. Therefore, by this definition, one way to determine significance is to repeat the process of making Great Circles from polarization directions, calculating the alignment function $\overline{\eta}$(H), and finding $\overline{\eta}_{\min}$, all for randomly directed vectors. One such process makes a “random run”.

The most reliable method of determining significance that we consider is called “Direct Method A”. Following the definition of significance, one generates many random runs with randomly directed transverse vectors assigned to the sources. A histogram of the random-based results for $\overline{\eta}_{\min}$ is then approximated by a suitable fitting function. Aside from a scale factor that normalizes the distribution, the fitting function of the histogram is the probability distribution of the random results $\overline{\eta}_{\min}$. Having found a function that approximates the probability distribution, one estimates the likelihood that random runs return better results than the observed

$\overline{\eta}_{\min}{}^{\rm obs}$ and that is the significance of $\overline{\eta}_{\min}{}^{\rm obs}$. Similar comments apply for avoidance.

It would be terribly inconvenient to apply Direct Method A for all the regions in the survey. Instead we introduce a “Library” of data that can be used to reconstitute the probability distributions for a range of samples with various number of sources and with various sizes. The Library data is used in two ways, as “Interpolation Method B” and as “Function Method C”, to develop values of significance for the alignments of the many regions considered in the survey. The Methods are discussed briefly where they are introduced in Sec. A7. For a more complete discussion, see Ref. 8.

3. Setting up the regions

The Hub Test needs enough information to draw great circles outward from the sources in the directions of the polarization vectors. Thus the required data includes the location of the sources as well as the polarization direction at each source. Also, the uncertainty in the measured polarization direction is needed to estimate the error bars on the calculated quantities. In this report, the uncertainty in the locations is taken to be insignificant.

The JVAS1450 catalog and the JVAS/CLASS 8.4-GHz catalog gives us all the locations of the sources in J2000.0 equatorial coordinates so we can find right ascension and declination of the sources in decimal degrees and the polarization position angle and its uncertainty in decimal degrees. The data also includes other interesting quantities such as the redshift. Such extra data is not needed here.

The computer program in Part II the Appendix assumes the needed data is collected in a table, called “data00”, in a prescribed order. One hopes the program in the Appendix can serve as a kind of template. If one possesses other data from other sources in another experimental campaign, then, by putting the other data in the same form as the data00 table here, one can run the program and get alignment and avoidance maps for the other data.

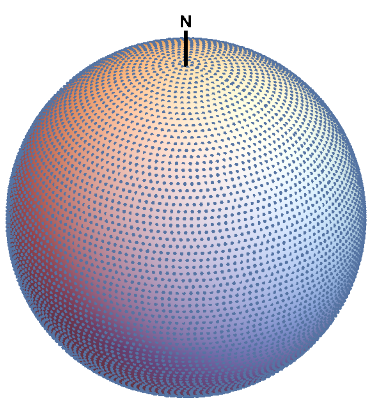

Figure 5. The grid. At a constant declination (latitude), the right ascension (longitude) of the grid points are spaced by 2°. The circles of constant declination are separated by 2°. Each of the regions analyzed is centered on one of the grid points.

The program constructs a mesh of grid points. The spacing of points on the grid can be adjusted as one wishes. See Figs. 5 for a plot of the grid for this article. We use a grid spacing of 2°, taking into account the shrinking radii of circles of constant declination as they approach the poles.

The computer program Part II the Appendix has another user-definable quantity, the region radius. Regions in this report have 5° radii. Not all the 5° radius regions have sources, only a few sources are in the South. And the number of sources in a region must be at least 7 for the significance estimates to be sufficiently valid.

The problem is with probability distributions, like Gaussians, that assign nonzero probability to non-acute alignment angles. With the distributions in this article and for samples with fewer than 7 sources, the probability of negative alignment angles is non-negligible. But alignment angles are acute and never negative, so any assignment of likelihood to negative alignment angles is an artifact. Similarly, the avoidance angle cannot be larger than 90°, yet the probability distribution continues to infinity with nonzero likelihood. We can ignore these effects for samples with more than seven sources, $N \geq 7$.

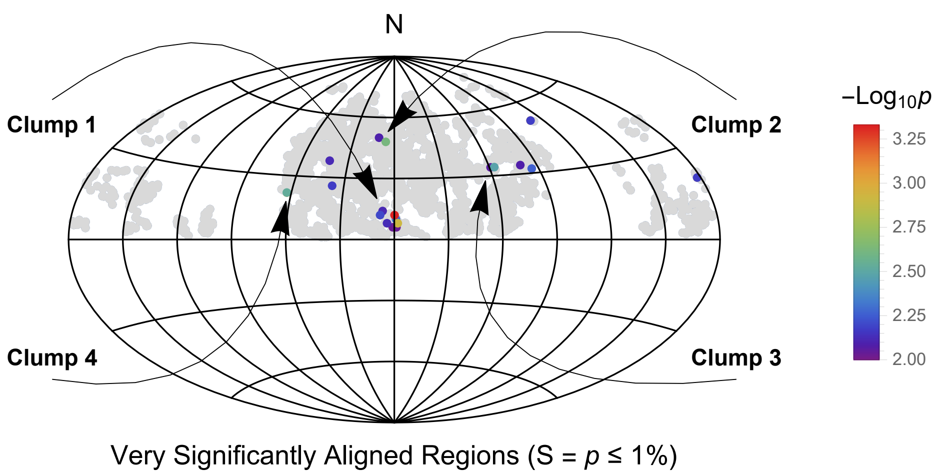

Figure 6. Regions with sources that are very significantly aligned are shaded in color. The centers of 5° radius regions that have at least 7 sources are plotted as gray points, a total of 1811 regions. The 19 regions whose polarization directions align with a significance less than 1%, meaning that $p \leq 0.01$, are considered "very significantly aligned" and are shaded in color. The range of significance runs from $p = 0.00047$ to $p = 0.01$ (very significant limit), and, since $-\text{Log}_{10}\, 0.00047 = 3.3$, we have $2.0 \leq -\text{Log}_{10}\, p \leq 3.3$. As shown in Fig. A4, the uncertainty in $-\text{Log}_{10}\, p$ is about ±0.4, running from ±0.25 to ±0.5.

4. The significance of the regions' alignments

With the Hub Test, the alignment of a sample is gauged by the smallest average alignment angle $\overline{\eta}_{\text{min}}$, the minimum value of the function $\overline{\eta}$(H), Eq. (1). For example, Fig. 3 plots the function $\overline{\eta}$(H) for the sample of 13 QSOs in Ref. 6. Similarly, for each of the 1811 qualifying regions, one finds the function $\overline{\eta}$(H) and then determine the smallest alignment angle $\overline{\eta}_{\text{min}}$. Thus, we get 1811 results $\overline{\eta}_{\text{min}}$, one for each region.

By assuming the significance of the alignment indicated by $\overline{\eta}_{\text{min}}$ depends mainly on the number of sources N and the root-mean-radius ρRMS of the region, one can find significance by Interpolation Method B and Function Method C, Ref. 8, based on an archived Library. The Library has a collection of parameters to generate probability distributions that can be utilized to obtain the significance of the alignment of the sources in a region.

Here, with polarization data from JVAS1450, we find that just 19 regions have very significant alignment. These are plotted in color in Fig. 6. Since there are 1811 regions, 19 regions is very close to 1% of the total number of regions. However, the most significantly aligned region has a significance of $p = 0.00047$ which means its alignment is better than all but one in 2100 regions with randomly directed polarizations. The most significantly aligned region lies just above the center of Fig. 6.

If the most significantly aligned region was the only region aligned better than 2100 randomly directed regions, one could argue that the alignment was consistent with pure chance. But there are other well-aligned regions in Fig. 6, so the likelihood that all are aligned by chance is much smaller than 1/1811. Furthermore, by collecting the sources in different regions, one can find larger collections of well-aligned samples and these may have significances that make the number 1811 of regions seem small. For example, by combining the 8 sources in the most significantly aligned region with 19 sources from overlapping very significantly aligned regions, one has a sample of 27 sources that has been studied in Ref. 5. There, the sample is shown to be aligned better than one in about 70,000 randomly directed samples. Collecting the 13 sources in two regions near (RA,dec) = (170°, 50°), one obtains a sample that is better aligned than one in 55,000 randomly directed samples. So the alignment of these samples is unlikely to be due to randomly oriented polarization directions.

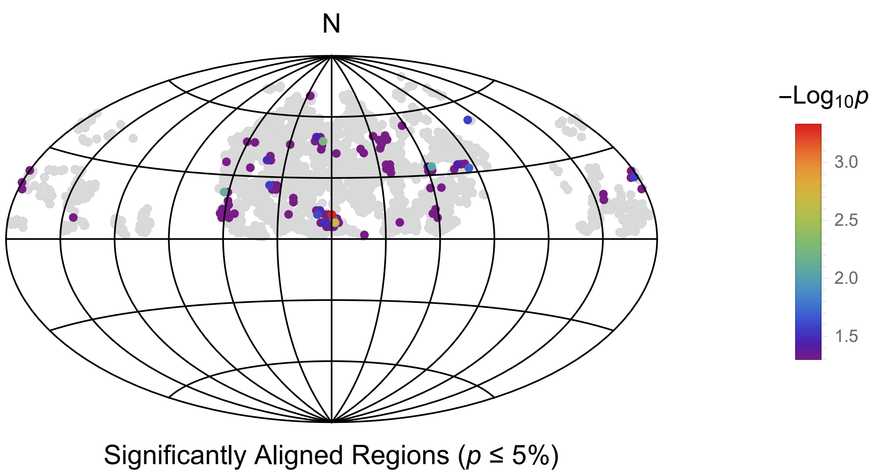

Figure 7. Regions with significant alignment are shaded in color. There are 96 colored dots compared with the 19 in Fig. 6. That makes sense, because to qualify as "significant" as many as one in twenty randomly directed samples need be better aligned than the region in question. For the "very significantly" aligned regions of Fig. 6, only one in a hundred randomly directed regions can be better aligned.

The less selective one is, the more regions are collected. In Fig. 7, the required significance is weakened to $p \leq 5\%$ from the $p \leq 1\%$ in Fig. 6. Thus the regions in color in Fig. 7 are aligned 'significantly', $p \leq 5\%$, but not 'very significantly' which would require $p = 1\%$ or less. One sees that the very significantly aligned regions displayed in Fig. 6 are surrounded by other regions that are merely significantly aligned.

5. Combining regions to make samples

We have previously studied two samples of polarized radio QSOs from the JVAS1450 catalog, Refs. 5 and 6. In this section we discuss the steps that determine the two samples that were studied and published. For details, see Sec. A9 "Selecting sources to analyze" in Part II the Appendix. The samples studied previously are labeled Clump 1 and Clump 2 in Figs. 6.

Clump 1 consists of eight overlapping 5° radius regions with alignments that are very significant, $p \leq 0.01 = 1\%$. The most significantly aligned region has eight sources and is aligned with significance $p_0 = 4.7\times10^{-4}$. Combining the sources from the eight regions yields a sample with $N = 27$ QSO sources and a root-mean-square radius ρRMS of 6.8°, which is roughly equivalent to a 10° radius circular region since taking the root-mean-square introduces a factor of about 0.7. One finds in Ref. 5, that the smallest alignment angle $\overline{\eta}_{\min}$ for the sample is $\overline{\eta}_{\min} = 21.1°$ which has a significance of $p = (0.44 \text{ to } 4.5)\times 10^{-5} \lesssim 0.1\, p_0$. Thus, by combining the eight regions one finds a sample that is better aligned than any of the individual regions.

Clump 2 in Fig. 6 has just two 5° regions with 13 QSOs when combined. The root-mean-square radius is about 5°, ρRMS = 4.7°. The significance of the alignment of the polarization directions of the 13 QSOs is $p = (1.7 \text{ to } 2.3)\times 10^{-5}$, compared with $p_0 = 2.4\times10^{-3}$ for one of the two regions. Again, we find that combining the two very significant regions yields a sample with more significant alignment than either region separately.

We found that simply combining neighboring very significantly aligned regions produced samples that were worth studying. One supposes that there are other, more careful, ways to find a sample to study. However, the point of making the survey is to find order in the jumble and to locate well aligned sources, thereby identifying neighborhoods that may worthy of further research.

6. Concluding Remarks

When confronted with a jumble of transverse vectors like that in Fig. 1, generating a survey that maps the significance of the alignment in regions may help organize the data and identify areas to investigate further. Conducting a survey like that in this report can be a first step to finding well-aligned polarization directions of a large catalog.

There are no guarantees, of course. Some overlooked small area may contain sources with interesting alignment properties, overlooked because the alignment is diluted in a region that is too big. A 5° radius region survey might be blind to a well-aligned collections of sources confined to 1° samples. Conversely, one suspects that a 1° region survey might miss some of the alignments that a 5° survey uncovers. Maybe the answer is to conduct more surveys.

Since finding the smallest alignment angle $\overline{\eta}_{\min}$ and the largest avoidance angle $\overline{\eta}_{\max}$ are such similar processes, this article treats only alignment in any detail. Yet avoidance may be the more important property of polarization directions for some sets of data. What if the polarization direction is perpendicular to some local structure. Then correlations of perpendiculars, *i.e.* avoidance and $\overline{\eta}_{\max}$, take center stage. Part II the Appendix treats alignment and avoidance equally.

References

1. R. Shurtleff, the ready-to-run Mathematica version of this notebook is available at the following URLs:
https://www.wolframcloud.com/obj/shurtleffr/Published/20211221Survey1450QSOsMapb.nb
https://www.dropbox.com/s/6bqy56vazlfuuu6/20211221Survey1450QSOsMapb.nb?dl=0
2. Wolfram Research, Inc., Mathematica, Version 12.1, Champaign, IL (2020).
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4.(13) Jackson, N., Battye, R. A., Browne, I. W. A., Joshi, S., Muxlow, T. W. B., and Wilkinson, P. N., A survey of polarization in the JVAS/CLASS flat-spectrum radio source surveys - I. The data and catalogue production, MNRAS, 376, 371-377, doi: 10.1111/j.1365-2966.2007.11442.x , arXiv:astro-ph/0703273 (2007).
5. Shurtleff, R., "Evaluating the Alignment of the Polarized Radio Waves from 27 QSOs in a Region near the NGP", https://vixra.org/abs/2105.0091 (2021).

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Part II the Appendix

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A1. Introduction

The Appendix is the computer program, a “notebook” written in the Wolfram Mathematica language. The inputs to the program in Sec. A1 can be changed so the survey can deal with new data.

The Appendix treats alignment and avoidance equally, what it finds for one, it finds for the other. Avoidance may be important if the polarization direction turns out to be perpendicular to some feature such as a jet or some other structure. Then it would be important to find correlations of directions that are perpendicular to the polarization vectors and that would be revealed by gauging avoidance.

A2. User Input

This notebook may be used as a template to evaluate new data.

1. The new data should conform to the format of the table “data00” displayed below.
2. You may want to furnish a home directory so the program can find and save data files.
3. The grid spacing can be chosen by the user below in this section.

4. The regions to be analyzed are circular with a radius that can be chosen by the user in this section.

Definitions:

homeDirectory a place on the computer to store and retrieve files.
gridSpacing in degrees, the angular separation of grid points along a circle of constant latitude and the angular separation of the circles of constant latitude. See Sec. A4 Grid.
rgnRadius radius of regions in degrees

data00 This data table contains the information about the sources that produces the rest of the notebook.
1.Object # 2. Ra (rad) 3. Dec (rad) 4. ψ (rad) 5. $\sigma\psi$ (rad)

In[1]:=
```
homeDirectory =
 "C:\\Users\\shurt\\Dropbox\\HOME_DESKTOP-0MRE5OJ\\SendXXX_CJP_CEJPetc\\SendViXra\\
   20200715AlignmentMethod\\20211221MapsOfSignificance"
```

Out[1]=
```
C:\Users\shurt\Dropbox\HOME_DESKTOP-0MRE5OJ\SendXXX_CJP_CEJPetc\
  SendViXra\20200715AlignmentMethod\20211221MapsOfSignificance
```

The following cell has the data00 table with the information about the sources. It is very large and, therefore, it is hidden from view. To see it go to “Cell Properties” and click “Open”.

In[3]:=
```
(*The table data00 can be uploaded from a file.*)
(*SetDirectory[homeDirectory]
   data00=Get["20200718data08JVAS1450.dat"];*)
```

In[4]:=
```
gridSpacing = 2 (*grid spacing in degrees*);
rgnRadius = 5.(*degrees*);
```

A3. Preliminary

Definitions:

er, eN, eE are unit vectors in a 3D Cartesian coordinate system from Origin to Source,
(α,δ) = RA and Dec of the source. We use degrees for the angles.
er(α,δ) = unit vectors from Origin to Source
eN(α,δ) = local North at Source
eE(α,δ) = local East at Source
αFROMr(er) = RA determined by radial unit vector er
δFROMr(er) = Dec determined by radial unit vector er

Aitoff Plot Functions
αH(α,δ) , xH(α,δ) , yH(α,δ), where xH, yH is centered on α = 0.
xH180(α,δ) , yH180(α,δ), where xH is centered on α = 180°.

In[6]:=
```
Print["The date and time that this statement was evaluated: ", Now]
```

```
The date and time that this statement was evaluated:  Wed 12 Jan 2022 14:05:57 GMT-5.
```

In[7]:=
```
(*We work with degrees, so define convenient functions.*)
cos[θ_] := Cos[θ (2. π/360.)];
sin[θ_] := Sin[θ (2. π/360.)];
tan[θ_] := Tan[θ (2. π/360.)];
arccos[x_] := ArcCos[x] (360./2. π); arcsin[x_] := ArcSin[x] (360./2. π);
arctan[x_] := ArcTan[x] (360./2. π)
```

In[10]:=
```
(* For a Source at (RA,dec) = (α,δ): er, eN,
eE are unit vectors from Origin to Source, local North, local East, resp.  *)
er[α_, δ_] := er[α, δ] = {cos[α] × cos[δ], sin[α] × cos[δ], sin[δ]}
eN[α_, δ_] := eN[α, δ] = {-cos[α] × sin[δ], -sin[α] × sin[δ], cos[δ]}
eE[α_, δ_] := eE[α, δ] = {-sin[α], cos[α], 0}
{"Check er.er = 1, er.eN = 0, er.eE = 0, eN.eN
   = 1, eN.eE = 0,eE.eE = 1, erXeE = eN, eEXeN = er, eNXer = eE: ",
 {0} == Union[Flatten[Simplify[{er[α, δ].er[α, δ] - 1, er[α, δ].eN[α, δ], er[α, δ].eE[α, δ],
      eN[α, δ].eN[α, δ] - 1, eN[α, δ].eE[α, δ], eE[α, δ].eE[α, δ] - 1, Cross[er[α, δ], eE[α, δ]] -
       eN[α, δ], Cross[eE[α, δ], eN[α, δ]] - er[α, δ], Cross[eN[α, δ], er[α, δ]] - eE[α, δ]}]]]}
```

Out[13]=
```
{Check er.er = 1, er.eN = 0, er.eE = 0, eN.eN = 1,
   eN.eE = 0,eE.eE = 1, erXeE = eN, eEXeN = er, eNXer = eE: , True}
```

Get (α,δ) in degrees from radial vector r:

In[14]:=
```
αFROMr[r_] := (N[arctan[Abs[r[[2]]/r[[1]]]]] /; (r[[2]] ≥ 0 && r[[1]] > 0))
αFROMr[r_] := (N[180. - arctan[Abs[r[[2]]/r[[1]]]]] /; (r[[2]] ≥ 0 && r[[1]] < 0))
αFROMr[r_] := (N[180. + arctan[Abs[r[[2]]/r[[1]]]]] /; (r[[2]] < 0 && r[[1]] < 0))
αFROMr[r_] := (N[360. - arctan[Abs[r[[2]]/r[[1]]]]] /; (r[[2]] < 0 && r[[1]] > 0))
αFROMr[r_] := (90. /; (r[[2]] ≥ 0 && r[[1]] == 0))
αFROMr[r_] := (270. /; (r[[2]] < 0 && r[[1]] == 0))
```

In[20]:=
```
δFROMr[r_] := (N[arctan[r[[3]]/√(r[[1]]^2 + r[[2]]^2)]] /; (√(r[[1]]^2 + r[[2]]^2) > 0))
δFROMr[r_] := (Sign[r[[3]]] 90. /; (√(r[[1]]^2 + r[[2]]^2) == 0))
```

The following Aitoff Plot formulas can be found in, for example, Ref. 9.

Imagine the Sources are plotted on the Celestial sphere and we are looking down on the sphere from the outside.

In[22]:=
```
αH[α_, δ_] := αH[α, δ] = arccos[cos[δ] × cos[α / 2.]]
```

$$xH[\alpha\_, \delta\_] := xH[\alpha, \delta] = \frac{2.\cos[\delta]\times\sin[\alpha/2.]}{\mathrm{Sinc}[\alpha H[\alpha, \delta]]}$$

$$yH[\alpha\_, \delta\_] := yH[\alpha, \delta] = \frac{\sin[\delta]}{\mathrm{Sinc}\left[\left(\frac{2.\pi}{360.}\right)\alpha H[\alpha, \delta]\right]}$$

Using the following functions produces an Aitoff Plot that is centered on α = 180°.
Imagine the Sources are plotted on the Celestial sphere and we are looking down on the sphere from the outside.

In[25]:=

$$xH180[\alpha\_, \delta\_] := xH180[\alpha, \delta] = \frac{2.\cos[\delta]\times\sin[(\alpha - 180.)/2.]}{\mathrm{Sinc}\left[\left(\frac{2.\pi}{360.}\right)\alpha H[(\alpha - 180.), \delta]\right]}$$

$$yH180[\alpha\_, \delta\_] := yH180[\alpha, \delta] = \frac{\sin[\delta]}{\mathrm{Sinc}\left[\left(\frac{2.\pi}{360.}\right)\alpha H[(\alpha - 180.), \delta]\right]}$$

A4. Sources

The source data table "data00" was input above in Sec. A2.

Definitions:

data00 - This data table contains the information that produces the rest of the notebook.

data00:

1.Object # 2. Ra (rad) 3. Dec (rad) 4. ψ (rad) 5. $\sigma\psi$ (rad) [6. z 7. p (%) 8. σp (%)]

Items 6,7,8 are not used in this notebook.

from data00:

rai(i) RA of ith source (radians)
deci(i) dec of ith source (radians)
ψi(i) position angle
$\sigma\psi$i(i) uncertainty in ψ

calculated:

ri(i) unit vector from Origin to ith Source
vNi(i) Local North at the ith Source, a 3D unit vector
vEi(i) Local East at the ith Source, a 3D unit vector
vψi(i) unit vector in direction of PA ψ in tangent plane at the ith Source
nSxψi(i) cross product of ri(i) and vψi(i) = r × vψ , perpendicular to both, a unit vector in tangent plane at the ith Source

plot

xyAitoffSources source coordinates on Aitoff projection of the Celestial Sphere
crossesOverPlus, Minusthe polarization vectors of some sources cross over the edge of the Aitoff projection
noCrossing sources with polarization vectors contained in the Aitoff projection
rPlusψ[i,d] endpoints of the polarization vector for the i^{th} source (d positive and negative)
polarLinesNoCrossing[d] polarization vectors for sources with no crossing problem
polarLinesCrossingPlus[d] polarization vectors for sources with d positive crossing beyond
polarLinesCrossingMinus[d]polarization vectors for sources with d negative crossing beyond
mapOfSources Aitoff plot of the data, sources and their polarization vectors

In[27]:=
```
(*Example of a data00 record.*)data00[[16]]
```
Out[27]=
```
{16, 0.0502079, 0.016832, 2.99673, 0.0147697, 1.4904, 3.02524, 0.0893572}
```

In[28]:=
```
(*From data00. CONVERT DATA TO DEGREES*)
rai[i_] := rai[i] = data00[[i, 2]] (360./(2. π)) (*RA of ith source*)
deci[i_] := deci[i] = data00[[i, 3]] (360./(2. π)) (*dec*)
ψi[i_] := ψi[i] = data00[[i, 4]] (360./(2. π)) (*PPA,
polarization position angle: clockwise from North with East to the right. *)
σψi[i_] := σψi[i] = data00[[i, 5]] (360./(2. π))
```

In[32]:=
```
(*Convenient functions*)
ri[i_] := ri[i] = er[rai[i], deci[i]]
(*unit vector from Origin to ith Source on Celestial Sphere*)
vNi[i_] := vNi[i] = eN[rai[i], deci[i]] (*North at i^th source*)
vEi[i_] := vEi[i] = eE[rai[i], deci[i]] (*East at i^th source*)
vψi[i_] := vψi[i] = cos[ψi[i]] × vNi[i] + sin[ψi[i]] × vEi[i] (*unit vector in direction of PPA*)
nSxψi[i_] := nSxψi[i] = sin[ψi[i]] × vNi[i] - cos[ψi[i]] × vEi[i] (* r Cross vψ *)
```

In[37]:=
```
(*Plot sources*)
xyAitoffSources =
  Table[{xH180[ rai[i], deci[i] ], yH180[ rai[i], deci[i] ]}, {i, Length[data00]}];
```

In[38]:=
```
(*Plot polarization directions*)
rPlusψ[i_, d_] := (ri[i] + d vψi[i]) / ((ri[i] + d vψi[i]).(ri[i] + d vψi[i]))^(1/2)
crossesOverPlus = {}; crossesOverMinus = {};
For[i = 1, i ≤ Length[data00], i++,
 If[ αFROMr[ rPlusψ[i, 0.05]] - rai[i]  < -200, AppendTo[crossesOverPlus, i]];
 If[ αFROMr[ rPlusψ[i, -0.05]] - rai[i]  > 200, AppendTo[crossesOverMinus, i]]]
noCrossing = Complement[Range[Length[data00]], Union[crossesOverPlus, crossesOverMinus]];
```

In[42]:=
```
(*Plot polarization directions*)
polarLinesNoCrossing[d_] :=
 Table[Line[{{xH180[αFROMr[ rPlusψ[i, d]], δFROMr[ rPlusψ[i, d]]],
     yH180[αFROMr[ rPlusψ[i, d]], δFROMr[ rPlusψ[i, d]]]},
    {xH180[αFROMr[ rPlusψ[i, -d]], δFROMr[ rPlusψ[i, -d]]],
     yH180[αFROMr[ rPlusψ[i, -d]], δFROMr[ rPlusψ[i, -d]]]}}], {i, noCrossing}]
polarLinesCrossingPlus[d_] := Table[Line[{{xH180[rai[i], deci[i]],
     yH180[rai[i], deci[i]]}, {xH180[αFROMr[ rPlusψ[i, -d]], δFROMr[ rPlusψ[i, -d]]],
     yH180[αFROMr[ rPlusψ[i, -d]], δFROMr[ rPlusψ[i, -d]]]}}], {i, crossesOverPlus}]
polarLinesCrossingMinus[d_] := Table[Line[{{xH180[αFROMr[ rPlusψ[i, d]],
      δFROMr[ rPlusψ[i, d]]], yH180[αFROMr[ rPlusψ[i, d]], δFROMr[ rPlusψ[i, d]]]},
    {xH180[rai[i], deci[i]], yH180[rai[i], deci[i]]}}],
  {i, crossesOverMinus(*noCrossing*)}]
```

In[45]:=
```
(*Construct the map of η̄(H).*)
mapOfSources =
 Show[{Table[ParametricPlot[{xH180[α, δ], yH180[α, δ]},
     {δ, -90, 90}, PlotStyle → {Black, Thickness[0.002]}, PlotPoints → 60,
     PlotRange → {{-4.0, 3.5}, 7.5/11.0 {-3, 3}}, Axes -> False, Frame → False], {α, 0, 360, 30}],
   Table[ParametricPlot[{xH180[α, δ], yH180[α, δ]}, {α, 0, 360},
     PlotStyle → {Black, Thickness[0.002]}, PlotPoints → 60], {δ, -60, 60, 30}], Graphics[
    {PointSize[0.004], Text[StyleForm["N", FontSize -> 14, FontWeight -> "Plain"], {0, 1.85}],
     Text[StyleForm["Equatorial Coordinate System", FontSize -> 14, FontWeight -> "Plain"],
      {0, -1.85}], Black, {(*Thick,*)polarLinesNoCrossing[0.05]},
     Black, {(*Thick,*)polarLinesCrossingPlus[0.05]}, Black, {(*Thick,*)
      polarLinesCrossingMinus[0.05]}, (*Sources S:*)Orange, Point[ xyAitoffSources ]
      }]}, ImageSize → 1.2×432]
Print["Figure A1. Pine needle plot of the transverse vectors of the sources. There are ",
 Length[data00], " sources."]
```

Out[45]=

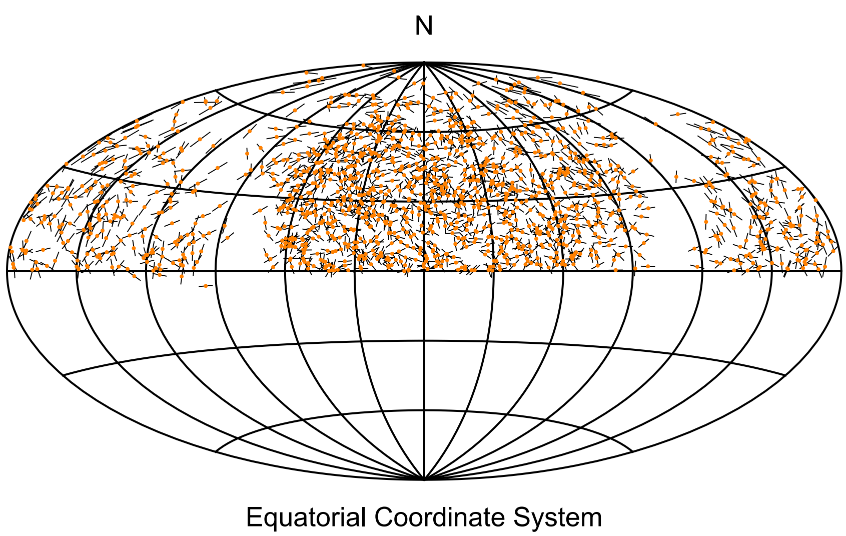

Figure A1. Pine needle plot of the transverse vectors of the sources. There are 1450 sources.

A5. Build a Grid

Make a grid for the Northern hemisphere, then the Southern hemisphere, then combine them.

Definitions:

dθ1	separation in degrees between grid points on a constant latitude circle and separation of constant latitude circles.
gridN, gridS	North and South hemisphere grids
nGrid	number of grid points

rHj(j) unit radial vector to jth grid point H_j
αHj[j], δHj[j] RA and dec of the j^{th} grid point in degrees
vHij(i,j) unit vector tangent to the great circle connecting the ith source with Hj in tangent space of the ith source
nSxHij(i,j) unit vector perpendicular to the plane of the great circle containing the ith source and the jth grid point Hj
ηiHj(i,j) alignment angle between the PPA direction and the great circle toward Hj in the tangent space at the ith source.

gridN and ***gridS*** and ***grid***

1. sequential point # 2. RA index 3. dec index 4. RA (range: 0 - 360°) 5. dec (range: -90° - +90°) 6. Cartesian coordinates of the point

In[47]:=
```
dθ1 = gridSpacing ;(* grid Spacing in degrees*)
```

Let’s get the grid. With “gridSpacing” = 2°, it is a 2°x2° grid.

In[48]:=
```
(*KEEP this cell - DO NOT DELETE*)
gridN = {}; idN = 1;
For[δj = 0., δj < 90./dθ1, δj++, δpointH = δj dθ1;
 For[ ai = 0., ai < Ceiling[360./dθ1 (cos[δpointH] + 0.01)],
  ai++, αpointH = ai dθ1/(cos[δpointH] + 0.01);
  AppendTo[gridN, {idN, ai, δj, αpointH, δpointH, er[αpointH, δpointH]}];
  idN = idN + 1
 ]]
```

In[50]:=
```
(*KEEP this cell - DO NOT DELETE*)
gridS = {}; idN = 1;
For[δj = 1., δj < 90./dθ1, δj++, δpointH = -δj dθ1;
 For[ ai = 0., ai < Ceiling[360./dθ1 (cos[δpointH] + 0.01)],
  ai++, αpointH = ai dθ1/(cos[δpointH] + 0.01);
  AppendTo[gridS, {idN, ai, δj, αpointH, δpointH, er[αpointH, δpointH]}];
  idN = idN + 1
 ]]
```

In[52]:=
```
(*KEEP this cell - DO NOT DELETE*)
grid = {}; j = 1;
For[jN = 1, jN ≤ Length[gridN], jN++, AppendTo[grid,
   {j, gridN[[jN, 2]], gridN[[jN, 3]], gridN[[jN, 4]], gridN[[jN, 5]], gridN[[jN, 6]]}];
 j = j + 1]
For[jS = 1, jS ≤ Length[gridS], jS++, AppendTo[grid,
   {j, gridS[[jS, 2]], gridS[[jS, 3]], gridS[[jS, 4]], gridS[[jS, 5]], gridS[[jS, 6]]}];
 j = j + 1]
```

In[55]:=
```
nGrid = Length[grid];
rHj[j_] := rHj[j] = grid[[j, 6]] (*unit radial vector to grid point H*)
αHj[j_] := αFROMr[rHj[j]]
δHj[j_] := δFROMr[rHj[j]]
```

In[59]:=
```
(* iᵗʰ Source and jᵗʰ grid point*)
(*vHij: unit vector tangent to the great circle connecting
  the ith source with Hj in tangent space of the ith source*)
(*nSxHij: unit vector perpendicular to the plane of the
  great circle containing the ith source and Hj*)
 (*ηiHj: alignment angle between the PPA direction ψ and the great
  circle toward Hj in the tangent space at the ith source. See Fig. 2.*)
(* The two unit vectors nSxψi and nSxHij are perpendicular to vψ and vHij,
but the angle between them is the same*)
vHij[i_, j_] := vHij[i, j] = (rHj[j] - (rHj[j].ri[i]) ri[i]) /
    (√((rHj[j] - (rHj[j].ri[i]) ri[i]).(rHj[j] - (rHj[j].ri[i]) ri[i])))
nSxHij[i_, j_] := nSxHij[i, j] = Cross[ri[i], rHj[j]] / √((Cross[ri[i], rHj[j]]).(Cross[ri[i], rHj[j]]))
ηiHj[i_, j_] := ηiHj[i, j] = arccos[Abs[nSxψi[i].nSxHij[i, j]]]
```

In[62]:=
```
(*Check (α,δ) range for the grid*)
(*ListPlot[{Sort[Table[grid[[j,4]],{j,nGrid}]],Sort[Table[grid[[j,5]],{j,nGrid}]]}];*)
```

In[63]:=
```
(*See the grid points*)
Show[{Graphics3D[{Sphere[{0, 0, 0}], Thick, Line[{{0, 0, -1.2}, {0, 0, 1.2}}],
     Text[Style["N", Bold], {0, 0, 1.25}]}, Boxed → False], ListPointPlot3D[
    Table[rHj[j], {j, nGrid}], PlotStyle → {PointSize[0.007]}]}, ImageSize → 72 × 4]
Print["Figure A2. The grid. There are ", nGrid, " grid points."]
```

Out[63]=

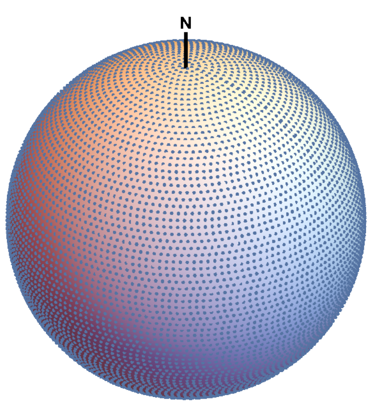

Figure A2. The grid. There are 10518 grid points.

A6. Setting up circular regions to analyze

(a) Collect the sources in circular regions centered on the grid points. (b) Drop the regions with too few sources. (c) Some regions may have duplicate source lists, meaning two regions have the same sources but different center points. Keep one of the two regions.

Definitions:

rgnPOPnumMIN = minimum number of sources needed for a region to be included in the analysis, “POP” - population
rgnCntrAndSrcId0 A region for every grid point is included. No regions cut.
rgnCntrAndSrcId2 This table has only sufficiently populated regions, $N \geq$ rgnPOPnumMIN (minimum)
sortRgnNsrc sort table “rgnCntrAndSrcId2” by number of sources in each region
1. region ID # 2. number of sources
NsrcMIN Fewest number of sources in any regions (must be at least rgnPOPnumMIN)
NsrcMAX Maximum number of sources in any region
rgnIDsWithnSrc[n]Id #s in rgnCntrAndSrcId2 of regions with exactly “n” sources
duplicatk Pairs of regions {k1,k2} that contain the same sources
dropDupk list of the second region in each pair in duplicatk. These are to be dropped.
rgnCntrAndSrcId No two regions have the same sources. Each region has at least the minimum number of sources.
1. sequential Id# 2. grid ID for region’s center point 3. Source data00 id#s for sources contained in the region
nSrck[k] number of sources in the kth region
nSrcTable list of the number of sources in the regions
srcIDrgnk[k] list of data00 ID #s for sources in the kth region
αSrcRgnk[k] RAs of the sources in the kth region
δSrcRgnk[k] decs of the sources in the kth region
xyHSrcRgnk[k] Aitoff plot coordinates for the sources in the kth region
rAVEk[k] unit radial vector to average location of the sources in the kth region
αAVEk[k] Right Ascension at the average location of the sources in the kth region
δAVEk[k] Declination at the average location of the sources in the kth region
ρSrcToAVEk[k] angles between the i^{th} sources and the average location in the kth region
ρRMSk[k] root mean square radius of the kth region
ηBarHkj(k,j) average alignment angle at Hj for the sources in the kth region
kNjηMinjηMax:
1. region ID# in “rgnCntrAndSrcId” table 2. N = number of sources in the region 3. {j, ηmin} : j = grid point ID# where $\overline{\eta}$ is minimum ηmin 4. {j, ηmax} : j = grid point ID# where $\overline{\eta}$ is maximum ηmax
(ηmink(k), ηmaxk(k)) In degrees. The min and max angles η for the sources in the kth region to align with any grid point Hj.

In[65]:=
```
rgnPOPnumMIN = 7. (*minimum number of sources*);
```

```
In[66]:= (*Identify sources in each region whose center is on the grid. Collect results. *)
rgnCntrAndSrcId0 = {};
For[j = 1, j ≤ Length[grid], j++, rgnCntr = grid[[j, 6]];
 ρrgn = rgnRadius;  (* region radius in degrees*)
 rgnSrcId = {};
 For[i = 1, i ≤ Length[data00], i++,
  If[er[rai[i], deci[i]].rgnCntr ≥ cos[ρrgn], rgnSrcId = AppendTo[rgnSrcId, i]]];
 AppendTo[rgnCntrAndSrcId0, {j, rgnSrcId}]  ]

In[68]:= (*Get a table with only sufficiently populous regions.*)
rgnCntrAndSrcId2 = {};
j = 0;
For[igrid = 1, igrid ≤ Length[grid], igrid++,
 If[Length[  rgnCntrAndSrcId0[[igrid, 2]]  ] ≥ rgnPOPnumMIN, (j = j + 1;
   AppendTo[rgnCntrAndSrcId2, {j, rgnCntrAndSrcId0[[igrid, 1]], rgnCntrAndSrcId0[[igrid, 2]]}])]]

In[71]:= sortRgnNsrc = Sort[Table[{k, Length[  rgnCntrAndSrcId2[[k, 3]]  ]},
    {k, Length[rgnCntrAndSrcId2]}], #1[[2]] < #2[[2]] &];
{sortRgnNsrc[[1]], sortRgnNsrc[[-1]]};
NsrcMIN = sortRgnNsrc[[1, 2]];
NsrcMAX = sortRgnNsrc[[-1, 2]];

In[75]:= For[n = 1, n ≤ 2 NsrcMAX, n++, rgnIDsWithnSrc0[n] = {}]
rgnIDsWithnSrc0[NsrcMAX];
(*Collect the IDs*)
For[k = 1, k <= Length[rgnCntrAndSrcId2], k++,
 AppendTo[rgnIDsWithnSrc0[ Length[rgnCntrAndSrcId2[[k, 3]]] ], k]]


In[78]:= duplicatk = {};
For[n = NsrcMIN, n ≤ NsrcMAX, n++,  For[k1 = 1, k1 ≤ Length[rgnIDsWithnSrc0[n]] - 1,
  k1++ ,  For[k3 = k1 + 1, k3 ≤ Length[rgnIDsWithnSrc0[n]], k3++,
   If[ Length[Union[rgnCntrAndSrcId2[[ rgnIDsWithnSrc0[n][[k1]], 3 ]] -
        rgnCntrAndSrcId2[[rgnIDsWithnSrc0[n][[k3]], 3]]]] == 1, AppendTo[duplicatk,
     {rgnIDsWithnSrc0[n][[k1]], rgnIDsWithnSrc0[n][[k3]]}]    ]          ]        ]           ]

In[79]:= (*For example, the regions in  duplicatk[[2]]
 have the same sources in rgnCntrAndSrcId2[[k,3]] item 3.*)
Print["Region ", duplicatk[[2, 1]], " and region ", duplicatk[[2, 2]],
 " have the same sources. The data00 IDs are ",
 rgnCntrAndSrcId2[[ duplicatk[[2, 1]], 3 ]], " and ",
 rgnCntrAndSrcId2[[ duplicatk[[2, 2]], 3 ]], "." ]

Region 19 and region 20 have the same sources. The data00 IDs are
 {198, 203, 204, 205, 206, 208, 209} and {198, 203, 204, 205, 206, 208, 209}.

In[80]:= (*Get the second region in each pair in duplicatk. These will be dropped. *)
dropDupk = Union[Table[duplicatk[[d2, 2]], {d2, Length[duplicatk]}]];
```

Remove duplicate populations.

```
In[81]:= rgnCntrAndSrcId = {}; k = 1;
For[ ka = 1, ka ≤ Length[rgnCntrAndSrcId2], ka++,   If[ Not[MemberQ[dropDupk, ka]] ,
   (AppendTo[  rgnCntrAndSrcId, {k, rgnCntrAndSrcId2[[ka, 2]], rgnCntrAndSrcId2[[ka, 3]]}];
    k = k + 1)            ]                                  ]

In[83]:= For[n = 1, n ≤ 2 NsrcMAX, n++, rgnIDsWithnSrc[n] = {}]
rgnIDsWithnSrc[NsrcMAX];
(*Collect the IDs*)
For[k = 1, k <= Length[rgnCntrAndSrcId],  k++,
 AppendTo[rgnIDsWithnSrc[ Length[rgnCntrAndSrcId[[k, 3]]] ], k]]

In[86]:= ListPlot[Table[{n, Length[rgnIDsWithnSrc[n] ]}, {n, 1, NsrcMAX + 5}],
 PlotRange → {{0, NsrcMAX + 5}, All}, PlotLabel → "Number of regions with n sources ",
 GridLines → Automatic, Frame → True, FrameLabel → {"n", "Number"}, ImageSize → 72 × 4]
Print["Figure A3. There are ", Length[rgnIDsWithnSrc[NsrcMIN]], " regions with ",
 NsrcMIN, " sources, the minimum number. There are ", Length[rgnIDsWithnSrc[NsrcMAX]],
 " regions with the maximum number of sources, ", NsrcMAX, "."]
```

Out[86]=

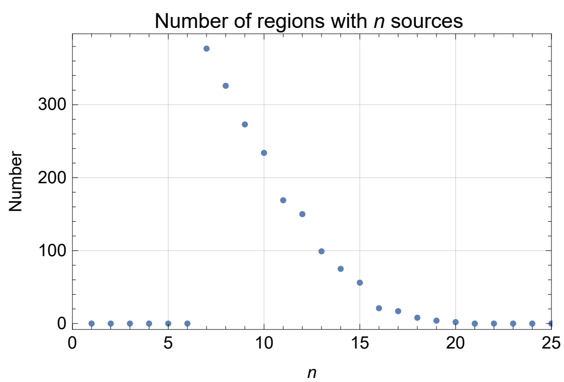

Figure A3. There are 377 regions with 7 sources, the minimum number. There are
 2 regions with the maximum number of sources, 20.

```
In[88]:= nSrck[k_] := nSrck[k] = Length[ rgnCntrAndSrcId[[k, 3]] ]
(*number of sources in the kth region*)
nSrcTable = Sort[Table[nSrck[k], {k, Length[ rgnCntrAndSrcId]}]];
srcIDrgnk[k_] := srcIDrgnk[k] = rgnCntrAndSrcId[[k, 3]]
(* data00 id numbers of the sources in the kth region*)
αSrcRgnk[k_] := Table[data00[[  id08, 2  ]] * (360. / (2. π)), {id08, srcIDrgnk[k]}];
(* RAs in degrees for the sources in the kth region*)
δSrcRgnk[k_] := Table[data00[[  id08 , 3 ]] * (360. / (2. π)), {id08, srcIDrgnk[k]}]; (* decs  *)
xyHSrcRgnk[k_] := Table[{ xH180[ αSrcRgnk[k][[i]], δSrcRgnk[k][[i]] ],
   yH180[ αSrcRgnk[k][[i]], δSrcRgnk[k][[i]] ] }, {i, Length[ αSrcRgnk[k] ] } ]
 (*Aitoff coordinates for the locations of the sources in the kth region*)

In[94]:= rAVEk0[k_] := rAVEk0[k] =
  Sum[ri[ rgnCntrAndSrcId[[k, 3, n1]] ], {n1, Length[ rgnCntrAndSrcId[[k, 3]] ]}] / nSrck[k]
rAVEk[k_] := rAVEk0[k] / (rAVEk0[k].rAVEk0[k])^(1/2)
(*unit radial vector to average location of the sources in the kth region*)
αAVEk[k_] := αAVEk[k] = αFROMr[rAVEk[k]]
δAVEk[k_] := δAVEk[k] = δFROMr[rAVEk[k]]
```

In[98]:=
```
(*We need the RMS radius of the k^th region to determine significances.*)
ρSrcToAVEk[k_] := Table[arccos[ ri[ rgnCntrAndSrcId[[k, 3, n1]] ].rAVEk[k] ],
   {n1, Length[ rgnCntrAndSrcId[[k, 3]] ]}]
ρRMSk[k_] := (1/Length[rgnCntrAndSrcId[[k, 3]] ]
     Sum[ρSrcToAVEk[k][[i]]^2, {i, Length[ ρSrcToAVEk[k] ]}])^(1/2)
```

In[100]:=
```
(*ηBarHkj: average alignment angle at Hj for the sources in the kth region, Eq. (1).*)
ηBarHkj[k_, j_] :=
 ηBarHkj[k, j] = Sum[ηiHj[i, j], {i, srcIDrgnk[k]}] / Length[srcIDrgnk[k]]
```

The following cell has the kNjηMinjηMax table. It is very large and, therefore, it is hidden from view. To see it go to "Cell Properties" and click "Open".

kNjηMinjηMax ***angles in degrees***

1. region ID# in "rgnCntrAndSrcId" table 2. N = number of sources in the region 3. {j, ηmin} : j = grid point ID# where $\overline{\eta}$ is minimum ηmin 4. {j, ηmax} : j = grid point ID# where $\overline{\eta}$ is maximum ηmax

In[102]:=
```
(*KEEP THIS CELL to generate the kNjηMinjηMax table.*)
(*t1=TimeUsed[]
  kNjηMinjηMax={};
For[k=1,k≤Length[rgnCntrAndSrcId],k++,ηBark=Table[{j,ηBarHkj[k,j]},{j,Length[grid]}];
   sortηBark=Sort[ηBark,#1[[2]]<#2[[2]]&];
   jηMin=sortηBark[[1]];
   jηMax=sortηBark[[-1]];
   AppendTo[kNjηMinjηMax,{k,nSrck[k],jηMin,jηMax}]]
  t2=TimeUsed[]
   t2-t1*)
(*This cell takes some time. On Dec. 30,2021, it took 1445.39 seconds.*)
```

In[103]:=
```
(*Save kNjηMinjηMax*)
(*SetDirectory[homeDirectory]
 Put[kNjηMinjηMax,"20211230kNjEtaMinjEtaMax1450a.dat"]
*)
```

In[104]:=
```
(*Get kNjηMinjηMax, if you've got it.*)
(*SetDirectory[homeDirectory] ;
kNjηMinjηMax=Get["20211226kNjEtaMinjEtaMax1450.dat"];
*)
```

In[105]:=
```
ηmink[k_] := ηmink[k] = kNjηMinjηMax[[k, 3, 2]]
(*In degrees. The smallest alignment angle ηmin determines how well the sources in
 the kth region align with any point Hj on the grid, i.e. anywhere on the sphere.*)
ηmaxk[k_] := ηmaxk[k] = kNjηMinjηMax[[k, 4, 2]] (*In degrees. The largest avoidance
   angle ηmax gives a measure of avoidance from any point Hj on the sphere*)
```

Section Summary

```
Initially, a total of 10518
  regions are created, each centered on one of the 10518 grid points which are
 2 degrees apart. The regions are circular, each with a radius of 5. degrees.
Regions with duplicate lists of sources
  are dropped. Regions must have a minimum number of sources.
There are 1811 regions with sufficient populations and duplicates dropped.

The min number of sources in a region is 7 and the max number is 20.
The median number of sources in a region is 9.
The arithmetic average number of sources in a region is 9.
```

A7. Probability Distributions and Significance of the Regions

The problem of “significance” is to determine the likelihood that random polarizations directions would have better alignment or avoidance than the observed polarization directions. Suppose we are given a region with a smallest alignment angle $\overline{\eta}_{\min}$ and a largest avoidance angle $\overline{\eta}_{\max}$. The most reliable method of finding the significance of either value is to creates many copies of the region but assign the sources randomly directed polarizations. Collect the angles $\overline{\eta}_{\min}$ and $\overline{\eta}_{\max}$ for each randomly directed copy and make a probability distribution for the collection of $\overline{\eta}_{\min}$ and a probability distribution for the collection of $\overline{\eta}_{\max}$. Fit the distributions with suitable functions and integrate to find the significances. This process is “Direct Method A”. It takes a lot of time and effort and would not be practical for a survey with hundreds, or more, of regions.

To avoid Direct Method A, we apply a combination of Interpolation Method B and Function Method C. Both are based on a “Library” of random run data. One finds that the probability distributions for smallest alignment angle $\overline{\eta}_{\min}$ with random runs can be fit by a function with just two free parameters, called the location $\eta 0$ of the peak and the half-width σ. Avoidance distributions take two more parameters. For details see Ref. 8.

We assume that just two properties of a region determine its the significance of its values of $\overline{\eta}_{\min}$ and $\overline{\eta}_{\max}$. The two properties are the number of sources nSrc and the root-mean-square radius ρRMS of the sources about their mean location. Thus the Library has tables of the distribution parameters $\eta 0$ and σ for many combinations of nSrc and ρRMS.

Interpolating the Library data to get $\eta 0$ and σ is called Interpolation Method B. The Library data can be fit with suitable functions. Substituting nSrc and ρRMS in those functions to get $\eta 0$ and σ is called Function Method C. Again, for details, see Ref. 8.

Definitions:

norm	a constant used to normalize the distribution so the integral of probability is 1.
probMIN0, probMAX0	probability distributions for alignment (MIN) and avoidance (MAX), functions of η, η_0, σ

probMIN0[η, $\eta 0$, σ], probMAX0	probability distributions for probability of η using given values of η_0, σ
signiMIN0[η, $\eta 0$, σ], signiMAX0	significance of η using given values of η_0, σ

In[113]:=
```
(* y = ((η - η0)/σ); dy = dη/σ *)
(* The normalization factor "norm" is needed for the probability density *)
```
$$norm = \left(\left(1/(2\pi)^{1/2}\right) NIntegrate\left[\left(1 + e^{4\,(y-1)}\right)^{-1} e^{-\frac{y^2}{2}}, \{y, -\infty, \infty\}\right]\right)^{-1};$$
```
norm;(*Constant needed to make the probability distributions integrate to unity.*)
```

In[115]:=
$$probMIN0[\eta\_, \eta0\_, \sigma\_] := \left(norm/\left(\sigma\,(2\pi)^{1/2}\right)\right)\left(1 + e^{4\,\frac{(\eta-\eta0-\sigma)}{\sigma}}\right)^{-1} e^{-\frac{1}{2}\left(\frac{\eta-\eta0}{\sigma}\right)^2}$$
```
signiMIN0[η_, η0_, σ_] := NIntegrate[probMIN0[η1, η0, σ], {η1, -∞, η}]
```

In[117]:=
$$probMAX0[\eta\_, \eta0\_, \sigma\_] := \left(norm/\left(\sigma\,(2\pi)^{1/2}\right)\right)\left(1 + e^{-4\,\frac{(\eta-\eta0+\sigma)}{\sigma}}\right)^{-1} e^{-\frac{1}{2}\left(\frac{\eta-\eta0}{\sigma}\right)^2}$$
```
signiMAX0[η_, η0_, σ_] := NIntegrate[probMAX0[η1, η0, σ], {η1, η, ∞}]
```

In[119]:= Print["The significance signiMIN0[η,η0,σ] is the integral of probMIN0, i.e. signiMIN0 = $\int_{-\infty}^{\eta} P_{MIN}{}^{0}(\eta_i)\,d\eta_i$: "]

Print["The significance signiMAX0[η,η0,σ] is the integral of probMAX0, i.e. signiMAX0 = $\int_{\eta}^{\infty} P_{MAX}{}^{0}(\eta_i)\,d\eta_i$: "]

The significance signiMIN0[η,η0,σ] is the integral of probMIN0, i.e. signiMIN0 = $\int_{-\infty}^{\eta} P_{MIN}{}^{0}(\eta_i)\,d\eta_i$:

The significance signiMAX0[η,η0,σ] is the integral of probMAX0, i.e. signiMAX0 = $\int_{\eta}^{\infty} P_{MAX}{}^{0}(\eta_i)\,d\eta_i$:

A7a. The Library data

Definitions:

fitData Parameters of the alignment (min) and avoidance (max) random run distributions. Originally in radians, converted to degrees after it is inputted below.

fitData:

1a. nSrci[i] Number of sources 1b. ρNomi[i] Nominal radius, deg. 1c. ρRMSi[i] RMS radius, deg.

2a. η0mini[i] peak alignment distribution 2b. dη0mini[i] standard error

3a. σmini[i] half-width alignment distr. 3b. dσmini[i] standard error

4a. η0maxi[i] peak alignment distribution 4b. dη0maxi[i] standard error

5a. σmaxi[i] half-width alignment distr. 5b. dσmaxi[i] standard error

wi[i] inverse square root of the number of sources, $w = 1/N^{1/2}$

τRMSi[i] inverse RMS radius, in deg.$^{-1}$

In[121]:=
```
fitData = {{{9., 0.004363, 0.0043, 10000.}, {0.598, 0.0013}, {0.1127, 0.0016},
    {0.977, 0.001700}, {0.1128, 0.002}}, {{9., 0.005818, 0.005734, 10000.},
    {0.5885, 0.0016}, {0.1118, 0.0019}, {0.9868, 0.001}, {0.1107, 0.001200}},
   {{9., 0.008727, 0.008601, 10000.}, {0.5707, 0.0011}, {0.1076, 0.0013},
```

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 {0.727354, 0.000099}, {0.0199, 0.00012}, {0.84565, 0.00013}, {0.0200400, 0.00016}},
{{225., 0.139626, 0.104116, 10000.}, {0.72498, 0.00013}, {0.020100, 0.00016},
 {0.84702, 0.0001500}, {0.0202100, 0.00018}}, {{225., 0.733038, 0.561218, 10000.},
 {0.72336, 0.00016}, {0.019770, 0.00019}, {0.84781, 0.0001400}, {0.02023, 0.00017}}};
```

In[122]:=
```
(*Identify the items in the fitData table with functions
 having recognizable names. Convert fitData radians to DEGREES:*)
nSrci[i_] := fitData[[i, 1, 1]]
```

In[123]:=
```
ρNomi[i_] := fitData[[i, 1, 2]] (360./(2. π)) (*The nominal radius in degrees*)

ρRMSi[i_] := fitData[[i, 1, 3]] (360./(2. π)) (*The RMS radius *)
```

In[125]:=
```
η0mini[i_] := fitData[[i, 2, 1]] (360./(2. π))

dη0mini[i_] := fitData[[i, 2, 2]] (360./(2. π))
```

In[127]:=
```
σmini[i_] := fitData[[i, 3, 1]] (360./(2. π))

dσmini[i_] := fitData[[i, 3, 2]] (360./(2. π))
```

In[129]:=
```
η0maxi[i_] := fitData[[i, 4, 1]] (360./2. π)
dη0maxi[i_] := fitData[[i, 4, 2]] (360./2. π)
```

In[131]:=
```
σmaxi[i_] := fitData[[i, 5, 1]] (360./2. π)
dσmaxi[i_] := fitData[[i, 5, 2]] (360./2. π)
```

In[133]:=
```
wi[i_] := 1/nSrci[i]^(1/2) (*w = 1/N^(1/2)*)
τRMSi[i_] := 1/ρRMSi[i] (* inverse RMS radius in inverse degrees*)
```

A7b. Interpolation Method B

The Library constructed for Method B in Sec. A2 is essentially a table of the values of the four parameters, $\eta_0{}^{\min}$, $\sigma^{\min}$, $\eta_0{}^{\max}$, and $\sigma^{\max}$, needed to determine the probability distributions and significances in Eqs. (A1-A4).

Instead of the variables N and ρRMS, the number of sources and the root-mean-square radius, we choose to consider the four parameters as functions of w and τRMS, the inverse square root of N and the inverse of the radius ρRMS,

$$w = N^{-1/2} \quad \text{and} \quad \tau\text{RMS} = \rho\text{RMS}^{-1}. \qquad \text{(A5)}$$

A change of variables from (N , ρRMS) to (w , τRMS).

Definitions:

Tables: wτη0minLib, wτdη0minLib, wτη0maxLib, wτdη0maxLib, wτσminLib, wτdσminLib, wτσmaxLib, wτdσmaxLib
The tables wτη0minLib ... have Library data in the form [(w, τRMS), quantity] were “quantity” is one of the parameters or their standard errors: $\eta_0{}^{\min}$, $d\eta_0{}^{\min}$, $\sigma^{\min}$, $d\sigma^{\min}$, $\eta_0{}^{\max}$, $d\eta_0{}^{\max}$, $\sigma^{\max}$, $d\sigma^{\max}$

The associated interpolation functions are η0minBint, dη0minBint , η0maxBint , dη0maxBint, σminBint, dσminBint, σmaxBint , dσmaxBint

Setting up the interpolations takes two steps. First a tables of the data are constructed. Each table has the form {w, τRMS, parameter}. Second, the interpolation for each parameter is defined. There are four parameters $\eta_0{}^{\min}$, $\sigma^{\min}$, $\eta_0{}^{\max}$, and $\sigma^{\max}$ and each one has a standard error $d\eta_0{}^{\min}$, $d\sigma^{\min}$, $d\eta_0{}^{\max}$, and $d\sigma^{\max}$ developed in the fitting process that gives fitData from random run data.

In[135]:=
```
wτη0minLib = Table[{{wi[i], τRMSi[i]}, η0mini[i]}, {i, Length[fitData]}];
wτdη0minLib = Table[{wi[i], τRMSi[i], dη0mini[i]}, {i, Length[fitData]}];
wτη0maxLib = Table[{wi[i], τRMSi[i], η0maxi[i]}, {i, Length[fitData]}];
wτdη0maxLib = Table[{wi[i], τRMSi[i], dη0maxi[i]}, {i, Length[fitData]}];
wτσminLib = Table[{wi[i], τRMSi[i], σmini[i]}, {i, Length[fitData]}];
wτdσminLib = Table[{wi[i], τRMSi[i], dσmini[i]}, {i, Length[fitData]}];
wτσmaxLib = Table[{wi[i], τRMSi[i], σmaxi[i]}, {i, Length[fitData]}];
wτdσmaxLib = Table[{wi[i], τRMSi[i], dσmaxi[i]}, {i, Length[fitData]}];
```

```
In[143]:= η0minBint = Interpolation[wτη0minLib];(* int - interpolation function*)
dη0minBint = Interpolation[wτdη0minLib];
η0maxBint = Interpolation[wτη0maxLib];
dη0maxBint = Interpolation[wτdη0maxLib];
σminBint = Interpolation[wτσminLib];
dσminBint = Interpolation[wτdσminLib];
σmaxBint = Interpolation[wτσmaxLib];
dσmaxBint = Interpolation[wτdσmaxLib];
```

Interpolation: Interpolation on unstructured grids is currently only supported for InterpolationOrder->1 or InterpolationOrder->All. Order will be reduced to 1.

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By the rules of interpolations, when the variables w and τ are in the range of the Library data, then Mathematica finds an average value from the surrounding Library data points. In terms of the variables w and τRMS, the ranges are

$$\frac{1}{15} \leq w \leq \frac{1}{3} \quad \text{and} \quad 0.024\ \text{deg}^{-1} \lesssim \tau\text{RMS} \lesssim 4\ \text{deg}^{-1} \qquad \text{(Ranges of Interpolation Variables)} \qquad \text{(A6)}$$

Note: The values for τRMS are only approximate because the limits shown are values of $1/\rho$Nominal and the nominal values ρNominal only approximate the root-mean-square values ρRMS, $\rho\text{Nominal} \approx \rho\text{RMS}$.

A7c. Fit the Library Data with Functions, Function Method C

Applying Interpolation Method B when one or both sample's variables are outside the Library data set, results in extrapolation, not interpolation. Instead of interpolating Library data points that surround the sample's variables, Mathematica guesses what lies beyond the Library's boundaries. In such a case or for other situations that arise, one can apply the following alternative Formula Method C to find the distribution parameters η_0^{min}, σ^{min}, η_0^{max}, and σ^{max} and , with Eqs. (A4, A5), the significances desired.

Formula Method C finds formulas to fit the four distribution parameters $\eta_0^{\text{min}}(w,\ \tau\text{RMS})$, $\sigma^{\text{min}}(w,\ \tau\text{RMS})$, $\eta_0^{\text{max}}(w,\ \tau\text{RMS})$, and $\sigma^{max}(w, \tau RMS)$. Best, Big and Small are just copied from 20211112InterpolateAndFormula2a which are copied from 20211116AlternateRandomRunStatsDegrees.nb

Definitions:

Alignment:

Library data fitting functions:
eta0minFit[w,τ], eta0minFitbig[w,τ], eta0minFitsmall[w,τ], deta0minFit[w,τ]
sigmaminFit[w,τ], sigminFitBig[w,τ], sigminFitSmall[w,τ], dsigmaminFit[w,τ]
Plots of the Library data fitting function for the iN^{th} value of w:
plotTauEtamin[iN], plotTauEtaminbig[iN], plotTauEtaminsmall[iN],
plotTausigmamin[iN], ...
Display of the fitting functions for all values of w:
eta0MinVSTauFit (Big, Best, Small) and eta0MinVSTauFit0 (Best only)

sortPercentDiffEta0minfit	percent differences between the Library data and the relevant fitting function, here for η_0^{min}.
sortPercentDiffSigmaminfit	Same, but for σ^{min}

Avoidance:
REPEAT ALL OF THE ABOVE AGAIN, BUT THIS TIME WITH “MAX”, NOT “MIN”, *i.e.* eta0maxFit[w,τ], ..., sortPercentDiffSigmamaxfit

```
In[151]:= (*Equation (A7), 20211112InterpolateAndFormula2a.nb  *)
eta0minFit[w_, τ_] :=
 45.0269 - w (47.386 + 7.32 w - 17.789 Tanh[(0.7096 - 0.3488 w) (-0.5348 + τ)])

In[152]:= (*Equation (A8)*) eta0minFitbig[w_, τ_] :=
 45.0434 - w (47.031 + 6.83 w + (-17.789 + 0.302 Sign[(-0.7096 + 0.3488 w) (-0.5348 + τ)])
      Tanh[(-0.5348 + τ + 0.0254 Sign[0.7096 - 0.3488 w])
        (0.7096 + w (-0.3488 + 0.0321 Sign[-0.5348 + τ]) + 0.0137 Sign[-0.5348 + τ])])

In[153]:= (*Equation (A9)*) eta0minFitsmall[w_, τ_] :=
 45.0103 - w (47.741 + 7.81 w + (-17.789 - 0.302 Sign[(-0.7096 + 0.3488 w) (-0.5348 + τ)])
      Tanh[(-0.5348 + τ - 0.0254 Sign[0.7096 - 0.3488 w])
        (0.7096 + w (-0.3488 - 0.0321 Sign[-0.5348 + τ]) - 0.0137 Sign[-0.5348 + τ])])

In[154]:= deta0minFit[w_, τ_] := eta0minFitbig[w, τ] - eta0minFit[w, τ]

In[155]:= (*Equation (A10)*)
sigmaminFit[w_, τ_] := 0.25 w (73.570 - 8.29 w + (3.093 + 10.658 w) Tanh[1.2161 (-1.6072 + τ)])

In[156]:= (*Equation (A11)*)
sigminFitBig[w_, τ_] := 0.25 w
  (73.679 - 7.86 w + (3.093 + w (10.658 + 0.508 Sign[-1.6072 + τ]) + 0.126 Sign[-1.6072 + τ])
     Tanh[(-1.6072 + τ + 0.0202 Sign[3.093 + 10.658 w]) (1.2161 + 0.0441 Sign[-1.6072 + τ])])
```

```
In[157]:= (*Equation (A12)*)
sigminFitSmall[w_, τ_] := 0.25 w
  (73.460 - 8.73 w + (3.093 + w (10.658 - 0.508 Sign[-1.6072 + τ]) - 0.126 Sign[-1.6072 + τ])
    Tanh[(-1.6072 + τ - 0.0202 Sign[3.093 + 10.658 w]) (1.2161 - 0.0441 Sign[-1.6072 + τ])])

In[158]:= dsigmaminFit[w_, τ_] := sigminFitBig[w, τ] - sigmaminFit[w, τ]

In[159]:= (*Equation (A13)*)
eta0maxFit[w_, τ_] := 45.1455 + w (44.230 + 8.35 w - 14.632 Tanh[0.6808 (-0.8608 + τ)])

In[160]:= (*Equation (A14)*)
eta0maxFitbig[w_, τ_] := 45.1632 + w (44.483 + 8.85 w + (-14.632 + 0.179 Sign[-0.8608 + τ])
     Tanh[(-0.8777 + τ) (0.6808 + 0.0106 Sign[0.8608 - τ])])

In[161]:= (*Equation (A15)*)
eta0maxFitsmall[w_, τ_] := 45.1279 + w (43.977 + 7.85 w + (-14.632 - 0.179 Sign[-0.8608 + τ])
     Tanh[(-0.8439 + τ) (0.6808 - 0.0106 Sign[0.8608 - τ])])

In[162]:= deta0maxFit[w_, τ_] := eta0maxFitbig[w, τ] - eta0maxFit[w, τ]

In[163]:= (*Equation (A16)*)
sigmamaxFit[w_, τ_] := 0.25 w (73.287 - 8.11 w + (2.773 + 11.126 w) Tanh[1.2850 (-1.6242 + τ)])

In[164]:= (*Equation (A17)*)
sigmaxFitBig[w_, τ_] := 0.25 w
  (73.400 - 7.66 w + (2.773 + w (11.126 + 0.521 Sign[-1.6242 + τ]) + 0.129 Sign[-1.6242 + τ])
    Tanh[(-1.6242 + τ + 0.0210 Sign[2.773 + 11.126 w]) (1.2850 + 0.0494 Sign[-1.6242 + τ])])

In[165]:= (*Equation (A18)*)
sigmaxFitSmall[w_, τ_] := 0.25 w
  (73.174 - 8.567 w + (2.773 + w (11.126 - 0.521 Sign[-1.6242 + τ]) - 0.129 Sign[-1.6242 + τ])
    Tanh[(-1.6242 + τ - 0.0210 Sign[2.773 + 11.126 w]) (1.2850 - 0.0494 Sign[-1.6242 + τ])])

In[166]:= dsigmamaxFit[w_, τ_] := sigmaxFitBig[w, τ] - sigmamaxFit[w, τ]
```

A7d. Combine Interpolation Method B and Function Method C

Apply Interpolation Method B when both sample's variables are within the range of the Library data set. Otherwise, one can apply the following alternative Formula Method C to find the distribution parameters $\eta_0^{\min}$, $\sigma^{\min}$, $\eta_0^{\max}$, and $\sigma^{\max}$.

Definitions:

Alignment:

Methods B,C combination parameter functions:

η0min[w,τ], dη0min[w,τ]   peak alignment distribution, standard error
σmin[w,τ], dσmin[w,τ]   half-width alignment distr., standard error
η0max[w,τ], dη0max[w,τ]   peak avoidance distribution, standard error

σmax[w,τ], dσmax[w,τ]   half-width avoidance distr., standard error

probMIN[η, nSrc, ρRMS], probMAX   probability distributions for probability of η using a sample's values of N, ρRMS
signiMIN[η,nSrc,ρRMS], signiMAX   significance of η using a sample's values of N, ρRMS

In[167]:=
```
η0min[w_, τ_] := If[(1/15. ≤ w ≤ 1/3.) && (0.025 ≤ τ ≤ 3.9), η0minBint[w, τ], eta0minFit[w, τ]];
dη0min[w_, τ_] :=
  If[(1/15. ≤ w ≤ 1/3.) && (0.025 ≤ τ ≤ 3.9), dη0minBint[w, τ], deta0minFit[w, τ]];
η0max[w_, τ_] := If[(1/15. ≤ w ≤ 1/3.) && (0.025 ≤ τ ≤ 3.9), η0maxBint[w, τ], eta0maxFit[w, τ]];
dη0max[w_, τ_] :=
  If[(1/15. ≤ w ≤ 1/3.) && (0.025 ≤ τ ≤ 3.9), dη0maxBint[w, τ], deta0maxFit[w, τ]];
σmin[w_, τ_] := If[(1/15. ≤ w ≤ 1/3.) && (0.025 ≤ τ ≤ 3.9), σminBint[w, τ], sigmaminFit[w, τ]];
dσmin[w_, τ_] :=
  If[(1/15. ≤ w ≤ 1/3.) && (0.025 ≤ τ ≤ 3.9), dσminBint[w, τ], dsigmaminFit[w, τ]];
σmax[w_, τ_] := If[(1/15. ≤ w ≤ 1/3.) && (0.025 ≤ τ ≤ 3.9), σmaxBint[w, τ], sigmamaxFit[w, τ]];
dσmax[w_, τ_] :=
  If[(1/15. ≤ w ≤ 1/3.) && (0.025 ≤ τ ≤ 3.9), dσmaxBint[w, τ], dsigmamaxFit[w, τ]];
```

In[175]:=
```
probMIN[η_, nSrc_, ρRMS_] := probMIN0[ η, η0min[nSrc^-1/2, ρRMS^-1], σmin[nSrc^-1/2, ρRMS^-1] ]
signiMIN[η_, nSrc_, ρRMS_] := signiMIN0[η, η0min[nSrc^-1/2, ρRMS^-1], σmin[nSrc^-1/2, ρRMS^-1]]
probMAX[η_, nSrc_, ρRMS_] := probMAX0[ η, η0max[nSrc^-1/2, ρRMS^-1], σmax[nSrc^-1/2, ρRMS^-1] ]
signiMAX[η_, nSrc_, ρRMS_] := signiMAX0[η, η0max[nSrc^-1/2, ρRMS^-1], σmax[nSrc^-1/2, ρRMS^-1]]
```

A7e. Significance of alignment and avoidance for the regions

To get significance formulas for each region, we use the number of sources and the ρRMS for each region, *i.e.* nSrck[k] and ρRMSk[k] for the k^{th}region. One has

In[179]:=
```
sigMINk[k_] := signiMIN[ ηmink[k], nSrck[k], ρRMSk[k] ]
sigMAXk[k_] := signiMAX[ηmaxk[k], nSrck[k], ρRMSk[k] ]
```

```
In[181]:= (*Get the ID#s k for Regions with very significant alignment.*)
ηMINVerySigkList = {};
For[k = 1, k ≤ Length[kNjηMinjηMax], k++,
 If[0.01 ≥ sigMINk[k], AppendTo[ηMINVerySigkList, {k, sigMINk[k]}]]]
ηMINVerySigkList;
Length[ηMINVerySigkList];

In[185]:= sortηMINVerySigkList = Sort[ηMINVerySigkList, #1[[2]] < #2[[2]] &];
Table[sortηMINVerySigkList[[i]], {i, 10}];
Length[sortηMINVerySigkList];

In[188]:= (*Get the ID#s k for Regions with ( 5%, NOT Very)  significant alignment.*)
ηMINSigkList = {};
For[k = 1, k ≤ Length[kNjηMinjηMax], k++,
 If[0.05 ≥ sigMINk[k], AppendTo[ηMINSigkList, {k, sigMINk[k]}]]]
ηMINSigkList;
Length[ηMINSigkList];

In[192]:= (*Regions with ( 5%, NOT Very)  significant alignment.*)
sortηMINSigkList = Sort[ηMINSigkList, #1[[2]] < #2[[2]] &];
Table[sortηMINSigkList[[i]], {i, 10}];
Length[sortηMINSigkList];

In[195]:= (*Get the ID#s k for Regions with very significant avoidance.*)
ηMAXVerySigkList = {};
For[k = 1, k ≤ Length[kNjηMinjηMax], k++,
 If[0.01 ≥ sigMAXk[k], AppendTo[ηMAXVerySigkList, {k, sigMAXk[k]}]]]
ηMAXVerySigkList;
Length[ηMAXVerySigkList];

In[199]:= sortηMAXVerySigkList = Sort[ηMAXVerySigkList, #1[[2]] < #2[[2]] &];
Table[sortηMAXVerySigkList[[i]], {i, Length[ηMAXVerySigkList]}];
Length[sortηMAXVerySigkList];

In[202]:= (*Regions with ( 5%, NOT Very)  significant alignment.*)
ηMAXSigkList = {};
For[k = 1, k ≤ Length[kNjηMinjηMax], k++,
 If[0.05 ≥ sigMAXk[k], AppendTo[ηMAXSigkList, {k, sigMAXk[k]}]]]
ηMAXSigkList;
Length[ηMAXSigkList];

In[206]:= (*Regions with ( 5%, NOT Very)  significant alignment.*)
sortηMAXSigkList = Sort[ηMAXSigkList, #1[[2]] < #2[[2]] &];
Table[sortηMAXSigkList[[i]], {i, Length[ηMAXSigkList]}];
Length[sortηMAXSigkList];
```

In[209]:=
```
(*Uncertainty of –Log₁₀S for ηmink with uncertainty for ηmink = ±1° . *)
negLogSigηminBest[k_] := –Log[10,
   signiMIN0[ ηmink[k], η0min[nSrck[k]^-1/2, ρRMSk[k]^-1], σmin[nSrck[k]^-1/2, ρRMSk[k]^-1]]]
negLogSigηminBig[k_] := –Log[10, signiMIN0[ ηmink[k] - (1),
    η0min[nSrck[k]^-1/2, ρRMSk[k]^-1] + dη0min[nSrck[k]^-1/2, ρRMSk[k]^-1],
    σmin[nSrck[k]^-1/2, ρRMSk[k]^-1] - dσmin[nSrck[k]^-1/2, ρRMSk[k]^-1]]]
negLogSigηminSmall[k_] := –Log[10, signiMIN0[ ηmink[k] + (1),
    η0min[nSrck[k]^-1/2, ρRMSk[k]^-1] - dη0min[nSrck[k]^-1/2, ρRMSk[k]^-1],
    σmin[nSrck[k]^-1/2, ρRMSk[k]^-1] + dσmin[nSrck[k]^-1/2, ρRMSk[k]^-1]]]
```

In[212]:=
```
(*Uncertainty of –Log₁₀S for ηmaxk with uncertainty for ηmaxk = ±1° . *)
negLogSigηmaxBest[k_] := –Log[10,
   signiMAX0[ ηmaxk[k], η0max[nSrck[k]^-1/2, ρRMSk[k]^-1], σmax[nSrck[k]^-1/2, ρRMSk[k]^-1]]]
negLogSigηmaxBig[k_] := –Log[10, signiMAX0[ ηmaxk[k] + (1),
    η0max[nSrck[k]^-1/2, ρRMSk[k]^-1] - dη0max[nSrck[k]^-1/2, ρRMSk[k]^-1],
    σmax[nSrck[k]^-1/2, ρRMSk[k]^-1] - dσmax[nSrck[k]^-1/2, ρRMSk[k]^-1]]]
negLogSigηmaxSmall[k_] := –Log[10, signiMAX0[ ηmaxk[k] - (1),
    η0max[nSrck[k]^-1/2, ρRMSk[k]^-1] + dη0max[nSrck[k]^-1/2, ρRMSk[k]^-1],
    σmax[nSrck[k]^-1/2, ρRMSk[k]^-1] + dσmax[nSrck[k]^-1/2, ρRMSk[k]^-1]]]
```

In[215]:=
```
negLogVerySigηmin =
  Table[Around[negLogSigηminBest[k], {negLogSigηminBest[k] – negLogSigηminSmall[k],
     negLogSigηminBig[k] – negLogSigηminBest[k]}],
   {k, Table[sortηMINVerySigkList[[i, 1]], {i, Length[sortηMINVerySigkList]}]}];
Print["For the very significantly aligned regions, S = p ≤ 10^-2, the –Log₁₀S values are"]
negLogVerySigηmin
```

For the very significantly aligned regions, S = $p \leqslant 10^{-2}$, the $-\text{Log}_{10}$S values are

Out[217]= $\{3.3^{+0.4}_{-0.4}, 2.92^{+0.4}_{-0.34}, 2.62^{+0.31}_{-0.29}, 2.53^{+0.4}_{-0.34}, 2.48^{+0.4}_{-0.32}, 2.30^{+0.29}_{-0.27}, 2.26^{+0.35}_{-0.32}, 2.20^{+0.33}_{-0.30}, 2.18^{+0.34}_{-0.31}, 2.17^{+0.28}_{-0.25}, 2.17^{+0.29}_{-0.26}, 2.13^{+0.34}_{-0.30}, 2.12^{+0.31}_{-0.28}, 2.12^{+0.34}_{-0.30}, 2.09^{+0.32}_{-0.29}, 2.08^{+0.33}_{-0.30}, 2.06^{+0.33}_{-0.30}, 2.03^{+0.31}_{-0.28}, 2.03^{+0.27}_{-0.25}\}$

In[218]:=
```
negLogVerySigηmax =
  Table[Around[negLogSigηmaxBest[k], {negLogSigηmaxBest[k] – negLogSigηmaxSmall[k],
     negLogSigηmaxBig[k] – negLogSigηmaxBest[k]}],
   {k, Table[sortηMAXVerySigkList[[i, 1]], {i, Length[sortηMAXVerySigkList]}]}];
Print["For the regions with very significant avoidance,
   S = p ≤ 10^-2, the –Log₁₀S values are"]
negLogVerySigηmax
```

For the regions with very significant avoidance, S = $p \leqslant 10^{-2}$, the $-\text{Log}_{10}$S values are

Out[220]= $\{4.7^{+0.5}_{-0.5}, 3.6^{+0.4}_{-0.4}, 2.96^{+0.35}_{-0.32}, 2.94^{+0.34}_{-0.31}, 2.9^{+0.4}_{-0.4}, 2.49^{+0.34}_{-0.31}, 2.49^{+0.31}_{-0.28}, 2.43^{+0.35}_{-0.32}, 2.43^{+0.30}_{-0.27}, 2.36^{+0.34}_{-0.31}, 2.22^{+0.33}_{-0.30}, 2.19^{+0.31}_{-0.28}, 2.18^{+0.33}_{-0.30}, 2.14^{+0.32}_{-0.29}, 2.05^{+0.27}_{-0.25}, 2.03^{+0.28}_{-0.26}, 2.02^{+0.31}_{-0.28}, 2.01^{+0.29}_{-0.26}, 2.01^{+0.31}_{-0.28}, 2.01^{+0.31}_{-0.28}\}$

In[221]:=
```
lpNegLogVerySigAlign = ListPlot[negLogVerySigηmin, PlotRange → {{0, 20}, {0, 5.5}},
   PlotLabel → " –Log₁₀p, Alignment  ", GridLines → Automatic,
   Frame → True, FrameLabel → {"Rank", "–Logp"}, ImageSize → 72 × 4];
```

In[222]:=
```
lpNegLogVerySigAvoid =
  ListPlot[negLogVerySigηmax, PlotRange → All, PlotLabel → " -Log10p, Avoidance  ",
   GridLines → Automatic, Frame → True, FrameLabel → {"Rank", "-Logp"}, ImageSize → 72 × 4];
```

In[223]:=
```
GraphicsRow[{lpNegLogVerySigAlign, lpNegLogVerySigAvoid}]
Print[
 "Figure A4. The negative log of the significance p for regions with very significant
   alignment (left) and avoidance (right). The most significant region has rank
   1, the next most significant has rank 2, etc. Most of the uncertainty is
   due to the experimental uncertainty in the polarization position angles ψ."]
```

Out[223]=

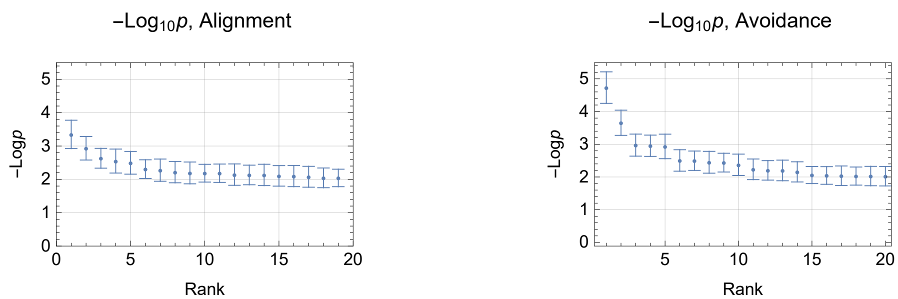


Figure A4. The negative log of the significance p for regions with very significant alignment (left) and avoidance (right). The most significant region has rank 1, the next most significant has rank 2, etc. Most of the uncertainty is due to the experimental uncertainty in the polarization position angles ψ.

The number of regions with very significant alignment is 19 regions, i.e. S = $p \le 10^{-2} = 0.01$.

The number of regions with significant alignment is 96 regions, i.e. S = $p \le 10^{-2} = 0.05$.

The number of regions with very significant avoidance is 20 regions, i.e. S = $p \le 10^{-2} = 0.01$.

The number of regions with significant avoidance is 88 regions, i.e. S = $p \le 10^{-2} = 0.05$.

The region with the most significant alignment is region number 393, which has S = p = 0.000467833.

The region with the most significant avoidance is region number 260, which has S = p = 0.000019276.

A8. Mapping the significance of the regions

In[232]:=
```
raDEClogSigForAllVerySigMin =
  Table[{αHj[rgnCntrAndSrcId[[sortηMINVerySigkList [[j, 1]], 2]]],
    δHj[rgnCntrAndSrcId[[sortηMINVerySigkList [[j, 1]], 2]] ] ,
    -Log[10, sortηMINVerySigkList [[j, 2]]]}, {j, Length[sortηMINVerySigkList]}];
```

In[233]:=
```
sortraDEClogSigForAllVerySigMin = Sort[raDEClogSigForAllVerySigMin, #1[[3]] > #2[[3]] &];
```

In[234]:=
```
lp1 = ListPlot[
   {Table[Style[{xH180[ αAVEk[k], δAVEk[k] ], yH180[ αAVEk[k], δAVEk[k] ]}, LightGray],
     {k, Length[rgnCntrAndSrcId]}],
    Table[Style[{xH180[sortraDEClogSigForAllVerySigMin[[-j]][[1]],
         sortraDEClogSigForAllVerySigMin[[-j]][[2]]],
       yH180[sortraDEClogSigForAllVerySigMin[[-j]][[1]],
         sortraDEClogSigForAllVerySigMin[[-j]][[2]]]},
      ColorData["Rainbow"][(sortraDEClogSigForAllVerySigMin[[-j]][[3]] - 2.) /
         (sortraDEClogSigForAllVerySigMin[[1]][[3]] - 2.)]],
     {j, Length[sortraDEClogSigForAllVerySigMin]}]},
   PlotRange → {{-4., 4.}, {-2.2, 2.2}}, PlotStyle → PointSize[Medium],
   PlotLegends → BarLegend[{"Rainbow", {2.0, sortraDEClogSigForAllVerySigMin[[1]][[3]]}},
     LegendLabel → "-Log10p"], Axes → False];
```

In[235]:=
```
lp2 = Show[{lp1, Table[ParametricPlot[{xH180[α, δ], yH180[α, δ]}, {δ, -90, 90},
      PlotStyle → {Black, Thickness[0.002]}, PlotPoints → 60], {α, 0, 360, 30}],
    Table[ParametricPlot[{xH180[α, δ], yH180[α, δ]}, {α, 0, 360},
      PlotStyle → {Black, Thickness[0.002]}, PlotPoints → 60], {δ, -60, 60, 30}],
    Graphics[{PointSize[0.004], Text[StyleForm["N", FontSize -> 14, FontWeight -> "Plain"],
        {0, 1.85}], Text[StyleForm["Very Significantly Aligned Regions (S = p ≤ 1%)",
         FontSize -> 14, FontWeight -> "Plain"], {0, -1.85}],
      Text[StyleForm["Clump 1", FontSize → 12, FontWeight -> "Bold"], {-3.3, 1.0}],
      {Arrow[BezierCurve[{{-3.3, 1.2}, {-1.3, 2.5}, {xH180[170, 20], yH180[170, 20]}}]]},
      Text[StyleForm["Clump 2", FontSize → 12, FontWeight -> "Bold"], {3.3, 1.0}],
      {Arrow[BezierCurve[{{3.3, 1.2}, {1.3, 2.5}, {xH180[175, 53], yH180[175, 53]}}]]},
      Text[StyleForm["Clump 3", FontSize → 12, FontWeight -> "Bold"], {+3.3, -1.0}], {Arrow[
        BezierCurve[{{+3.3, -1.2}, {+0.3, -1.5}, {xH180[235, 28], yH180[235, 28]}}]]},
      Text[StyleForm["Clump 4", FontSize → 12, FontWeight -> "Bold"], {-3.3, -1.0}],
      {Arrow[BezierCurve[{{-3.3, -1.2}, {-1.3, -1.5},
          {xH180[118, 18], yH180[118, 18]}}]]}}]}, ImageSize → 432]
Print["Figure A5. The very significantly aligned regions are shaded in color.
   The regions in grey have sources that are not very significantly aligned."]
```

Out[235]=

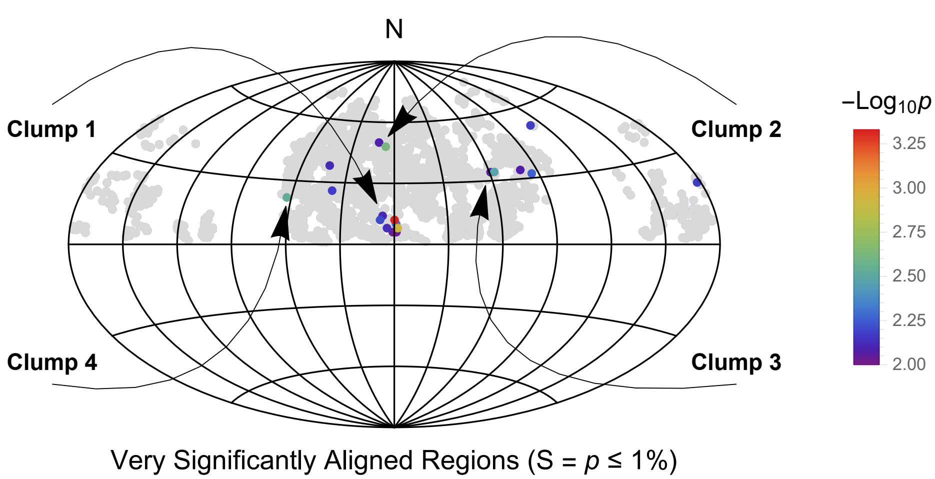


Figure A5. The very significantly aligned regions are shaded in color.
The regions in grey have sources that are not very significantly aligned.

In[237]:=
```
(*Regions with ( 5%, NOT Very)  significant alignment.*)
raDEClogSigForAllSigMin = Table[{αHj[rgnCntrAndSrcId[[sortηMINSigkList [[j, 1]], 2]]],
    δHj[rgnCntrAndSrcId[[sortηMINSigkList [[j, 1]], 2]] ] ,
    -Log[10, sortηMINSigkList [[j, 2]]]}, {j, Length[sortηMINSigkList]}];
```

In[238]:=
```
sortraDEClogSigForAllSigMin = Sort[raDEClogSigForAllSigMin, #1[[3]] > #2[[3]] &];
```

In[239]:=
```
lp3 = ListPlot[
   {Table[Style[{xH180[ αAVEk[k], δAVEk[k] ], yH180[ αAVEk[k], δAVEk[k] ]}, LightGray],
     {k, Length[rgnCntrAndSrcId]}],
    Table[Style[{xH180[sortraDEClogSigForAllSigMin[[-j]][[1]],
        sortraDEClogSigForAllSigMin[[-j]][[2]]], yH180[sortraDEClogSigForAllSigMin[[
          -j]][[1]], sortraDEClogSigForAllSigMin[[-j]][[2]]]},
      ColorData["Rainbow"][(sortraDEClogSigForAllSigMin[[-j]][[3]] - 2.) /
        (sortraDEClogSigForAllSigMin[[1]][[3]] - 2.)]],
     {j, Length[sortraDEClogSigForAllSigMin]}]}, PlotRange → {{-4., 4.}, {-2.2, 2.2}},
   PlotStyle → PointSize[Medium], PlotLegends →
    BarLegend[{"Rainbow", {-Log[10, 0.05], sortraDEClogSigForAllSigMin[[1]][[3]]}},
     LegendLabel → "-Log10p"], Axes → False];
```

In[240]:=
```
(*Regions with ( 5%, NOT Very)  significant alignment.*)
lp4 = Show[{lp3, Table[ParametricPlot[{xH180[α, δ], yH180[α, δ]}, {δ, -90, 90},
     PlotStyle → {Black, Thickness[0.002]}, PlotPoints → 60], {α, 0, 360, 30}],
   Table[ParametricPlot[{xH180[α, δ], yH180[α, δ]}, {α, 0, 360},
     PlotStyle → {Black, Thickness[0.002]}, PlotPoints → 60], {δ, -60, 60, 30}],
   Graphics[{PointSize[0.004], Text[StyleForm["N", FontSize -> 14, FontWeight -> "Plain"],
      {0, 1.85}], Text[StyleForm["Significantly Aligned Regions (p ≤ 5%)",
       FontSize -> 14, FontWeight -> "Plain"], {0, -1.85}]}]}, ImageSize → 432]
Print["Figure A6. Significantly aligned regions, this map has 96 significantly
   aligned regions, compared to 19 very significantly aligned regions in Fig. A5."]
```

Out[240]=

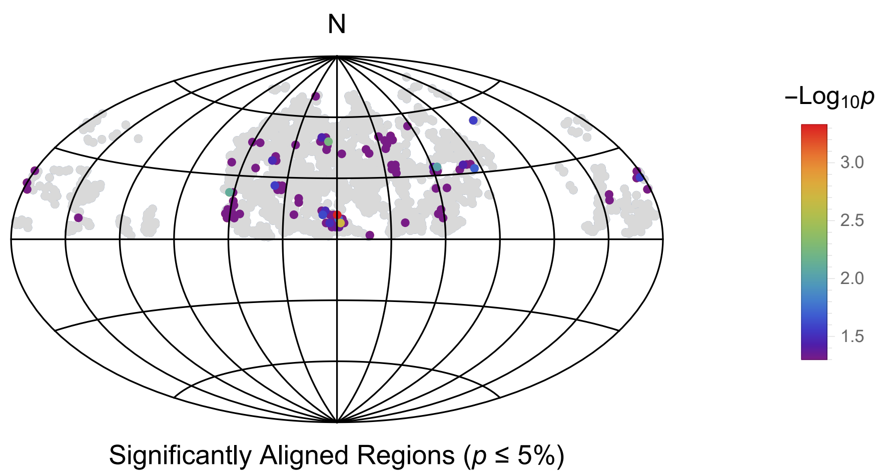


Figure A6. Significantly aligned regions, this map has 96 significantly
aligned regions, compared to 19 very significantly aligned regions in Fig. A5.

In[242]:=
```
raDEClogSigForAllVerySigMax =
  Table[{αHj[rgnCntrAndSrcId[[sortηMAXVerySigkList [[j, 1]], 2]]],
    δHj[rgnCntrAndSrcId[[sortηMAXVerySigkList [[j, 1]], 2]] ] ,
    -Log[10, sortηMAXVerySigkList [[j, 2]]]}, {j, Length[sortηMAXVerySigkList]}];
sortraDEClogSigForAllVerySigMax = Sort[raDEClogSigForAllVerySigMax, #1[[3]] > #2[[3]] &];
lp5 = ListPlot[
   {Table[Style[{xH180[ αAVEk[k], δAVEk[k] ], yH180[ αAVEk[k], δAVEk[k] ]}, LightGray],
     {k, Length[rgnCntrAndSrcId]}],
    Table[Style[{xH180[sortraDEClogSigForAllVerySigMax[[-j]][[1]],
        sortraDEClogSigForAllVerySigMax[[-j]][[2]]],
       yH180[sortraDEClogSigForAllVerySigMax[[-j]][[1]],
        sortraDEClogSigForAllVerySigMax[[-j]][[2]]]},
      ColorData["Rainbow"][(sortraDEClogSigForAllVerySigMax[[-j]][[3]] - 2.) /
        (sortraDEClogSigForAllVerySigMax[[1]][[3]] - 2.)]],
     {j, Length[sortraDEClogSigForAllVerySigMax]}]},
   PlotRange → {{-4., 4.}, {-2.2, 2.2}}, PlotStyle → PointSize[Medium],
   PlotLegends → BarLegend[{"Rainbow", {2.0, sortraDEClogSigForAllVerySigMax[[1]][[3]]}},
     LegendLabel → "-Log10p"], Axes → False];
```

In[245]:=
```
lp6 = Show[{lp1, Table[ParametricPlot[{xH180[α, δ], yH180[α, δ]}, {δ, -90, 90},
      PlotStyle → {Black, Thickness[0.002]}, PlotPoints → 60], {α, 0, 360, 30}],
    Table[ParametricPlot[{xH180[α, δ], yH180[α, δ]}, {α, 0, 360},
      PlotStyle → {Black, Thickness[0.002]}, PlotPoints → 60], {δ, -60, 60, 30}],
    Graphics[{PointSize[0.004], Text[StyleForm["N", FontSize -> 14, FontWeight -> "Plain"],
       {0, 1.85}], Text[StyleForm["Regions with Very Significant Avoidance (p ≤ 1%) ",
        FontSize -> 14, FontWeight -> "Plain"], {0, -1.85}]}]}, ImageSize → 432]
Print["Figure A7. Regions whose polarization directions very
   significantly avoid some place on the Celestial Sphere."]
```

Out[245]=

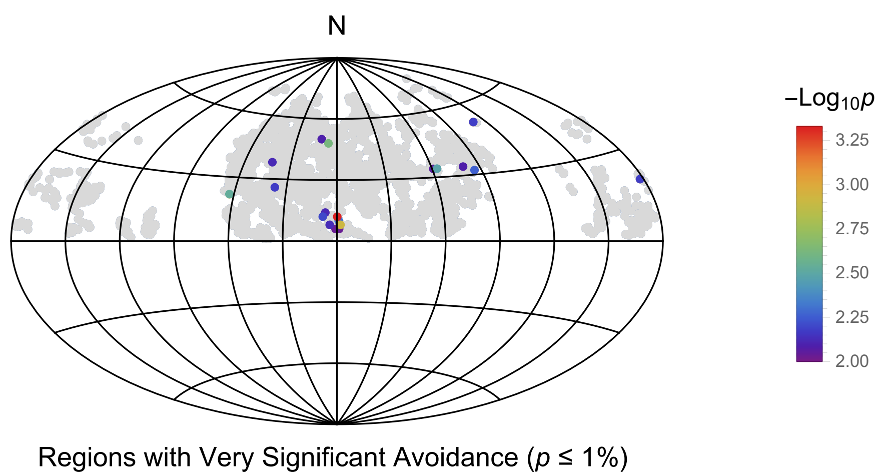
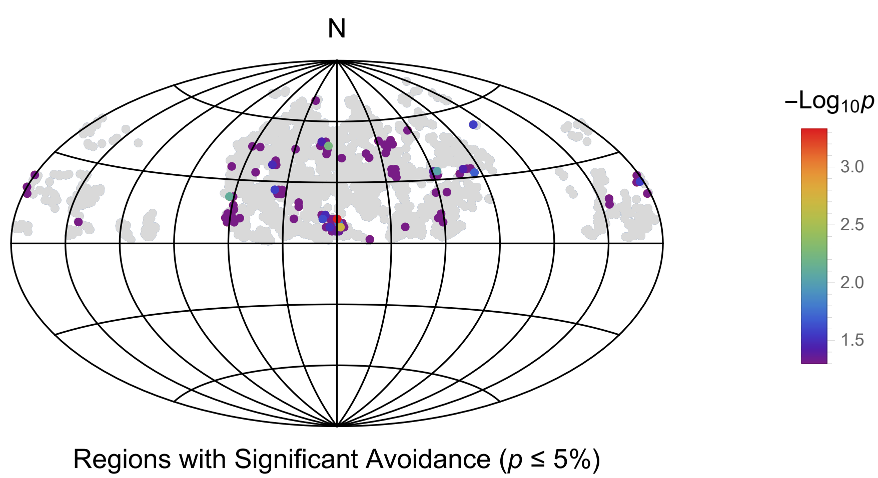


Figure A7. Regions whose polarization directions
  very significantly avoid some place on the Celestial Sphere.

In[247]:=
```
(*Regions with ( 5%, NOT Very)  significant avoidance.*)
raDEClogSigForAllSigMax = Table[{αHj[rgnCntrAndSrcId[[sortηMAXSigkList [[j, 1]], 2]]],
    δHj[rgnCntrAndSrcId[[sortηMAXSigkList [[j, 1]], 2]] ] ,
    -Log[10, sortηMAXSigkList [[j, 2]]]}, {j, Length[sortηMAXSigkList]}];
sortraDEClogSigForAllSigMax = Sort[raDEClogSigForAllSigMax, #1[[3]] > #2[[3]] &];
lp7 = ListPlot[
  {Table[Style[{xH180[ αAVEk[k], δAVEk[k] ], yH180[ αAVEk[k], δAVEk[k] ]}, LightGray],
    {k, Length[rgnCntrAndSrcId]}],
   Table[Style[{xH180[sortraDEClogSigForAllSigMax[[-j]][[1]],
       sortraDEClogSigForAllSigMax[[-j]][[2]]], yH180[sortraDEClogSigForAllSigMax[[
         -j]][[1]], sortraDEClogSigForAllSigMax[[-j]][[2]]]},
     ColorData["Rainbow"][(sortraDEClogSigForAllSigMax[[-j]][[3]] - 2.) /
       (sortraDEClogSigForAllSigMax[[1]][[3]] - 2.)]],
    {j, Length[sortraDEClogSigForAllSigMax]}]}, PlotRange → {{-4., 4.}, {-2.2, 2.2}},
  PlotStyle → PointSize[Medium], PlotLegends →
   BarLegend[{"Rainbow", {-Log[10, 0.05], sortraDEClogSigForAllSigMax[[1]][[3]]}},
    LegendLabel → "-Log10p"], Axes → False];
```

In[250]:=
```
(*Regions with ( 5%, NOT Very)  significant avoidance.*)
lp8 = Show[{lp3, Table[ParametricPlot[{xH180[α, δ], yH180[α, δ]}, {δ, -90, 90},
     PlotStyle → {Black, Thickness[0.002]}, PlotPoints → 60], {α, 0, 360, 30}],
   Table[ParametricPlot[{xH180[α, δ], yH180[α, δ]}, {α, 0, 360},
     PlotStyle → {Black, Thickness[0.002]}, PlotPoints → 60], {δ, -60, 60, 30}],
   Graphics[{PointSize[0.004], Text[StyleForm["N", FontSize -> 14, FontWeight -> "Plain"],
      {0, 1.85}], Text[StyleForm["Regions with Significant Avoidance (p ≤ 5%)",
       FontSize -> 14, FontWeight -> "Plain"], {0, -1.85}]}]}, ImageSize → 432]
Print["Figure A8. Regions whose polarization directions significantly avoid
   some place on the Celestial Sphere. This map has 88 regions with significant
   avoidance, compared to 20 regions with very significant avoidance in Fig. A7."]
```

Out[250]=




```
Figure A8. Regions whose polarization directions significantly avoid
  some place on the Celestial Sphere. This map has 88 regions with significant
  avoidance, compared to 20 regions with very significant avoidance in Fig. A7.
```

A9. Selecting sources to analyze

Definitions:

firstClumpjsForSortηMINVerySigkList1450 List of item #s in table "sortηMINVerySigkList" in this clump
raDEClogSigForVery***1st***Clump1450 Table of 1. RA 2. dec 3. significance of the smallest alignment angle $\overline{\eta}_{\min}$
firstVeryClumpksForRgnCntrAndSrcId1450 List of region ID #s in this clump. Most region data is in the table "rgnCntrAndSrcId"
firstVeryClumpQsosIDinData001450 List of source ID #s in the table "data00"

Replace *first* and *1st* by second and 2nd, third and 3rd, etc for the other clumps.

Clump 1

```
In[252]:= firstClumpjsForSortηMINVerySigkList1450 = {};
Table[If[(165. ≤ raDEClogSigForAllVerySigMin[[i, 1]] ≤ 200.) &&
    (0 ≤ raDEClogSigForAllVerySigMin[[i, 2]] ≤ 30.),
   AppendTo[firstClumpjsForSortηMINVerySigkList1450, i]],
  {i, Length[raDEClogSigForAllVerySigMin]}];
Length[firstClumpjsForSortηMINVerySigkList1450];

In[255]:= raDEClogSigForVery1stClump1450 =
  Table[{αHj[rgnCntrAndSrcId[[sortηMINVerySigkList [[j, 1]], 2]]],
    δHj[rgnCntrAndSrcId[[sortηMINVerySigkList [[j, 1]], 2]] ] ,
    - Log[10, sortηMINVerySigkList [[j, 2]]]},
   {j, firstClumpjsForSortηMINVerySigkList1450}];
firstVeryClumpksForRgnCntrAndSrcId1450 = Table[rgnCntrAndSrcId[[
    sortηMINVerySigkList [[j, 1]], 1]], {j, firstClumpjsForSortηMINVerySigkList1450}];

In[257]:= firstVeryClumpQsosIDinData001450 =
 Union[Flatten[Table[rgnCntrAndSrcId[[sortηMINSigkList [[j, 1]], 3]],
    {j, firstClumpjsForSortηMINVerySigkList1450}]]]
Length[firstVeryClumpQsosIDinData001450];
Print["Clump 1 combines the sources in ",
 Length[firstClumpjsForSortηMINVerySigkList1450], " regions, for a total of ",
 Length[firstVeryClumpQsosIDinData001450], " sources."]

Out[257]= {659, 660, 663, 667, 674, 680, 682, 690, 695, 696, 698, 707, 712,
 714, 718, 720, 721, 727, 728, 731, 734, 744, 746, 751, 752, 762, 764}

Clump 1 combines the sources in 8 regions, for a total of 27 sources.
```

Clump 2

```
In[260]:= secondClumpjsForSortηMINVerySigkList1450 = {};
Table[If[(150. ≤ raDEClogSigForAllVerySigMin[[i, 1]] ≤ 190.) &&
     (30 ≤ raDEClogSigForAllVerySigMin[[i, 2]] ≤ 60.),
    AppendTo[secondClumpjsForSortηMINVerySigkList1450, i]],
   {i, Length[raDEClogSigForAllVerySigMin]}];

In[262]:= raDEClogSigForVery2ndClump1450 =
   Table[{αHj[rgnCntrAndSrcId[[sortηMINVerySigkList [[j, 1]], 2]]],
     δHj[rgnCntrAndSrcId[[sortηMINVerySigkList [[j, 1]], 2]] ],
     -Log[10, sortηMINVerySigkList [[j, 2]]]},
    {j, secondClumpjsForSortηMINVerySigkList1450}];
secondVeryClumpksForRgnCntrAndSrcId1450 = Table[rgnCntrAndSrcId[[
     sortηMINVerySigkList [[j, 1]], 1]], {j, secondClumpjsForSortηMINVerySigkList1450}];

In[264]:= secondVeryClumpQsosIDinData001450 =
  Union[Flatten[Table[rgnCntrAndSrcId[[sortηMINVerySigkList [[j, 1]], 3]],
     {j, secondClumpjsForSortηMINVerySigkList1450}]]]
Length[secondClumpjsForSortηMINVerySigkList1450];
Length[secondVeryClumpQsosIDinData001450];
Print["Clump 2 combines the sources in ",
  Length[secondClumpjsForSortηMINVerySigkList1450], " regions, for a total of ",
  Length[secondVeryClumpQsosIDinData001450], " sources."]

Out[264]= {618, 624, 638, 657, 661, 666, 668, 672, 697, 699, 708, 713, 719}

Clump 2 combines the sources in 2 regions, for a total of 13 sources.
```

Clump 3

```
In[268]:= thirdClumpjsForSortηMINVerySigkList1450 = {};
Table[If[(230. ≤ raDEClogSigForAllVerySigMin[[i, 1]] ≤ 250.) &&
     (25. ≤ raDEClogSigForAllVerySigMin[[i, 2]] ≤ 40.),
    AppendTo[thirdClumpjsForSortηMINVerySigkList1450, i]],
   {i, Length[raDEClogSigForAllVerySigMin]}];

In[270]:= raDEClogSigForVery3rdClump1450 =
   Table[{αHj[rgnCntrAndSrcId[[sortηMINSigkList [[j, 1]], 2]]],
     δHj[rgnCntrAndSrcId[[sortηMINSigkList [[j, 1]], 2]] ] ,
     -Log[10, sortηMINSigkList [[j, 2]]]}, {j, thirdClumpjsForSortηMINVerySigkList1450}];
thirdVeryClumpksForRgnCntrAndSrcId1450 = Table[rgnCntrAndSrcId[[
     sortηMINVerySigkList [[j, 1]], 1]], {j, thirdClumpjsForSortηMINVerySigkList1450}];
```

In[272]:=
```
thirdVeryClumpQsosIDinData001450 =
 Union[Flatten[Table[rgnCntrAndSrcId[[sortηMINSigkList [[j, 1]], 3]],
    {j, thirdClumpjsForSortηMINVerySigkList1450}]]]
Length[thirdClumpjsForSortηMINVerySigkList1450];
Length[thirdVeryClumpQsosIDinData001450];
Print["Clump 3 combines the sources in ",
 Length[thirdClumpjsForSortηMINVerySigkList1450], " regions, for a total of ",
 Length[thirdVeryClumpQsosIDinData001450], " sources."]
```

Out[272]= {1063, 1070, 1081, 1093, 1094, 1098, 1106, 1113, 1133}

```
Clump 3 combines the sources in 2 regions, for a total of 9 sources.
```

Clump 4

In[276]:=
```
fourthClumpjsForSortηMINVerySigkList1450 = {};
Table[If[(105. ≤ raDEClogSigForAllVerySigMin[[i, 1]] ≤ 125.) &&
    (10. ≤ raDEClogSigForAllVerySigMin[[i, 2]] ≤ 30.),
   AppendTo[fourthClumpjsForSortηMINVerySigkList1450, i]],
  {i, Length[raDEClogSigForAllVerySigMin]}];
```

In[278]:=
```
raDEClogSigForVery4thClump1450 =
  Table[{αHj[rgnCntrAndSrcId[[sortηMINSigkList [[j, 1]], 2]]],
    δHj[rgnCntrAndSrcId[[sortηMINSigkList [[j, 1]], 2]] ] ,
    - Log[10, sortηMINSigkList [[j, 2]]]}, {j, fourthClumpjsForSortηMINVerySigkList1450}];
fourthVeryClumpksForRgnCntrAndSrcId1450 = Table[rgnCntrAndSrcId[[
    sortηMINVerySigkList [[j, 1]], 1]], {j, fourthClumpjsForSortηMINVerySigkList1450}];
```

In[280]:=
```
fourthVeryClumpQsosIDinData001450 =
 Union[Flatten[Table[rgnCntrAndSrcId[[sortηMINSigkList [[j, 1]], 3]],
    {j, fourthClumpjsForSortηMINVerySigkList1450}]]]
Length[fourthClumpjsForSortηMINVerySigkList1450];
Length[fourthVeryClumpQsosIDinData001450];
Print["Clump 4 consists of the sources in ",
 Length[fourthClumpjsForSortηMINVerySigkList1450], " region, for a total of ",
 Length[fourthVeryClumpQsosIDinData001450], " sources."]
```

Out[280]= {275, 284, 289, 292, 295, 311, 314, 315}

```
Clump 4 consists of the sources in 1 region, for a total of 8 sources.
```

URLs:

https://www.wolframcloud.com/obj/shurtleffr/Published/20211221Survey1450QSOsMapb.nb

https://www.dropbox.com/s/6bqy56vazlfuuu6/20211221Survey1450QSOsMapb.nb?dl=0